Multiagent Resource Allocation with Sharable Items: Simple Protocols and Nash Equilibria

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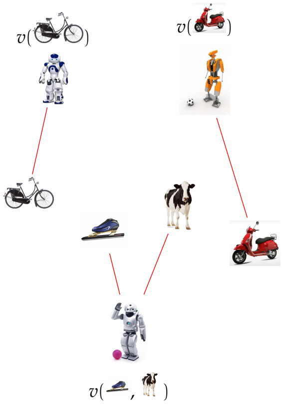
MultiAgent Resource Allocation (MARA)



non-sharable resources:
allocations are partitions.

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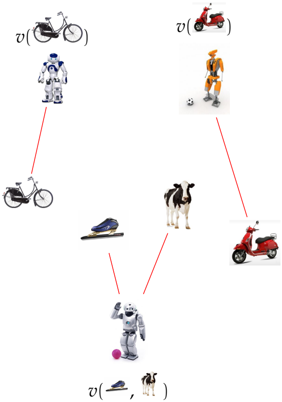

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Distributed protocols converging to optimal allocations.

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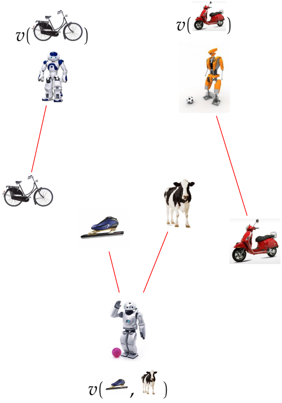

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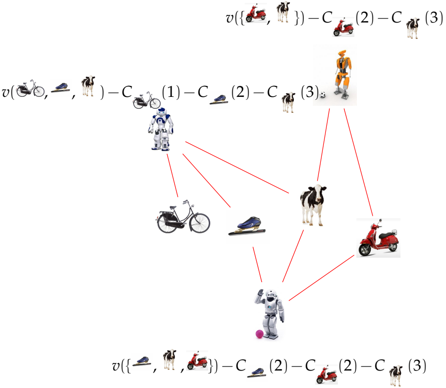

sharable resources.

Distributed protocols converging to optimal allocations.

📖 Study distributed resource allocation problems where synergies between resources may exist and where resources can be shared.

outline

- **Control:** to start using a resource, an agent must receive the consent of the current users. Side payments are necessary.
- **No control:** agents are free to use any resource they want. Relation with congestion games and Nash equilibria.

MARA with indivisible and sharable resources

A **MARA** problem with indivisible **sharable** items is $\langle \mathcal{N}, \mathcal{R}, (\Sigma_i)_{i\in\mathcal{N}}, (d_{i,r})_{i\in\mathcal{N}, r\in\mathcal{R}}, (v_i)_{i\in\mathcal{N}} \rangle$ with

- $\mathcal{N} = \{1, 2, \dots, n\}$ is a finite set of n **agents**.
- $\mathcal{R}$ is a finite set of m **resources**.
- Σ_i is the set of **bundles** of agent i.
- $d_{i,r} : \{1, \dots, n\} \to \mathbb{R}$ is the **delay** perceived by agent i when using resource r.
- $v_i : \Sigma_i \to \mathbb{R}$ is the **valuation function** for agent i: for a bundle $\sigma \in \Sigma_i$, $v_i(\sigma)$ is the value of using the resources in the bundle σ_i, irrespective of the congestion.

Notations and Assumptions

- σ is an **allocation**.
- The **utility** of agent i in profile σ is defined as
$$u_i(\sigma) = v_i(\sigma_i) - \sum_{r \in \sigma_i} d_{i,r}(n_r(\sigma)).$$
- $n_r(\sigma)$ the number of agents that use resource r in allocation σ, i.e., $n_r(\sigma) = |\{i \in \mathcal{N} \mid r \in \sigma_i\}|$.
- ➩ $d_{i,r}(n_r(\sigma))$ is the delay of using resource r experienced by agent i in allocation σ.

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- $d_{i,r}(n_r(\sigma))$ is the delay of using resource r experienced by agent i in allocation σ.
- A MARA problem is **symmetric** when the delay is the same for all agents (but resource-dependent).
- **Assumption:** the delay is a **nondecreasing** function in the number of agents using the resource.
- **Assumption:** all valuation functions are **normalised**, i.e., $v_i(\emptyset) = 0$ for all agents $i \in \mathcal{N}$.

Definition (deal)

A $\delta = (\sigma, \sigma')$ is a transformation from an allocation σ to an allocation σ'.

Definition (individual rational deal)

A deal $\delta = (\sigma, \sigma')$ is **individually rational (IR)** if there exists a payment function p such that $\forall i \in \mathcal{N}$, $u_i(\sigma') - u_i(\sigma) > p_i$, except for agents i **unaffected** by δ and for whom $p_i = 0$ is also permitted.

An agent i is **unaffected** by a deal $\delta = (\sigma, \sigma')$ if $\sigma(i) = \sigma'(i)$ and $|\{j \in \mathcal{N} \mid r \in \sigma(j)\}| = |\{j \in \mathcal{N} \mid r \in \sigma'(j)\}|$ for all $r \in \sigma(i)$.

In an IR deal, an agent i that does not change its bundle may be affected and hence, i may

- receive a payment (from agents starting to use a resource i uses) or
- make a payment (to agents that stop using a resource i uses)

General **convergence**

Theorem

Any sequence of IR deals will eventually result in an allocation of resources with maximal social welfare.

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Any sequence of IR deals will eventually result in an allocation of resources with maximal social welfare.

However, an IR-deal may be quite complex (involving many agents and many resources at the same time) and hard to find.

Simple Deals

- **ADD**(i,r)**:** agent i adds to its bundle a single resource it is not currently using. For $r \notin \sigma_i$, agent i will have $\sigma_i \cup \{r\}$ after the ADD(i,r) action.
- **DROP**(i,r)**:** agent i drops a resource it currently uses. i.e., after the drop, agent i will use $\sigma_i \setminus \{r\}$.
- **SWAP**(i,j,r)**:** agent i swaps the use of resource r with agent j, i.e., agent i drops the use of r and agent j adds the resource.
- **1-deal:** a deal that concerns a single item, but possibly **multiple** agents.

Example of a **convergence** result

A valuation function is modular iff
for all $\sigma, \sigma' \subseteq \mathcal{N}$, $v(\sigma \cup \sigma') = v(\sigma) + v(\sigma') - v(\sigma \cap \sigma')$

Theorem

> **If** all valuation functions are **modular**,
> **then any** sequence of IR **1-deals** will eventually result in an allocation with maximal social welfare.

However, a 1-deal may still be **complex**, as it may involve **many** agents.

SWAP-deals may be needed: it is not always possible to decompose a deal into a sequence of ADD-deals or DROP-deals.

2-agent 1-resource symmetric example: $v_i(r) = 4$, $v_j(r) = 6$, $d_r(1) = 2$ and $d_r(2) = 5$. Imagine 1 uses r. ADD(j,r) is not rational. Only SWAP(i,j,r) is rational.

Example of an **existence** result

Theorem

If all valuation functions are **modular** and all delay functions are **nondecreasing** and **convex**,
then there exists a sequence of IR ADD-deals leading from the empty allocation to an allocation with maximal social welfare.

Convexity is necessary

$\mathcal{N} = \{1,2,3\}$, same valuation function $v_i(r) = 5$ and $v_i(\emptyset) = 0$ symmetric concave delay function d_r: $d_r(1) = 0$ and $d_r(k) = 3$ for $k > 1$.

The full allocation (which is optimal) cannot be reached from the empty allocation. $0 \Rightarrow 5 \Rightarrow 2(5-3) = 4 \Rightarrow 3(5-3) = 6$.

Existence of sequence of ADD from empty allocation

Theorem

If all valuation functions are modular and all delay functions are nondecreasing and convex,
then there exists a sequence of IR ADD-deals leading from the empty allocation to an allocation with maximal social welfare.

Convexity is necessary

$\mathcal{N} = \{1,2,3\}$, same valuation function $v_i(r) = 5$ and $v_i(\emptyset) = 0$
symmetric concave delay function d_r: $d_r(1) = 0$ and $d_r(k) = 3$ for $k > 1$.

The full allocation (which is optimal) cannot be reached from the empty allocation. $0 \Rightarrow 5$ ✗ $2(5-3) = 4 \Rightarrow 3(5-3) = 6$.

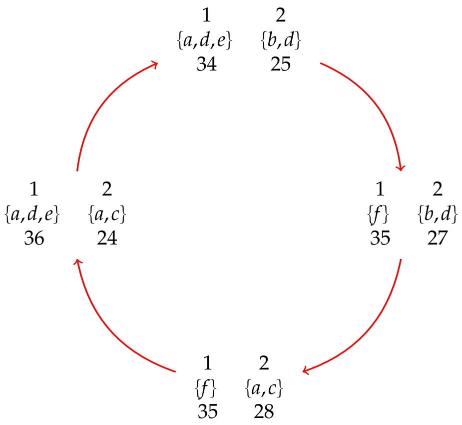
Summary

MARA with indivisible and sharable resources with control

(a new user must receive the consent from current users before starting to use a resource)

Theorem	Result	Valuation	Delay	Symmetry	Deals	Init. Alloc.	Control
4	convergence	any	any	no	all	any	none
5	convergence	modular	any	no	1-deals	any	none
7	existence	modular	n.d.+convex	no	ADD	empty	none
9	existence	modular	n.d.+convex	no	DROP	full	none
10	convergence	modular	n.d.+convex	yes	ADD-DROP-SWAP	any	none
12	convergence	modular	n.d.+convex	yes	ADD-SWAP	empty	precedence
13	convergence	modular	n.d.+convex	yes	ADD-SWAP	empty	greedy

Absence of Control: **no** NE in pure strategy



resource	a	b	c	d	e	f
$d_{1,r}(1)$	20	45	48	20	16	65
$d_{2,r}(1)$	24	45	48	28	32	130
$d_{i,r}(2)$	28	45	48	30	48	195

$v_1(\{a,d,e\}) = 100$

$v_1(\{f\}) = 100$

$v_2(\{b,d\}) = 100$

$v_2(\{a,c\}) = 100$

Lemma

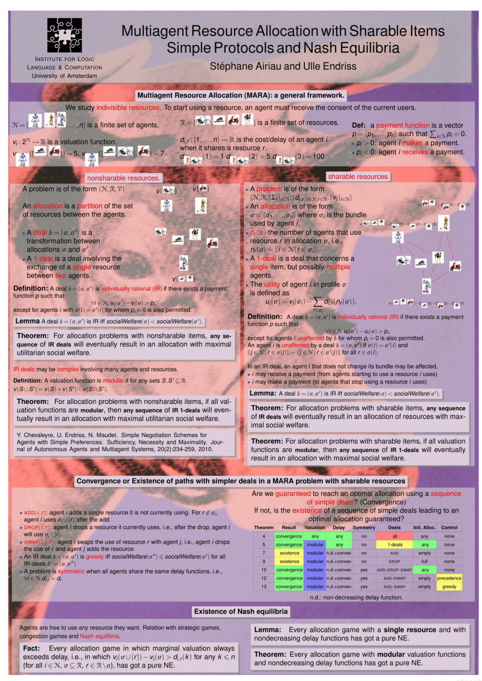
Every allocation game with a **single** resource and with nondecreasing delay functions has got a pure NE.

Theorem

Every allocation game with **modular valuation functions** and nondecreasing delay functions has got a pure NE.

Conclusion

- We studied MARA for sharable resources.
- We obtained convergence and existence results for protocols leading to allocations that maximize utilitarian social welfare.
- We used results from congestion games to determine some classes of MARA problems possessing a pure Nash equilibrium.
- Many results assume modular valuation function. Can we say something about other classes?
- Can we say something about protocols leading to optimal egalitarian social welfare or to envy-free allocation?



Multiagent Resource Allocation with Sharable Items
Simple Protocols and Nash Equilibria

Institute for Logic Language & Computation
University of Amsterdam

Stéphane Airiau and Ulle Endriss

Multiagent Resource Allocation (MARA): a general framework.

We study indivisible resources. To start using a resource, an agent must receive the consent of the current users.

$\mathcal{N} = \{ , , , \ldots, n\}$ is a finite set of agents,

$v_i : 2^{\mathcal{N}} \to \mathbb{R}$ is a valuation function.

$v(\{ , \}) = 5$, $v(\{ , \},) = 7$

$\mathcal{R} = \{ , , , \}$ is a finite set of resources.

$d_{i,r} : \{1, \ldots, n\} \to \mathbb{R}$ is the cost/delay of an agent i when it shares a resource r.

$d(1) = 1$ $d(2) = 5$ $d(3) = 100$

Def: a payment function is a vector $p = \langle p_1, \ldots, p_n \rangle$ such that $\sum_{i \in \mathcal{N}} p_i = 0$.

- $p_i > 0$: agent i makes a payment.
- $p_i < 0$: agent i receives a payment.

nonsharable resources.

A problem is of the form $(\mathcal{N}, \mathcal{R}, \mathcal{V})$

An allocation is a partition of the set of resources between the agents.

- A deal $\delta = (\sigma, \sigma')$ is a transformation between allocations σ and σ'.
- A 1-deal is a deal involving the exchange of a single resource between two agents.

Definition: A deal $\delta = (\sigma, \sigma')$ is individually rational (IR) if there exists a payment function p such that

$$\forall i \in \mathcal{N},\ v_i(\sigma') - v_i(\sigma) > p_i,$$

except for agents i with $\sigma(i) = \sigma'(i)$ for whom $p_i = 0$ is also permitted.

Lemma A deal $\delta = (\sigma, \sigma')$ is IR iff $socialWelfare(\sigma) < socialWelfare(\sigma')$.

Theorem: For allocation problems with nonsharable items, **any sequence** of **IR deals** will eventually result in an allocation with maximal utilitarian social welfare.

IR deals may be complex involving many agents and resources.

Definition: A valuation function is modular if for any sets $S, S' \subseteq \mathcal{R}$ $v(S \cup S') = v(S) + v(S') - v(S \cap S')$.

Theorem: For allocation problems with nonsharable items, if all valuation functions are **modular**, then **any sequence** of **IR 1-deals** will eventually result in an allocation with maximal utilitarian social welfare.

Y. Chevaleyre, U. Endriss, N. Maudet. Simple Negotiation Schemes for Agents with Simple Preferences: Sufficiency, Necessity and Maximality. Journal of Autonomous Agents and Multiagent Systems, 20(2):234-259, 2010.

sharable resources

- A problem is of the form $\langle \mathcal{N}, \mathcal{R}, (\Sigma_i)_{i \in \mathcal{N}}, (d_{i,r})_{i \in \mathcal{N}, r \in \mathcal{R}}, (v_i)_{i \in \mathcal{N}} \rangle$
- An allocation is of the form $\sigma = \langle \sigma_1, \ldots, \sigma_n \rangle$ where σ_i is the bundle used by agent i.
- $n_r(\sigma)$ the number of agents that use resource r in allocation σ, i.e., $n_r(\sigma) = |\{i \in \mathcal{N} \mid r \in \sigma_i\}|$.
- A 1-deal is a deal that concerns a single item, but possibly multiple agents.
- The utility of agent i in profile σ is defined as

$$u_i(\sigma) = v_i(\sigma_i) - \sum_{r \in \sigma_i} d_{i,r}(n_r(\sigma)).$$

Definition: A deal $\delta = (\sigma, \sigma')$ is individually rational (IR) if there exists a payment function p such that

$$\forall i \in \mathcal{N}\ u_i(\sigma') - u_i(\sigma) > p_i,$$

except for agents i unaffected by δ for whom $p_i = 0$ is also permitted.
An agent i is unaffected by a deal $\delta = (\sigma, \sigma')$ if $\sigma(i) = \sigma'(i)$ and $|\{j \in \mathcal{N} \mid r \in \sigma(j)\}| = |\{j \in \mathcal{N} \mid r \in \sigma'(j)\}|$ for all $r \in \sigma(i)$.

In an IR deal, an agent i that does not change its bundle may be affected,

- i may receive a payment (from agents starting to use a resource i uses)
- i may make a payment (to agents that stop using a resource i uses)

Lemma: A deal $\delta = (\sigma, \sigma')$ is IR iff $socialWelfare(\sigma) < socialWelfare(\sigma')$.

Theorem: For allocation problems with sharable items, **any sequence** of **IR deals** will eventually result in an allocation of resources with maximal social welfare.

Theorem: For allocation problems with sharable items, if all valuation functions are **modular**, then **any sequence** of **IR 1-deals** will eventually result in an allocation with maximal social welfare.

Convergence or Existence of paths with simpler deals in a MARA problem with sharable resources

- ADD(i, r): agent i adds a single resource it is not currently using. For $r \notin \sigma_i$, agent i uses $\sigma_i \cup \{r\}$ after the add.
- DROP(i, r): agent i drops a resource it currently uses. i.e., after the drop, agent i will use $\sigma_i \setminus \{r\}$.
- SWAP(i, j, r): agent i swaps the use of resource r with agent j, i.e., agent i drops the use of r and agent j adds the resource.
- An IR deal $\delta = (\sigma, \sigma')$ is greedy iff $socialWelfare(\sigma'') \leqslant socialWelfare(\sigma')$ for all IR-deals $\delta' = (\sigma, \sigma'')$
- A problem is symmetric when all agents share the same delay functions, i.e., $\forall i \in \mathcal{N}\ d_{i,r} = d_r$

Are we guaranteed to reach an optimal allocation using a sequence of simple deals? (Convergence)
If not, is the existence of a sequence of simple deals leading to an optimal allocation guaranteed?

Theorem	Result	Valuation	Delay	Symmetry	Deals	Init. Alloc.	Control
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n.d.: non-decreasing delay function.

Existence of Nash equilibria

Agents are free to use any resource they want. Relation with strategic games, congestion games and Nash equilibria.

Fact: Every allocation game in which marginal valuation always exceeds delay, i.e., in which $v_i(\sigma \cup \{r\}) - v_i(\sigma) > d_{i,r}(k)$ for any $k \leqslant n$ (for all $i \in \mathcal{N}$, $\sigma \subseteq \mathcal{R}$, $r \in \mathcal{R} \setminus \sigma$), has got a pure NE.

Lemma: Every allocation game with a **single resource** and with nondecreasing delay functions has got a pure NE.

Theorem: Every allocation game with **modular** valuation functions and nondecreasing delay functions has got a pure NE.

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