Introduction to topics on preference modelling

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1. Introduction

As can be seen from the origin and background of the authors of the eighteen papers collected in this volume, the theme of "preference modelling" is common to people coming from many different disciplines. Over the years, important contributions to this field have been made by economists, mathematicians, operations researchers, philosophers, political scientists and psychologists. Indeed, once it is accepted that "people make decisions" and that their "values" somehow interfere with the decisions they take, studying "preferences" as the expression of values and relating them to decisions seems inevitable. Of course, people coming from different disciplines will, in general, have different points of view on "preference modelling". Psychologists will be inclined to study the process by which judgements of preference are formed, transformed and influenced. Economists will be interested in specifying models of preference for the participants to a market that will allow them to derive nice equilibrium results and/or useful comparative statics propositions. Philosophers will concentrate on the links between "ethics" and preferences and investigate the notion of "rationality". Political scientists will study decisions made by groups of people and design electoral systems. Operations researchers will develop tools and models that might help an individual or a group to come to a decision and justify it. This list is not exhaustive: computer scientists (incorporating notions of "value" in knowledge-based systems), statisticians (testing a hypothesis on collected preference information and building tools to represent it) or marketers (optimising the characteristics of a product to increase its market share given what is known of the preferences of various segments of consumers) have also often contributed to the topic of preference modelling. A useful – although not universal – guide to this variety of points of view consists of distinguishing a *normative* perspective (central questions in this perspective are: How to behave "rationally" in a specific situation? What are the preference models allowing us to reach this goal?), a *descriptive* perspective (How do people make decisions in a specific situation?) and a *prescriptive* one (How to help someone to reach a decision in a specific situation?). The borderlines between these various perspectives are, however, often fuzzy and many preference models have been used in more than one of them.

In the face of such a diversity, one may wonder if there is anything like a field of "preference modelling". Although coming from different disciplines and having different objectives in mind, people working on "preferences" share a number of tools and preoccupations. With a clear emphasis on formal aspects, we recall in section 2 what can be called the "classical theory" of preference modelling, which serves as a "benchmark" of most of the works in the area. We outline in section 3 how the papers in this volume extend, modify or enrich this "classical theory".

2. Basic tools and results

Although preference modelling has a long history, most of the basic tools and results in the area were elaborated after World War II. Several books published in the 70's (see [14, 26, 27, 48]) consolidated what can now be called the "classical theory" of preference modelling. Ignoring details, this classical theory may be seen as characterised by:

- the use of a specific *language*,
- the use of a specific *syntax*, and
- the emphasis put on a number of *particular situations*.

We briefly review each of these points below.

2.1. The language

Studies in preference modelling usually take as a starting point a set X of "objects" to be compared or evaluated. Depending on the context, this set may be finite (X consists in a set of candidates applying for a job) or infinite (X consists in a set of consumption levels of, perfectly divisible, commodities). Consider an ordered pair (x, y) of objects. In the classical theory, it is supposed that the answer to the question "is x at least as good as y?" can have two and only two possible answers: "yes" or "no". Asking such a question for all possible ordered pairs of objects defines a binary relation $\succcurlyeq$ on X (i.e. a subset of X^2 – we use the notation $x \succcurlyeq y$ instead of $(x, y) \in \succcurlyeq$) in the following way:

$x \succcurlyeq y$ iff the answer to the question "is x at least as good as y?" is "yes".

It is easy to see that considering a pair $\{x, y\}$ of objects, four and only four mutually exclusive cases arise:

(i) $[x \succcurlyeq y$ and $y \succcurlyeq x]$ denoted by $x \sim y$, which reads "x is *indifferent* to y",

(ii) [Not $(x \succcurlyeq y)$ and Not $(y \succcurlyeq x)$] denoted by $x \mathrel{?} y$, which reads "x is *incomparable* to y",

(iii) [$x \succcurlyeq y$ and Not $(y \succcurlyeq x)$] denoted by $x \succ y$, which reads "x is *strictly preferred* to y", and

(iv) [Not $(x \succcurlyeq y)$ and $y \succcurlyeq x$] denoted by $y \succ x$, which reads "y is *strictly preferred* to x".

By construction, it is clear that $\sim$ and ? are symmetric (i.e. $x \sim y \Rightarrow y \sim x$) and $\succ$ is asymmetric (i.e. $x \succ y \Rightarrow$ Not$(y \succ x)$). Furthermore, if, as seems rather indisputable, $\succcurlyeq$ is supposed to be reflexive (i.e. $x \succcurlyeq x$, for all x), then $\sim$ is reflexive and ? is irreflexive (Not(x ? x) for all x).

The language of the "classical theory" is that of *binary relations*. Although the use of this language may seem obvious and inevitable, it raises several problems. Among the most important ones, let us mention:

- The *observability* problem. If preferences are to be studied, it should be possible to "observe" them in a safe and reliable way. Asking a question like "is x at least as good as y?" may not offer such a tool because it would base the entire theory on a purely *declarative basis* without obvious and direct connections with observable phenomena. A classical remedy to this problem is to take as primitive "observed choices", i.e. choices made between several possible objects belonging to a subset Y of X. This is the core of the so-called "revealed preference" theory in which $\succcurlyeq$ is inferred from observed choices. This is not always possible, however: choices allowing such an inference are essentially "binary" in that choices made over pairs of objects govern choices made over larger sets. Conditions guaranteeing the possibility of "rationalising" choices by a preference relation are classical, see [48, 49]. They have recently been severely criticised (see [33, 51, 52]), thus weakening the appeal of the language of the classical theory.
- The *informational* problem. Sticking with the definition of $\succcurlyeq$ via the answer to the question "is x at least as good as y?", one could envisage answers different from just "yes" or "no", for example,
  - allowing for "I do not know" answers;
  - allowing for answers including information on the "strength" of the preference, e.g., "x is vastly – moderately, slightly – preferred to y";
  - allowing for answers including information on the "credibility" of the proposition "x is at least as good as y", e.g. "the credibility of the proposition 'x is at least as good as y' is larger than the credibility of the proposition 'z is at least as good as w'", or even "the degree of credibility with which 'x is at least as good as y' is 0.3".
- The *replication* problem. The language of "classical preference modelling" implicitly assumes that the answer to the question "is x at least as good as y?" is stable over time (or, at the least, does not study the problem of the evolution of preferences over time). If this appears not to be so, the pertinent premises might

well be different. One could envisage the use of the frequency (or probability) of each of the answers when the question is repeated several times or the frequency (or probability) with which an object is chosen in a set of objects when this set is presented several times (see [30]).

2.2. *The syntax*

The "classical theory" of preference modelling not only comes with a specific language but also with a specific syntax. In addition to the reflexivity of $\succcurlyeq$ this syntax amounts to imposing that:

- $\succcurlyeq$ is complete (for all x, y with $x \neq y$, Not $(x \succcurlyeq y) \Rightarrow y \succcurlyeq x$), and
- $\succcurlyeq$ is transitive ($x \succcurlyeq y$ and $y \succcurlyeq z \Rightarrow x \succcurlyeq z$).

These two properties, which define what is usually called a *weak order*, have far-reaching consequences. They imply that

- there is no incomparability (? is empty),
- indifference ($\sim$) is transitive,
- preference ($\succ$) is transitive, and
- indifference and preference combine in a "nice" way (e.g. [$x \succ y$ and $y \sim z \Rightarrow x \succ z$] and [$x \sim y$ and $y \succ z \Rightarrow x \succ z$]).

When $\succcurlyeq$ is a weak order, indifference is an equivalence (reflexive, symmetric and transitive relation) and the set of equivalence classes of X under $\sim$ is totally ordered by $\succ$.

Using the language and the syntax of the "classical theory" allows answers to be given to a number of basic problems. We briefly sketch three of them here.

2.2.1. *Problem 1: Choosing from a preference relation*

Suppose that preferences have been modelled according to a weak order $\succcurlyeq$ on a set X. If, for some reason, a choice is to be made among a subset $Y \subseteq X$ of objects, how can $\succcurlyeq$ be used to guide this choice?

An obvious way of defining the set $C(Y, \succcurlyeq)$ of chosen objects (note that since we allow $C(Y, \succcurlyeq)$ to contain more than one object, speaking of "choosable" objects may be more appropriate) in Y given $\succcurlyeq$ is

$$C(Y, \succcurlyeq) = \{x \in Y \colon y \succ x \text{ for no } y \in Y\},$$

an object belonging to the choice set if there is no object that is strictly preferred to it. It is not difficult to see that $C(Y, \succcurlyeq)$ is always non-empty when Y is finite (the case in which Y is infinite raises some difficulties due to the possible existence of "infinite preference chains"; see [5]) and $\succcurlyeq$ is a weak order. It should, however, be noted that $\succcurlyeq$ being a weak order is a sufficient but not a necessary condition for $C(Y, \succcurlyeq)$ to be non-empty. A classical result in graph theory (see [48]) shows that $C(Y, \succcurlyeq)$ will be non-empty on a finite set Y as soon as $\succ$ has no circuits in Y (i.e. it is never true that, for some k and some $x_1, x_2, \ldots, x_k$ belonging to Y, $x_1 \succ x_2$, $x_2 \succ x_3$, ..., $x_{k-1} \succ x_k$ and $x_k \succ x_1$). Thus, using more general preference structures than weak orders still allows us to give simple and satisfactory answers to problem 1. Finally, let us mention that, in some situations (think of a competitive exam), it is not only necessary to be able to choose objects from a subset $Y \subseteq X$, but also to rank order the objects in Y. The syntax used in the classical theory gives an obvious answer to that problem since the restriction of a weak order on X to a subset Y of X is clearly a weak order.

2.2.2. Problem 2: Representing preferences using numbers

Manipulating a preference relation $\succcurlyeq$ on X can be cumbersome. Representing $\succcurlyeq$ using numbers will allow us to simplify the manipulation of $\succcurlyeq$ and to use optimisation techniques to give answers to problem 1.

It is clear that the completeness and transitivity of $\succcurlyeq$ are necessary conditions for the existence of a real-valued function f on X such that, for all $x, y \in X$,

$$x \succcurlyeq y \Leftrightarrow f(x) \geq f(y). \tag{1}$$

When X is finite or countably infinite, these conditions are also sufficient to obtain such a numerical representation (see e.g. [14, 27]). The situation is slightly more complex when X is uncountable since (1) imposes that the structure of X should not be too different from the structure of $\mathbb{R}$ and that $\succcurlyeq$ should behave on X much like $\geq$ behaves on $\mathbb{R}$. The syntax of the classical theory remains, however, at the heart of the existence of numerical representation of preferences.

2.2.3. Problem 3: Aggregating preferences

Suppose that $n \geq 2$ preference relations $\succcurlyeq_1, \succcurlyeq_2, \ldots, \succcurlyeq_n$ have been collected on X, e.g. because objects are evaluated according to several points of view (voters, criteria, experts, etc.). In such a situation, one may want to use this information in order to build a "collective" preference relation $\succcurlyeq$ aggregating the information contained in $\succcurlyeq_1, \succcurlyeq_2, \ldots, \succcurlyeq_n$. Most often, what is in fact looked for is a "mechanism" that would be able to aggregate *any* n-tuple of preference relations into a "collective" relation (this defines an "electoral system" or an "aggregation method"). Using the syntax of the classical theory, such a mechanism may be seen as an aggregation function F from $WO(X)^n$ into $WO(X)$, where $WO(X)$ denotes the set of all weak orders on X. Since the classic work of Arrow (see [2]), it is well known that such an aggregation raises serious problems; imposing a small number of apparently innocuous conditions on F often precipitates an impossibility result: no aggregation mechanism can satisfy them simultaneously (for a review of these various impossibility results, see [50]).

A good example of the difficulties uncovered by "Arrow-like" results is obtained with the "method of majority decisions". It consists in declaring that "x is collectively at least as good as y" if "x is at least as good as y" for at least 50% of the weak orders $\succcurlyeq_1, \succcurlyeq_2, \dots, \succcurlyeq_n$. Although such a method seems very reasonable and perfectly in line with our usual conception of "democracy", it does not always lead to a collective preference relation that is a weak order. The strict part of this relation may even contain circuits: this is the celebrated "Condorcet Effect"; taking $X = \{x, y, z\}$, $n = 3$ and $x \succ_1 y \succ_1 z$, $z \succ_2 x \succ_2 y$ and $y \succ_3 z \succ_3 x$ gives a prototype example of such a situation. Using a collective preference relation which contains circuits in its asymmetric part in order to choose a subset of candidates and/or to rank order them is far from being an easy task. It has generated numerous studies (see [47]).

The syntax of the classical theory is not without problems. We have already mentioned that it is overly restrictive with respect to problem 1 and that it is not well-suited to deal with aggregation problems. Additional problems are

- the possibility of obtaining consistent and replicable violations of transitivity of indifference (see [29]) and even of strict preference (see [36, 54]) in contexts having nothing to do with aggregation problems;
- the frequent occurrence of incomparability as suggested by common sense and the practice of decision-aid (see [44, 45, 55]).

Keeping intact the language of the "classical theory" but changing its syntax results in two different types of extensions:

- Classical ones (semi-orders, interval orders, partial orders, sub-orders, etc.) allow for intransitive indifference and/or incomparability while maintaining the ban on circuits of strict preference. This leads to models of preferences in which the link between choices and preferences remains simple. Numerical representations of type (1) are modified either by the introduction of thresholds or by the replacement of an "iff" representation by a representation which goes only "one way", e.g. of the type $x \succ y \Rightarrow f(x) > f(y)$ (on these classical extensions, see [16, 37, 42, 43]). Let us finally note that allowing the collective preference relation to be of such types does very little to solve the "aggregation problem" uncovered by Arrow-like results, see [50].
- Non-classical ones admitting circuits in strict preference (see [6, 15, 18–20, 54, 56]). Such models clearly raise many questions. In particular, using them to choose a subset of objects or to rank order objects is far from being obvious. They have often been criticised for allowing various kinds of "irrational behaviour" (as in the famous "money-pump" argument, see [41]). We strongly recommend [18] as a radical antidote to these considerations. Besides their unsurprising occurrence in aggregation problems, this paper shows not only that circuits of strict preference may occur and may correspond to a seemingly rational behaviour, but that many arguments showing the irreducible irrationality of such preferences are based on much less stable foundations than what is usually thought (a whole volume of this journal was devoted to this topic – see vol. 23, 1990).

2.3. Particular situations

Specifying the language and the syntax of the classical theory can be done more or less independently of the nature of the set X of objects. Special structures for X have received much attention in the literature. Supplementing the language and the syntax of the classical theory with additional conditions taking advantage of the structure of X allows us to obtain more specific preference models. Among the structures that have received much attention, let us mention:

- "decision-making with multiple criteria", where elements of X are characterised by a vector of evaluations on several dimensions, attributes or criteria,
- "decision-making under risk", where elements of X are seen as probability distributions on a set of consequences,
- "decision-making under uncertainty", where elements of X are characterised by the consequences they produce contingently to the occurrence of various states of nature.

In these three situations (note that in all these cases, the structure of X has the flavour of a Cartesian product), it is tempting to add to the classical theory additional conditions that will allow to obtain more specific models:

- *mutual preference independence* in the case of *multiple criteria*,
- *independenc*e in the case of *risk*,
- the s*ure thing principle* in the case of *uncertainty*.

When these additional conditions are supplemented with more technical ones, mainly amounting to introducing some "richness" in X and ensuring that $\succcurlyeq$ behaves consistently in this rich structure, well-known specialisations of the "classical theory" are obtained (see [14 or 60]):

- *additive utilities* in the case of multiple attributes,
- *Expected Utility* in the case of risk,
- *Subjective Expected Utility* in the case of uncertainty.

One of the main interests of these specialised theories is that they allow us to build numerical representations that are much more specific than (1). Note that in (1), any strictly increasing transformation applied to f leads to an alternative numerical representation of $\succcurlyeq$ (f defines what is usually called an "ordinal scale"). The use of the additional conditions mentioned above implies that f can be decomposed additively when the structure of X is rich. The numerical representations then define "interval scales" (unique up to the choice of an origin and a unit). Many methods have been devised to assess them (see [26, 27]).

These additional conditions have been submitted to extensive experimental tests. In particular, in the area of risk and uncertainty, the conditions underlying the Expected

Utility and the Subjective Expected Utility models were shown to be consistently and replicably violated in many experiments (classic papers are [1, 12, 25, 35]). This gave rise to an extremely active field of research (see [17, 31, 39, 40, 59] in the case of risk and [21, 22, 46, 60] in the case of uncertainty). "Dutch book type" arguments have often been used to criticise these extensions of the classical theory on normative grounds (see [41]). As in the case of intransitive preferences, it seems that the validity of this type of argument deserves close scrutiny (see [32, 34] for a criticism of the arguments related to "dynamic consistency" in the case of risk).

Let us finally mention that other types of structures for X may be of interest. In some instances, X has a topological structure which allows us to introduce notions such as "continuity" for numerical representations. In other cases, X may be endowed with a binary operation allowing us to combine objects (examples are the addition of two commodity bundles or the even-chance mixture of two objects) and hypotheses are made on the relations between the preference relation and this binary operation.

This very brief and incomplete introduction of the "classical theory" leaves room for at least three types of extensions:

- *changing the language*, e.g. having recourse to different primitives and/or to different logics,
- *changing the syntax*, e.g. abandoning transitivity or completeness,
- *modifying the additional conditions* when X has a special structure.

These three types of extensions may be combined, giving rise to more complex models (note that if the language is changed, elements of the syntax will have to be redefined).

Our brief and incomplete survey of the "classical theory" concentrated on formal aspects. It leaves aside many important features of this theory. Let us note that it may be seen as being essentially

- *static*, i.e. giving little or no attention to genuinely dynamic considerations such as, for example, the preference for flexibility (see [28] for a pioneering work on the subject) or evolution in time of preferences,
- *individual*, i.e. conceived in a context in which strategic interactions between several individuals or groups are not essential,
- "*formulation-free*", i.e. giving no or little attention to the process of modelling the set of objects and elaborating a model of preference.

Finally, it should be noted that many important topics related to the classical theory have been left aside. Among the most important ones, we should mention

- the "art and science" of collecting preference information from subjects (see [57]),

- the links between “preference modelling” and the problem of “meaningfulness” in measurement theory (see [42]),
- the statistical analysis of preference information (see [10, 23]),
- philosophical themes on the links between preferences and “values” and on the nature of “values” (see [7, 11, 53, 58]).

3. Presentation of the papers

The eighteen papers collected in this volume do not, of course, cover all possible extensions of the “classic model” alluded to in section 2. They nevertheless address some of its limitations and/or explore more in depth some of its points.

Four papers are mainly concerned with the relations between choices and preferences. The paper by Josep E. Peris, Carmen M. Sanchez and Begoña Subiza, *Numerical representation of choice functions*, shows that the language of preference relations can be entirely dispensed with while still arriving at “numerical representations”. This allows us to reformulate classical results on numerical representations in purely choice-theoretic terms. The two papers by Andrey V. Malishevski, *Generalized utility based on values of opportunity sets*, and by Klaus Nehring and Clemens Puppe, *Extended partial orders: A unifying structure for abstract choice theory*, study choices that cannot be “rationalised” by a preference relation. They both consider models in which “subsets of objects” play a central role, for example, because some objects interact (e.g. items on the card of a restaurant) or because “having the choice” has an intrinsic value (e.g. allowing room for “flexibility”). The paper by Taradas Bandyopadhyay, *Choice procedures and rational selection*, is devoted to sequential choice procedures. It describes and characterises mechanisms of choice that, in many respects, are more realistic than the one-stage optimisation of a single function. Stephen A. Clark, *Dynamically consistent choice and the revelation of Strotz–Pollack equilibrium*, takes as basic objects of study “decision trees without chance nodes”. This allows him to define and characterize various versions of “dynamically consistent behaviour” using purely choice-theoretic tools.

Antoine Billot, *Auto-biased choice theory*, deals with a surprisingly neglected problem, that of the formation and the evolution in time of preferences (for a recent alternative theory, see [13]). He presents a theory of the evolution in time of stochastic preferences in which the desirability of an object is learnt step by step through the organisation of “competitions” – some of which being more informative than others.

The paper by Bartel van de Walle, Bernard de Baets and Etienne Kerre, *Characterizable fuzzy preference structures*, explores a radical extension of the language of the classical theory replacing “crisp” by “fuzzy” binary relations, i.e. fuzzy subsets of X^2. The manipulation of such relations raises several difficult problems (the use of fuzzy sets does not allow to keep together all the “nice” properties of a Boolean lattice). This paper shows how the use of a two-parameter family of fuzzy preference structures provides an efficient and elegant solution to these problems. Patrice Perny, *Multi-criteria filtering methods based on concordance and non-discordance principles*, uses a similar language in the context of multiple criteria evaluations. He presents several models allowing to assign objects to categories (which may or may not be ordered) defined by several examples on the basis of multiple criteria. Such a problem frequently arises in many situations (granting credit, selecting students for a programme, diagnosing patients) and has strong connections with recent research in computer science.

Five papers in this volume are devoted to situations involving risk and uncertainty. The paper by James S. Dyer and Jianmin Jia, *Preference conditions for utility models: A risk-value perspective*, deals with decision under risk within the classical framework of Expected Utility theory. It gives various conditions allowing such a model to be written in a "risk-value" form while restricting the functional form for the utility function. These results, extending classical ones (see [3, 38]), are likely to be of much help for the practice of Decision Analysis. Peter C. Fishburn and Irving H. LaValle, *Subjective expected lexicographic utility: Axioms and assessment*, use the classical framework of [4] for decision making under uncertainty. They keep intact the ordering and independence side of the classical theory while severely weakening the "Archimedean" (or "continuity") condition, a condition that is always rather difficult to test in practice. This gives rise to the "Subjective Expected Lexicographic Utility" model, which includes two important features:

- the use of vectors of linear functionals that are compared lexicographically,
- the use of "matrix probabilities" instead of the classical "subjective probabilities".

The authors present a rather exhaustive investigation of the foundations of this model and give some hints on its possible use in practice.

The paper by Christophe Gonzales and Jean-Yves Jaffray, *Imprecise sampling and direct decision-making*, extends the classical setting of decision making under uncertainty in a way that is likely to be well adapted to the treatment of uncertainty in large databases. They study decision problems under uncertainty in which information is available only through *finite sampling* in a "data base" in which information may be *incomplete*. They present a fairly complete axiomatic analysis of a preference model adapted to this situation that combines aspects of Expected Utility generalised to cope with belief functions (see [24]) and Direct Decision Making (see [9]), a model generalizing Expected Utility to the case of frequentist information.

Two other papers on the same topic have a more experimental flavour. Mohammed Abdellaoui and Bertrand Munier, *The risk-structure dependence effect: Experimenting with an eye on decision-aiding*, present an experimental test of several models of decision making under risk by estimating indifference curves or different regions in the so-called "Marshack–Machina" triangle. Their sophisticated protocol, which makes use of computer-assisted individual interviews, allows them to show the sensibility of preferences on the "risk structure", i.e. the set of prospects under consideration. This gives many hints for the practice of Decision Analysis as well as shedding a new light on several recent experimental findings in the area. Although also dealing with experimental issues, the paper by Paul Anand, *Should we use simultaneous equations to model decision-making? – A methodological note*, is more conceptual in nature. In view of recent experimental findings in decision making under risk showing that most of the existing models are not able to satisfactorily explain observed data, he suggests a radical change in our statistical habits, forcefully arguing in favour of the use of simultaneous equation models in order to analyse such data.

Two papers in this volume focus on situations in which the set of objects has a special structure, and numerical representations exploiting this special structure are looked for. Gianni Bosi, *A note on the existence of continuous representations of homothetic preferences on a topological vector space*, studies the case in which X is a positive cone in a topological vector space. He gives conditions allowing us to obtain a continuous numerical representation of a preference relation on such a set that is homogeneous of degree one (i.e. such that $f(tx) = tf(x)$), this property having some importance in several aggregation problems in economics (see [8]). Juan Candeal, Esteban Induráin and Esteban Olóriz, *Existence of additive utility on semigroups: An elementary proof*, suppose that a binary operation $*$ is defined on X which makes $(X, *)$ a (positive) semigroup. They give conditions guaranteeing the existence of a numerical representation of a preference relation on X that is additive with respect to $*$ (i.e. such that $f(x * y) = f(x) + f(y)$). This generalises well-known results about ordered Archimedean groups (e.g. Hölder's theorem) which underlie many important constructions in extensive, difference and conjoint measurement (see [27]).

Two papers tackle problems related to the aggregation of preferences. Irène Charon and Olivier Hudry, *Lamarckian genetic algorithms applied to the aggregation of preferences*, deal with the classical problems of Slater (approximating a tournament by a linear order) and Kemeny (approximating a valued tournament by a linear order). They devise special heuristics – combining Simulated Annealing and Genetic Algorithms – and present extensive numerical results showing their usefulness in providing solutions to these computationally challenging problems. The paper by Pavel Y. Chebotarev and Elena Shamis, *Characterizations of scoring methods for preference aggregation*, studies "scoring methods" allowing the derivation of choices and rankings on the basis of conflicting information (e.g. information obtained after aggregating preferences). They show the interest of a general property – "self consistency" – in the study and characterisation of such methods and review a large number of papers using scoring methods for various purposes.

The final paper by John T. Buchanan, Erez J. Henig and Mordecai I. Henig, *Objectivity and subjectivity in the decision process*, is more conceptual and prescriptively-oriented. It discusses at length the interest and the philosophical justification for separating as much as possible "objective" from "subjective" elements in a decision-aid process. This clarifies the role of preference modelling in a decision-aiding process.

Although incomplete, we do hope that this collection of papers will allow the reader to have a good overview of recent developments in the vast and fascinating field of “preference modelling”.

References

[1] M. Allais, Le comportement de l’homme rationnel devant le risque, critique des postulats et axiomes de l’école américaine, Econometrica 21(1953)503–46.

[2] K.J. Arrow, *Social Choice and Individual Values*, 2nd ed., Wiley, 1963.

[3] K.J. Arrow, *Aspects of the Theory of Risk-Bearing*, Academic Bookstore, Helsinki, 1965.

[4] F.J. Anscombe and R.J. Aumann, A definition of subjective probability, Annals of Mathematical Statistics 34(1963)199–205.

[5] T.X. Bergstrom, Maximal elements of acyclic relations on compact sets, Journal of Economic Theory 10(1975)403–404.

[6] D. Bouyssou, Some remarks on the notion of compensation in MCDM, European Journal of Operational Research 26(1986)150–160.

[7] J. Broome, *Weighting Goods*, Basil Blackwell, 1991.

[8] J.S. Chipman, Homothetic preferences and aggregation, Journal of Economic Theory 8(1974) 26–38

[9] Y. Coignard and J.-Y. Jaffray, Direct decision making, in: *Decision Theory and Decision Analysis: Trends and Challenges*, ed. S. Ríos, Kluwer Academic, 1994, pp. 81–90.

[10] C.H. Coombs, *A Theory of Data*, Wiley, 1964.

[11] T.A. Cowan and P.C. Fishburn, Foundations of preference, in: *Essays in Honor of Werner Leinfellner*, eds. G. Eberlein and H. Berghel, D. Reidel, 1988, pp. 261–271.

[12] D. Ellsberg, Risk, ambiguity and the savage axioms, Quarterly Journal of Economics 75(1961)643–669.

[13] J.-C. Falmagne and J.-P. Doignon, Stochastic evolution of rationality, Theory and Decision 43(1997) 107–138.

[14] P.C. Fishburn, *Utility Theory for Decision-Making*, Wiley, 1970.

[15] P.C. Fishburn, Nontransitive measurable utility, Journal of Mathematical Psychology 26(1982) 31–67.

[16] P.C. Fishburn, *Interval Orders and Interval Graphs*, Wiley, 1985.

[17] P.C. Fishburn, *Nonlinear Preference and Utility Theory*, Johns Hopkins University Press, 1988.

[18] P.C. Fishburn, Nontransitive preferences in decision theory, Journal of Risk and Uncertainty 4(1991) 113–134.

[19] P.C. Fishburn, Nontransitive additive conjoint measurement, Journal of Mathematical Psychology 35(1991)1–40.

[20] P.C. Fishburn, Additive differences and simple preference comparisons, Journal of Mathematical Psychology 36(1992)21–31.

[21] I. Gilboa, Expected utility with purely subjective non-additive probabilities, Journal of Mathematical Economics 16(1987)65–68.

[22] I. Gilboa and D. Schmeidler, Maxmin expected utility with non-unique prior, Journal of Mathematical Economics 18(1989)141–153.

[23] P.E. Green, D.S. Tull and G. Albaum, *Research for Marketing Decisions*, Englewood Cliffs, 1988.

[24] J.-Y. Jaffray, Linear utility theory for belief functions, Operations Research Letters 8(1989) 107–112.

[25] D. Kahneman and A. Tversky, Prospect theory: An analysis of decision under risk, Econometrica 47(1979)263–291.

[26] R.L. Keeney and H. Raiffa, *Decisions with Multiple Objectives – Preferences and Value Tradeoffs*, Wiley, 1976.

[27] D.H. Krantz, R.D. Luce, P. Suppes and A. Tversky, *Foundations of Measurement, Vol. 1: Additive and Polynomial Representation*, Academic Press, 1971.

[28] D.M. Kreps, A representation theorem for “preference for flexibility”, Econometrica 47(1979) 565–577.

[29] R.D. Luce, Semiorders and a theory of utility discrimination, Econometrica 24(1956)178–191.

[30] R.D. Luce, *Individual Choice Behavior: A Theoretical Analysis*, Wiley, 1959.

[31] M. Machina, Expected ultility without the independence axiom, Econometrica 50(1982)277–323.

[32] M.J. Machina, Dynamic consistency and non-expected utility models of choice under uncertainty, Journal of Economic Literature 27(1989)1622–1688.

[33] A.V. Malishevski, Criteria for judging the rationality of decisions in the presence of vague alternatives, Mathematical Social Sciences 26(1993)205–247.

[34] E.L. McClennen, *Rationality and Dynamic Choice: Foundational Explorations*, Cambridge University Press, 1990.

[35] K.R. McCrimmon and S.Larsson, Utility theory: Axioms versus “paradoxes”, in: *Expected Utility Hypotheses and the Allais Paradox*, eds. M. Allais and O. Hagen, Reidel, 1979, pp. 27–145

[36] K.O. May, Intransitivity, utility and the aggregation of preference patterns, Econometrica 22(1954) 1–13.

[37] M. Pirlot and Ph. Vincke, *Semiorders. Properties, Representations, Applications*, Kluwer, 1997.

[38] J.W. Pratt, Risk aversion in the small and in the large, Econometrica 3(1964)122–136.

[39] J. Quiggin, A theory of anticipated utility, Journal of Economic Behaviour and Organization 3(1982) 323–343.

[40] J. Quiggin, *Generalized Expected Utility Theory: The Rank Dependent Model*, Kluwer, 1993.

[41] H. Raiffa, *Decision Analysis – Introductory Lectures on Choices Under Uncertainty*, 2nd ed., Addison-Wesley, 1970.

[42] F.S. Roberts, *Measurement Theory with Applications to Decision Making, Utility and the Social Sciences*, Addison-Wesley, 1979.

[43] M. Roubens and Ph. Vincke, *Preference Modelling*, Springer, 1985.

[44] B. Roy, *Méthodologie Multicritère d’Aide à la Décision*, Economica, 1985 (English translation: *Multicriteria Methodology for Decision Aiding*, Kluwer, 1996).

[45] B. Roy, The outranking approach and the foundations of ELECTRE methods, Theory and Decision 31(1991)49–73.

[46] D. Schmeidler, Subjective probability and expected utility without additivity, Econometrica 57 (1989)571–587.

[47] Th. Schwartz, *The Logic of Collectice Choice*, Columbia University Press, 1986.

[48] A.K. Sen, *Collective Choice and Social Welfare*, Holden Day, 1970.

[49] A.K. Sen, Social choice theory: A re-examination, Econometrica 45(1977)53–89.

[50] A.K. Sen, Social choice theory, in: *Handbook of Mathematical Economics III*, eds. K.J. Arrow and M.D. Intriligator, North-Holland, 1986, pp. 1073–1181.

[51] A.K. Sen, Internal consistency of choice, Econometrica 61(1993)495–521.

[52] R. Sugden, Why be consistent? A critical analysis of consistency requirements in choice theory, Economica 52(1985)167–183

[53] A. Tsoukiàs and Ph. Vincke, A survey on nonconventional preference modelling, Ricerca Operativa 61(1992)5–49.

[54] A. Tversky, Intransitivity of preferences, Psychological Review 76(1969)31–48.

[55] Ph. Vincke, *Multicriteria Decision-Aid*, Wiley, 1992.

[56] K. Vind, Independent preferences, Journal of Mathematical Economics 20(1991)119 – 135.
[57] D. von Winterfeld and W. Edwards, *Decision Analysis and Behavioral Research*, Cambridge University Press, 1986.
[58] G.H. von Wright, *The Logic of Preference*, Edinburgh University Press, 1963.
[59] M.E. Yaari, The dual theory of choice under risk, Econometrica 55(1987)95 – 115.
[60] P.P. Wakker, *Additive Representations of Preferences – A New Foundation of Decision Analysis*, Kluwer, 1989.

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