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Using DEA as a tool for MCDM: some remarks

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The purpose of this paper is to study proposals to use Data Envelopment Analysis (DEA) as a tool for Multiple Criteria Decision Making (MCDM). We first recall, using a simple model, the equivalence between the concept of 'efficiency' in DEA and that of 'convex efficiency' in MCDM. Examples are then used to show that various techniques that have been proposed in the DEA literature to deal with MCDM problems violate simple normative properties that are commonly accepted. We conclude with some remarks on the possible areas of interaction between DEA and MCDM.

Keywords: MCDM; DEA; efficiency; convex efficiency

Introduction and motivation

Twenty years after the publication of the founding paper of Charnes *et al*,[1] Data Envelopment Analysis (DEA) can safely be considered as one of the recent 'success stories' in OR: several hundreds of papers have been published since 1978 and, in the best tradition of OR, real-world applications of DEA have lead to new theoretical developments and *vice versa*.[2,3]

DEA deals with the evaluation of the performance of Decision Making Units (DMU) performing a *transformation process* of several *inputs* several *outputs*. Relying on a technique based on Linear Programming (LP) and without having to introduce any subjective or economic parameters (weights, prices, etc.), DEA provides a 'measure of efficiency' of each DMU allowing, in particular, to separate *efficient* from *non-efficient* DMU and to indicate for each non-efficient DMU its 'efficient peers'.

The success of DEA in the area of performance evaluation together with the formal analogies existing between DEA and Multiple Criteria Decision Making (MCDM) (which become clear replacing DMU with alternatives, outputs with criteria to be maximised, inputs with criteria to be minimised, etc.) have lead some authors to propose to use DEA as a tool for MCDM.[4–6] A number of recent papers[7–9] have begun the analysis of the links between DEA and MCDM. We shall concentrate here, using simple examples, on the potential usefulness of DEA for MCDM.

This paper is organised as follows. We first recall, using a simple model, the links existing between the notions of efficiency in DEA and in MCDM. We proceed by giving examples showing that various attempts to use DEA as a tool for MCDM raise serious problems. We then summarise our findings and conclusions.

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Efficiency in MCDM and in DEA

The equivalence between the notion of 'efficiency' in DEA and that of 'convex efficiency' in MCDM is not a new fact.[7–9] It is however worth recalling here since it is crucial for our purposes. Furthermore we shall use a simple model which, in our opinion, makes the result more transparent than in previous analyses.

Let $X = \{a_1, a_2, \ldots, a_\ell\}$ be a finite set of alternatives that have been evaluated on a set of n real-valued criteria. Contrary to previous works in the area but in line with most works in the area of MCDM, we shall suppose throughout the paper that preference increases with all criteria. Although it is not difficult to generalise the analysis in order to include criteria to be 'minimised', this hypothesis will allow us to keep things simple considering DEA models having only 'outputs' and therefore to neglect the 'return to scale' problem. In order to avoid unnecessary complications, we shall also suppose that the evaluations of the alternatives on the criteria are strictly positive. We denote by $y_{jk} > 0$ the evaluation of alternative a_k on criterion j.

Alternative a_i is said to *dominate* alternative a_k if $y_{ji} \geqslant y_{jk}$ for $j = 1, 2, \ldots, n$, at least one of these inequalities being strict. An alternative $a \in X$ is said to be *efficient* in X if no alternative in X dominates it. It is clear, under very mild conditions on preferences, that efficient alternatives should receive special consideration in MCDM.

A 'folk theorem' in MCDM goes as follows:[10] if it is possible to find a set of strictly positive weights $w_1, w_2, \ldots, w_n$ such that the weighted sum of the criteria for alternative a_i is larger or equal than the weighted sum for any other alternative in X, then a_i is efficient (in X). It should be noted that the weights used in this 'folk theorem' may be normalised in any convenient way, for example letting $\Sigma_{j=1}^{n} w_j = \alpha > 0$ or $\Sigma_{j=1}^{n} w_j y_{jk} = \beta > 0$ for some alternative a_k.

A 'folk remark'[10] is the following: not all efficient alternatives in X can be characterised through the use of weighted sums if the image of X in the space of criteria is not convex (see Figure 1).

In the area of MCDM, alternatives that maximise (in X) a weighted sum of all criteria for some strictly positive weights are called convex efficient (CE) in X; alternatives that are not CE are said to be convex dominated (CD). LP offers a powerful tool for testing if an alternative is CE; many different formulations can be used for this purpose. Let us consider the following LP designed to test if $a_* \in X = \{a_1, a_2, \ldots, a_\ell\}$ is CE:

$$\text{(P)} \qquad \min D$$

subject to

$$\sum_{j=1}^{n} (y_{j*} - y_{jk}) w_j + D \geqslant 0, \qquad k = 1, 2, \ldots, \ell$$

$$\sum_{j=1}^{n} w_j y_{j*} = 1$$

$$w_j \geqslant \varepsilon, \qquad j = 1, 2, \ldots, n,$$

where ε is an arbitrarily small positive number and y_{j*} denotes the evaluation of alternative a_* on criterion j. It should be observed that since $a_* \in X$ and a_* is compared to all alternatives in X, *including itself*, negative values of D are infeasible. It is easily seen that a_* is CE (in X) if and only if the optimal value of the objective function of (P) is 0. In fact, if a_* is CE, we have, be definition, $\Sigma_{j=1}^{n} y_{j*} w'_y \geqslant \Sigma_{j=1}^{n} y_{ik} w'_j, k = 1, 2, \ldots, \ell$ for some strictly positive weights $w'_1, w'_2, \ldots, w'_n$. Now taking $w_j = w'_j / \Sigma_{j=1}^{n} w'_j y_{j*}$ and $D = 0$ gives a feasible solution of (P) with a suitably chosen ε. This is also an optimal solution since negative values of D are infeasible. Conversely if we have an optimal solution $w_1^*, w_2^*, \ldots, w_n^*, D^*$ of (P) with $D^* = 0$, then $\Sigma_{j=1}^{n} y_{j*} w_j^* \geqslant \Sigma_{j=1}^{n} y_{jk} w_j^*, k = 1, 2, \ldots, \ell$, so that a_* is CE since $w_j^* \geqslant \varepsilon > 0$.

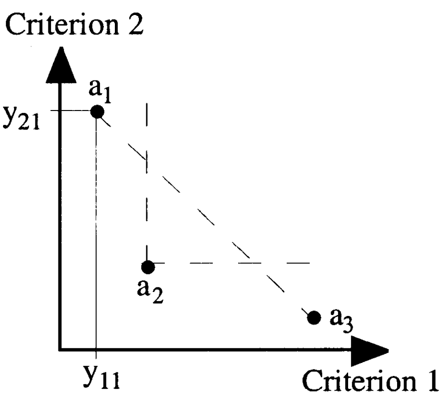

Figure 1 In $X = \{a_1, a_2, a_3\}$ all alternatives are efficient. Alternative a_2 cannot maximise a weighted sum of the two criteria.

In view of Figure 1, it is clear that when a_* is CE, the optimal solution of (P) will not, in general, be unique: more than one set of 'optimal weights' w_j^* will be compatible with $D^* = 0$.

Taking the dual of (P) leads to:

$$\text{(D)} \qquad \max M + \varepsilon \sum_{j=1}^{n} s_j$$

subject to

$$y_{j^*} M + \sum_{k=1}^{\ell} (y_{j^*} - y_{jk}) \lambda_k + s_j = 0, \qquad j = 1, 2, \ldots, n$$

$$\sum_{k=1}^{\ell} \lambda_k = 1$$

$$\lambda_k \geqslant 0, s_j \geqslant 0, \qquad M \text{ unrestricted},$$

which is easily seen to be equivalent to the output oriented BCC (primal) version of DEA[2] when there are no inputs (or equivalently when all alternatives have common values on all inputs). Loosely speaking and ignoring slacks, $E = 1 + M$ can be considered as a 'measure of inefficiency'. It is a kind of 'radial distance' from a_* to the CE frontier, that is the coefficient ($\geqslant 1$) by which the evaluations of a_* on all criteria should be multiplied in order to make it CE.[9]

Let us finally note that the 'Free Disposal Hull'[11] (FDH), a variant of DEA that rests on different assumptions on the technology underlying the transformation process of inputs into outputs, gives rise to a notion of efficiency that exactly coincides with the one used in MCDM.

DEA applied to MDCM problems

Several ways of using DEA as a tool for MCDM have been proposed in the literature. Relying on the analysis of the previous section we shall analyse three of them here.

A 'folk' technique

A 'folk' technique[4] amounts to using the opposite of the optimal value of the objective function of (P) for each alternative as a 'utility function' and to choose and/or to rank order alternatives according to this function. The intuition behind this technique is clear in view of the interpretation of (P) proposed above: $D^* = 0$ if the alternative is CE, $D^* > 0$ of the alternative is CD and $1 + D^*$ can be interpreted as a measure of inefficiency of an alternative.

For choice problems, this technique is not without difficulties. We mention here what we consider to be the two most serious ones:

(1) in most real-world problems *all* alternatives to be evaluated are likely to be efficient; many of them are

likely to be convex efficient. Hence most, if not all alternatives will be chosen. This is the classical'discrimination problem' in DEA.[4]

(2) although *all* efficient alternatives may be considered as candidates for choice (many efforts have been devoted to the creation of MCDM tools in line with this principle[12,13]) this technique excludes from the choice set all efficient alternatives that are not CE (for example, alternative a_2 in Figure 1).

When this technique is used to rank order alternatives, some new problems appear:

(1) the ranking of CD alternatives is somewhat arbitrary: equivalent formulations of (P) (for example, if the BCC–DEA model is replaced by an additive DEA model[2], this amounts to using a different metric to measure the distance to the CE frontier) for characterising CE alternatives may lead to a different ranking of CD alternatives;

(2) *all* CE alternatives are ranked before *all* CD alternatives. This appears disturbing since it is well-known that some CD alternatives may be sufficiently attractive to deserve to be ranked before most CE alternatives (see Figure 2).

Although this not specific to this technique, let us finally mention that the meaningfulness, in the measurement-theoretic sense of the term,[14] of the manipulation of the optimal solution of (P) for CD alternatives when criteria are supposed to be measured on interval (or ratio) scales raises serious conceptual and computational difficulties.[15,16]

Andersen and Petersen's technique

The technique proposed by Andersen and Petersen[17] allows to discriminate between CE alternatives by modifying the computation of the 'inefficiency measure' now based on a 'radial distance' to a CE frontier obtained *without* taking into account the alternative being evaluated. In our framework, this amounts to modifying (P) by removing the constraint comparing a_* with itself. This defines problem (P′). It is clear, using the same arguments as before, that a non positive optimal value of the objective function of (P′) is equivalent to convex efficiency. The technique of Andersen and Petersen amounts to using the opposite of the optimal value of the objective function of (P′) for each alternative as a 'utility function'.

This technique gives results identical to the 'classical' DEA for CD alternatives but allows to 'discriminate' between CE ones.[17] It should be clear that it does not solve any of the problems encountered with the 'folk' technique when trying to rank order alternatives: all CE alternatives are ranked above all CD ones; changing the formulation of the model (for example, using an additive DEA model) may now alter the *whole* ranking.

When used to choose alternatives, this technique allows to discriminate between CE alternatives. In order to understand how this discrimination is performed let us consider the example of Figure 3 (in which alternative e has 3.9 on both criteria).

In $\{a, b, c\}$, applying the ranking technique of Andersen and Petersen gives (using obvious notations) the ranking $b \succ (a \sim c)$ which could be interpreted as a 'bonus' given to 'central alternatives' (we omit the details of our numerical examples; they are simple enough to be treated without the use of any DEA-dedicated software. Simple geometric considerations and some familiarity with a LP package should suffice in all cases). Using a similar technique on $\{a, b, c, d\}$ gives $b \succ (a \sim c) \succ d$, confirming our previous analysis. However on $\{a, b, c, d, e\}$ we obtain $(a \sim c) \succ b \succ e \succ d$. This shows that the ranking obtained is 'set dependent' and therefore allows possible 'rank reversals', This is not surprising since convex efficiency is obviously a set dependent concept. However what appears more disturbing is that this set dependency occurs in a rather non intuitive and arbitrary way: the addition of a close variant (e) of a top ranked alternative

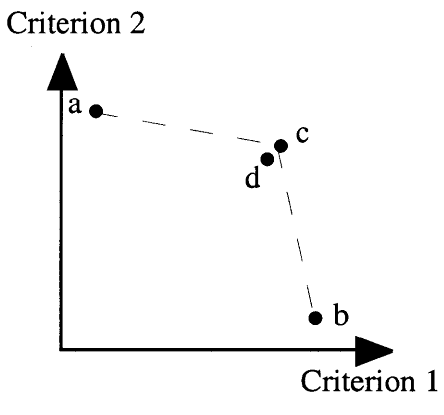

Figure 2 Alternatives a, b and c are CE in $X = \{a, b, c, d\}$; alternative d is always ranked below a and b with the folk technique.

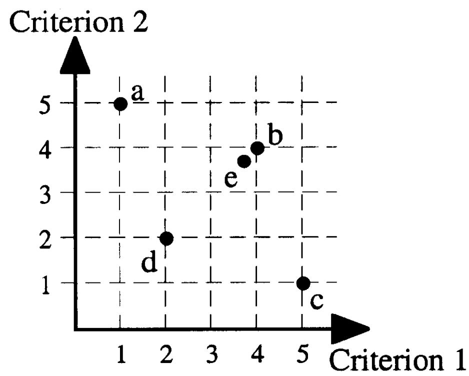

Figure 3 Illustration of the technique of Andersne and Petersen.

(*b*) decreases the position of the latter in the ranking. Therefore the 'bonus' appears to be given not to 'central' but to 'isolated' alternatives. This appears most unsatisfactory if the 'language' created by the various criteria is used as an incentive to create new alternatives.

The cross-evaluation technique

The cross-evaluation technique was developed by DEA researchers[4,18–20] in order to overcome the 'discrimination problem' (that is the fact that most alternatives are likely to be efficient in DEA). It is based on the following simple idea. Testing the CE status of an alternative using (P) implies finding weights for the criteria. Using these weights, it is possible to compute the weighted sum for the remaining alternatives so that each alternative 'rates' all others according to its own 'point of view'. Let us denote by z_{ki} the weighted sum of the criteria obtained for alternative a_i when using the weights obtained in the optimal solution of (P) with $a_* = a_k$. The $\ell \times \ell$ square matrix $\mathbf{Z} = [z_{ki}]$ (having ones on its main diagonal) is called the 'cross-evaluation matrix'. The column averages of the cross-evaluation matrix (that is the mean rating of an alternative when rated by all other alternatives) are then used as a 'utility function' for choosing and/or ranking; other ways for exploiting the cross-evaluation matrix can be envisaged[18,19] such as using the principal (right) eigen vector of $\mathbf{Z}$.

Note that since the 'optimal weights' of the criteria in (P) are, in general, not unique, the cross-evaluation matrix $\mathbf{Z}$ is not unique either. Various solutions have been proposed[18,19] to overcome this difficulty. The targeted aggressive (TA) formulation amounts to defining z_{ki} using weights among the optimal solutions of (P) for $a_* = a_k$ that minimise $\Sigma_{j=1}^{n} y_{ji} w_j$. Similarly the 'blanketed aggressive' (BA) (resp. The 'targeted benevolent' (TB) and the 'blanketed benevolent' (BB) formulation defines z_{ki} using weights among the optimal solutions of (P) for $a_* = a_k$ that minimise $\Sigma_{i \neq k} \Sigma_{j=1}^{n} y_{ji} w_j$ (resp. That maximise $\Sigma_{j=1}^{n} y_{ji} w_j$, that maximise $\Sigma_{i \neq k} \Sigma_{j=1}^{n} y_{ji} w_j$). It is not easy to find intuitive arguments favouring the choice of one of these four formulations. This is all the more disturbing that they may well give different results. On the example of Figure 3, we obtain on $\{a, b, c, e\}$ the ranking $b \succ e \succ (a \sim c)$ using TA and $b \succ (a \sim c) \succ e$ using TB. Furthermore, it should be noted that the use of any of these four formulations does not guarantee the uniqueness of the set of weights used to defined the values z_{ki}. On the example of Figure 3 using BA, we may obtain on $\{a, b, c, e\}$ either of the three rankings $b \succ e \succ (a \sim c), b \succ a \succ e \succ e$ or $b \succ c \succ e \succ a$, depending of the arbitrary choice of a particular set of 'optimal weights'.

It should be noted that the cross-evaluation technique used for ranking problems allows, contrary to the previous techniques encountered, to rank CD alternatives before CE ones. On the example of Figure 3 (in which a, b and c are CE and d and e are CD) the ranking obtained on $\{a, b, c, d, e\}$ using TA is $b \succ e \succ (a \sim c) \succ d$; the same example shows that this technique allows to discriminate between CE alternatives in choice problems.

Concerning ranking problems, it is clear that the cross-evaluation technique (combined with any of the four formulations mentioned above) produces, by construction, a ranking that is set dependent. As for Andersen and Petersen's technique, this should not be considered as a problem *per se*. The example of Figure 4 shows however that this technique may exhibit non intuitive set dependencies. In the perfectly symmetric situation of Figure 4, all alternatives unsurprisingly end up tied using TA. Suppose now that the evaluation of c on the first criteria is decreased to 4. We now obtain (still using TA) $b \succ c \succ a$, which shows that the position of c vis-à-vis a improves when the evaluation of c is decreased. This *non monotonicity* is due to complex set dependency effects at work with the cross-evaluation technique (simple examples show that a similar phenomenon may occur using other formulations than TA and/or replacing column averages with the use of the principal eignen vector). Again this appears disturbing if multiple criteria are used as an incentive to create nice variants of alternatives. In view of this failure of monotonicity, we would not recommend the use of this technique either for choosing or for ranking alternatives.

Discussion

Summarising our findings leads to the following unsurprising conclusion: in the area of MCDM going beyond (convex) efficiency analysis inevitably implies the introduction of either preference information or arbitrariness. The reluctance to introduce properly modelled preference information (weights, trade-offs, utility functions, etc.) leads to methods the normative properties of which can easily be questioned. This is the case for the three DEA-based techniques analysed in this paper. Of course, this does not mean that DEA in its own sphere of application—performing evaluation—is useless and that research at the

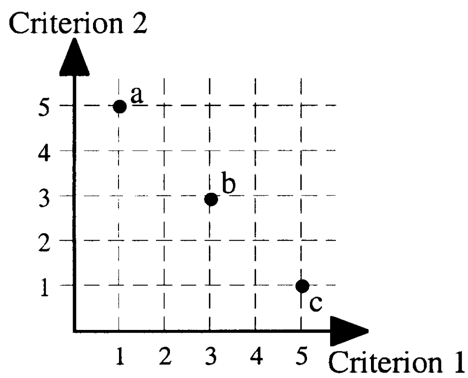

Figure 4 Illustration of the cross-evaluation technique.

intersection of the fields of DEA and MCDM should not be pursued. Our aim was simply to show that too direct a transposition to other fields of fashionable but otherwise useful tools may lead to unsatisfactory results.

Conclusions

The success of DEA will hopefully lead the MCDM community to adopt more problem-oriented lines of research and to realise that simple management-oriented tools are useful. It might also be the sign that the power of LP, although it has already been widely used in MCDM (especially in Goal Programming[21–24] and in interactive methods[12,13,25,26]) has still many things to offer. Alternatively, DEA could benefit from some ideas that have received much attention in the MCDM community, for example the distinction between efficiency and convex efficiency, the importance, and difficulty, of modelling preferences, the necessity to consider normative properties in order to guide the creation of new aggregation methods. Research on the links between DEA and MCDM is still in its early times. As shown by a series of recent papers[27,28] this field is likely to be active in the near future.

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