Dedication

Some days after the completion of this book, Jean-Yves Jaffray, who actively contributed to it, passed away, struck down by cancer. He was a pioneer of new approaches to decision theory. He was among the first to bridge the gap between expected utility and the maximin rule, thus enlarging the notion of subjective probability to encompass belief functions. He was a founding father of a group of international scientists (RUD) which actively pursues this line of investigation. The decision theory community at large will forever miss his outstanding creativity and vision, as well as his generosity, his integrity, his modesty, his sense of humor, his faithfulness and, in the end, his amazing courage. Several generations of researchers are indebted to him for their scientific vocation. His seminal works will certainly remain a major source of inspiration for future generations of scientists in decision theory. This book is dedicated to his memory.

Alain Chateauneuf,
Michèle Cohen,
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Patrice Perny,
and the Editors

Concepts and Methods of the Decision-making Process

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Contents

Preface

The idea of publishing a treatise presenting the state of the art on concepts and methods for decision-support stems from the fact that decision is a multidisciplinary topic of investigation. Indeed, it lies at the heart of various areas of science such as economics, psychology, political science, management, operational research and artificial intelligence. A recurrent temptation is, for any specific research community, to protect its identity against influences from other areas. It therefore seemed to us quite important to stand against such an attitude, and try to put the contributions of the various communities which have become involved in this topic in a unified perspective. In order to grasp all dimensions of decision-support, we have to think in terms of empirical studies, mathematical models and algorithms as well as logic-based and other types of computerized representation tools. Psychologists, economists, sociologists, mathematicians, computer scientists – and decision-makers – have every interest to get together and speak with one another, in order to implement decision-support tools that are at the same time useful and cogent.

The classical approach

In order to be convinced of the relevance of such a multidisciplinary standpoint, it is useful to briefly revisit the history of decision sciences. Right after the end of World War II, the landscape of this area looked empty. Operational research had inherited from the neo-classic economics tradition the idea of 'rational decision', and promoted an approach based on the optimization of a single objective function under constraints. The development of linear programming by George Dantzig (and later on, of non-linear programming and dynamic programming) provided efficient tools for implementing this approach on real-sized problems (encountered in military logistics, in production research and delivery management for industrial firms). In the same years, John von Neumann and Oscar Morgenstern, followed by Leonard Savage, broadened the scope of the constraint-based optimization paradigm to situations where consequences of decisions are risky or uncertain. Very soon the so-called 'Decision

Analysis School' (led by scholars such as Howard Raiffa and Ward Edwards) demonstrated the usefulness of such theoretical results on practical case studies in decision analysis. The extension of these approaches to the construction of objective functions accounting for several criteria was proposed by Gérard Debreu, Duncan Luce and later on by Ralph Keeney and Howard Raiffa in situations of certainty, uncertainty or risk. All in all, the mid-1970s witnessed a well-established, blossoming field which can be dubbed *classical decision theory*.

Questioning the classical theory

Considering such achievements, why bother advancing research in decision sciences? The point is that concurrently to contributions to the validation of the classical theory, radical criticisms of this theory appeared. Noticeably, the works of Herbert Simon cast doubts on the optimization-based approach to decision as being the unique admissible paradigm of rationality. In addition, classical methods of operational research could not tackle all large-sized optimization problems. The emergence of complexity theory, proposed by Jack Edmonds and Richard Karp, suggested that the perspective of more and more powerful computers was not sufficient to overcome this difficulty in the near future.

On the side of decision under risk and uncertainty, the Bayesian approach, stemming from the work of Savage, appeared like the prototype of a rigorous and elegant approach to rational decision. However, it was undermined due to empirical studies run by economists (Maurice Allais and Daniel Ellsberg) and psychologists (Daniel Kahneman, Amos Tversky and Paul Slovic). Results of such studies demonstrated that, in some cases, human behavior consistently violated the principles of expected utility theory when selecting best decisions. Probability measures seemed to lose their status of a unique rational tool for modeling uncertainty when information about decision consequences is missing. The fact that expected utility theory could not always account for the behavior of decision-makers triggered the search for new formal models relying on non-probabilistic representations of uncertainty. Following the pioneering works of David Schmeidler and John Quiggin new, more flexible and realistic mathematical models were proposed. Expected utility was replaced by another more general integral, proposed by Gustave Choquet in the 1950s.

New trends in operational research

In the area of operational research, scholars became more and more aware of the practical limitations of the optimization-based approach to all decision problems. It was not always crystal-clear that a theoretically optimal solution turned out to be an operationally good one from the viewpoint of the user. One of the reasons for such a discrepancy lies in the presence of more than one criterion to be taken into account in order to class a solution to a decision problem as sufficient. From this point of view, multiattribute utility theory was not entirely satisfying. It assumed the preferences of the decision-maker to be well structured. However, it was prone to neglect the practical difficulty of comparing two alternatives with respect to several dimensions. Following Bernard Roy, new decision analysis methods dedicated to the case of multiple criteria emerged in the 1970s and 1980s. They acknowledged the ordinal and ill-defined nature of information available in real-world problems, paving the way for more realistic methodologies for multicriteria decision-support. The idea that an optimization algorithm could force a decision-maker to use a computed solution was given up. It was acknowledged that the role of a formal method was to help the decision-maker build a satisfactory solution. Moreover, the need to consider several criteria presupposed a study of the dependencies between these criteria and the extent to which they can compensate each other, laying bare the possibility of incomparability between solutions. Finally, the analogy between voting theory (where information from voters is essentially ordinal) and multiple criteria decision-making triggered the development of new approaches designed to handle the latter problem.

The emergence of artificial intelligence

Some time passed before the field of artificial intelligence (AI) became concerned with decision problems. This occurred in the 1990s in connection with planning problems under uncertainty and partial observability found in robotics, and the design of user-centered computerized recommender systems and web services. Traditionally, since the late 1950s, artificial intelligence used to focus on declarative knowledge representation and automated reasoning methods, as well as general solving techniques that may apply to a large class of problems. A systematic use of propositional and first-order logics as knowledge representation or programming tools was promoted by scholars such as John McCarthy and Alain Colmerauer. It prompted the emergence of qualitative approaches, even if probability theory and the expected utility approach was also finally accepted, in more recent years. Qualitative approaches especially make sense when it is very time-consuming or costly to build fully fledged utility functions in some application at hand and a coarse representation of preference and uncertainty is good enough to come up with a reasonable decision. In recent years, AI tried to exploit the formal setting of the classical decision theory. The foundations of some ordinal representations were studied. Some of these works come very close to formal results in voting theory, albeit adapting them to specific representation frameworks. In such ordinal setting, possibility theory is the natural counterpart to probability theory in the classical decision theory. Formal similarities between voting theory, decision under uncertainty and multiple criteria decision-making can therefore be laid bare where voters, states of nature and criteria play the same role in each respective problem.

The emphasis of artificial intelligence on representation issues for decision problems makes sense particularly in problems where the set of states of the world is huge, so that the explicit description of a probability or possibility distribution on the state space becomes too burdensome a task. The recent years have therefore witnessed the emergence of powerful tools for the compact representation of uncertainty or preference, especially graphical representations such as Bayesian networks, influence diagrams and conditional preference networks (often called CP-nets). As well as computer languages for logic programming or constraint-based programming, generic problem-solvers for Boolean or combinatorial decision problems are provided: in such approaches, the user is simply requested to express the problem under concern in the appropriate declarative language. This elementary approach (from the standpoint of expressiveness) was enriched by the introduction of soft constraints, bridging the gap with more traditional quantified frameworks for decision analysis.

This treatise

By and large, the progressive questioning of the claims deriving from the classical theory led to a very wide and active research area devoted to decision analysis and decision science. Many new concepts and topics emerged from this critical assessment of the foundations of decision theory: non-additive frameworks for uncertainty representation, Choquet integral, possibility theory, bounded rationality models, non-transitive preference representations, incomparability, interactions and dependencies between criteria, processing of ordinal information and avoiding threshold effects in quantitative representations (among others). Recent developments are too scattered and numerous to be extensively described in these pages, let alone to be able to predict their evolution. Nevertheless, one specific feature of current trends deserves to be emphasized; these works now involve various disciplines. Economists and operational researchers were joined by psychologists, organizational sociologists and scholars in artificial intelligence and theoretical computer sciences. Interestingly, and as a hint to why this state is unsurprising *ex post*, let us highlight the key role played by John von Neumann, a mathematician who is a pioneer in decision theory and operational research as much as in computer science.

The goal of this treatise is to survey the main results and methods in decision theory and decision-support, in order to enable the reader to enter this area and grasp its extent beyond the specifics of the various areas that contributed to this problem. Each chapter provides a state-of-the-art overview of a particular approach to decision, in terms of modeling, representation or problem-solving tool. The book is composed of three parts. The first part is devoted to mathematical concepts useful for the modeling of decision problems, as well as compact representation techniques and combinatorial problem-solving methods. The second part focuses on decision under uncertainty and the third part reviews the various approaches to multiple criteria decision-making. The fact that all chapters of this book are written by French-speaking authors should not be surprising; research in France and French-speaking countries dealing with decision problems has been very active in the last 50 years, following a long tradition initiated by Borda and Condorcet more than two centuries ago. In the following, we provide a more extensive description of the contents of this book.

Chapters 1–7: Modeling tools

The first chapter, written by Alexis Tsoukias, places the current trends of decision theory in a historical perspective, stressing the interaction between this field and others such as cognitive sciences, organization theory, psychology and artificial intelligence, etc. It brings forward a clear distinction between the practice and the process of decision-making from the theory and the techniques used to 'solve' decision problems. The latter are simply tools that are instrumental within a general methodology for decision-support, whose basic features are discussed by the author. A general decision-support process is described that can serve as a guideline to practitioners independently of the specific tool to be employed.

A basic notion in decision theory, whether under uncertainty or multiple criteria, is that of a preference relation. This kind of construction naturally appears for pairwise comparison of alternatives that account for the decision-maker opinion. In Chapter 2, Denis Bouyssou and Philippe Vincke present the main mathematical structures instrumental in modeling preference (total order, weak order, interval order and semi-order) and discuss various ways of representing them (by graphs, matrices and numerical functions). A brief account of the problem of preference aggregation is provided, a crucial issue in several chapters of this treatise. This chapter in no way considers the issue of eliciting preferences from the decision-maker. This issue is dealt with in the third part of this treatise.

Uncertain information is pervasive in decision problems (as well as in many others). There is a recurrent confusion between two kinds of uncertainty, which is not always easy to resolve in the purely probabilistic setting and is at the origin of many difficulties and debates. In Chapter 3, Didier Dubois and Henri Prade make a careful distinction between these two forms of uncertainty: aleatory uncertainty (which results from the intrinsic variability of natural phenomena) and epistemic uncertainty (mainly due to a lack of information about the reality under concern). Of course, both types of uncertainty may be simultaneously present in a given problem. The authors show that specific representation frameworks are suitable for each kind of uncertainty: probability measures for aleatory uncertainty, sets (e.g. logic and intervals) for incomplete information and new uncertainty theories combining the two ingredients. Basic concepts useful for reasoning and decision are surveyed, especially conditioning and information fusion methods.

Decision-making is a human activity and, as such, influenced by psycho-physiological effects and subject to cognitive limitations of the human mind. In Chapter 4, Jean-Charles Pomerol interprets decision-making activity as driven by reasoning and emotion according to recent discoveries in neurobiology. Several basic concepts such as bounded rationality are introduced. Decision can be triggered by the recognition of patterns in the state of the world. As a consequence, the author presents the basic principles of case-based decision-making. He discusses cognitive biases related to the perception of probabilities.

Multiple criteria analysis is often distinguished from multiple criteria optimization. The main difference relates to the techniques for describing solutions. Multiple criteria analysis is characterized by a small number of well-defined options that can be explicitly enumerated. Multiple criteria optimization deals with intentionally described (possibly infinite) sets of options defined by means of constraints. In Chapter 5, Jacques Teghem provides an introduction to multiple criteria optimization, a field where technical difficulties relevant to optimization combine two conceptual difficulties inherent to reasoning with multiple criteria. The scope of the chapter is limited to multiple criteria linear programming, where both constraints and objective functions can be represented by linear expressions in terms of decision variables. After a refresher on basic notions, including efficient solutions, Teghem reviews the main techniques used in various multiple criteria linear programming problems according to whether variables are continuous, discrete or Boolean. Building on exact and interactive methods, the author also considers approximate methods with special focus on the use of meta-heuristics. The last two sections of this chapter are devoted to the case of imperfect information: aleatory data (subject to probabilistic randomness) and imprecise data (fuzzy multiple criteria linear programming).

Mathematical programming, whether linear or not, is no longer the only tool capable of solving decision problems where the set of solutions is implicitly described and may involve a large (possibly infinite) number of solutions. Constraint-based programming is a tool stemming from artificial intelligence, which strengthens and enriches the gamut of available decision optimization techniques. Many real-world problems such as scheduling, resource management, pattern recognition and diagnosis can be modeled as constraint satisfaction problems (CSPs). Chapter 6 provides an outline of current tools that address such problems. Gérard Verfaillie and Thomas Schiex also deal with extensions of CSPs where the satisfaction of constraints can be a matter of degree.

Traditionally, preferences are defined over sets of alternatives described by vectors of local evaluations along various attributes. Global preference is analytically determined by means of a formal preference aggregation model merging preferences according to each criterion. This process is more precisely described in the third part.

In Chapter 7, Jérôme Lang takes a more general point of view relying on logical representations of preference. He presents compact preference representation languages allowing a simpler expression of preferences between complex entities, expressed by the decision-maker in natural languages. These languages are supplemented with efficient algorithms that can compute whether one alternative is preferred to another and find optimal solutions. Such languages are the topic of extensive research in AI. The author shows how propositional logic-based formalisms can be instrumental for the representation of preference. Various logics of preference are surveyed, starting from so-called *ceteris paribus* preferences, ending with conditional preference logics and graphical representations such as CP nets. The chapter concludes with a brief insight into the potential of multiple-valued and paraconsistent logics.

Chapters 8–14: Decision under uncertainty

This part is a compendium of various mathematical or empirical models for decision under uncertainty. First, the various existing criteria for decision-making under uncertainty are reviewed (in the historical order of their appearance): expected utility, subjective expected utility, non-additive extensions thereof and qualitative criteria. One chapter is devoted to the empirical validity of such criteria from the viewpoint of cognitive psychology. The final two chapters of this part focus on mastering the combinatorial complexity of multistage decision-making problems under uncertainty: Bayesian networks, influence diagrams and Markov decision processes for planning under uncertainty.

Chapter 8, written by Alain Chateauneuf, Michèle Cohen and Jean-Marc Tallon, outlines the theory of decision under risk after von Neumann and Morgenstern. Assumption of decision under risk is that a probability distribution over the states of the world is available. In such a situation, any decision is a matter of choosing between lotteries. von Neumann and Morgenstern proposed necessary and sufficient conditions for the justification of:

1) the existence and uniqueness (up to a linear transformation) of a utility function quantifying the attractiveness of the various consequences of the tentative decisions, according to the decision-maker;

2) the criterion of expected utility, as *the* rational evaluation basis for ranking decisions.

This chapter puts some emphasis on the issue of representing the attitude of the decision-maker in the face of risk. Several approaches to model risk aversion, even if intuitively distinct, turn out to be equivalent in this setting which shows some limitation in expressiveness. Another limitation is highlighted by means of the Allais paradox whereby decision-makers, when faced with both sure and uncertain gains, may consistently violate the independence axiom. Modern approaches to decision under risk that weaken the independence axiom are surveyed.

Chapter 9, written by Alain Chateauneuf, Michèle Cohen and Jean-Yves Jaffray is a refresher on the classical theory of decision under uncertainty according to Savage, who axiomatically justified subjective expected utility as a criterion for ranking decisions. Contrary to the case of decision under risk, the decision-maker does not know the probabilities of the states of nature. Decisions are construed as functions (also called *acts*) from the state space to a set of consequences. Savage proposed a set of postulates on the preference between acts implying that this preference can be represented by an expected utility with respect to a subjective probability distribution. These postulates imply that the set of states is infinite. In other words, in this approach, even if the decision-maker is ignorant about the current situation, they behave as if they knew of a probability distribution on states and ranked decisions using the criterion proposed by von Neumann and Morgenstern. The chapter presents other settings where the same result was proved, especially the elegant proposal of Anscombe and Aumann where the state space is finite but decisions have random consequences modeled by lotteries on a finite set of possible results. The authors then recall the history of empirical criticisms of this approach, which also suffers from the Allais paradox and also from the Ellsberg paradox. The latter shows that many decision-makers are likely to violate the sure-thing principle (the key axiom of Savage theory) in the face of incomplete information, which is incompatible with a probabilistic representation of uncertainty.

Chapter 10, written by Alain Chateauneuf and Michèle Cohen, surveys the numerous decision models and criteria that were proposed as a consequence of the various critiques of the classical theory. These models were proposed to accommodate the Allais and Ellsberg paradoxes, and to offer refined views of risk and uncertainty aversion. This chapter considers both extensions of the classical theories under risk and under uncertainty. In the case of uncertainty, the main step was taken by David Schmeidler within the Anscombe–Aumann setting. He suggested that the independence axiom only applies to comonotonic decisions, no mixture of which can help hedging against the variability of consequences. Under this restriction, the criterion takes the form of a Choquet integral with respect to a monotonic set function representing uncertainty. Other extensions were later proposed to accommodate the Choquet integral, for instance in the Savage setting, and are surveyed in this chapter. Let us mention the multiprior model of Gilboa and Schmeidler, in which decisions are ranked according to their minimal expected utility with respect to a family of probabilities (coinciding with a Choquet integral w.r.t a lower envelope), and the generalization to belief functions of Hurwicz criterion taking a weighted average between the best and the worst consequences. These approaches use representations of uncertainty presented in Chapter 3. The last section of this chapter considers the generalization of the von Neuman-Morgenstern model proposed by Quiggin (the so-called 'rank-dependent' model). The basic idea is that the decision-maker has a subjective perception of objective probabilities. This is encoded by means of a function that models the subjective perception of objective probabilities, and the criterion is again Choquet integral with respect to this distorted probability measure. This model addresses the Allais paradox, distinguishing attitude towards risk (the probability distortion function) and towards sure consequences (the utility function). More recent variants of this approach are discussed.

Chapter 11, written by D. Dubois, H. Fargier, H. Prade and R. Sabbadin, studies what remains of the classical theory when utility and uncertainty functions are no longer numerical and are expressed by preference relations studied in Chapter 2. In the pure relational framework, rational decision rules are qualitative or possibility theory-based counterparts to Condorcet pairwise majority rule, and impossibility results similar to those in the voting framework are found. When a common qualitative value scale is used, criteria for decision under uncertainty extend the maximin and maximax criteria introduced by Shackle and Wald in the early 1950s in the case of total uncertainty. The most general one is the Sugeno integral, a qualitative counterpart to the Choquet integral. This chapter discusses in detail prioritized pessimistic and optimistic extensions of maximin and maximax criteria respectively, the underlying uncertainty theory being possibility theory. In this approach, the attitude of the decision-maker is only expressed by the choice of a set function representing uncertainty. In order to cope with the lack of discrimination of qualitative decision criteria, techniques to refine the obtained ranking of decisions are surveyed. Especially, it is shown that the prioritized pessimistic and optimistic extensions of maximin and maximax criteria can be refined by expected utility criteria with respect to a so-called big-stepped probability function, the utility functions being respectively concave or convex.

Chapter 12, written by Eric Raufaste and Dennis Hilton, considers decision under uncertainty from the viewpoint of cognitive psychology. They report on studies evaluating the extent to which normative decision theories faithfully account for the actual behavior of decision-makers. This chapter echoes Chapter 4, which discusses this issue in a broader context. The works of Daniel Kahneman and Amos Tversky demonstrated at length, in a series of experiments, that human beings may fail to comply with normative assumptions. The authors present the so-called Prospect Theory, somewhat akin, through the use of distortion functions, to the Quiggin decision model while using a bipolar value scale. Indeed, the bipolar behavior of human decision-makers can be examined, since a decision-maker does not have the same attitude in the face of gains and in the face of losses. The authors then broaden the scope of the chapter towards a global descriptive approach to human decision, including the case of several dimensions and the search for Pareto-dominance. An important cause of deviation between theoretical models and human behavior is the so-called attentional focusing, namely the fact that a decision-maker concerned with one aspect of the decision process tends to neglect other aspects. It makes the choice process prone to manipulation through the way possible choices are described to the user (framing effect). This chapter also highlights the systematic use of heuristics by decision-makers as shortcuts to speed up the choice process. The study of such heuristics, as carried out by Gigerenzer, enable some human errors to be predicted. This chapter also discusses the role of emotions in the decision process.

The next two chapters consider computational issues for large decision-making problems, especially within the classical expected utility approach. In Chapter 13, Jean-Yves Jaffray surveys the theoretical background of Bayesian networks and influence diagrams. A Bayesian network is a concise representation of a large joint probability distribution on a Cartesian product of finite universes. Its merit is to highlight the local conditional independence properties between groups of variables via the so-called *d-separation* property. A Bayesian net enables the determination of posterior probabilities to be carried out via local computations for the purpose of belief revision when some variables are instantiated. This technique is especially efficient when paths relating variables in the graph are unique. Bayesian nets are extended to the handling of decision trees. These are structures involving decision nodes and chance nodes; the decision-maker tries to find the best decision sequences (policies) in order to maximize the overall expected utility. The computation of optimal policies is made easier by folding the decision tree in order to do away with redundancies. Local computation techniques similar to those in Bayesian nets mentioned in this chapter are possible due to the additive separability of expected utility. The extension of such methods to more general decision rules such as the Choquet integral is problematic because this separability property no longer holds.

Chapter 14, written by Régis Sabbadin, also studies combinatorial aspects of decision under uncertainty for tree-like structures, such as those found in planning problems. Here a temporal dimension is added. The basic idea is that each decision causes the system state to evolve to another state, the aim being to reach a prescribed final state of interest to the user. Uncertainty lies first in the non-determinism of actions whose result is poorly known. These problems are formalized by means of so-called *Markov decision processes* (MDPs) where the result of actions only depends on the previous, supposedly known, state. The optimized criterion is again expected utility along trajectories, and the computation methods are based on dynamic programming which is again possible due to the additive separability of expected utility. This chapter surveys several methods for solving MDP problems, and also considers the more difficult case of *partially observed* MDPs (POMDPs) where the actual result of past actions cannot be completely known. An account is given of decision processes whose transition probabilities are only known via learning. The determination of an optimal policy is carried out simultaneously with the probability learning process. MDPs often contain redundant parts that can be factorized, in order to speed up computations. Concise representations of MDPs exist that are similar to those for decision trees presented in the previous chapter.

The last part of Chapter 14 is devoted to the qualitative counterpart of MDPs where the representation of non-deterministic actions and of the information about the current state is based on possibility theory. Optimistic and pessimistic qualitative criteria presented in Chapter 11 are taken advantage of. From a computational point of view, such criteria preserve a form of separability that makes them amenable to dynamic programming techniques such as the expected utility techniques. This conforms with the fact pointed out in Chapter 11, that possibilistic criteria can be refined by expected utility. Moreover, the increase in complexity when moving from MDPs to POMDPs is significantly smaller than in the probabilistic setting. Without denying the expressive power of probabilistic POMDPs, the qualitative approach to planning under uncertainty is therefore equally attractive.

Chapters 15–20: Multiple criteria decision

The third part of this treatise is devoted to multiple criteria decision analysis and, more generally, to aggregation methods for multiple and potentially conflicting judgements or pieces of advice. There are three groups of chapters.

The first group deals with decision-support methods based on conjoint measurement theory. This theory aims at determining conditions under which a binary relation, modeling preferences and defined product set can be represented numerically. The conditions that allow such a numerical representation lead generally to results having a constructive proof, and therefore provide guidelines for the elicitation and the structuring of preferences expressed by a decision-maker.

Chapter 15, prepared by Mohammed Abdellaoui and Christophe Gonzales, presents the classical theory of conjoint measurement which deals with the numerical representation of structures that are complete and transitive. An original aspect of this chapter is its unified presentation framework including (1) the case of sure information that mainly leads to additive value functions and (2) the situations under risk and uncertainty that lead to various decompositions (additive, multiplicative or multilinear) of utility functions according to von Neumann and Morgenstern. Particular attention is paid in this chapter to encoding methods, i.e. to procedures for the elicitation of preferences from a decision-maker. The extension of these methods to new decision models under uncertainty (as in rank-dependent expected utility) is also considered for situations under risk and uncertainty.

Chapter 16, written by Denis Bouyssou and Marc Pirlot, is devoted to the study of conjoint measurement models where transitivity or completeness are no longer assumed. It is shown that the use of different forms of the one-dimensional ‘traces’ of a binary relation on a Cartesian product is instrumental in the derivation of a numerical representation for non-necessarily transitive relations. Models thus obtained can be viewed as offering a general framework that encompasses different aggregation methods proposed in the literature. In particular, it is shown how these general models enable an axiomatic analysis of multicriteria methods based on outranking relations to be performed (as in the ELECTRE method and related approaches).

In the above two chapters, the different components of the Cartesian product of the evaluation domains were not assumed to be expressed on a common scale. The second group of chapters in this part deals with methods where it makes sense to evaluate objects according to different points of view on a common scale. Such a hypothesis may appear to be quite bold. However, it is frequently made when grading student's work, or more generally when different experts on a panel use the same evaluation scale.

Chapter 17, written by Jean-Luc Marichal, presents an overview of the various existing models for aggregating scores belonging to a common scale. In particular, it includes a survey of different types of averages, associative aggregation functions and aggregations based on the use of non-additive measures (in particular, Choquet and Sugeno integrals). Particular attention is paid to the question of the meaningfulness of such aggregation schemes, i.e. to the question of knowing whether the algebraic manipulations performed by the aggregation may lead to opposite conclusions if the evaluation unit on the common scale is changed.

The second chapter of this group (Chapter 18), authored by Michel Grabisch, is devoted to the practical use and interpretation of the aggregation schemes presented in the Chapter 17. It is shown, using the MACBETH approach, how it is possible in practice to evaluate objects on a common scale. Particular emphasis is put on the distinction between unipolar and bipolar scales. For bipolar scales, there exists a neutral point expressing indifference inside the scale, and the aggregation of ratings above the neutral level can be made according to a different logic from that used for ratings below this level. It leads to considering aggregation schemes that are more general than those considered in the previous chapter (e.g. based on an integral with respect to a bi-capacity). The interpretation of parameters involved in such representations (especially in terms of interaction between criteria) and their practical elicitation are discussed in detail.

The third and last group of chapters is devoted to the links between multiple criteria analysis and social choice theory. It should indeed be clear that the aggregation of evaluations according to different criteria is not unrelated to the aggregation of individual opinions regarding various candidates to an election.

Chapter 19, written by Denis Bouyssou, Thierry Marchant and Patrice Perny, offers a simple introduction to social choice theory. It is shown, by means of various examples, why the aggregation methods proposed in social choice theory do not satisfy all the expectations that we might have. These problems are related to more general results (such as the famous Arrow theorem) that examine the difficulty of designing a purely ordinal aggregation of various points of view. This chapter provides a brief survey of the literature, and stresses the relevance of many classical results in social choice theory for the design or the use of a multiple criteria method for decision analysis.

Chapter 20, authored by Olivier Hudry, Bruno Leclerc, Bernard Monjardet and Jean-Pierre Barthélémy, concludes this book. It deals with the analysis of aggregation methods based on the search for medians of a collection of binary relations, i.e. relations that minimize a distance to preference relations in such a collection. This type of method is very old in social choice theory (and dates back at least to Condorcet). These techniques are analyzed in great detail according to the type of relations to be aggregated and the nature of the result under concern. Different algorithmic formulations of these medians are proposed and their complexity studied. The problem of determining a median of a collection of relations is an example of a more general one: that of finding medians in a lattice or in a semi-lattice. The results that are thus obtained at the more general level shed a powerful light on the former problem of aggregation of preference relations.

Hopefully, the collection of survey articles gathered in this book offers a broad overview on the representation and the computational aspects of decision problems and the foundations of decision under uncertainty and multiple points of view. It will guide the reader trough the abundant literature that exists on the topic. The authors of surveys proposed here are renowned contributors to the study of the questions covered in this volume. This volume also demonstrates that research on the use of formal methods for the study of decision problems is active in French-speaking countries.

Denis Bouyssou, Didier Dubois, Marc Pirlot and Henri Prade