Title: Probability and Entropy Compression in discrete mathematics (Winter 2024)

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One of the powerful tools in Discrete Mathematics is the use of probability theory to prove deterministic results. Often to prove the existence of a structure, one can define a probability space on a general set of objects and show that an object chosen uniformly at random from this space has the desired structure with positive probability.

One of the key tools in probabilistic methods is a simple lemma in probability theory due to Lovasz and Erdos (Lovasz Local Lemma). In a relatively recent breakthrough result, Moser and Tardos (2009) provided a constructive proof of the lemma, thus allowing many of the existence proofs to be turned into polynomial time algorithms, significantly impacting algorithmic theory. The idea is founded on Shannon's classical theorem that a random bit string cannot be losslessly compressed.

Pre-requisites:
The knowledge equivalent to a first course both in graph theory and probability theory.

Topics:

Part 1: The probabilistic method
- the basic method, basic examples from graph theory and elsewhere;
  linearity of expectation, the first moment method (examples from Ramsey theory, Random graphs)
- Sharpening our tools - Concentration: the Azuma, Talagrand and McDiarmid inequalities
- Existence of graphs with counter-intuitive properties
- Large independent sets in triangle-free graphs
- The Lovasz Local Lemma and its applications

Part 2: The Entropy Compression Method - Algorithmizing the Lovasz Local Lemma

- Introduction: Shannon's Entropy Theorem
- k-SAT with bounded intersection of clauses; A constructive proof of the
Local Lemma by Moser-Tardos (2009).
- Acyclic-edge colorings using entropy compression (Esperet - Parreau) (2013)
- Molloy's elegant proof of Johansson's theorem on coloring triangle-free graphs (2017)

References:
1. N. Alon, J. Spencer, The probabilistic method (book)
2. M. Molloy and B. Reed, Graph Colouring and the Probabilistic Method (book)
3. The three papers mentioned above due to Moser-Tardos, Esperet-Parreau and Molloy