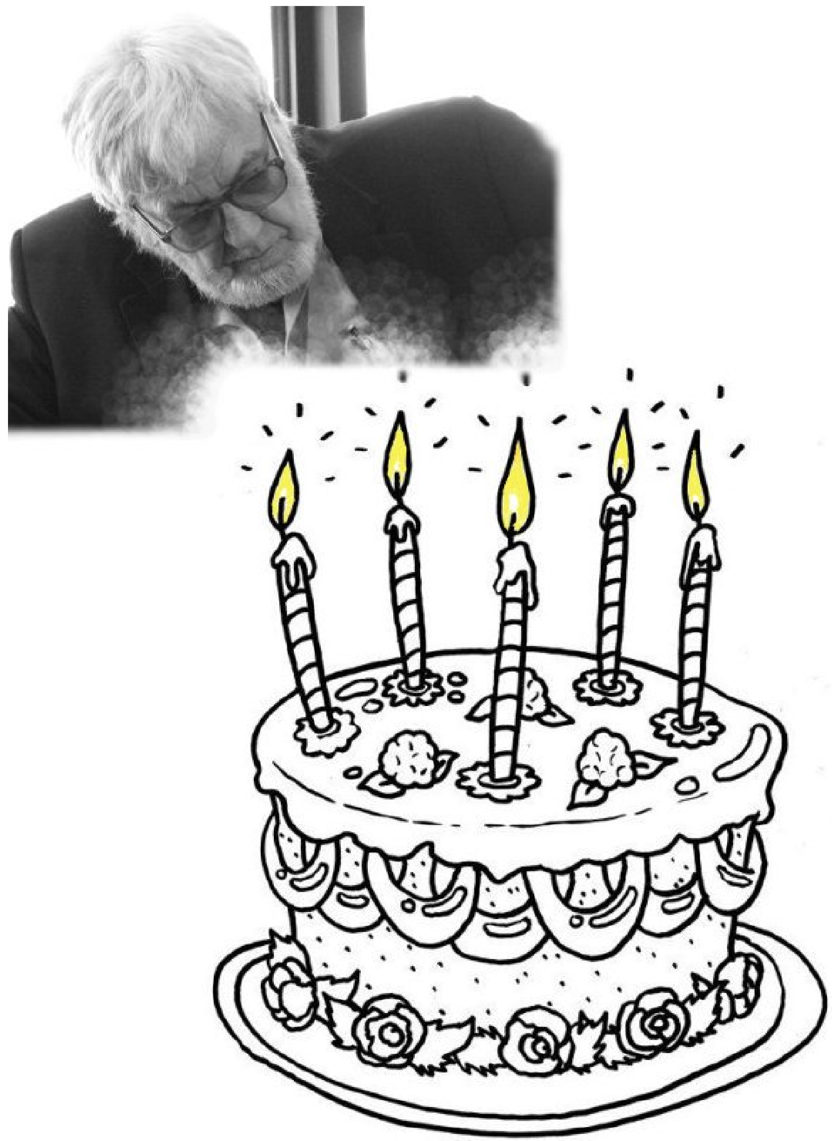
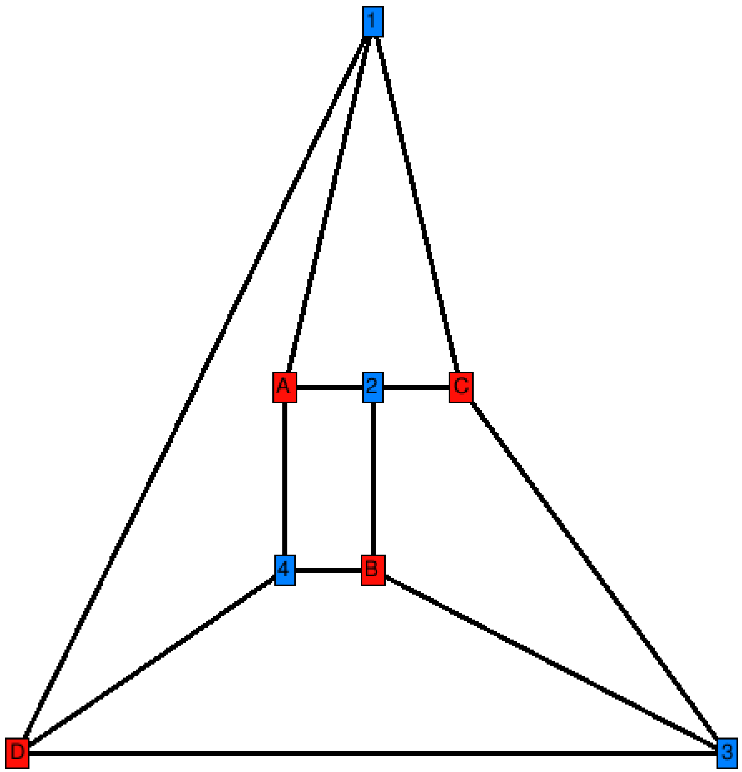
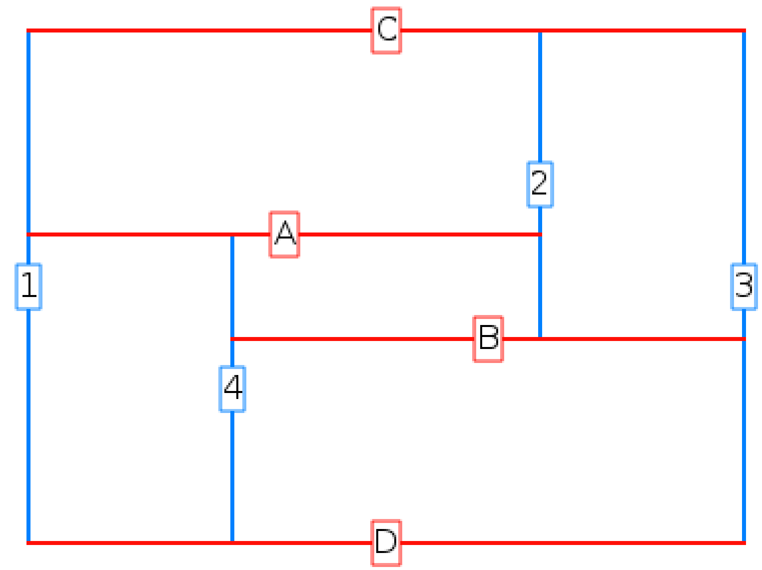
Geometric Representations of Graphs



H. de Fraysseix, P. Ossona de Mendez, J. Pach, P. Rosenstiehl

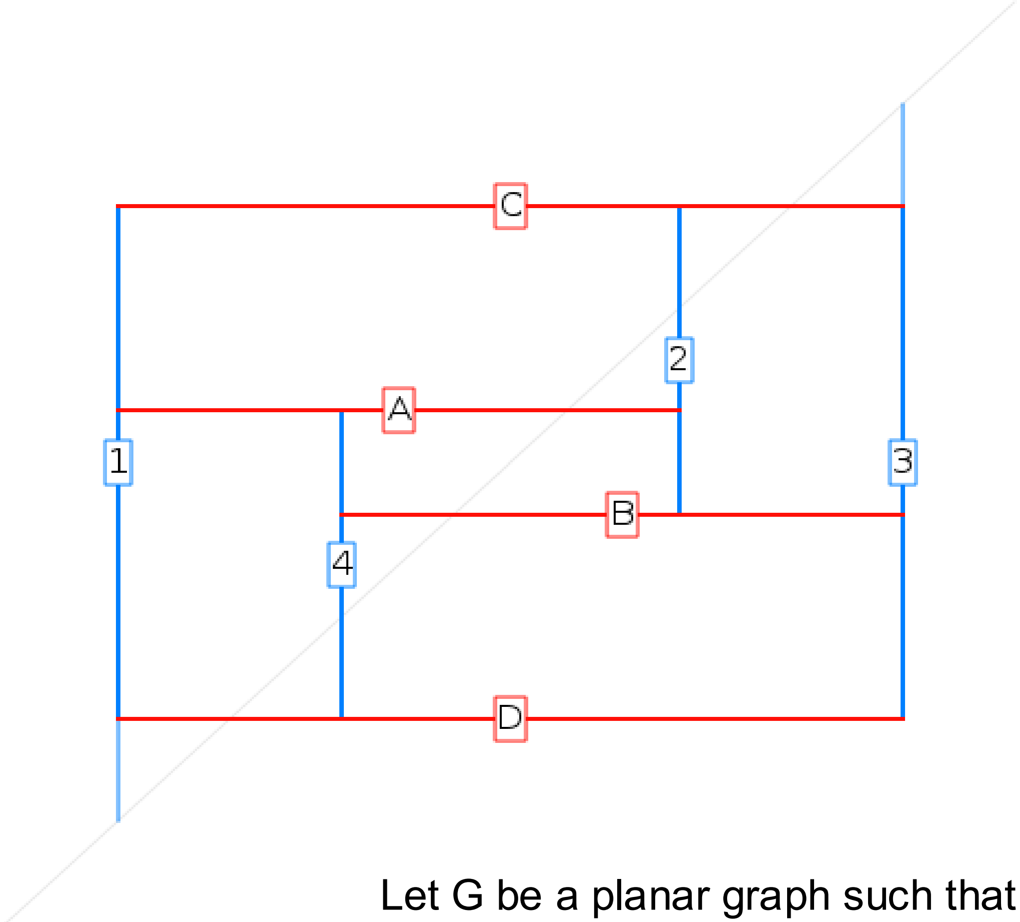
Bipartite planar graphs: representation by contacts of segments

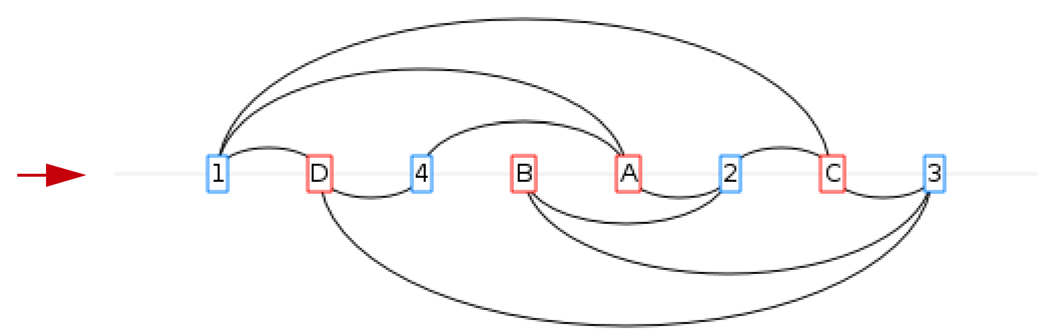




vertices → segments
edges → contact points
same topology

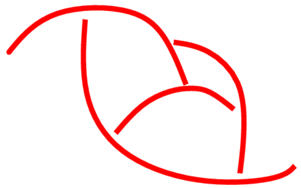
Bipartite planar graphs:
Contacts of segments → 2 trees on 2 pages





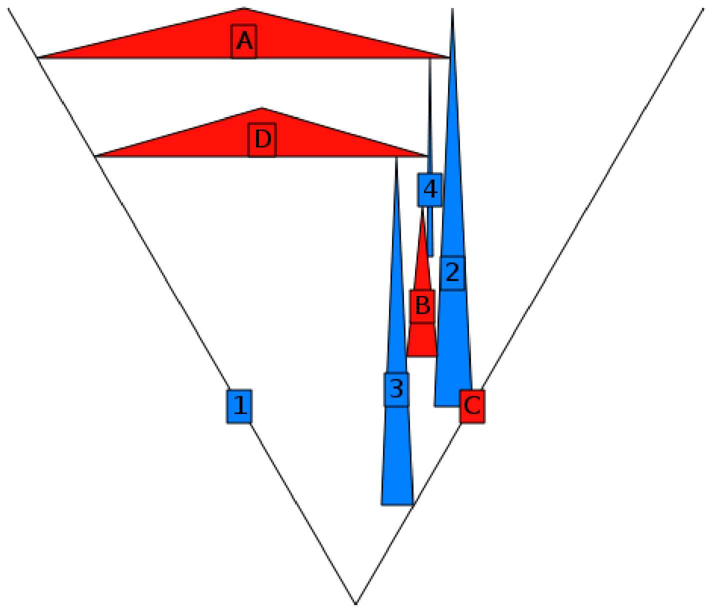
Let G be a planar graph such that for any subgraph H of G (with n(H) > 1):

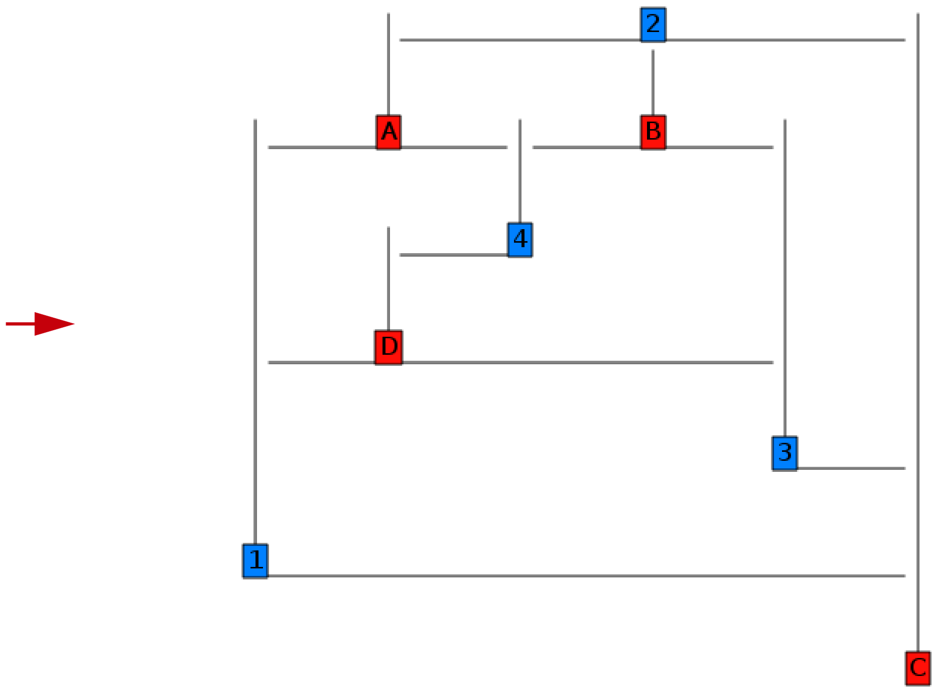
- m(H) ≤ 2*n(H) – 2 then G is representable by a contact family of pseudo-segments.
- m(H) ≤ 2*n(H) – 3 then G is representable by a contact family of segments.



K4 is not representable by a contact family of segments

Planar graphs:
Representation by contacts of triangles → contacts of **T**



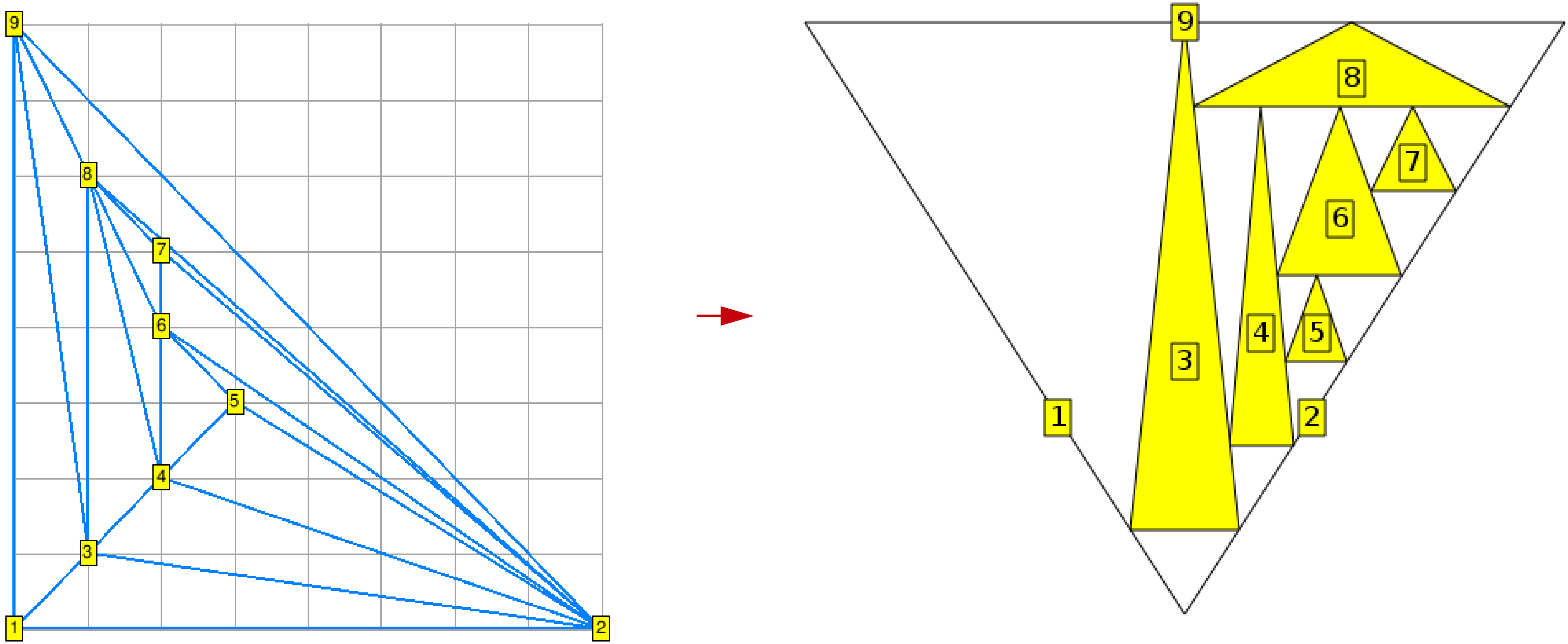


Exponential size

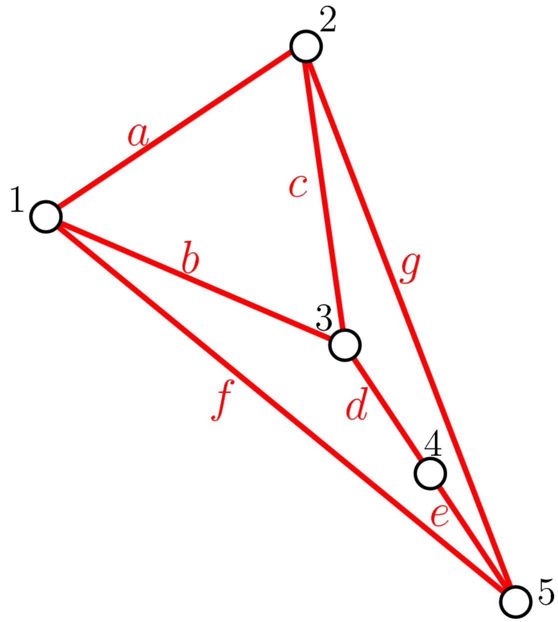
Linear size

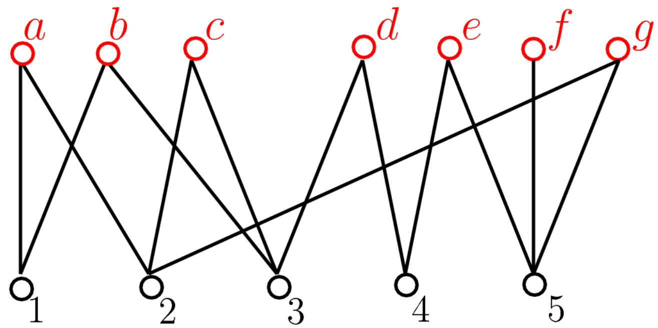
Vertex packing algorithm →

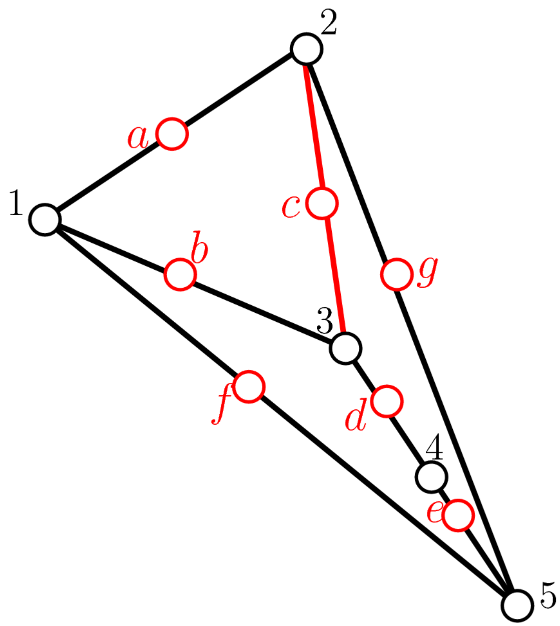
- straight line drawing on a linear size grid
- representation by contacts of triangles

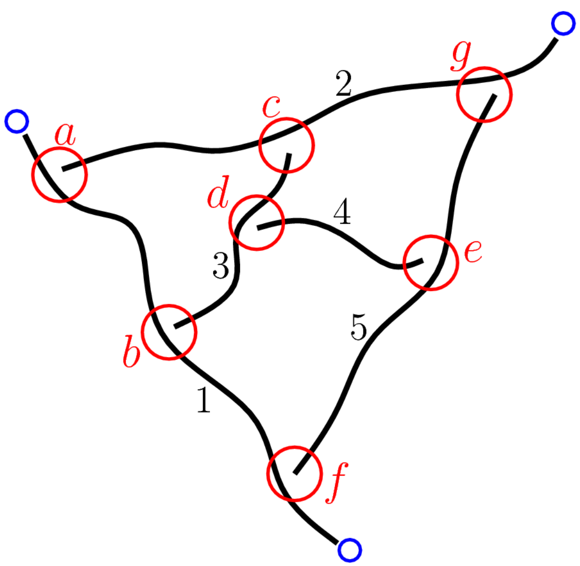


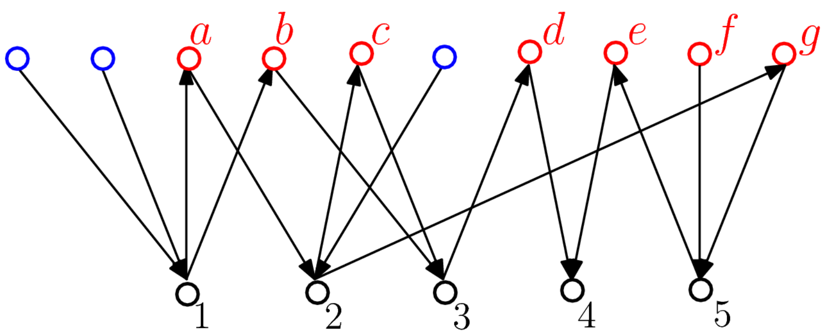
Incidence graph of a graph / a contact system











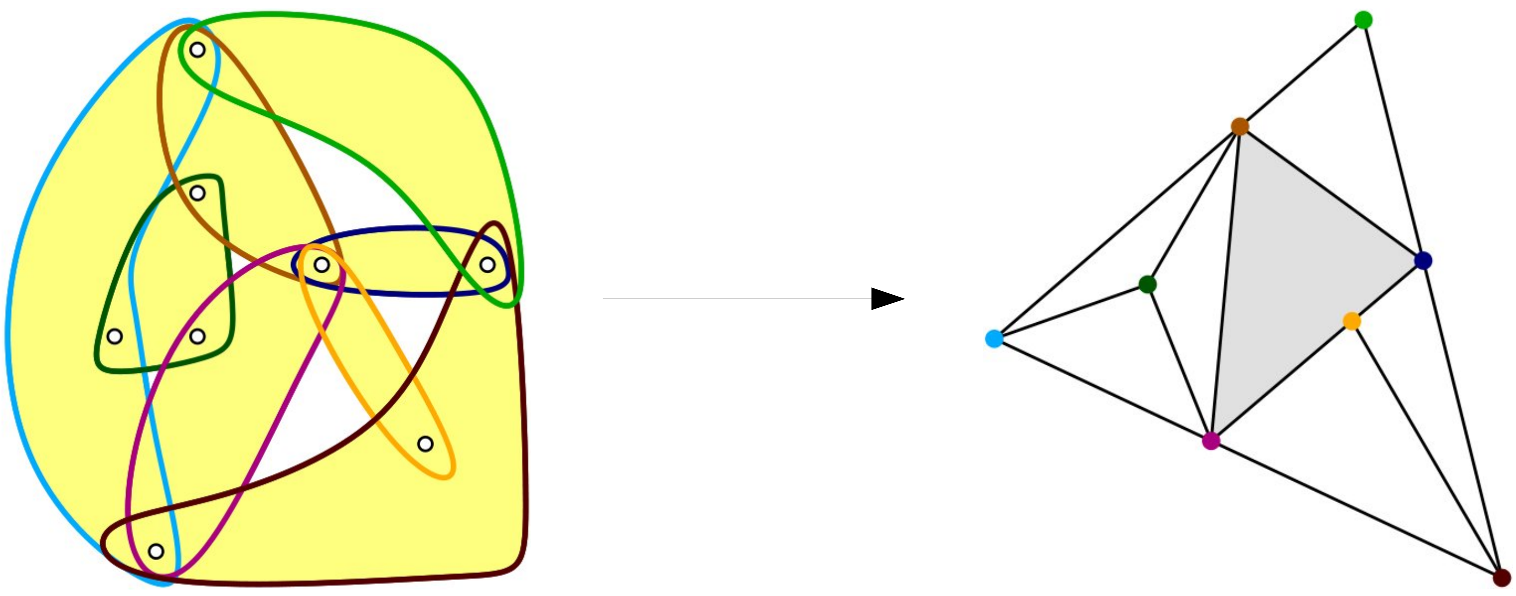
Indegree ≤ 1

Indegree = 2

(2,≤1) orientation

Planar Linear Hypergraphs:
Representation by contacts of segments and/or triangles

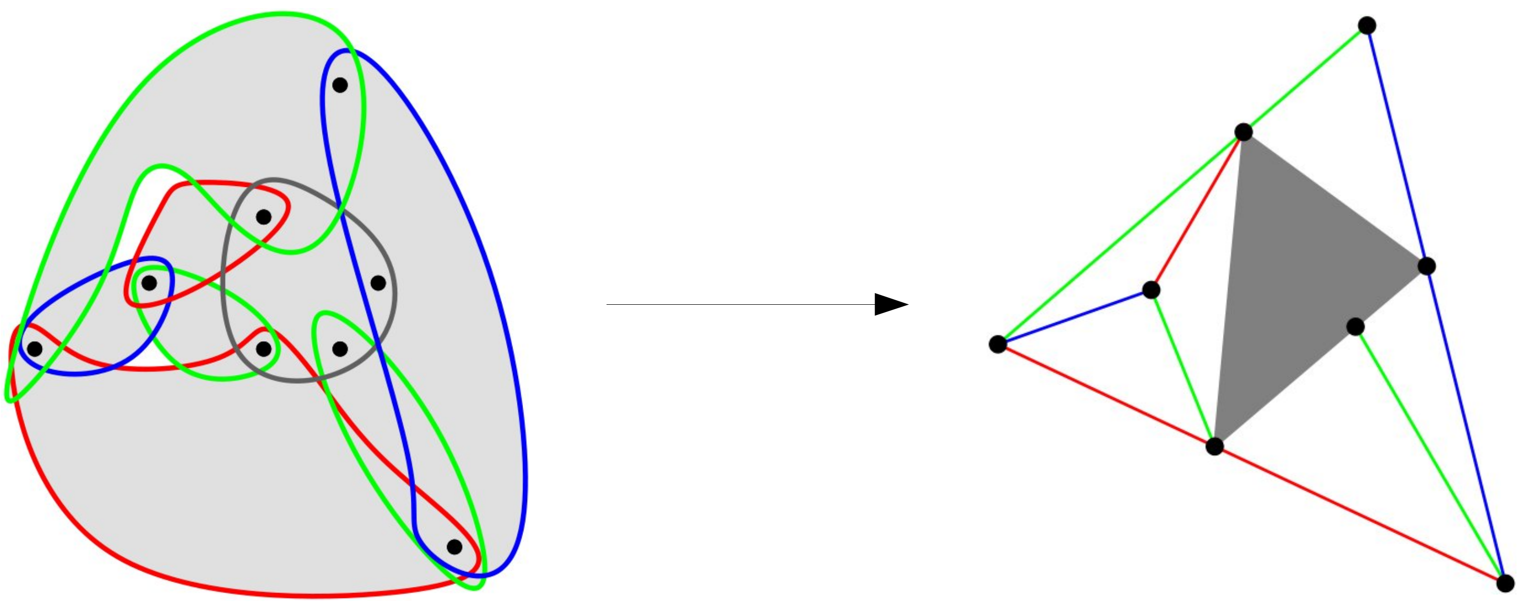
(Vertices are represented by segments or triangles
Edged by contact points)



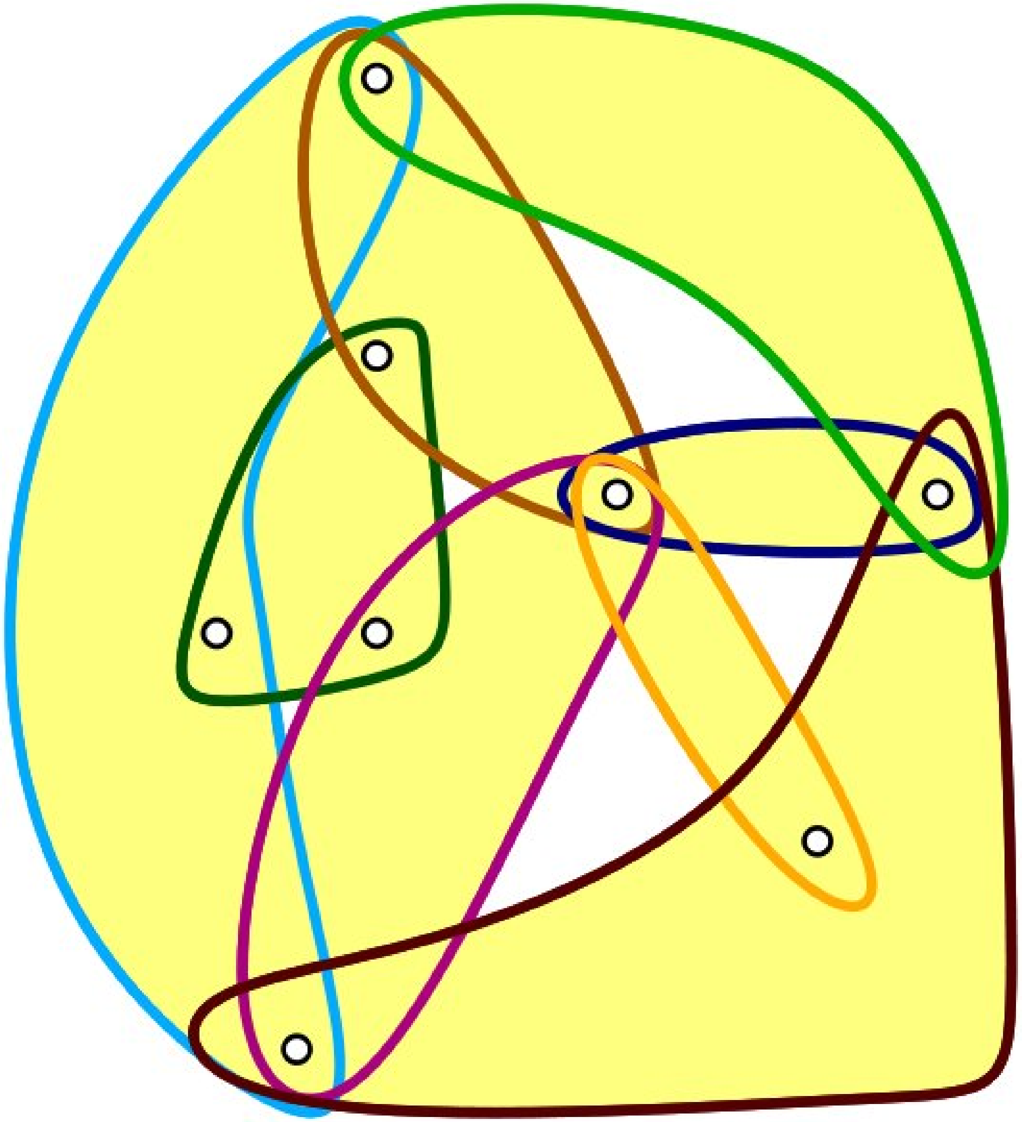
H linear ⇔ 2 edges share at most 1 vertex

Planar Linear Hypergraphs: Representation by contacts of segments and/or triangles

(Edges are represented by segments or triangles
Vertices by contact points)

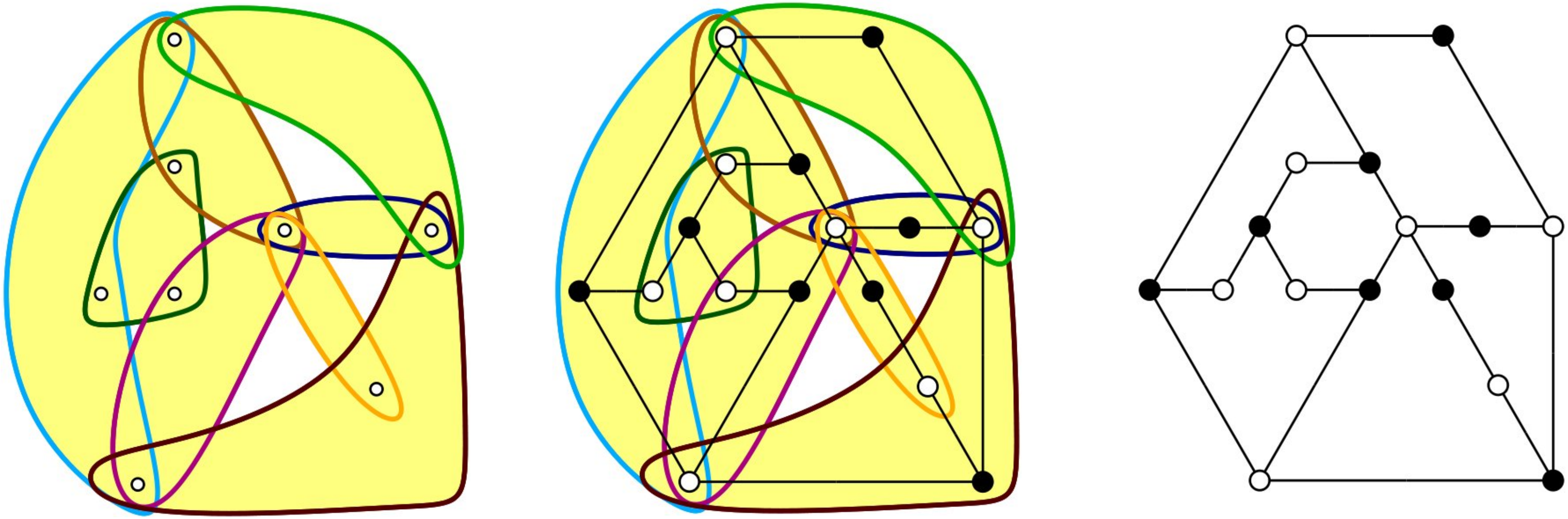


Our Hypergraph H
(linear, planar)



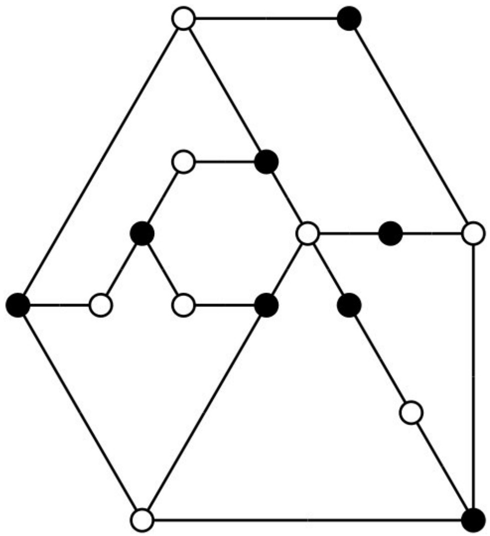
Incidence graph **R** of a planar linear hypergraph **H**: planar bipartite graph without cycle of lenght 4

(white vertex → triangle/segment
black vertex → contact point)

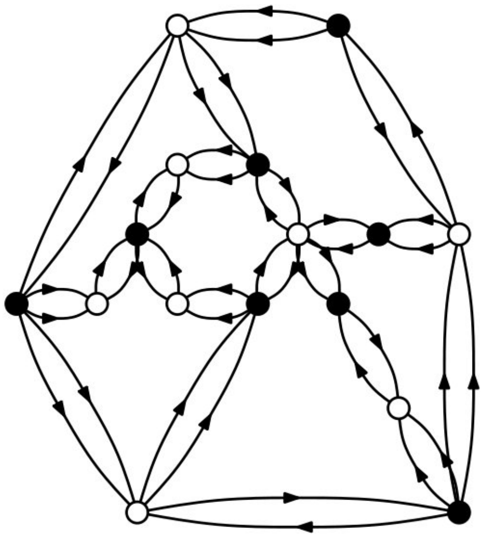


H planar ⇔ R planar

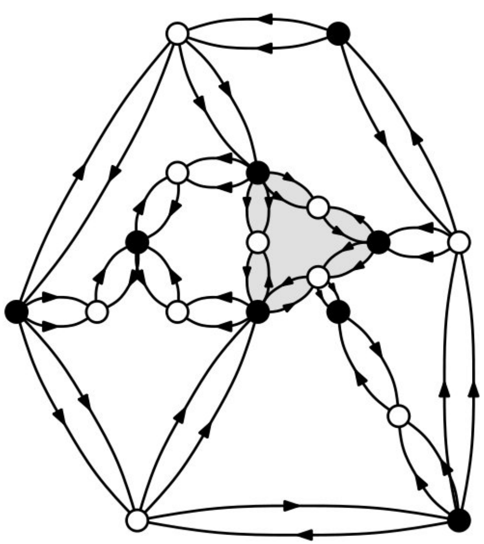
Incidence graph



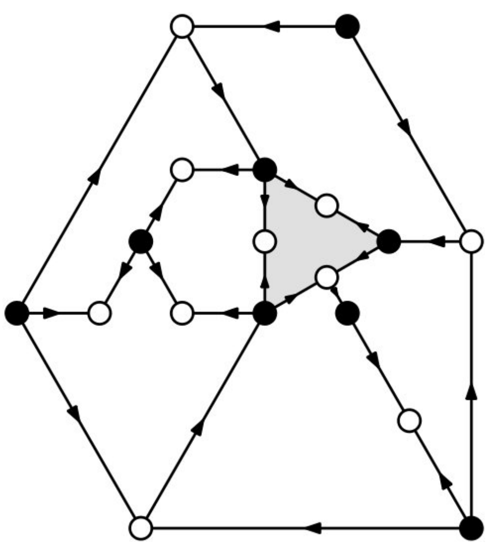
3-Orientation



Splitting some vertices



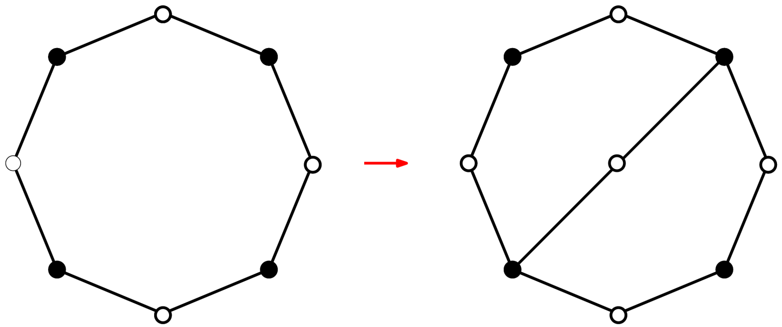
Symplifying → (2,≤1) Orientation



Constuction of a (2,≤ 1)-orientation:

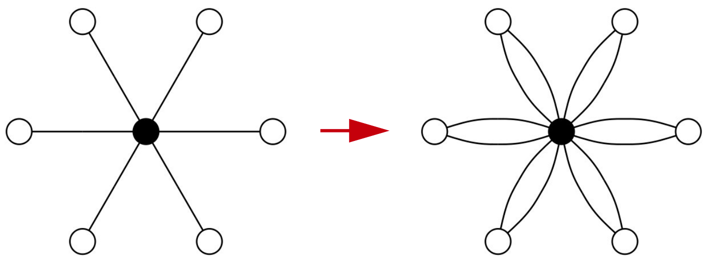
- white vertices will get exactly 2 incoming edges
- black vertices will get at most 1 incoming edge

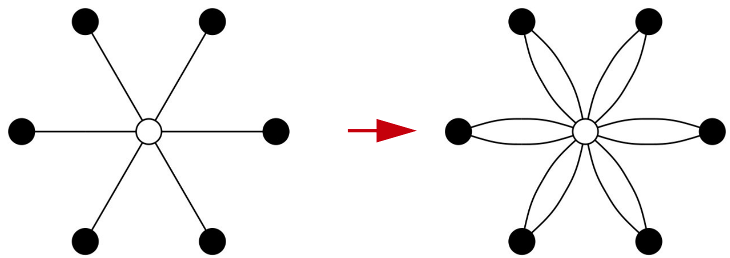
Make all faces of length 6



Add a vertex *r* incident to the black vertices of the external face

Double all edges





λ-orientation of a multigraph

Lemma:
Let G be a multigraph, let λ be a mapping from V(G) to N.
Then there exists an orientation of G such that each vertex $v \in V(G)$ has indegree bounded by $\lambda(v)$ if and only if

$$\forall A \subseteq V(G) \;:\; |E(G[A])| \leq \sum_{v \in A} \lambda(v)$$

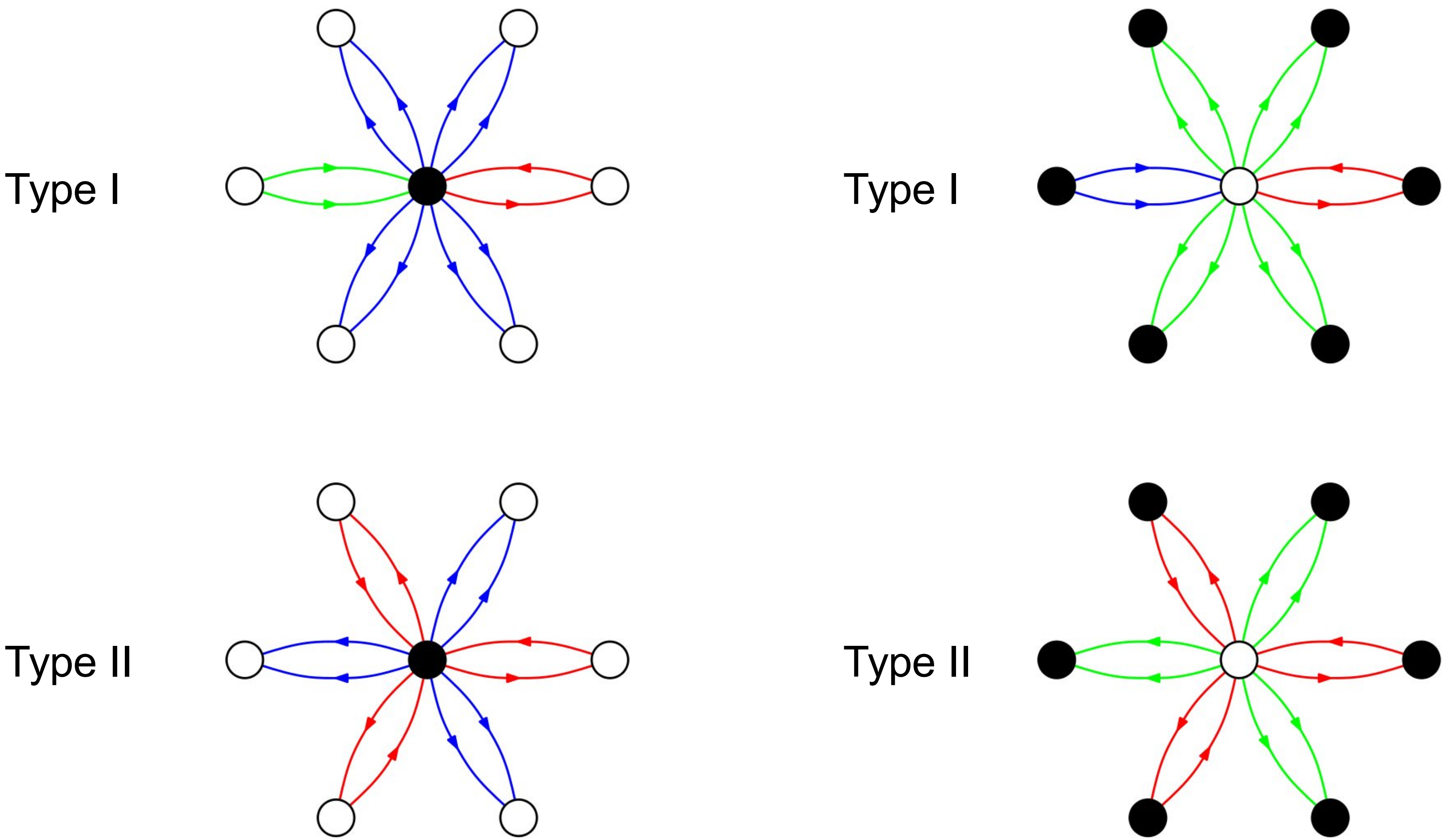
Moreover, this orientation is such that each vertex v has indegree $\lambda(v)$ if and only if we also have the global condition

$$|E(G)| = \sum_{v \in V(G)} \lambda(v).$$

3-orient the graph

We define $\lambda(v)=3$ for the original vertices and
$\lambda(r)=0$ for the extra vertex.
Using Euler formula, the previous lemma applies.

Types of Vertices

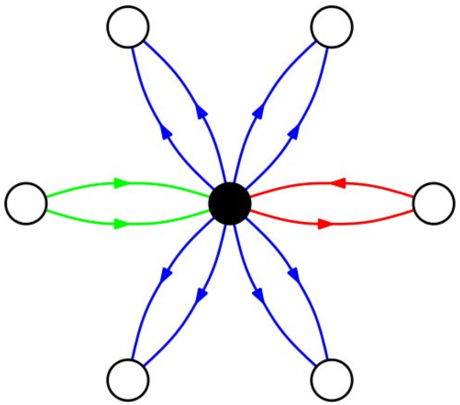


both incoming to white
both incoming to black
otherwise

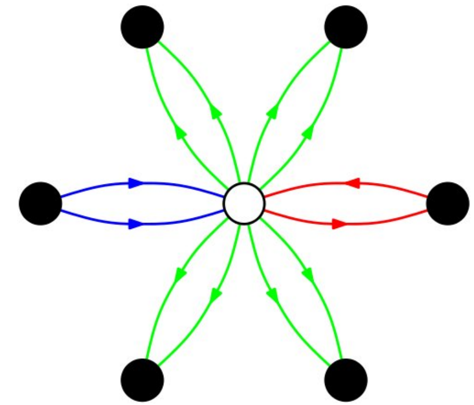
one incoming to white
one incoming to black
one incoming to white

Split white vertices of type 2

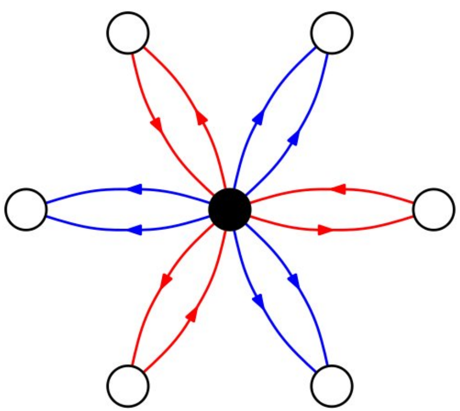
Type I



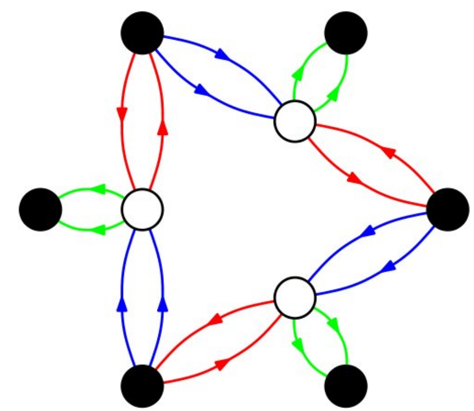
Type I



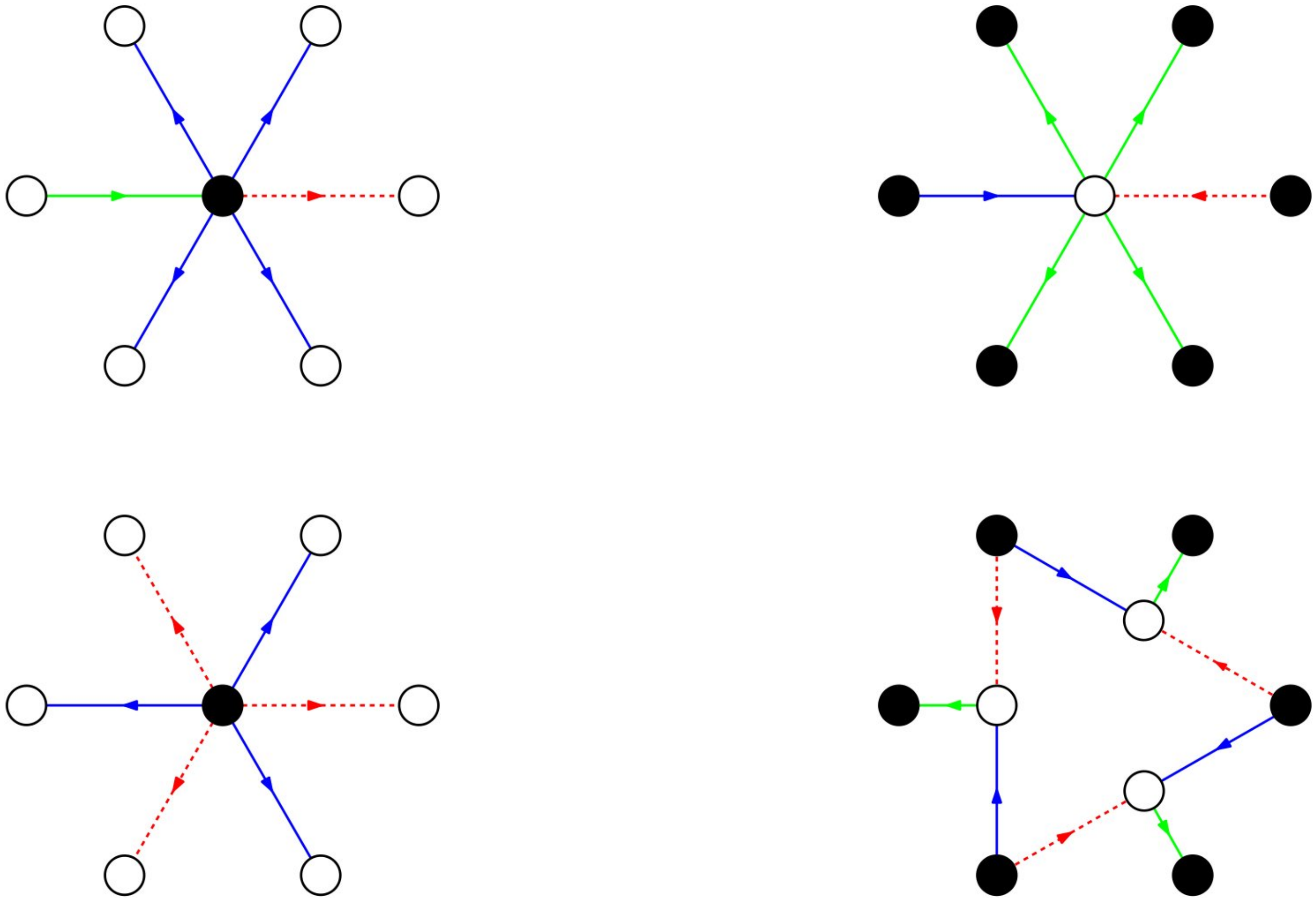
Type II



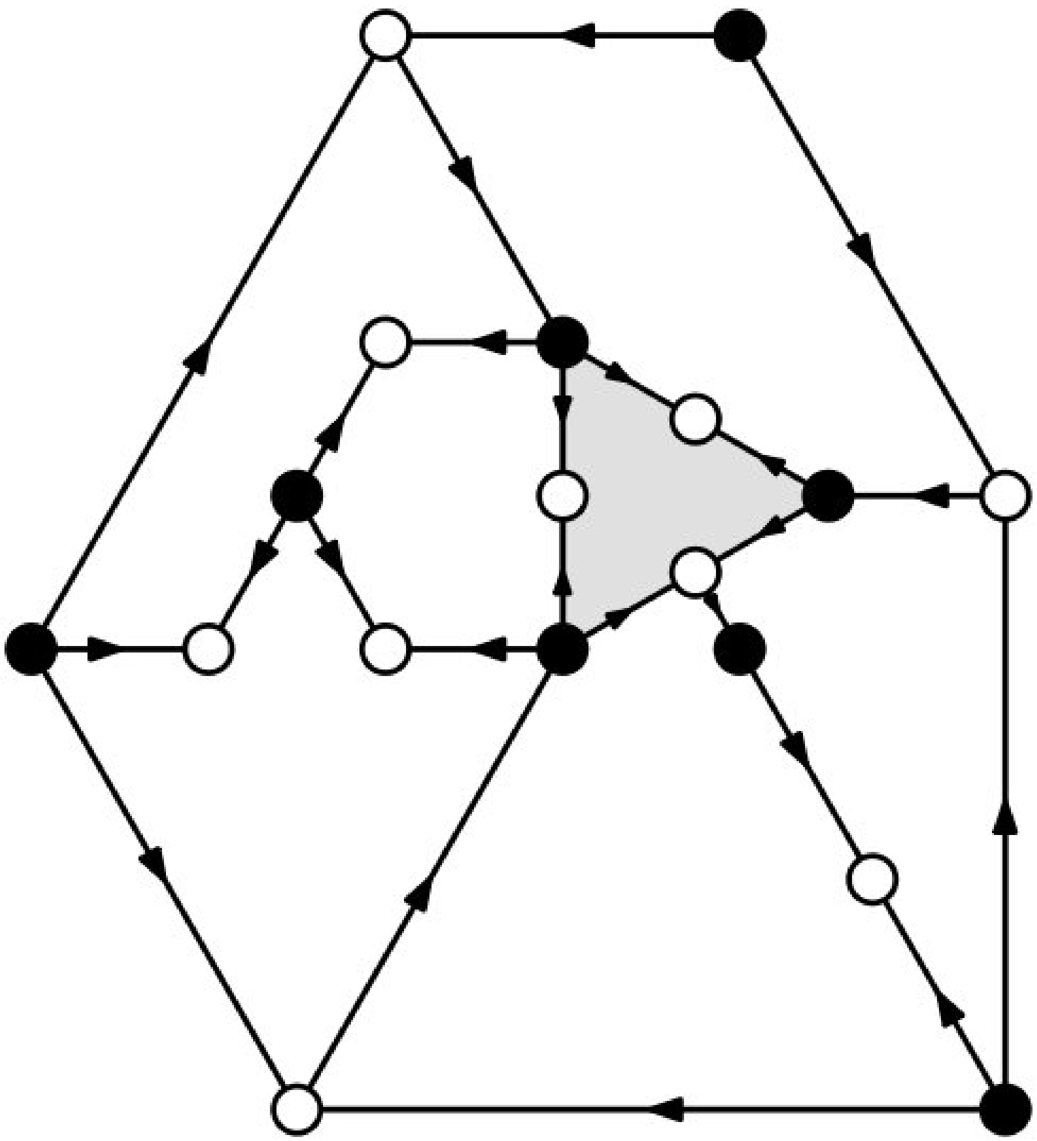
SPLIT!



Finally we get a (2,≤1)-Orientation

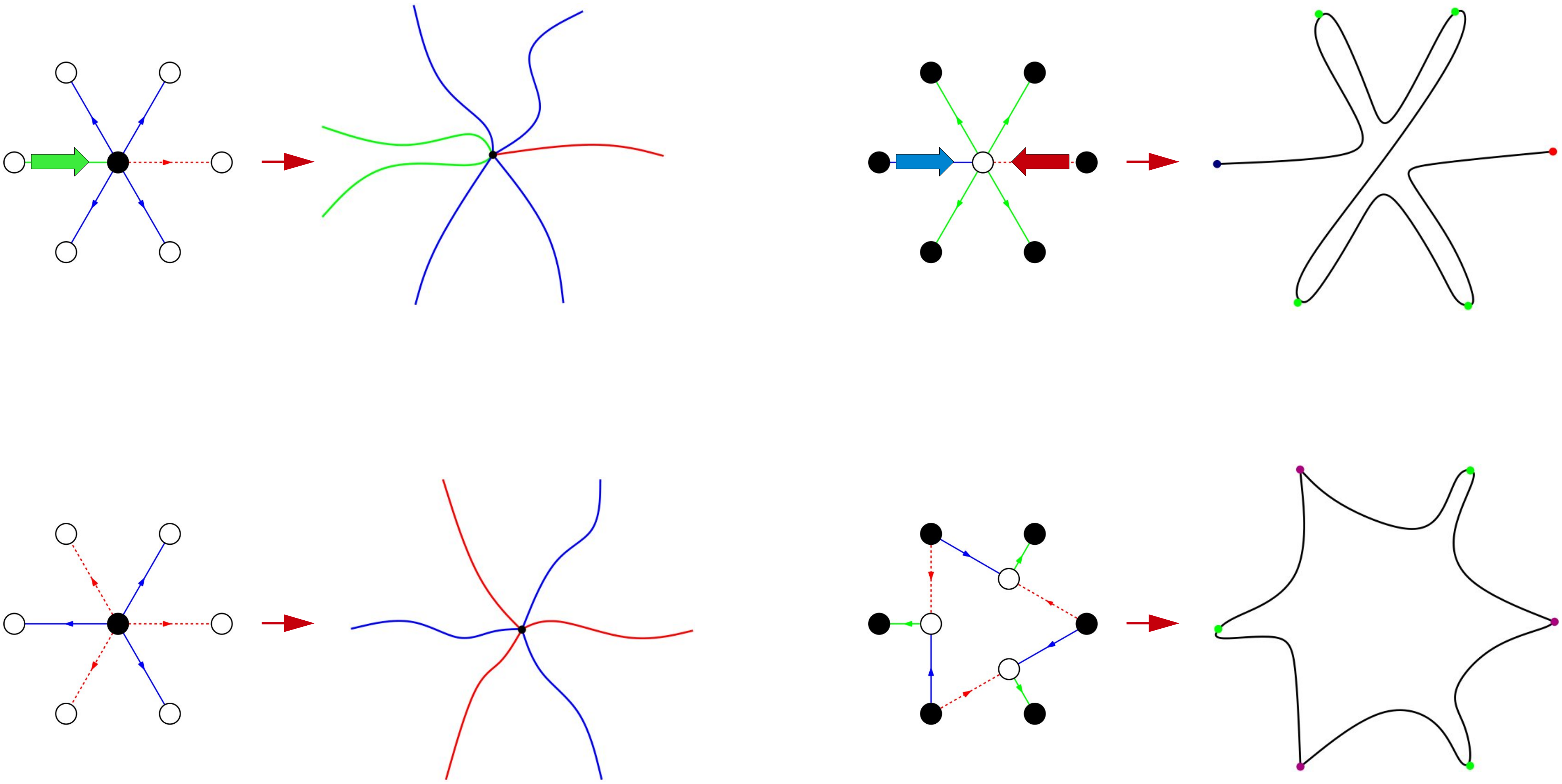


(2,≤1)-Orientation

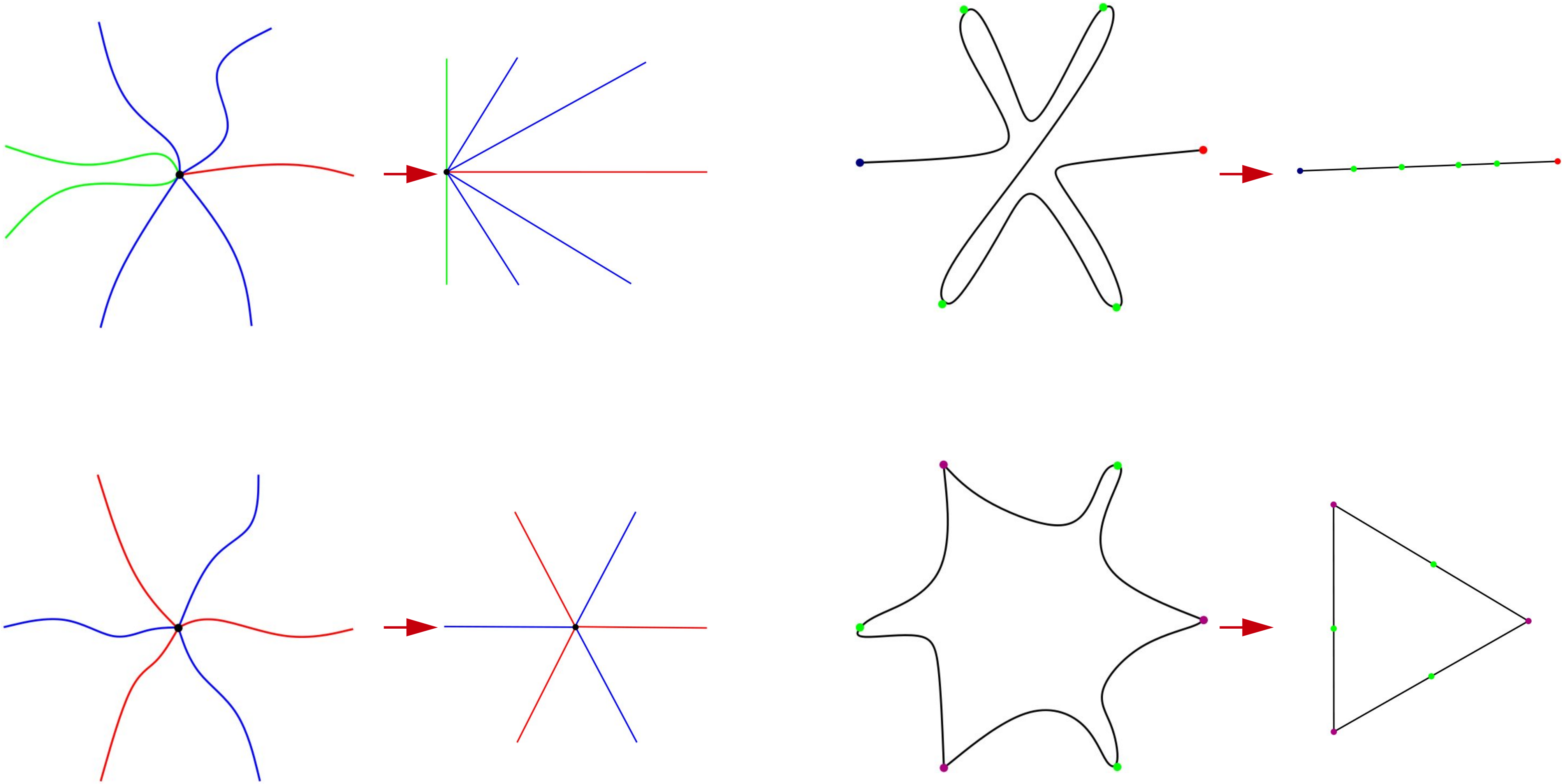


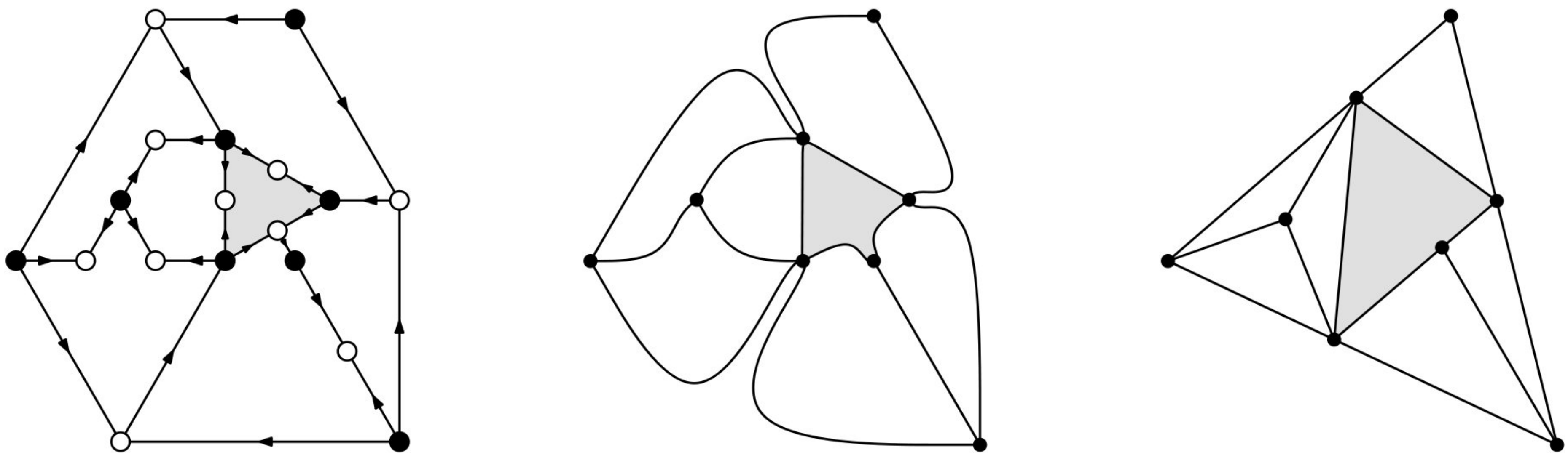
White: indegree = 2 Black: indegree ≤ 1

Contacts of Pseudo-Segments

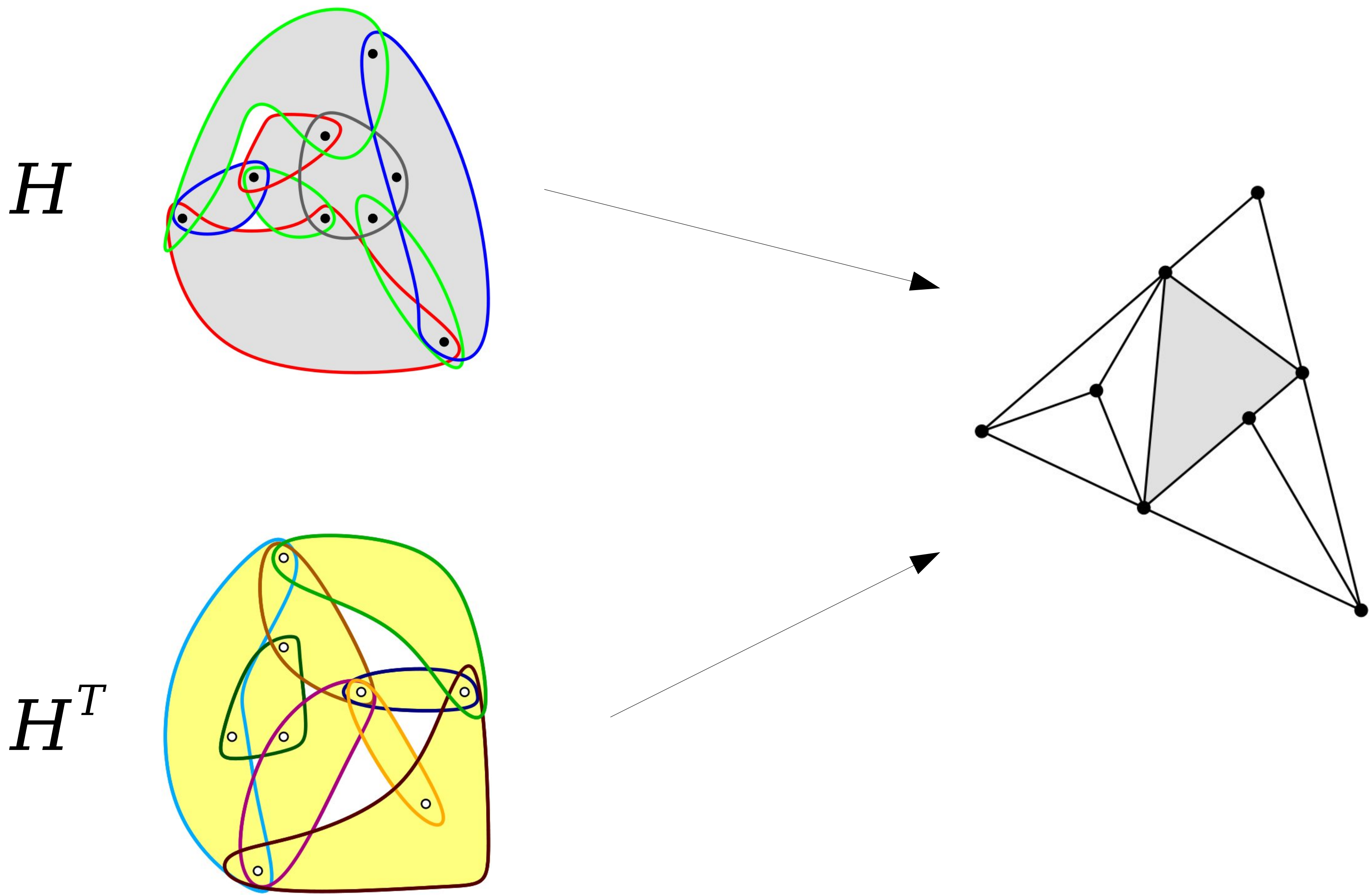


Stretching the Pseudo-Segments





Eventually...



□

Thank you for your attention...