International Doctoral School on
Multiple Criteria Decision Analysis (MCDA), Data Mining and Rough Sets

THE MACBETH APPROACH

METHOD, APPLICATIONS AND SOFTWARE

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DECISION ANALYSIS

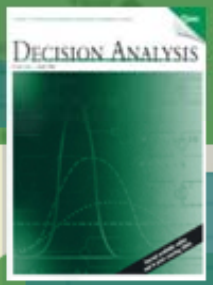
Development and use of logical methods
for the improvement of decision-making
in public and private enterprise.

Such methods include:

- models for decision-making under conditions of uncertainty or multiple objectives
- techniques of risk analysis and risk assessment;
- experimental and descriptive studies of decision-making behavior
- economic analysis of competitive and strategic decisions
- techniques for facilitating decision-making by groups
- computer modeling software and expert systems for decision support

http://decision-analysis.society.informs.org/

Decision Analysis Society



Welcome to **DAWeb**, the web site of the Decision Analysis Society of INFORMS. The Society promotes the development and use of logical methods for the improvement of decision-making in public and private enterprise. Such methods include models for decision-making under conditions of uncertainty or multiple objectives; techniques of risk analysis and risk assessment; experimental and descriptive studies of decision-making behavior; economic analysis of competitive and strategic decisions; techniques for facilitating decision-making by groups; and computer modeling software and expert systems for decision support. Our members include practitioners, educators, and researchers with backgrounds in engineering, business, economics, statistics, psychology, and other social and applied sciences.

The Decision Analysis Society is a subdivision of INFORMS, the Institute for Operations Research and the Management Sciences. INFORMS is the world's largest organization of operations researchers and management scientists, with over 12,000 members. The Decision Analysis Society is now the second largest scoiety within INFORMS, with over 940 members and 170 international members.

INFORMS was created in 1995 by the merger of the Operations Research Society of America (ORSA, founded in 1952) and The Institute for Management Sciences (TIMS, founded in 1953). The Decision Analysis Society was founded in 1980 as the ORSA Special Interest Group on Decision Analysis, becoming the INFORMS Section on Decision Analysis upon the merger of ORSA with TIMS. In February 1996, the Section on Decision Analysis became the Decision Analysis Society of INFORMS, reflecting the increasing scope of our activities.

NEWS & ANNOUNCEMENTS

- Decision Analysis Society Flyer NEW
- Decision Analysis Journal Flyer NEW
- Decision Analysis Journal
- INFORMS Practice Conf. 2007
- INFORMS International 2007
- INFORMS Seattle 2007

A Taxonomy of Decision Models

Problem dominated by

Uncertainty — Multiple Objectives

EXTEND conversation

- Event tree
- Fault tree
- Influence diagram

REVISE opinion

- Bayesian nets
- Bayesian statistics

SEPARATE into components

- Credence decomposition
- Risk analysis

CHOOSE option

- Payoff matrix
- Decision tree

EVALUATE options

- Multi-criteria decision analysis

ALLOCATE resources

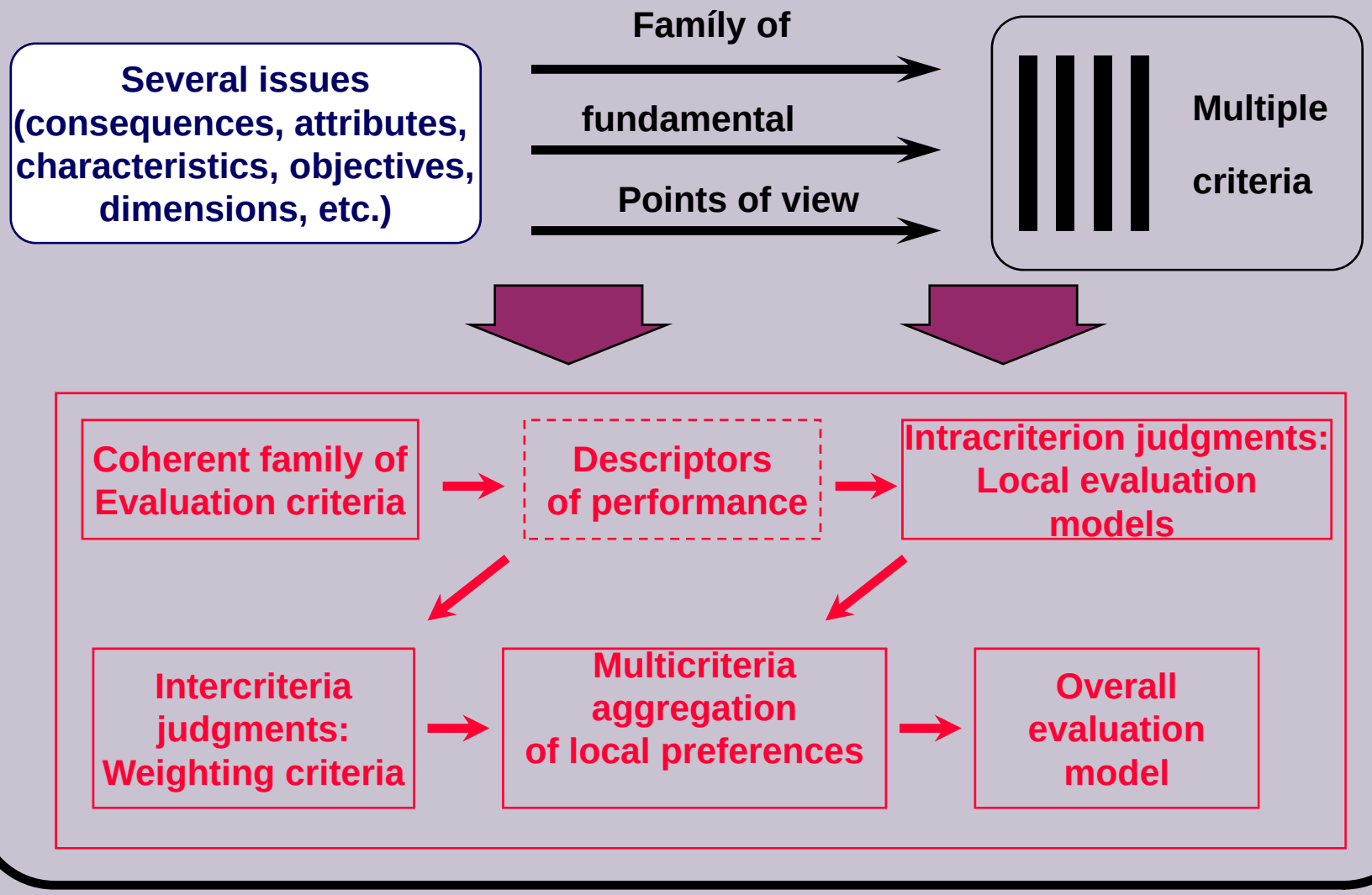
- Multi-criteria commons dilemma

NEGOTIATE

- Multi-criteria bargaining analysis

Reference: L.D.Phillips, Decision Analysis in 2005

MULTIPLE CRITERIA DECISION ANALYSIS



MULTI-CRITERIA VALUE MEASUREMENT

- Measuring the relative value of options in each criterion:

  Numerical (e.g. direct rating) and

  Non-numerical approaches (e.g. MACBETH)

- Criteria weighting procedures

  Numerical techniques (e.g. swing weighting)

  Non-numerical techniques (e.g. MACBETH)

MULTI-CRITERIA VALUE MEASUREMENT

Evaluation framework: Additive value model

$$V(a) = \sum_{j=1}^{n} k_j . v_j(a)$$

With:
$$\begin{cases} v_j(\text{upper anchor}_j) = 100, \forall j \\ v_j(\text{lower anchor}_j) = 0, \forall j \\ V(\text{all upper anchors}) = 100 \\ V(\text{all lower anchors}) = 0 \end{cases}$$

$V(a)$ overall value of option a

$v_j(a)$ local value (score) of option a against criterion j

k_j scaling constant (relative weight) of criterion j

$$\sum_{j=1}^{n} k_j = 1 \quad \textbf{and} \quad k_j > 0 \ (j = 1,\ldots,n)$$

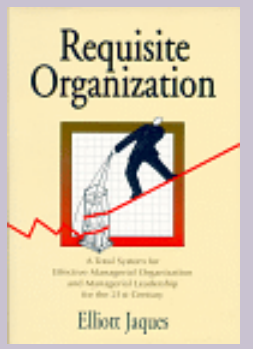
Non-numerical approach: MACBETH

Measuring **A**ttractiveness by a **C**ategorical **B**ased **E**valuation **Tech**nique

An interactive pairwise comparison approach
to guide the construction of a quantitative value model
from qualitative value judgments

When you take all non-verbal judgment out of a decision
it becomes a calculation and not a decision.

Elliot Jaques
Requisite Organization, 1988



Download the Demo Version of M-MACBETH
http://www.m-macbeth.com

How does it work?

MACBETH uses a simple question-answer protocol

that involves only two options in each question:

Ask the evaluator to pairwise compare options
by given a *qualitative* judgement
of the difference in attractiveness
between each two options

For x and y such that
x is preferred to y,
the difference in attractiveness
between x and y is:

MACBETH semantic categories of difference of attractiveness:

C_6

C_5

C_4

C_3

C_2

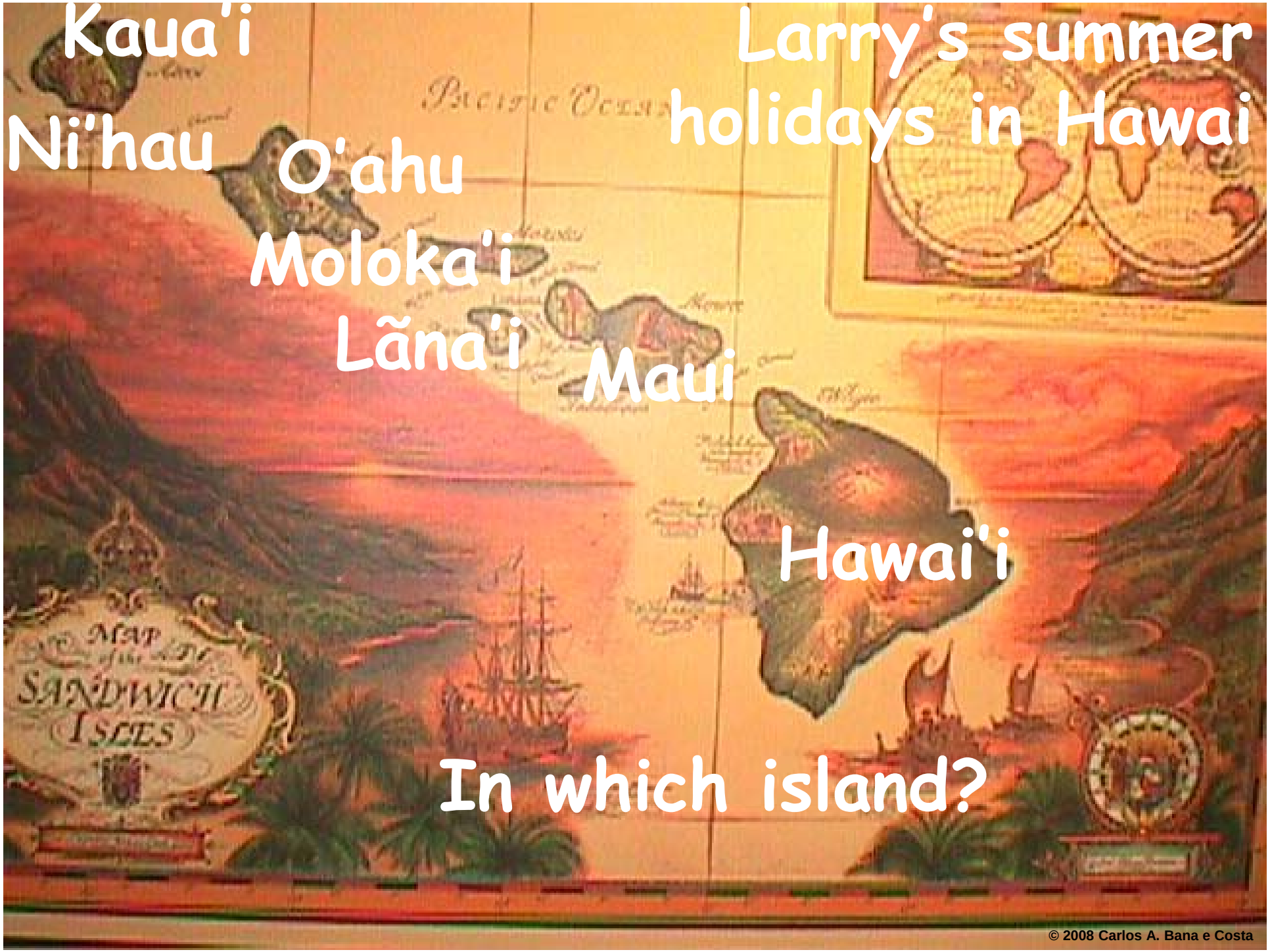
C_1

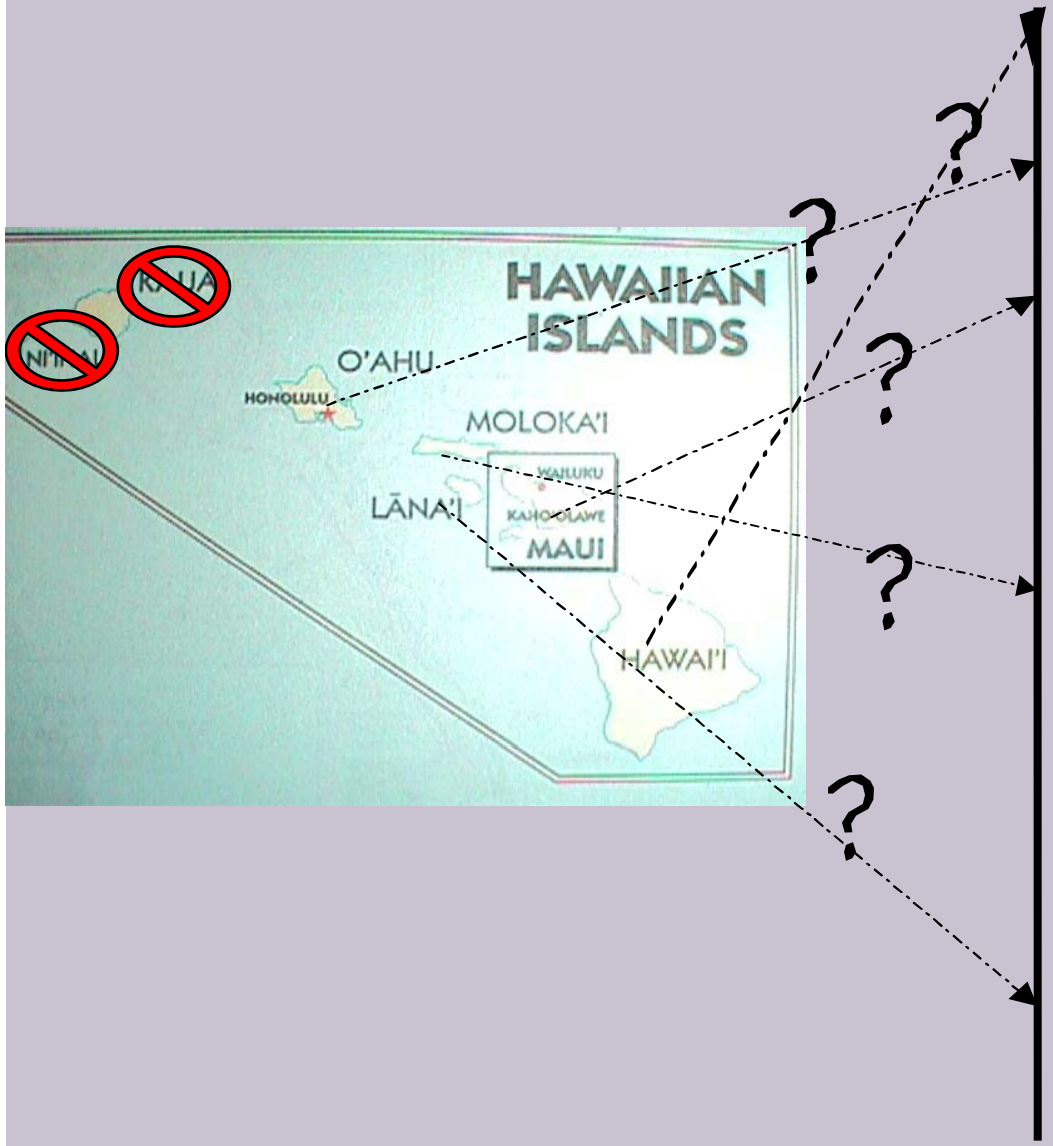
C_0

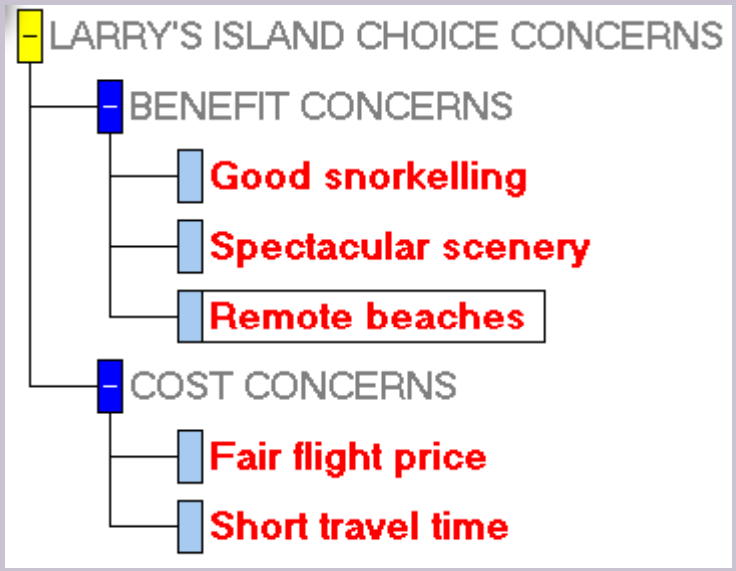
Note: the ‘weak’, ‘strong’ and ‘extreme’ were initially called the fundamental categories, but the M-MACBETH software that implements the MACBETH approach does not make this distinction and even allows for group judgments that do not distinguish between several consecutive categories, such as ‘strong or very strong’.

How many judgements?

For a set X of m options, the number of pairwise comparisons can vary from a maximum of m(m-1)/2 judgments, when all pairwise comparisons are made, to a minimum acceptable number of m–1 judgments, as when comparing only each two consecutive options in the ranking or one options with all of the other m–1 (however, it is recommended to ask for some additional judgments to perform several consistency checks).



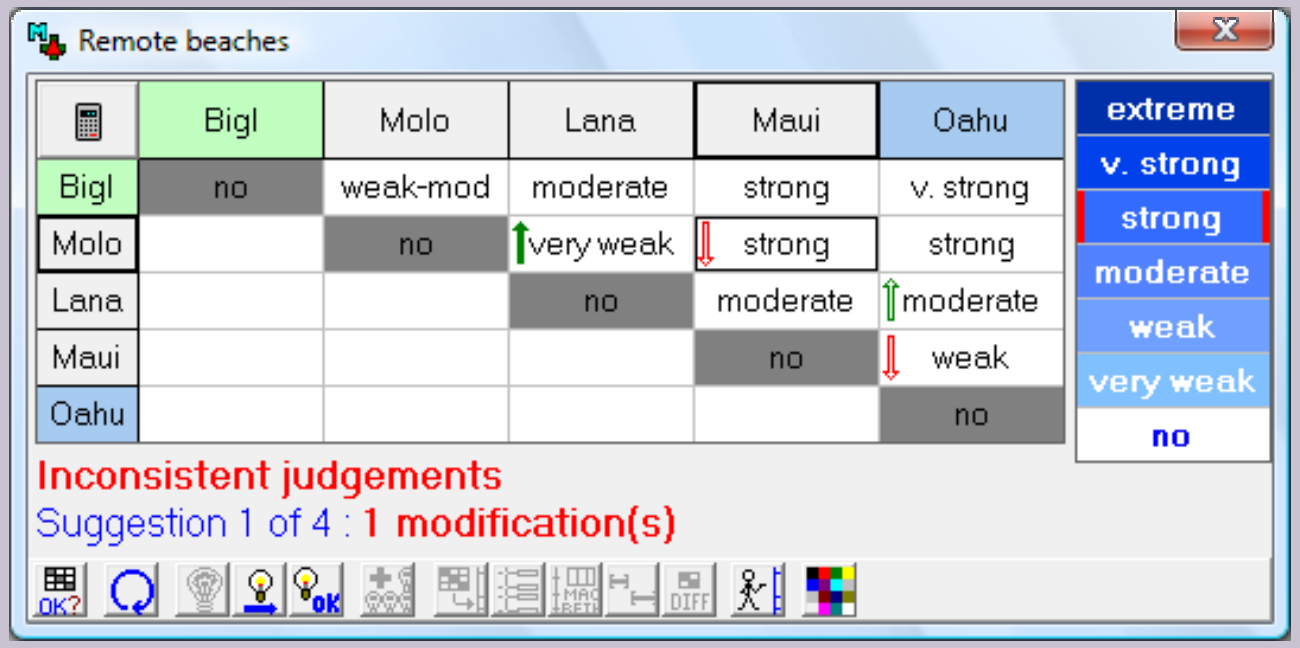




Assessing MACBETH intracriterion preference information

As each judgement is entered in the matrix, its consistency with the judgments already inserted is checked and possible inconsistencies are detected.

if an inconsistency is detected, suggestions to overcome it are presented. Technically, this is done by a mathematical programming algorithm (see Bana e Costa et al. 2005 for details).



	Bigl	Molo	Lana	Maui	Oahu
Bigl	no	weak-mod	moderate	strong	v. strong
Molo		no	very weak	strong	strong
Lana			no	moderate	moderate
Maui				no	weak
Oahu					no

Inconsistent judgements

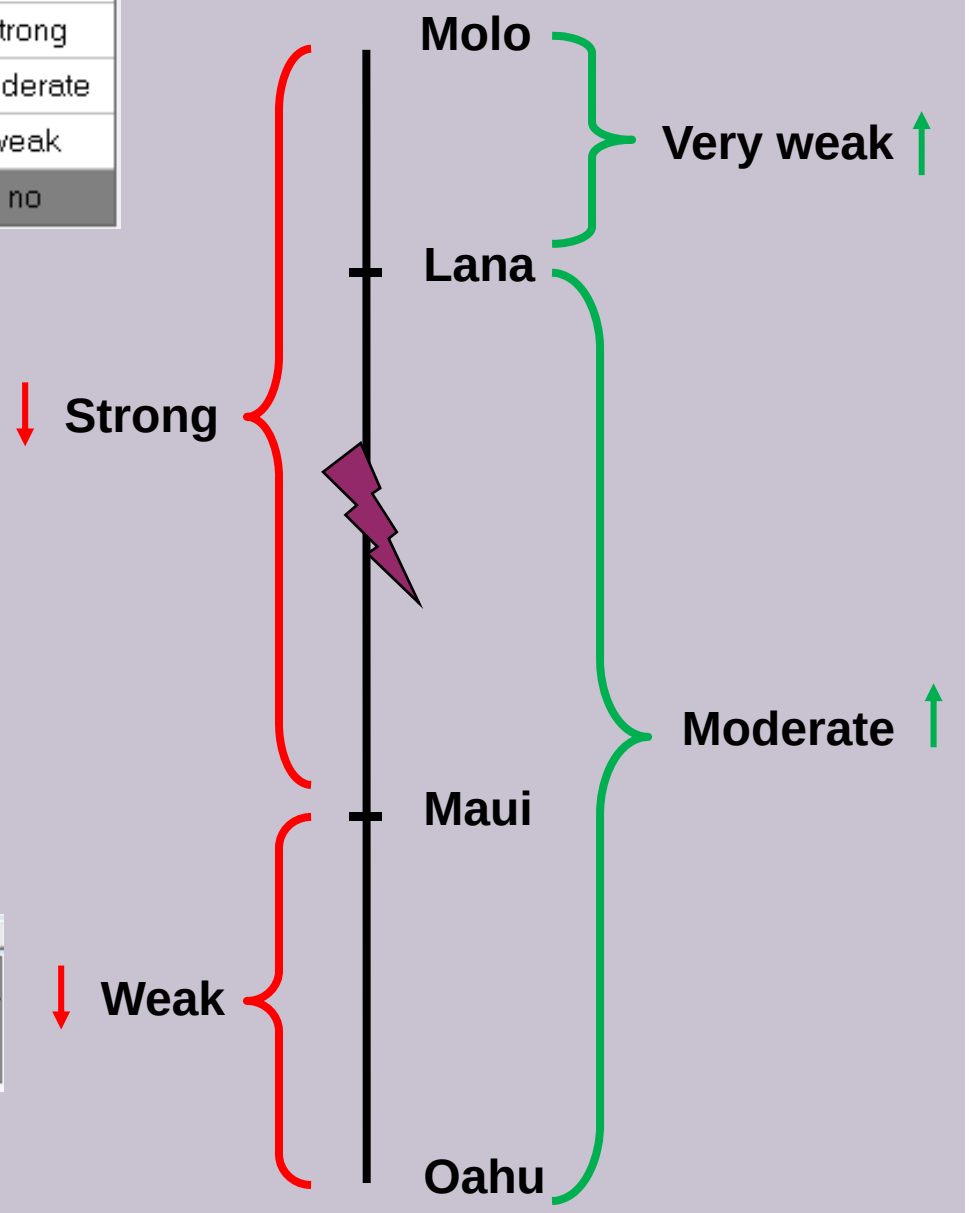
MACBETH has found 4 way(s),
requiring 1 category change(s),
to obtain consistent judgements.

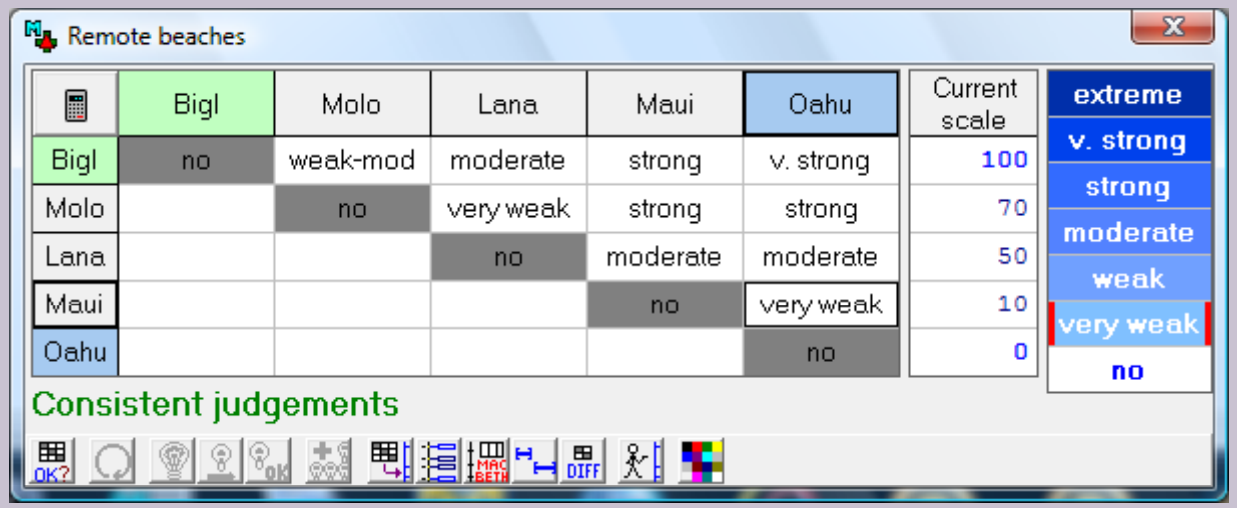
Use the " + " key or this button
to cycle through the suggestions.

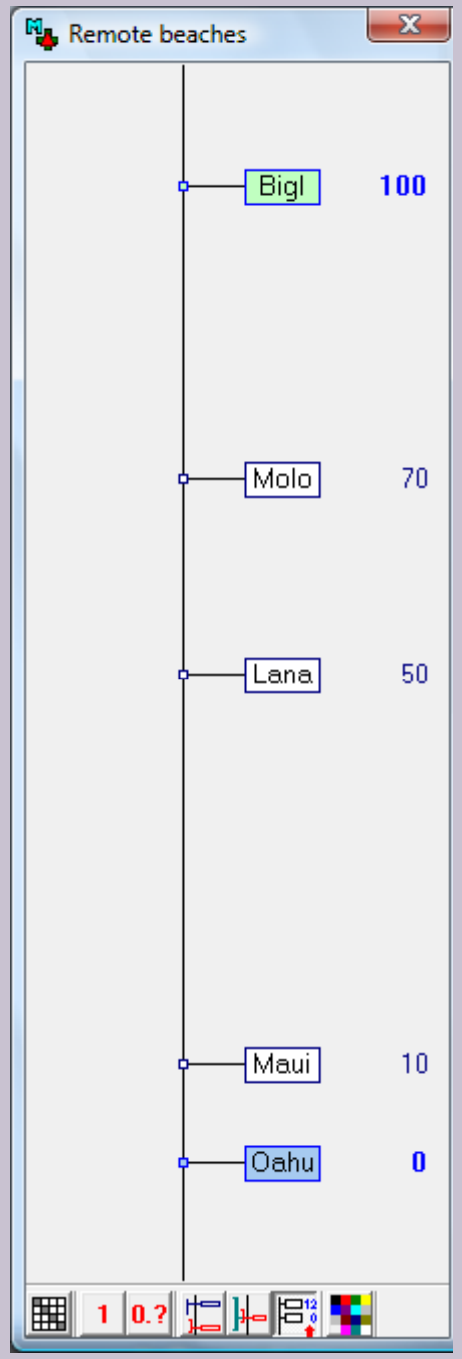
OK

Remote beaches

Problems	Diff.	Couples		Couples	Diff.
1	strong	Molo - Maui	>	Lana - Oahu	moderate
	weak	Maui - Oahu	>	**Molo - Lana**	very weak







For a set of consistent judgements, MACBETH suggests a <u>numerical scale</u> v on X that satisfies the following measurement rules:

<u>Rule 1</u>

$\forall\, x, y \in X : v(x) = v(y)$ *iff* x and y are equally attractive

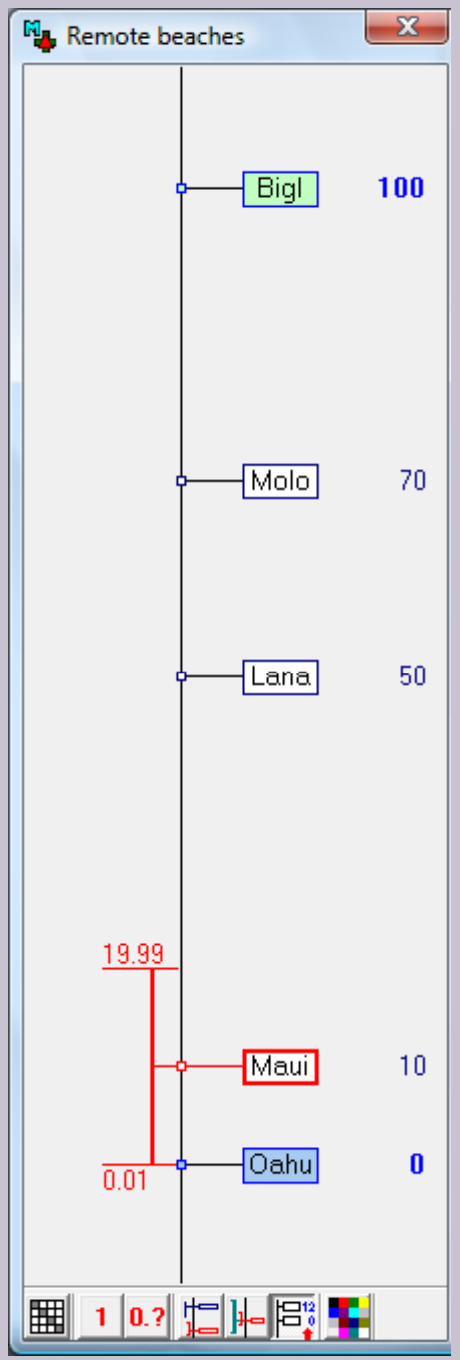
$\forall\, x, y \in X : v(x) > v(y)$ *iff* x is more attractive than y;

<u>Rule 2</u>

$\forall\, k, k' \in \{ 1, 2, 3, 4, 5, 6 \}, \forall\, x, y, w, z \in X,$

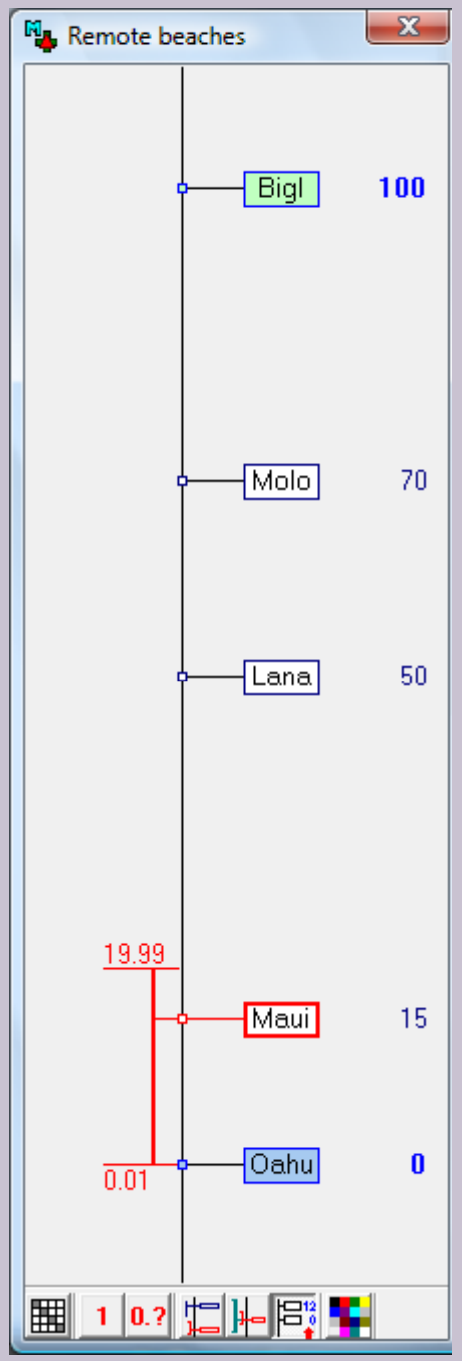
with $(x, y) \in C_k$ and $(w, z) \in C_{k'}$:

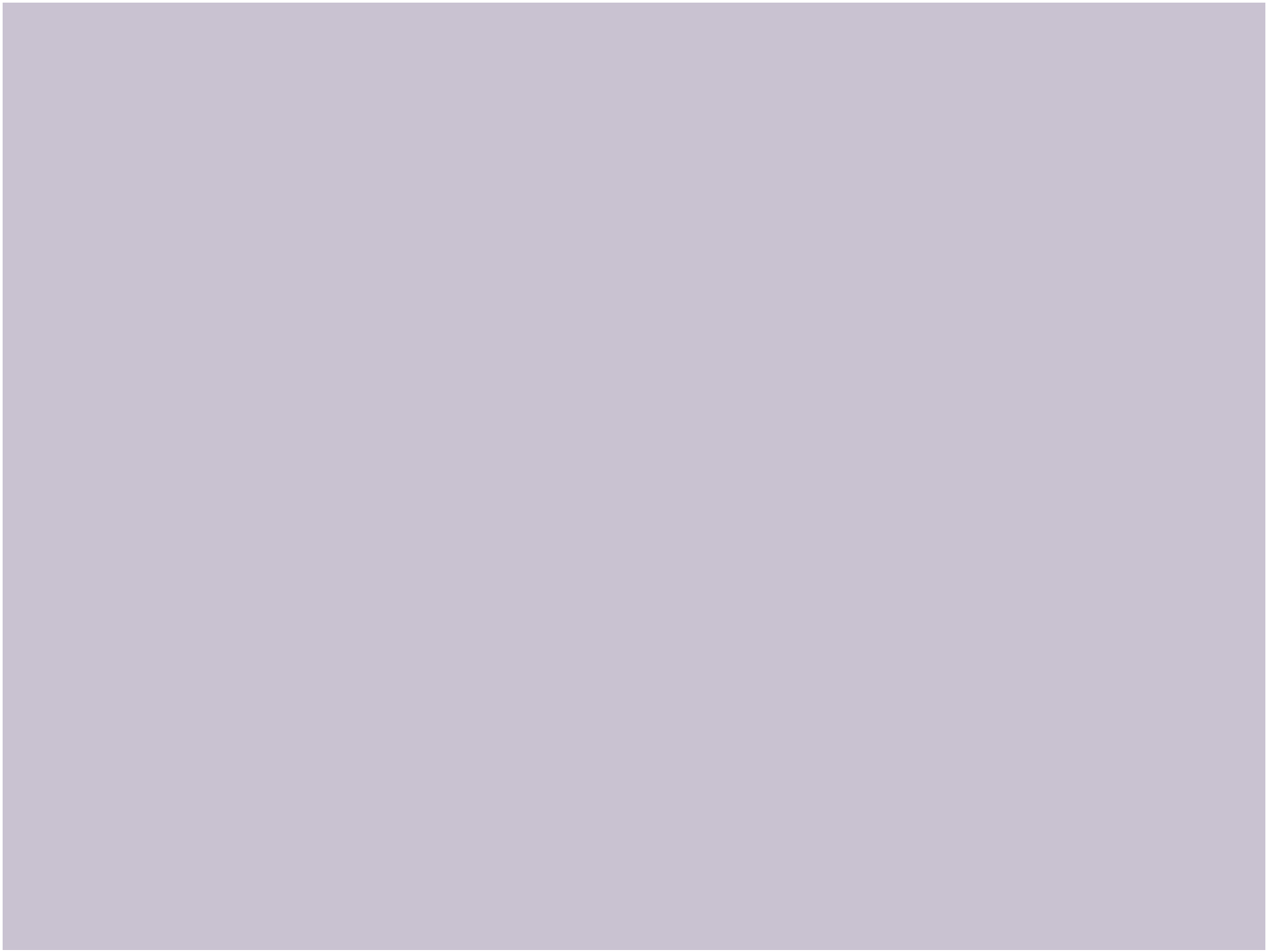
$$k \geq k' + 1 \Rightarrow v(x) - v(y) > v(w) - v(z)$$



The software determines the interval within which each score of each option can vary when the other *m*-1 scores are fixed and still remain compatible with the matrix of judgments.

This allows the adjustment of the scale by comparing differences of scores, to arrive to a cardinal scale.





“Behind” M-MACBETH

João Bana e Costa
Carlos Bana e Costa

"Behind" M-MACBETH

	A	B	C	D
A		Very Weak	Moderate	Very Strong
B			Moderate	Strong
C				Strong
D				

Extreme	6
Very Strong	5
Strong	4
Moderate	3
Weak	2
Very Weak	1

"Behind" M-MACBETH

	A	B	C	D
A		Very Weak 1	Moderate	Very Strong
B			Moderate 3	Strong
C				Strong 4
D				

Extreme	6
Very Strong	5
Strong	4
Moderate	3
Weak	2
Very Weak	1

"Behind" M-MACBETH

	A	B	C	D
A		Very Weak 1	Moderate 4	Very Strong
B			Moderate 3	Strong
C				Strong 4
D				

Extreme	6
Very Strong	5
Strong	4
Moderate	3
Weak	2
Very Weak	1

$$v(A)-v(C) = v(A)-v(B) + v(B)-v(C)$$

"Behind" M-MACBETH

	A	B	C	D
A		Very Weak 1	Moderate 4	Very Strong
B			Moderate 3	Strong
C				Strong 4
D				

Extreme	6
Very Strong	5
Strong	4
Moderate	3
Weak	2
Very Weak	1

$$v(A)-v(C) < v(C)-v(D)$$

"Behind" M-MACBETH

	A	B	C	D
A		Very Weak 1	Moderate 4	Very Strong
B			Moderate 3	Strong
C				Strong 5
D				

Extreme	7
Very Strong	6
Strong	5
Moderate	3-4
Weak	2
Very Weak	1

Increase v(C)-v(D) of 1

"Behind" M-MACBETH

	A	B	C	D
A		Very Weak 1	Moderate 4	Very Strong
B			Moderate 3	Strong 8
C				Strong 5
D				

Extreme	10
Very Strong	9
Strong	5-8
Moderate	3-4
Weak	2
Very Weak	1

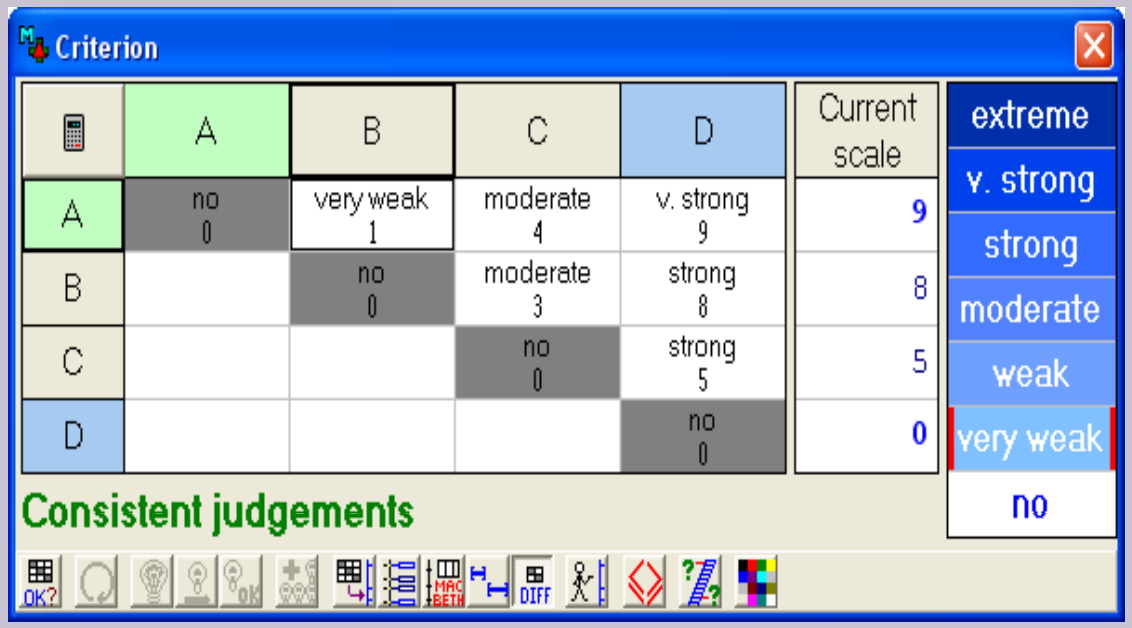
$$v(B)-v(D) = v(B)-v(C) + v(C)-v(D)$$

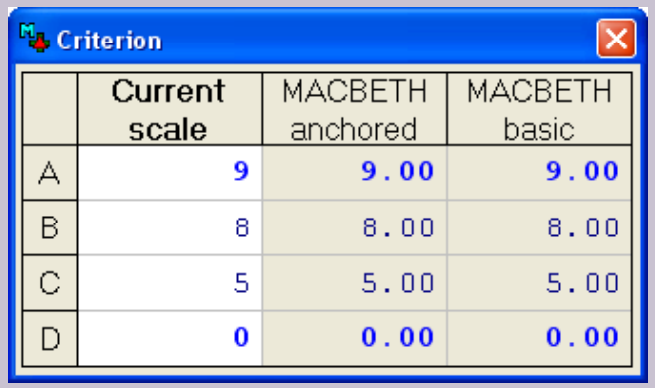
"Behind" M-MACBETH

	A	B	C	D
A	no	Very Weak 1	Moderate 4	Very Strong 9
B		no	Moderate 3	Strong 8
C			no	Strong 5
D				no

Extreme	10
Very Strong	9
Strong	5-8
Moderate	3-4
Weak	2
Very Weak	1

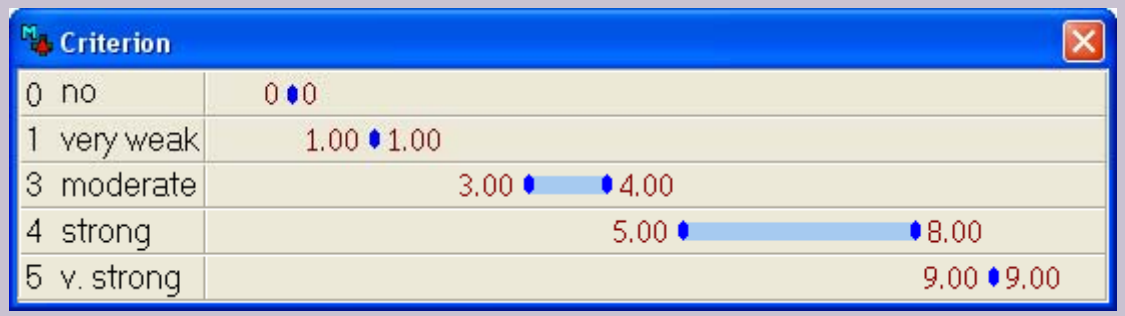
$$v(A)-v(D) = v(A)-v(B) + v(B)-v(C)+ + v(C)-v(D)$$

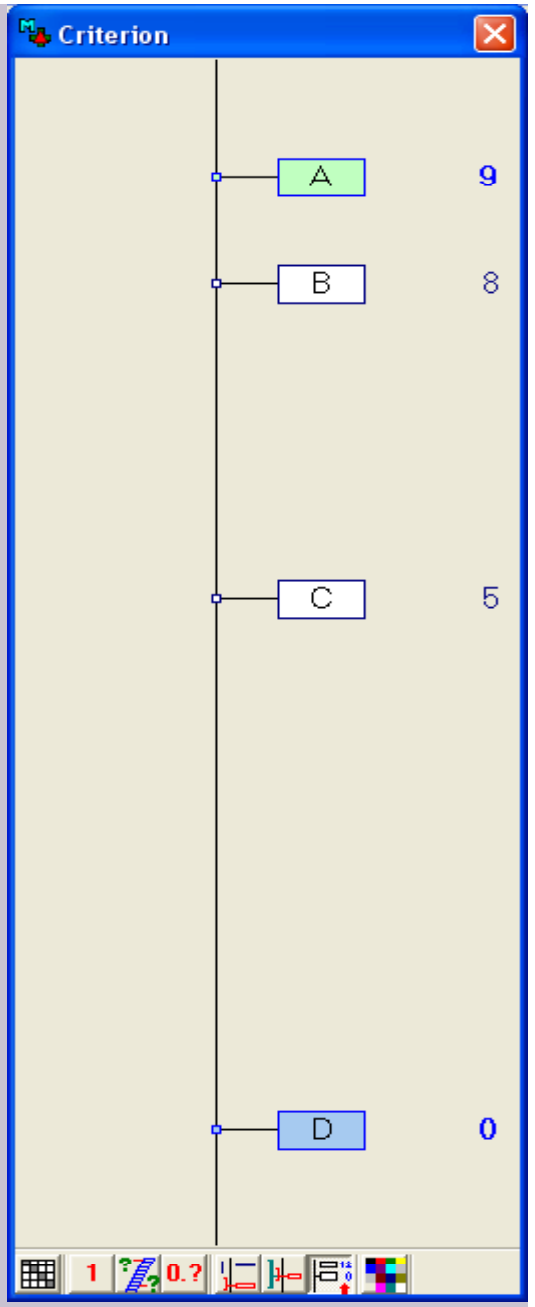




Criterion

	Current scale	MACBETH anchored	MACBETH basic
A	9	9.00	9.00
B	8	8.00	8.00
C	5	5.00	5.00
D	0	0.00	0.00





MACBETH weighting procedure

Reference levels

Glob. ref. sit.
[scenery]
[time]
[snorkelling]
[beaches]
[price]
{ all worst }

snorkelling	scenery	beaches	price	time
ExcNear	Lana	Bigl	0	0
GoodNear	Bigl	Molo	125	5
ExcOut	Oahu	Lana	250	20
GoodOut	Molo	Maui	375	40
	Maui	Oahu	500	100

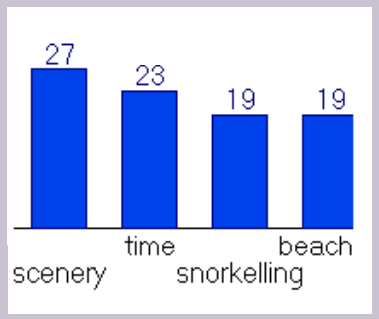
strong

strong to very strong

strong

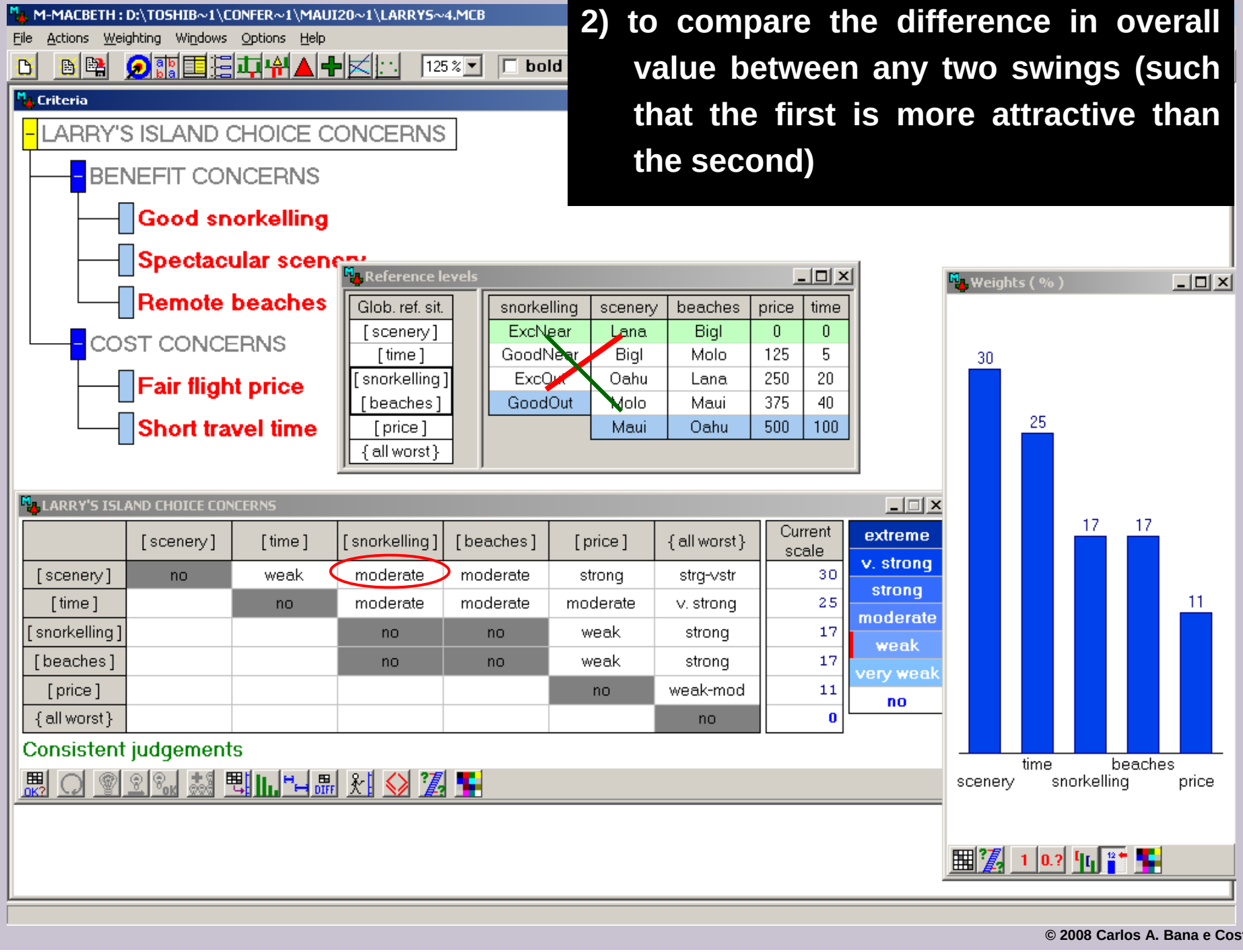
weak to moderate

very strong



SUM=1

Qualitative swing judgements



2) to compare the difference in overall value between any two swings (such that the first is more attractive than the second)

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