# Decisions with multiple attributes

## A brief introduction to conjoint measurement models

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# Introduction

**Aims**

mainly pedagogical

- present elements of the classical theory
- position some extensions w.r.t. this classical theory

# Introduction

## Comparing holiday packages

| | cost | # of days | travel time | category of hotel | distance to beach | Wifi | cultural interest |
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| $A$ | 200 € | 15 | 12 h | *** | 45 km | Y | ++ |
| $B$ | 425 € | 18 | 15 h | **** | 0 km | N | −− |
| $C$ | 150 € | 4 | 7 h | ** | 250 km | N | + |
| $D$ | 300 € | 5 | 10 h | *** | 5 km | Y | − |

## Central problems

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## Two different contexts

1. decision aiding
   - careful analysis of objectives
   - careful analysis of attributes
   - careful selection of alternatives
   - availability of the DM
2. recommendation systems
   - no analysis of objectives
   - attributes as available
   - alternatives as available
   - limited access to the user

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## Basic model

- additive value function model

$$x \succsim y \Leftrightarrow \sum_{i=1}^{n} v_i(x_i) \geq \sum_{i=1}^{n} v_i(y_i)$$

$x, y$ : alternatives

$x_i$ : evaluation of alternative $x$ on attribute $i$

$v_i(x_i)$ : number

- underlies most existing MCDM techniques

## Underlying theory: conjoint measurement

- Economics (Debreu, 1960)
- Psychology (Luce & Tukey, 1964)
- tools to help structure preferences

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# Outline: Classical theory

1. An aside: measurement in Physics
2. An example: even swaps
3. Notation
4. Additive value functions: outline of theory
5. Additive value functions: implementation

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# Part I

## Classical theory: conjoint measurement

# Outline

# Aside: measurement of physical quantities

**Lonely individual on a desert island**

- no tools, no books, no knowledge of Physics
- wants to rebuild a system of physical measures

**A collection of rigid straight rods**

- problem: measuring the length of these rods
  - pre-theoretical intuition
    - length
    - softness, beauty

**3 main steps**

- comparing objects
- creating and comparing new objects
- creating standard sequences

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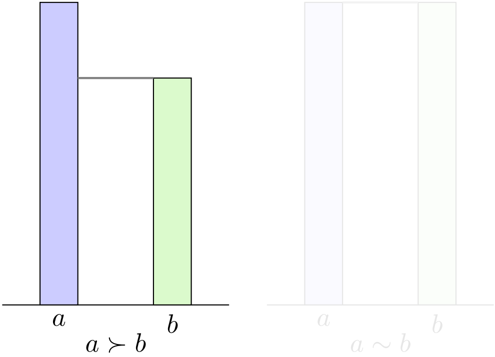

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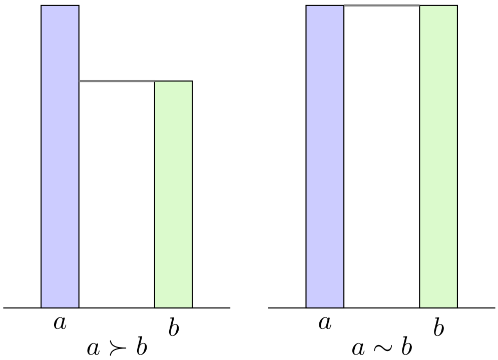

# Comparing objects

**Results**

- $a \succ b$: extremity of rod $a$ is higher than extremity of rod $b$
- $a \sim b$: extremity of rod $a$ is as high as extremity of rod $b$

**Expected properties**

- $a \succ b$, $a \sim b$ or $b \succ a$
- $\succ$ is asymmetric
- $\sim$ is symmetric
- $\succ$ is transitive
- $\sim$ is transitive
- $\succ$ and $\sim$ combine "nicely"
  - $a \succ b$ and $b \sim c \Rightarrow a \succ c$
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# Comparing objects

## Summary of experiments

- binary relation $\succsim = \succ \cup \sim$ that is a weak order
  - complete ($a \succsim b$ or $b \succsim a$)
  - transitive ($a \succsim b$ and $b \succsim c \Rightarrow a \succsim c$)

## Consequences

- associate a real number $\Phi(a)$ to each object $a$
- the comparison of numbers faithfully reflects the results of experiments

$$a \succ b \Leftrightarrow \Phi(a) > \Phi(b) \qquad a \sim b \Leftrightarrow \Phi(a) = \Phi(b)$$

- the function $\Phi$ defines an ordinal scale
  - applying an increasing transformation to $\Phi$ leads to a scale that has the same properties
  - any two scales having the same properties are related by an increasing transformation

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go faster

# Comments

## Nature of the scale

- $\Phi$ is quite far from a full-blown measure of length...
- useful though since it allows the experiments to be done only once

## Hypotheses are stringent

- highly precise comparisons
- several practical problems
  - any two objects can be compared
  - connections between experiments
  - comparisons may vary in time
- idealization of the measurement process

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- use the available objects to create new ones
- concatenate objects by placing two or more rods "in a row"




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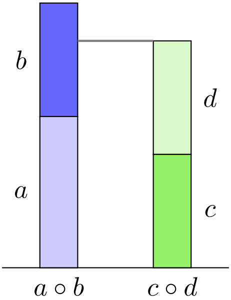


$$a \circ b \succ c \circ d$$

# Concatenation

- we want to be able to deduce $\Phi(a \circ b)$ from $\Phi(a)$ and $\Phi(b)$
- simplest requirement

$$\Phi(a \circ b) = \Phi(a) + \Phi(b)$$

- monotonicity constraints

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# Example

- five rods: $r_1, r_2, \ldots, r_5$
- we may only concatenate two rods (space reasons)
- we may only experiment with different rods
- data:

$$r_1 \circ r_5 \succ r_3 \circ r_4 \succ r_1 \circ r_2 \succ r_5 \succ r_4 \succ r_3 \succ r_2 \succ r_1$$

- all constraints are satisfied: weak ordering and monotonicity

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$$r_1 \circ r_5 \succ r_3 \circ r_4 \succ r_1 \circ r_2 \succ r_5 \succ r_4 \succ r_3 \succ r_2 \succ r_1$$

| | $\Phi$ | $\Phi'$ | $\Phi''$ |
|---|---|---|---|
| $r_1$ | 14 | 10 | 14 |
| $r_2$ | 15 | 91 | 16 |
| $r_3$ | 20 | 92 | 17 |
| $r_4$ | 21 | 93 | 18 |
| $r_5$ | 28 | 100 | 29 |

- $\Phi$, $\Phi'$ and $\Phi''$ are equally good to compare simple rods
- only $\Phi$ and $\Phi''$ capture the comparison of concatenated rods
- going from $\Phi$ to $\Phi''$ does not involve a "change of units"

- it is tempting to use $\Phi$ or $\Phi''$ to infer comparisons that have not been performed…
- disappointing

$$\Phi : r_2 \circ r_3 \sim r_1 \circ r_4 \quad \Phi'' : r_2 \circ r_3 \succ r_1 \circ r_4$$

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# Step 3: creating and using standard sequences

- choose a standard rod
- be able to build perfect copies of the standard
- concatenate the standard rod with its perfects copies




$$S(8) \succ a \succ S(7)$$
$$\Phi(s) = 1 \Rightarrow 7 < \Phi(a) < 8$$

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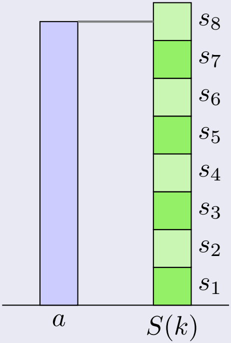


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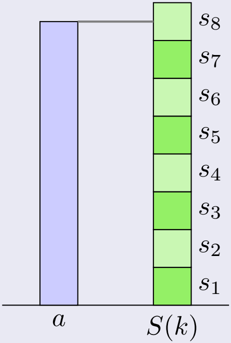


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# Convergence

**First method**

- choose a smaller standard rod
- repeat the process

**Second method**

- prepare a perfect copy of the object
- concatenate the object with its perfect copy
- compare the "doubled" object to the original standard sequence
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## Extensive measurement

- Krantz, Luce, Suppes & Tversky (1971, chap. 3)

## 4 Ingredients

1. well-behaved relations $\succ$ and $\sim$
2. concatenation operation $\circ$
3. consistency requirements linking $\succ$, $\sim$ and $\circ$
4. ability to prepare perfect copies of some objects in order to build standard sequences

## Neglected problems

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# Question

**Can this be applied outside Physics?**

- no concatenation operation (intelligence!)

# What is conjoint measurement?

## Conjoint measurement

- mimicking the operations of extensive measurement
  - when there are no concatenation operation readily available
  - when several dimensions are involved

## Seems overly ambitious

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# Example: Hammond, Keeney & Raiffa

## Choice of an office to rent

- five locations have been identified
- five attributes are being considered
  - *Commute* time (minutes)
  - *Clients*: percentage of clients living close to the office
  - *Services*: ad hoc scale
    - $A$ (all facilities), $B$ (telephone and fax), $C$ (no facility)
  - *Size*: square feet ($\simeq 0.1$ m$^2$)
  - *Cost*: \$ per month

## Attributes

- *Commute*, *Size* and *Cost* are natural attributes
- *Clients* is a proxy attribute
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| | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| *Commute* | 45 | 25 | 20 | 25 | 30 |
| *Clients* | 50 | 80 | 70 | 85 | 75 |
| *Services* | $A$ | $B$ | $C$ | $A$ | $C$ |
| *Size* | 800 | 700 | 500 | 950 | 700 |
| *Cost* | 1850 | 1700 | 1500 | 1900 | 1750 |

**Hypotheses and context**

- a single cooperative DM
- choice of a single office
- ceteris paribus reasoning seems possible

  *Commute*: decreasing *Clients*: increasing
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- dominance has meaning

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| *Commute* | 45 | 25 | 20 | 25 | 30 |
| *Clients* | 50 | 80 | 70 | 85 | 75 |
| *Services* | $A$ | $B$ | $C$ | $A$ | $C$ |
| *Size* | 800 | 700 | 500 | 950 | 700 |
| *Cost* | 1850 | 1700 | 1500 | 1900 | 1750 |

- $b$ dominates alternative $e$
- $d$ is "close" to dominating $a$
- divide and conquer: dropping alternatives
  - drop $a$ and $e$

| | $b$ | $c$ | $d$ |
|---|---|---|---|
| *Commute* | 25 | 20 | 25 |
| *Clients* | 80 | 70 | 85 |
| *Services* | $B$ | $C$ | $A$ |
| *Size* | 700 | 500 | 950 |
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- no more dominance
- assessing tradeoffs
- all alternatives except $c$ have a common evaluation on *Commute*
- modify $c$ in order to bring it to this level
  - starting with $c$, what is the gain on *Clients* that would exactly compensate a loss of 5 min on *Commute*?
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| | $c$ | $c'$ |
|---|---|---|
| *Commute* | 20 | **25** |
| *Clients* | 70 | $\mathbf{70} + \delta$ |
| *Services* | $C$ | $C$ |
| *Size* | 500 | 500 |
| *Cost* | 1500 | 1500 |

find $\delta$ such that $c' \sim c$

**Answer**

- for $\delta = 8$, I am indifferent between $c$ and $c'$
- replace $c$ with $c'$

| | $c$ | $c'$ |
|---|---|---|
| *Commute* | 20 | **25** |
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| | $b$ | $c'$ | $d$ |
|---|---|---|---|
| *Commute* | 25 | 25 | 25 |
| *Clients* | 80 | 78 | 85 |
| *Services* | $B$ | $C$ | $A$ |
| *Size* | 700 | 500 | 950 |
| *Cost* | 1700 | 1500 | 1900 |

- all alternatives have a common evaluation on *Commute*
- divide and conquer: dropping attributes
    - drop attribute *Commute*

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- check again for dominance
- unfruitful
- assess new tradeoffs
  - neutralize Service using *Cost* as reference

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## Questions

- what maximal increase in monthly cost would you be prepared to pay to go from $C$ to $B$ on service for $c'$?
  - answer: 250 \$
- what minimal decrease in monthly cost would you ask if we go from $A$ to $B$ on service for $d$?
  - answer: 100 \$

| | $b$ | $c'$ | $c''$ | $d$ | $d'$ |
|---|---|---|---|---|---|
| *Clients* | 80 | 78 | 78 | 85 | 85 |
| *Services* | $B$ | $C$ | **B** | $A$ | **B** |
| *Size* | 700 | 500 | 500 | 950 | 950 |
| *Cost* | 1700 | 1500 | 1500 + **250** | 1900 | 1900 − **100** |

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|---|---|---|---|---|---|
| *Clients* | 80 | 78 | 78 | 85 | 85 |
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- replacing $c'$ with $c''$
- replacing $d$ with $d'$
- dropping Service

| | $b$ | $c''$ | $d'$ |
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| *Clients* | 80 | 78 | 85 |
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- checking for dominance: $c''$ is dominated by $b$
- $c''$ can be dropped

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| | $b$ | $d'$ |
|---|---|---|
| *Clients* | 80 | 85 |
| *Size* | 700 | 950 |
| *Cost* | 1700 | 1800 |

- no dominance
- question: starting with $b$ what is the additional cost that you would be prepared to pay to increase size by 250?
  - answer: 250 $

| | $b$ | $b'$ | $d'$ |
|---|---|---|---|
| *Clients* | 80 | 80 | 85 |
| *Size* | 700 | **950** | 950 |
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- dropping $c''$

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- replace $b$ with $b'$
- drop *Size*

| | $b'$ | $d'$ |
|---|---|---|
| *Clients* | 80 | 85 |
| *Size* | 950 | 950 |
| *Cost* | 1950 | 1800 |

| | $b'$ | $d'$ |
|---|---|---|
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- check for dominance
- $d'$ dominates $b'$

## Conclusion

- Recommend $d$ as the final choice

- replace $b$ with $b'$
- drop *Size*

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# Summary

## Remarks

- very simple process
- process entirely governed by $\succ$ and $\sim$
- no question on "intensity of preference"
- notice that importance plays absolutely no rôle
- why be interested in something more complex?

## Problems

- set of alternative is small
  - many questions otherwise
- output is not a preference model
  - if new alternatives appear, the process should be restarted
- what are the underlying hypotheses?

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# Monsieur Jourdain doing conjoint measurement

## Similarity with extensive measurement

- $\succ$: preference, $\sim$: indifference
- we have implicitly supposed that they combine nicely

## Recommendation: $d$

- we should be able to prove that $d \succ a$, $d \succ b$, $d \succ c$ and $d \succ e$
- dominance: $b \succ e$ and $d \succ a$
- tradeoffs + dominance: $b \succ c''$, $c \sim c'$, $c' \sim c$, $d' \sim d$, $b' \sim b$, $d' \succ b'$

$$d \succ a, b \succ e$$

$$c'' \sim c', c' \sim c, b \succ c''$$

$$\Rightarrow b \succ c$$

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## OK... but where are the standard sequences?

- hidden... but really there!
- standard sequence for length: objects that have exactly the same length
- tradeoffs: preference intervals on distinct attributes that have the same length
  - $c \sim c'$
  - $[25, 20]$ on *Commute* has the same length as $[70, 78]$ on *Client*

| | $c$ | $c'$ | $f$ | $f'$ |
|---|---|---|---|---|
| *Commute* | 20 | **25** | *20* | **25** |
| *Clients* | 70 | **78** | *78* | **82** |
| *Services* | *C* | *C* | *C* | *C* |
| *Size* | 500 | 500 | 500 | 500 |
| *Cost* | 1500 | 1500 | 1500 | 1500 |

$[70, 78]$ has the same length $[78, 82]$ on *Client*

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| *Services* | *C* | *C* | *C* | *C* |
| *Size* | 500 | 500 | 500 | 500 |
| *Cost* | 1500 | 1500 | 1500 | 1500 |

$[70, 78]$ has the same length $[78, 82]$ on *Client*

# Monsieur Jourdain doing conjoint measurement

## OK... but where are the standard sequences?

- hidden... but really there!
- standard sequence for length: objects that have exactly the same length
- tradeoffs: preference intervals on distinct attributes that have the same length
  - $c \sim c'$
  - $[25, 20]$ on *Commute* has the same length as $[70, 78]$ on *Client*

| | $c$ | $c'$ | $f$ | $f'$ |
|---|---|---|---|---|
| *Commute* | 20 | **25** | *20* | **25** |
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# Outline

# Setting

- $N = \{1, 2, \dots, n\}$ set of attributes
- $X_i$: set of possible levels on the $i$th attribute
- $X = \prod_{i=1}^{n} X_i$: set of all conceivable alternatives
  - $X$ include the alternatives under study... and many others

- $J \subseteq N$: subset of attributes
- $X_J = \prod_{j \in J} X_j$, $X_{-J} = \prod_{j \notin J} X_j$
- $(x_J, y_{-J}) \in X$
- $(x_i, y_{-i}) \in X$

- $\succsim$: binary relation on $X$: "at least as good as"
- $x \succ y \Leftrightarrow x \succsim y$ and $Not[y \succsim x]$
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# Preference relations on Cartesian products

## Applications

- Economics: consumers comparing bundles of goods
- Decision under uncertainty: consequences in several states
- Inter-temporal decision making: consequences at several moments in time
- Inequality measurement: distribution of wealth across individuals

- Decision making with multiple attributes
  - in all other cases, the Cartesian product is homogeneous

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# What will be ignored today

**Ignored**

- structuring of objectives
- from objectives to attributes
- adequate family of attributes
- risk, uncertainty, imprecision

**Keeney's view: Value-focused thinking**

- fundamental objectives: why?
- means objectives: how?

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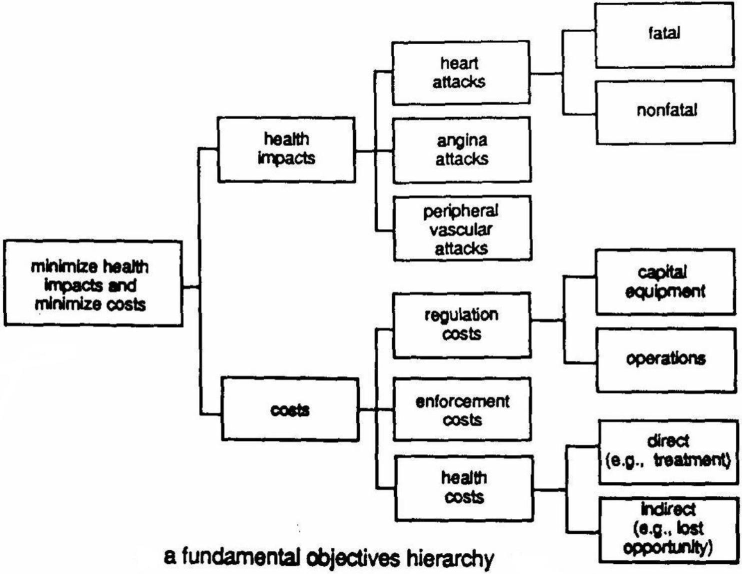
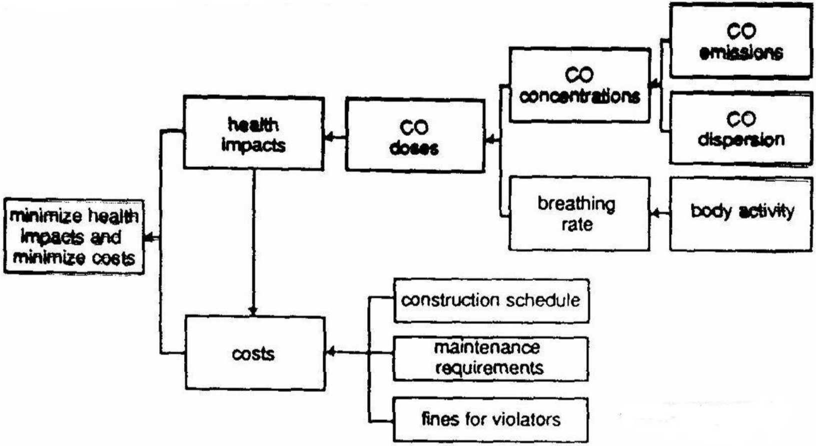


a fundamental objectives hierarchy




a means-ends objectives network

Table I. Preclosure Objectives and Performance Measures

| Objective | Performance measure |
|---|---|
| **Health-and-safety impacts** | |
| 1. Minimize worker health effects from radiation exposure at the repository | $X_1$: repository-worker radiological fatalities |
| 2. Minimize public health effects from radiation exposure at the repository | $X_2$: public radiological fatalities from repository |
| 3. Minimize worker fatalities from nonradiological causes at the repository | $X_3$: repository-worker nonradiological fatalities |
| 4. Minimize public fatalities from nonradiological causes at the repository | $X_4$: public nonradiological fatalities from repository |
| 5. Minimize worker health effects from radiation exposure in waste transportation | $X_5$: transportation-worker radiological fatalities |
| 6. Minimize public health effects from radiation exposure in waste transportation | $X_6$: public radiological fatalities from transportation |
| 7. Minimize worker fatalities from nonradiological causes in waste transportation | $X_7$: transportation-worker nonradiological fatalities |
| 8. Minimize public fatalities from nonradiological causes in waste transportation | $X_8$: public nonradiological fatalities from transportation |
| **Environmental impacts** | |
| 9. Minimize aesthetic degradation | $X_9$: constructed scale[a] |
| 10. Minimize the degradation of archaeological, historical, and cultural properties | $X_{10}$: constructed scale[a] |
| 11. Minimize biological degradation | $X_{11}$ constructed scale[a] |
| **Socioeconomic impacts** | |
| 12. Minimize adverse socioeconomic impacts | $X_{12}$: constructed scale[a] |
| **Economic impacts** | |
| 13. Minimize repository costs | $X_{13}$: millions of dollars |
| 14. Minimize waste-transportation costs | $X_{14}$: millions of dollars |

*Table 4.1.* A constructed attribute for public attitudes

| Attribute level | Description of attribute level |
|---|---|
| 1 | *Support:* No groups are opposed to the facility and at least one group has organized support for the facility. |
| 0 | *Neutrality:* All groups are indifferent or uninterested. |
| −1 | *Controversy:* One or more groups have organized opposition, although no groups have action-oriented opposition. Other groups may either be neutral or support the facility. |
| −2 | *Action-oriented opposition:* Exactly one group has action-oriented opposition. The other groups have organized support, indifference or organized opposition. |
| −3 | *Strong action-oriented opposition:* Two or more groups have action-oriented opposition. |

# Marginal preference and independence

## Marginal preferences

- $J \subseteq N$: subset of attributes
- $\succsim_J$ marginal preference relation induced by $\succsim$ on $X_J$

$$x_J \succsim_J y_J \Leftrightarrow (x_J, z_{-J}) \succsim (y_J, z_{-J}), \text{ for all } z_{-J} \in X_{-J}$$

## Independence

- $J$ is independent for $\succsim$ if

$$[(x_J, z_{-J}) \succsim (y_J, z_{-J}), \text{ for some } z_{-J} \in X_{-J}] \Rightarrow x_J \succsim_J y_J$$

- common levels on attributes other than $J$ do not affect preference

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# Independence

**Definition**

- for all $i \in N$, $\{i\}$ is independent, $\succsim$ is weakly independent
- for all $J \subseteq N$, $J$ is independent, $\succsim$ is independent

**Proposition**

Let $\succsim$ be a weakly independent weak order on $X = \prod_{i=1}^{n} X_i$. Then:

- $\succsim_i$ is a weak order on $X_i$
- $[x_i \succsim_i y_i$, for all $i \in N] \Rightarrow x \succsim y$
- $[x_i \succsim_i y_i$, for all $i \in N$ and $x_j \succ_j y_j$ for some $j \in N] \Rightarrow x \succ y$

for all $x, y \in X$

**Dominance**

- as soon as I have a weakly independent weak order
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# Independence in practice

## Independence

- it is easy to imagine examples in which independence is violated
  - Main course and Wine example
- it is nearly hopeless to try to work if weak independence (at least weak separability) is not satisfied
- some (e.g., R. L. Keeney) think that the same is true for independence
- in all cases if independence is violated, things get complicated
  - decision aiding vs AI

## May be excessive

- more on independence in part II

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# Outline

# Outline of theory: 2 attributes

## Question

- suppose I can "observe" $\succsim$ on $X = X_1 \times X_2$
- what must be supposed to guarantee that I can represent $\succsim$ in the additive value function model

$$v_1 : X_1 \rightarrow \mathbb{R}$$
$$v_2 : X_2 \rightarrow \mathbb{R}$$
$$(x_1, x_2) \succsim (y_1, y_2) \Leftrightarrow v_1(x_1) + v_2(x_2) \geq v_1(y_1) + v_2(y_2)$$

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# Uniqueness

**Important observation**

Suppose that there are $v_1$ and $v_2$ such that

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If $\alpha > 0$

$$w_1 = \alpha v_1 + \beta_1 \quad w_2 = \alpha v_2 + \beta_2$$

is also a valid representation

**Consequences**

- fixing $v_1(x_1) = v_2(x_2) = 0$ is harmless
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# Standard sequences

## Preliminaries

- choose arbitrarily two levels $x_1^0, x_1^1 \in X_1$
- make sure that $x_1^1 \succ_1 x_1^0$
- choose arbitrarily one level $x_2^0 \in X_2$
- $(x_1^0, x_2^0) \in X$ is the reference point (origin)
- the preference interval $[x_1^0, x_1^1]$ is the unit

## Building a standard sequence on $X_2$

- find a "preference interval" on $X_2$ that has the same "length" as the reference interval $[x_1^0, x_1^1]$
- find $x_2^1$ such that

$$(x_1^0, x_2^1) \sim (x_1^1, x_2^0)$$

$$v_1(x_1^0) + v_2(x_2^1) = v_1(x_1^1) + v_2(x_2^0) \text{ so that}$$
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# Going on

$$(x_1^0, x_2^1) \sim (x_1^1, x_2^0)$$
$$(x_1^0, x_2^2) \sim (x_1^1, x_2^1)$$
$$(x_1^0, x_2^3) \sim (x_1^1, x_2^2)$$
$$\dots$$
$$(x_1^0, x_2^k) \sim (x_1^1, x_2^{k-1})$$

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$$\Rightarrow v_2(x_2^2) = 2, v_2(x_2^3) = 3, \dots, v_2(x_2^k) = k$$

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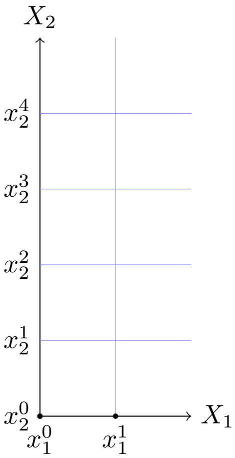

$X_2$
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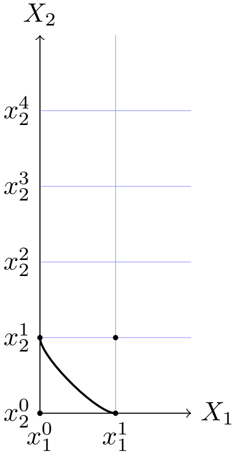
$x_1^0$
$x_1^1$



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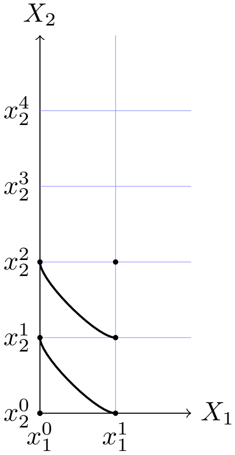

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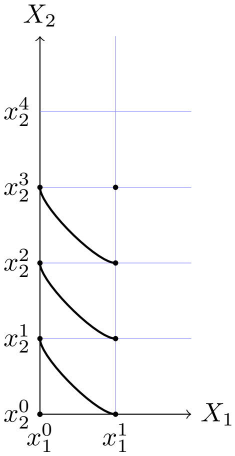

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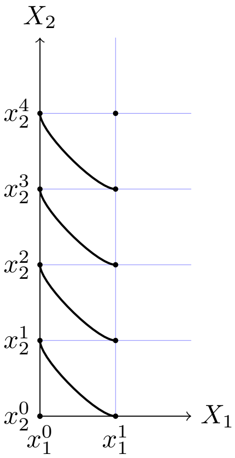

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$x_2^0$
$x_1^0$
$x_1^1$
$X_1$

# Standard sequence

## Archimedean

- implicit hypothesis for length
    - the standard sequence can reach the length of any object

$$\forall x, y \in \mathbb{R}, \exists n \in \mathbb{N} : ny > x$$

- a similar hypothesis has to hold here
- rough interpretation
    - there are not “infinitely” liked or disliked consequences

# Building a standard sequence on $X_1$

$$(x_1^2, x_2^0) \sim (x_1^1, x_2^1)$$
$$(x_1^3, x_2^0) \sim (x_1^2, x_2^1)$$
$$\dots$$
$$(x_1^k, x_2^0) \sim (x_1^{k-1}, x_2^1)$$

$$v_1(x_1^2) - v_1(x_1^1) = v_2(x_2^1) - v_2(x_2^0) = 1$$
$$v_1(x_1^3) - v_1(x_1^2) = v_2(x_2^1) - v_2(x_2^0) = 1$$
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$$v_1(x_1^2) = 2, v_1(x_1^3) = 3, \dots, v_1(x_1^k) = k$$

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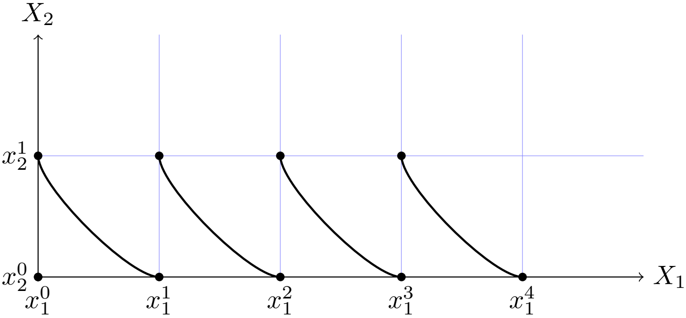
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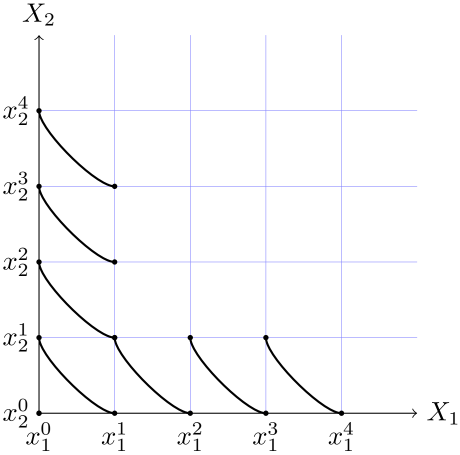

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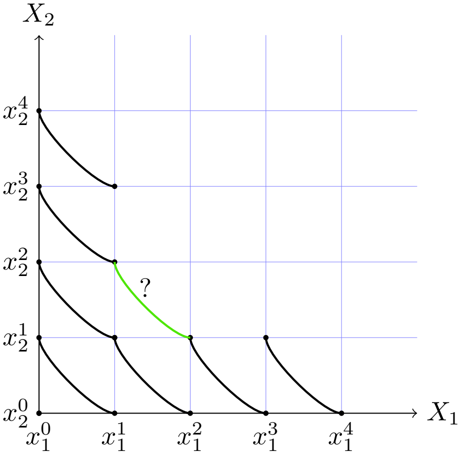
$X_2$
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$x_1^3$
$x_1^4$
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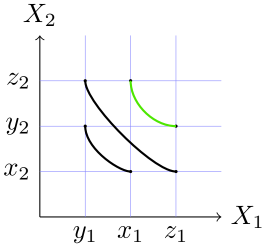
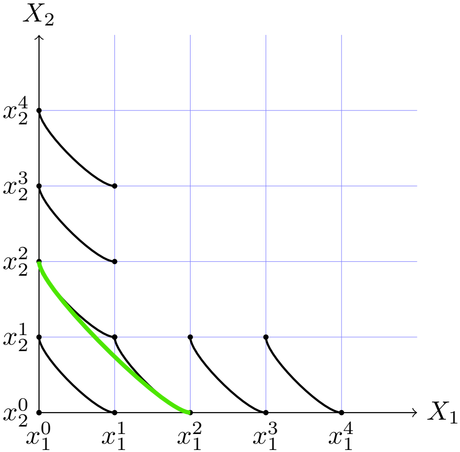
# Thomsen condition

$$\left.\begin{array}{c}(x_1, x_2) \sim (y_1, y_2)\\ \text{and}\\ (y_1, z_2) \sim (z_1, x_2)\end{array}\right. \Rightarrow (x_1, z_2) \sim (z_1, y_2)$$




**Consequence**

- there is an additive value function on the grid



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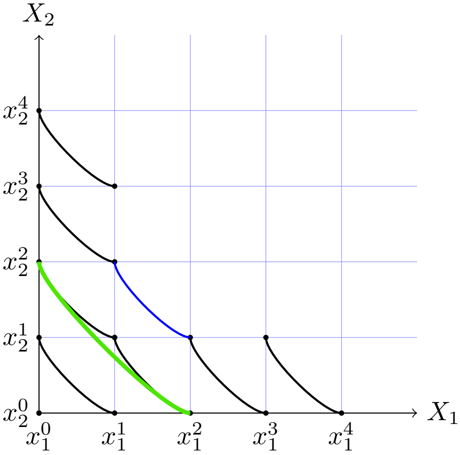

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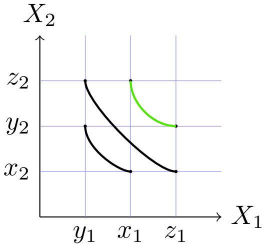


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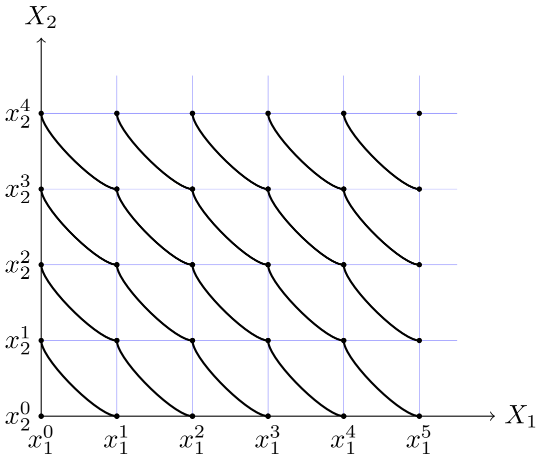
$X_2$
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# Summary

- we have defined a “grid”
- there is an additive value function on the grid

- iterate the whole process with a “denser grid”

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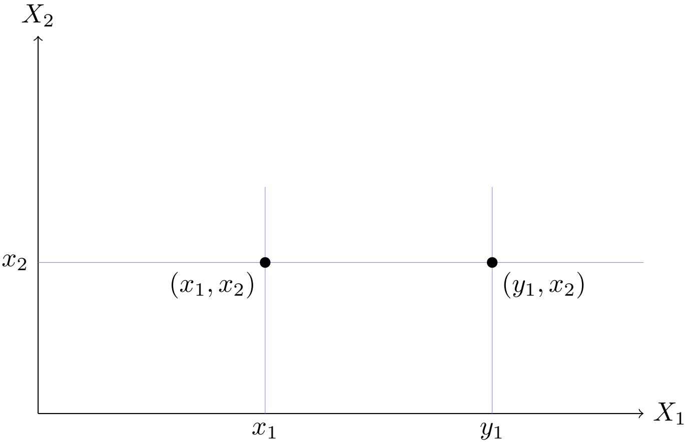
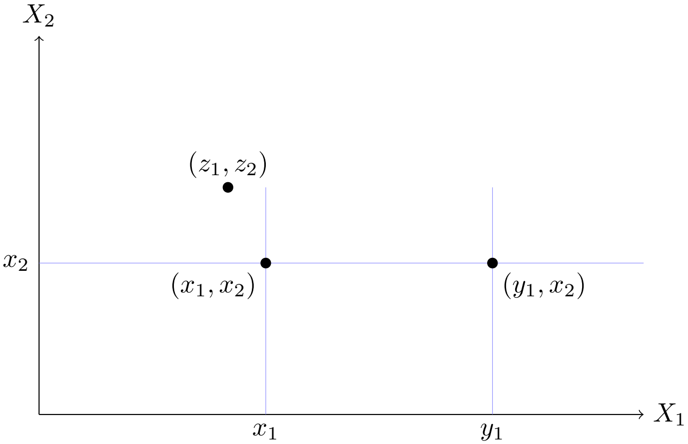
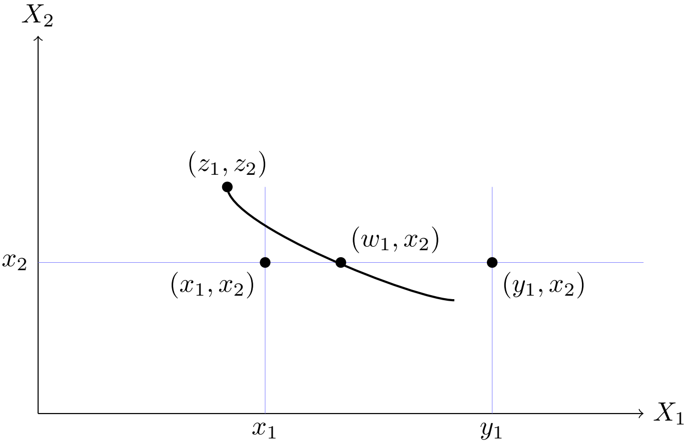
# Hypotheses

- Archimedean: every strictly bounded standard sequence is finite
- essentiality: both $\succ_1$ and $\succ_2$ are nontrivial
- restricted solvability


$X_2$
$x_2$
$(x_1, x_2)$
$(y_1, x_2)$
$X_1$
$x_1$
$y_1$




$$\left.\begin{array}{l}(y_1, x_2) \succ (z_1, z_2) \\ (z_1, z_2) \succ (x_1, x_2)\end{array}\right\}$$




$$\left.\begin{array}{c}(y_1, x_2) \succ (z_1, z_2) \\ (z_1, z_2) \succ (x_1, x_2)\end{array}\right\} \Rightarrow \exists w_1 \text{ such that } (z_1, z_2) \sim (w_1, x_2)$$

# Basic result

**Theorem (2 attributes)**

If

- restricted solvability holds
- each attribute is essential

then
the additive value function model holds
if and only if
$\succsim$ is an independent weak order satisfying the Thomsen and the Archimedean conditions

The representation is unique up to scale and location

# General case

## Good news

- entirely similar. . .
- with a very nice surprise: Thomsen can be forgotten
  - if $n = 2$, independence is identical with weak independence
  - if $n > 3$, independence is much stronger than weak independence

| | $X_1$ | $X_2$ | $X_3$ |
|---|---|---|---|
| $a$ | 75 | 10 | 0 |
| $b$ | 100 | 2 | 0 |
| $c$ | 75 | 10 | 40 |
| $d$ | 100 | 2 | 40 |

$X_1$: % of nights at home
$X_2$: attractiveness of city
$X_3$: salary increase
weak independence holds
$a \succ b$ and $d \succ c$ is reasonable

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# Outline

# Assessing value functions

## Standard technique

- check independence
- build standard sequences
  - importance has no rôle
  - do not even pronounce the word!!

## Problems

- many questions
- questions on fictitious alternatives
- rests on indifference judgments
- discrete attributes
- propagation of "errors"

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## Principle

- select a number of reference alternatives that the DM knows well
- rank order these alternatives
- test, using LP, if this information is compatible with an additive value function
    - if yes, present a central one
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- builds marginal value functions based on the comparison of differences of preference, considering each attribute at a time

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# Summary

## Conjoint measurement

- highly consistent theory
- together with practical assessment techniques

## Why consider extensions?

- hypotheses may be violated
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# Part II

## A glimpse at possible extensions

# Summary

## Additive value function model

- requires independence
- requires a finely grained analysis of preferences

## Two main types of extensions

1. models with interactions
2. more ordinal models

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# Outline

# Interactions

**Two extreme models**

- additive value function model
  - independence
- decomposable model
  - only weak independence

$$x \succsim y \Leftrightarrow \sum_{i=1}^{n} v_i(x_i) \geq \sum_{i=1}^{n} v_i(y_i)$$

$$x \succsim y \Leftrightarrow F[v_1(x_1), \ldots v_n(x_n)] \geq F[v_1(y_1), \ldots v_n(y_n)]$$

# Decomposable models

$$x \succsim y \Leftrightarrow F[v_1(x_1), \ldots v_n(x_n)] \geq F[v_1(y_1), \ldots v_n(y_n)]$$

$F$ increasing in all arguments

**Result**

Under mild conditions, any weakly independent weak order may be represented in the decomposable model

**Problem**

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# Two main directions

**Extensions**

1. work with the decomposable model
   - rough sets
2. find models “in between” additive and decomposable
   - CP-nets, GAI
   - fuzzy integrals

# Rough sets

## Basic ideas

- work within the general decomposable model
- use the same principle as in UTA
- replacing the numerical model by a symbolic one
- infer decision rules

IF

$x_1 \geq a_1, \ldots, x_i \geq a_i, \ldots, x_n \geq a_n$ and

$y_1 \leq b_1, \ldots, y_i \leq b_i, \ldots, y_n \leq b_n$

THEN

$x \succsim y$

- many possible variants
- Greco, Matarazzo, Słowiński

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# GAI: Example

## Choice of a meal: 3 attributes

$X_1 = \{\text{Steak}, \text{Fish}\}$
$X_2 = \{\text{Red}, \text{White}\}$
$X_3 = \{\text{Cake}, \text{sherBet}\}$

## Preferences

$$x^1 = (S, R, C) \quad x^2 = (S, R, B) \quad x^3 = (S, W, C) \quad x^4 = (S, W, B)$$
$$x^5 = (F, R, C) \quad x^6 = (F, R, B) \quad x^7 = (F, W, C) \quad x^8 = (F, W, B)$$

$$x^2 \succ x^1 \succ x^7 \succ x^8 \succ x^4 \succ x^3 \succ x^5 \succ x^6$$

- the important is to match main course and wine
- I prefer Steak to Fish
- I prefer Cake to sherBet if Fish
- I prefer sherBet to Cake if Steak

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$$x^2 \succ x^1 \succ x^7 \succ x^8 \succ x^4 \succ x^3 \succ x^5 \succ x^6$$

**Independence**

$$x^1 \succ x^5 \Rightarrow v_1(S) > v_1(F)$$
$$x^7 \succ x^3 \Rightarrow v_1(F) > v_1(S)$$

**Grouping main course and wine?**

$$x^7 \succ x^8 \Rightarrow v_3(C) > v_3(B)$$
$$x^2 \succ x^1 \Rightarrow v_3(B) > v_3(C)$$

# Example

$$x^1 = (S, R, C) \quad x^2 = (S, R, B) \quad x^3 = (S, W, C) \quad x^4 = (S, W, B)$$
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$$x^2 \succ x^1 \succ x^7 \succ x^8 \succ x^4 \succ x^3 \succ x^5 \succ x^6$$

**Independence**

$$x^1 \succ x^5 \Rightarrow v_1(S) > v_1(F)$$
$$x^7 \succ x^3 \Rightarrow v_1(F) > v_1(S)$$

**Grouping main course and wine?**

$$x^7 \succ x^8 \Rightarrow v_3(C) > v_3(B)$$
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$$x \succsim y \Leftrightarrow u_{12}(x_1, x_2) + u_{13}(x_1, x_3) \geq u_{12}(y_1, y_2) + u_{13}(y_1, y_3)$$

$$u_{12}(S, R) = 6 \quad u_{12}(F, W) = 4 \quad u_{12}(S, W) = 2 \quad u_{12}(F, R) = 0$$
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# Generalized Additive Independence

## GAI (Gonzales & Perny)

- axiomatic analysis
- if interdependences are known
  - assessment techniques
  - efficient algorithms (compactness of representation)

## What R. L. Keeney would probably say

- the attribute "richness" of meal is missing

## GAI

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# Fuzzy integrals

## Choquet and Sugeno

- Other types of interactions can be modelled by fuzzy integrals
  - Choquet integral
  - Sugeno integral
- Encompassed in the framework of the decomposable model
- Difficult to analyze due to the commensurability hypothesis

# Outline

# Observations

## Classical model

- deep analysis of preference that may not be possible
  - preference are not well structured
  - several or no DM
  - prudence

## Idea

- it is not very restrictive to suppose that levels on each $X_i$ can be ordered
- aggregate these orders
- possibly taking importance into account

## Social choice

- aggregate the preference orders of the voters to build a collective preference

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# Outranking methods

**ELECTRE**

$x \succsim y$ if

Concordance: a "majority" of attributes support the assertion

Discordance: the opposition of the minority is not "too strong"

$$x \succsim y \Leftrightarrow \begin{cases} \sum_{i : x_i \succsim_i y_i} w_i \geq s \\ Not[y_i \; V_i \; x_i], \forall i \in N \end{cases}$$

**Model**

Such relations can be analyzed within a conjoint measurement model

**Drawbacks**

- Condorcet paradox: $\succsim$ may have cycles
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# Ways out

## Accepting intransitivity

- find way to extract information in spite of intransitivity
  - ELECTRE I, II, III, IS
  - PROMETHEE I, II

## Do not use paired comparisons

- only compare $x$ with carefully selected alternatives
  - ELECTRE TRI
  - methods using reference points

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# Conclusion

## Explicit models

Advantage of considering explicit models:

- analysis of their expressivity (through axioms)
- their analysis may provide hints for elicitation

## General frameworks

- allows to compare models within the framework (common axioms)
- better understanding

## Drawback

Studying models takes time and energy

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