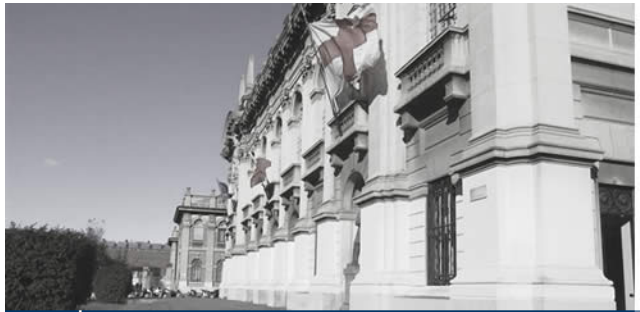

POLITECNICO DI MILANO

1°

Methods and Models for Decision Making

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Aims:

- introduction to the basics of decision theory
- discussion about decision making in design (and in other fields)
- presentation of risk analysis, multicriteria, group decision, …
- definition of possible research topics (in design area)

Outline:

- **(1) Introduction**
- **(2) Tools & frame**
- (3) Mental models
- (4) Design & decision
- (5) Classification
- (6) Ranking-1, risk analysis
- (7) Ranking-2, multicriteria
- (8) A tentative case (discuss.)
- (9) Seminar
- (10) Rating problems
- (11) Group decision
- (12) Genetic alg. + …
- (13) Research topics
- (14) Case results (if any …)
- (15) Conclusions

DM: an introduction

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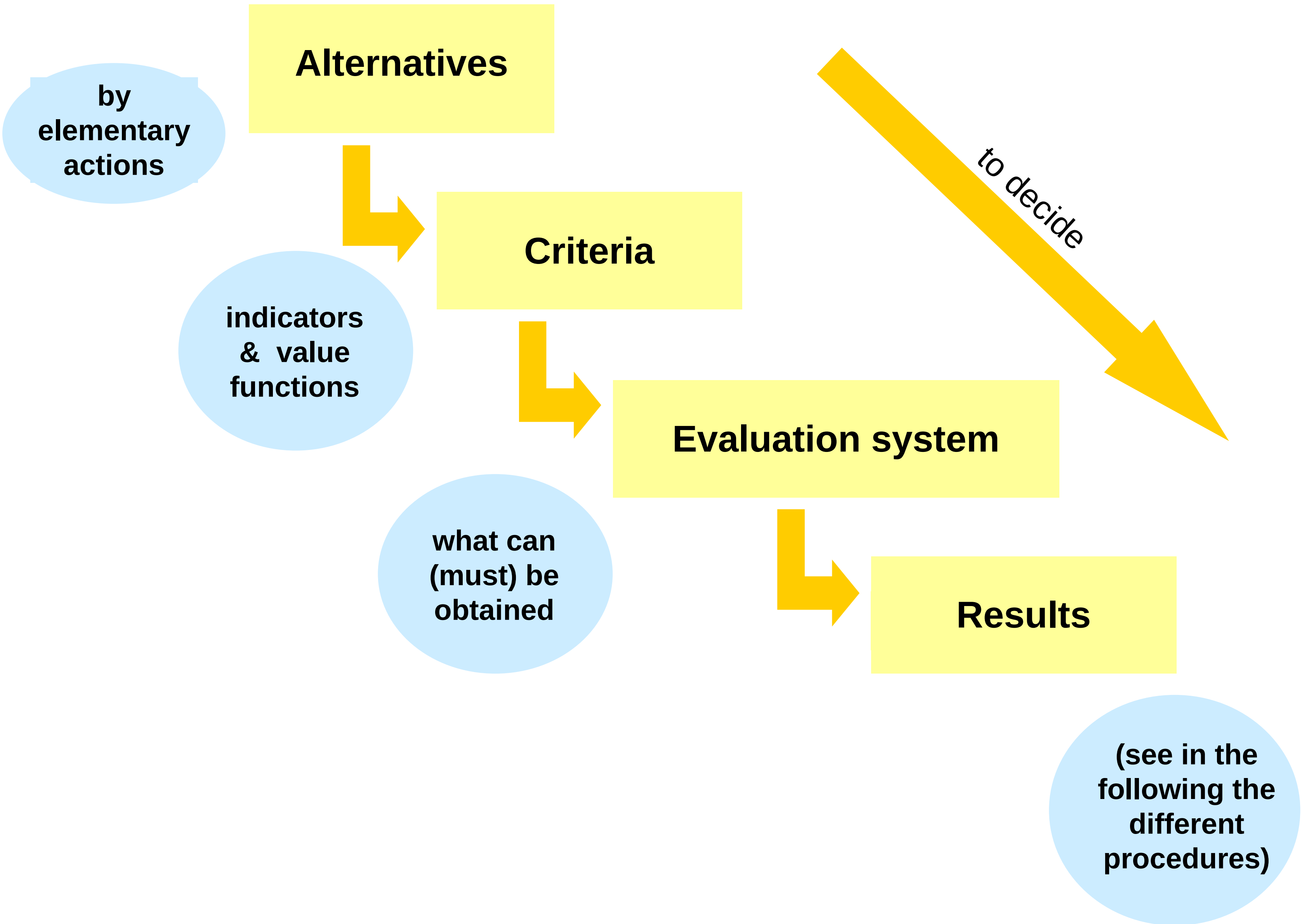
The steps of a decision



The different (4) levels of a decision process

You want to go outside to dinner with your wife, so …

i. **Information →** Let's go out for dinner.

ii. **Feedback →** Let's go out for dinner, do you agree ?

iii. **Discussion →** Let's go out for dinner, where can we go ?

iv. **Involvment →** Would you like to go out ? to do what ?

↳ different actors (Decision Makers, DM's)

↳ a (possibly pre-defined) procedure

Short history:

- 40's → Genesis (during the 2° war)
- 50-60's → ***Development [*]*** *(LP probl. & Combinatorics)*
- 60-70's → Specialization (non linear, integer, B&B, …)
- 70-80's → Multicriteria (the importance of trade-off)
- 50-90's → Multiple DM (the different points of view)
- 80-00's → Decision Aiding (sw supporting the process)

[*] ***max f(x), s.t. x Є X*** *(with X finite or infinite set)*

Links & references:

- http://www.informs.org (the INFORMS site)
- http://www.euro-online.org (the EURO site)
- http://www.airo2.org (the AIRO, Italian site)
- http://corsi.metid.polimi.it (the site of Center METID)
- A. Tsoukias, *From decision theory to decis. aiding method.*, EJOR, 2007

An “ideal” decision problem

- **Someone who decides**

  with respect to one clear **objective**

  with a set of well defined **constraints**

  with all the suitable **information**

  in presence of a finite / infinite **set of alternatives**

- **Two (ideal) examples**

Ideal example 1

Combinatorial optimization

Your chorus is defining the storyboard of a concert and you must choose between a set of mottetti (a "mottetto" is a choral musical composition).

Each mottetto (m_1, m_2, …, m_n) has a time of execution t_j and a level of success s_j ($j = 1,…,n$).

The total time of the exhibition is T min.

What can you do ?

If you want, consider this specific instance:

$n = 4$; $t = (10, 22, 37, 9)$; $s = (60, 55, 100, 15)$; $T = 45$

(i) What are the variables ?

(ii) How many solutions ?

(iii) What is the optimal choice ?

Ideal example 2

Linear programming

You must define the week production of a (small) firm that has only 2 products, PA and PB.

One item of PA needs 2 units of the resource R1 and 1 unit of the resource R2.

One item of PB needs 1 unit of the resource R1 and 3 units of the resource R2.

The net revenue for each item (PA or PB) is 500 €.

You have (weekly) 400 units of R1 and 900 units of R2.

You know that the maximum possible sale for PB is 250 items.

What can you do ?

(i) What are the variables ?

(ii) How many solutions ?

(iii) What is the optimal choice ? (you can solve with Excel …)

A <u>real</u> decision problem

- **Uncertainties** (non-deterministic context, data mining)
- **Complexity** (problem dimension, non linearity, …)
- **Several stakeholders** (distributed decision power)
- **Different rationalities** (criteria and preferences)
- **Various time horizons** (often)
- **Use of simulation models**

  ↳ what … if …

Tools

A formal decision process needs instruments for:

i. **abstraction**

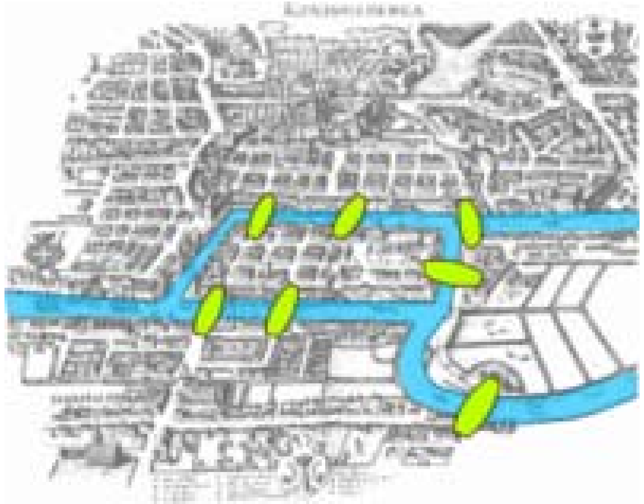
ii. **analysis**

iii. **synthesis**

(and more …)

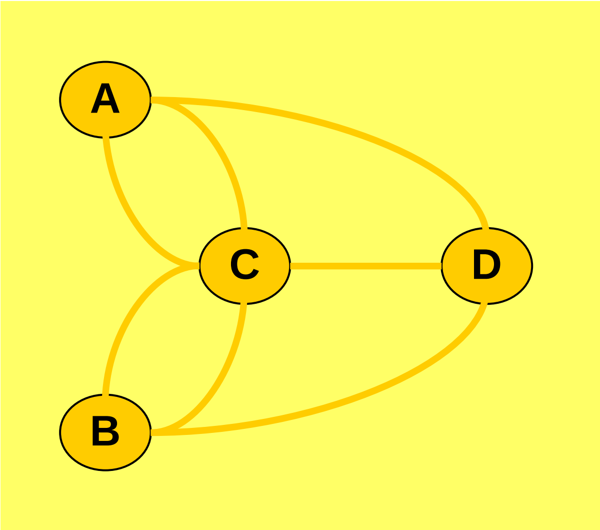
Tools for abstraction / 1

- 1736
- Konigsberg



- The 7 bridges
- A riddle

- Euler
- Graph theory



- The Euler model
- The answer (similar to …)

Tools for abstraction / 2

Sherlock Holmes & the death of count Kinskij

- The count drunk poisoned water (from one of his 7 lovers)
- All 7 lovers were in the castle the day of his death
- The murderer should have come to the castle twice (one for exploring, the other for killing), while the others only one.
- Statements of the 7 women:

Alice saw	B C E F
Barbara saw	A C D E G
Clara saw	A B D
Diana saw	B C E
Elena saw	A B D G
Francesca saw	A G
Gloria saw	B E F

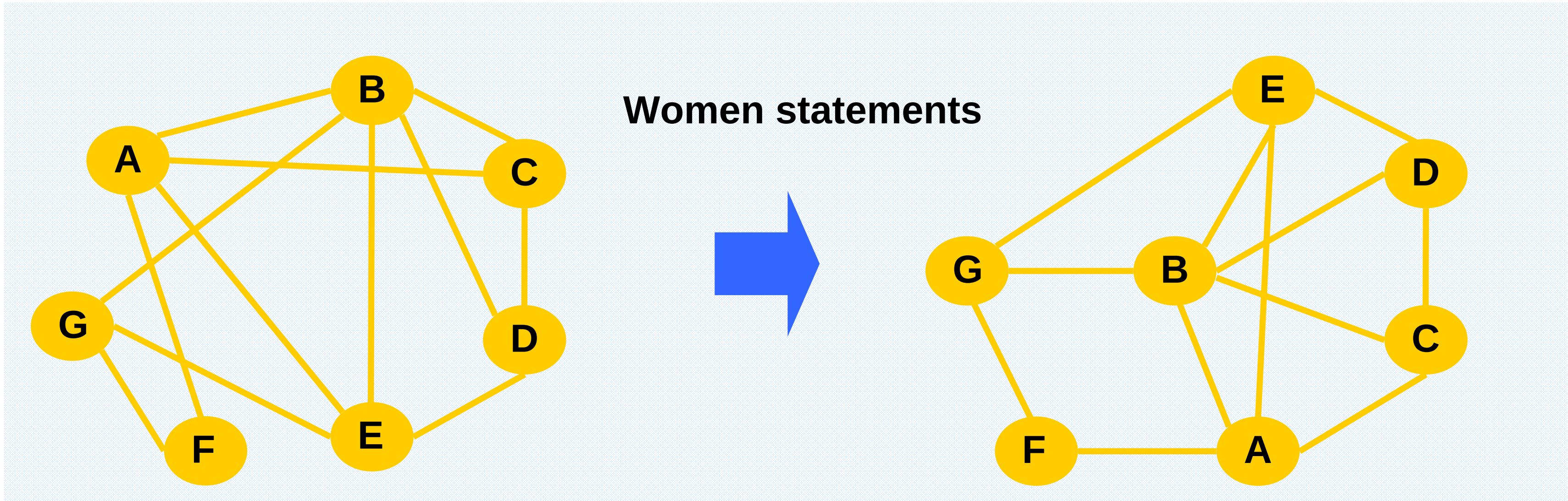
Elementary,

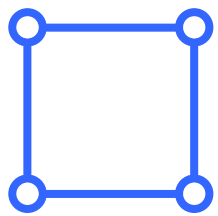
my dear Watson !

(said Sherlock H.)

The solution

S. H. & the death of count Kinskij





Impossible !

A E D C
A E G F
A B G F

$\nexists$ (so **A** lies)

- **General reports**
  - http://teoriadeigrafi.altervista.org/teoria_dei_grafi.pdf (a tutorial)
  - http://en.wikipedia.org/wiki/Graph_theory
  - http://en.wikipedia.org/wiki/Route_inspection_problem

- **Applications**
  - http://bla...
  - http://bla...
  - http://www.ratp.info/orienter/cv/cv_en/carteparis.php (the Paris metro)

search …

- **A famous problem – TSP**
  - http://www-e.uni-magdeburg.de/mertens/TSP/index.html
  - http://www.tsp.gatech.edu/index.html
  - http://www.densis.fee.unicamp.br/~moscato/TSPBIB_home.html

Tools for analysis / 1

- Sudoku (Corriere della Sera, 3 Sept. 2006)

		4				9		
	1	6	2		4	3	8	
	8						5	
4			6		2			1
3			9		8			4
	3						6	
	6	7	3		5	1	4	
		2				8		

- **Branching rules → a tree**
- **A lot of (small) subproblems**

Tools for analysis / …

Step 2

		4				9		
	1	6	2		4	3	8	
	8					4	5	
4			6		2			1
3			9		8			4
	3						6	
	6	7	3		5	1	4	
	4	2				8		

Step 4

		4				9	1	
	1	6	2		4	3	8	
	8	3				4	5	
4			6		2			1
3			9		8			4
	3						6	
	6	7	3		5	1	4	
	4	2				8		

Step 6

		4				9	1	
	1	6	2		4	3	8	7
	8	3				4	5	
4			6		2			1
3			9		8			4
	3						6	
	6	7	3		5	1	4	X
	4	2				8		

What number in position X ? *2 or 9*

branch (a) → X = 2

but if X = 2, there is no place for a 2 in the right-high block;
so X = 2 → NO

branch (b) → X = 9

in this case …

Tools for analysis / …

Step 8

		4				9	1	
	1	6	2		4	3	8	7
	8	3				4	5	
4			6		2			1
3			9		8			4
	3						6	
8	6	7	3	2	5	1	4	9
	4	2				8		

Step 9

		4				9	1	
	1	6	2	Y	4	3	8	7
	8	3				4	5	
4			6		2			1
3			9		8			4
	3						6	
8	6	7	3	2	5	1	4	9
	4	2				8		

What in the position Y ? *5 or 9*

branch (b1) → Y = 5

in this case …

Open situations (to be explored) are (b1) with Y = 5, and (b2) with Y = 9

Tools for analysis / …

Step 13 (of b1)

		4				9	1	
9	1	6	2	5	4	3	8	7
	8	3				4	5	
4			6		2			1
			5					
3			9		8			4
	3	9				2	6	
8	6	7	3	2	5	1	4	9
	4	2				8		

Step 26 (of b1)

	5	4	8			9	1	
9	1	6	2	5	4	3	8	7
	8	3	7			4	5	
4			6		2			1
6			5					
3			9		8			4
1	3	9	4	8	7	2	6	5
8	6	7	3	2	5	1	4	9
5	4	2	1			8	7	3

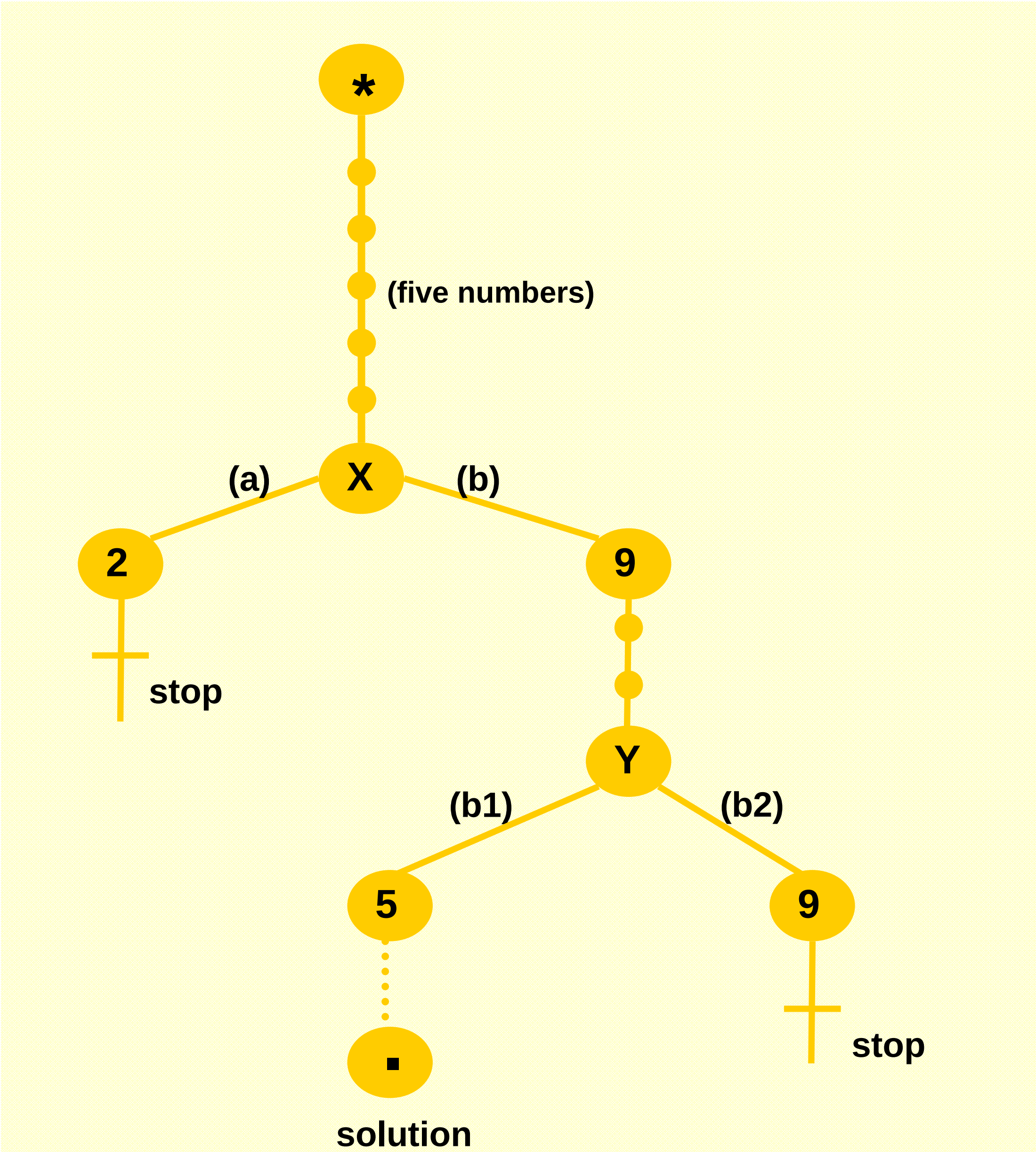
Step 53 (of b1)

7	5	4	8	3	6	9	1	2
9	1	6	2	5	4	3	8	7
2	8	3	7	9	1	4	5	6
4	9	8	6	7	2	5	3	1
6	2	1	5	4	3	7	9	8
3	7	5	9	1	8	6	2	4
1	3	9	4	8	7	2	6	5
8	6	7	3	2	5	1	4	9
5	4	2	1	6	9	8	7	3

Stop !
(the solution is unique)

so branch (b2) +

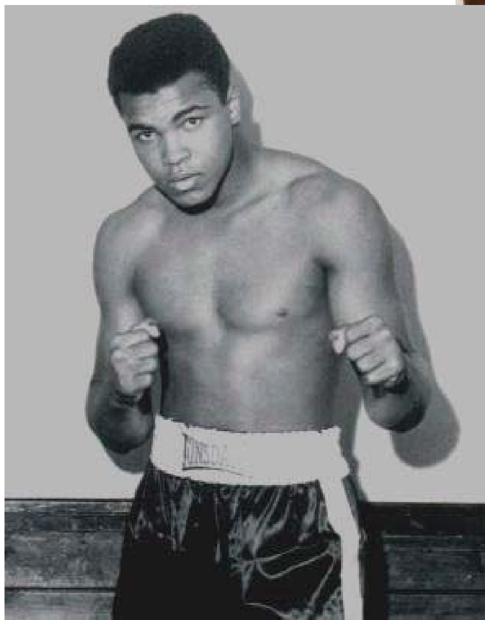
The solution (visualization)

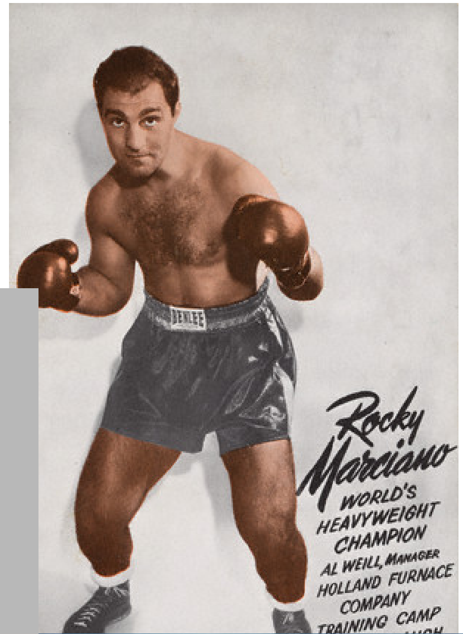


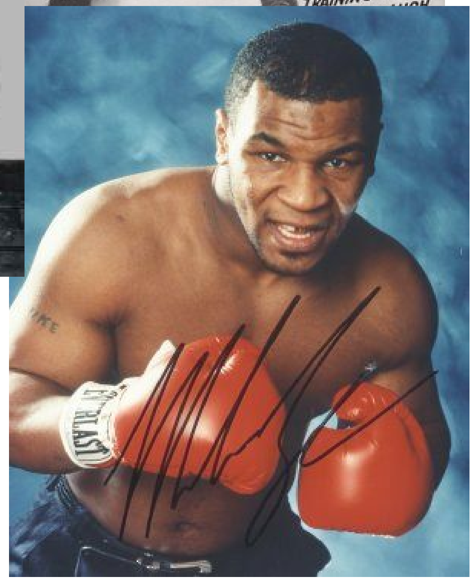
- Branching rules
- A lot of (easier) subproblems
- Stopping rules

Tools for synthesis

Who is the all time world's best boxeur ?







Indicators:

- strength
- speed
- n. of victories
- years of premiership
- …

We need a common framework to compare the alternatives !

Tools & frame

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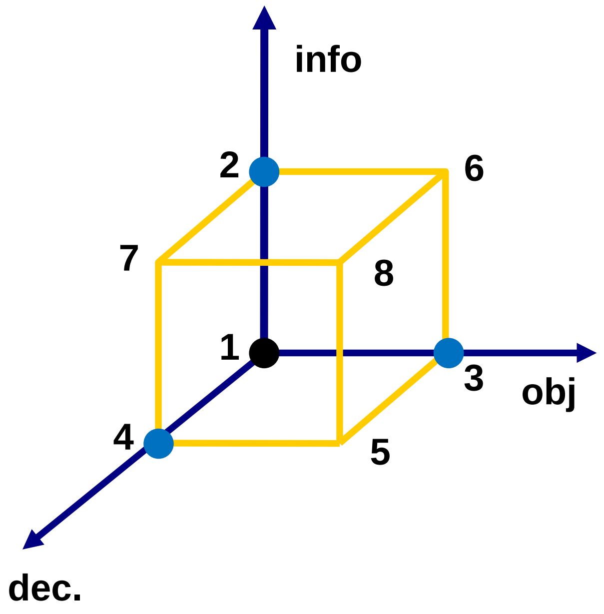
Decision processes: a frame



Information
- complete
- partial → state identific. & risk an.

Objectives
- one
- more → trade-off

Dec. makers
- one
- more → conflicts

1. Math. programming
2. **Risk analysis**
3. **Multiple criteria**
4. **Social choice**

5, 6, 7, 8 → Game theory, …

A real decision process

- **<u>Uncertainties</u>** (non deterministic context, …)
- Complexity (problem dimension, non linearity, …)
- Several stakeholders (distributed decision power)
- Different rationalities (criteria and preferences)
- Different time horizons (often)
- Use of simulation models

  what … if …

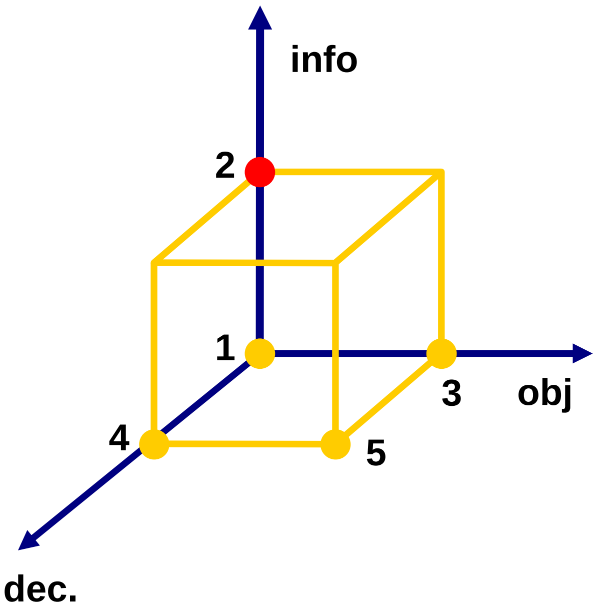
- The perception of the problem:

  differences between

  normative approach

  cognitive approach

Decision processes in a non-deterministic context



1. Math. programming
2. Risk analysis
3. Multi-objective (criteria)
4. Social choice

5, 6, 7, 8 → ….

Information < complete / partial **[*]**

Objectives < one / more

Dec. makers < one / more

[*] → non-deterministic context

perception & mental models

Two (opposite) theories

(a) Normative theory → what the DM **should do**
(prescriptive)

(b) Cognitive theory → what the DM **really does** → experimental tests
(descriptive)

When they are the same ? → if the (*single*) DM has all the information (***in a deterministic way***) and has clearly in mind *the (single)* criterion of evaluation

↓

optimization

$\boxed{\text{N-1°}}$ Principle of INVARIANCE

Equivalent (from the logical point of view) versions of the same problem **must** produce the same choice

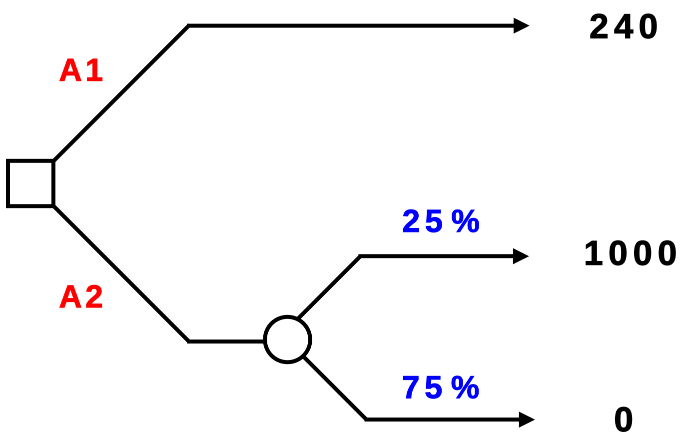
Examples

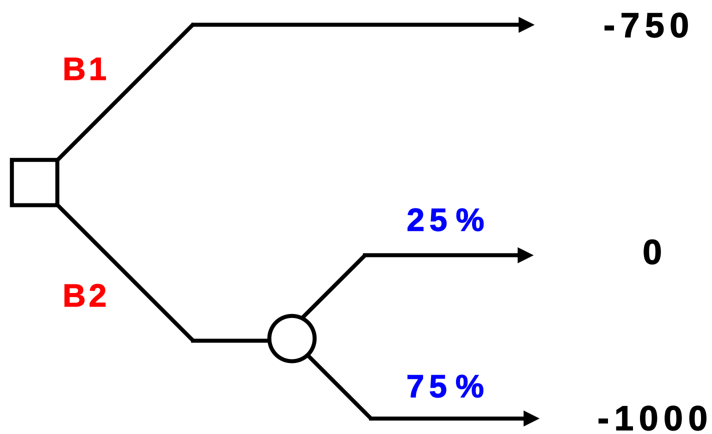
- Change names or positions for the options
- Change measure units
- Add a constant value for all the results

Counterexamples

- Lotteries (A, B, C)
- Ellsberg paradox (1961)

Lotteries (case A and case B)





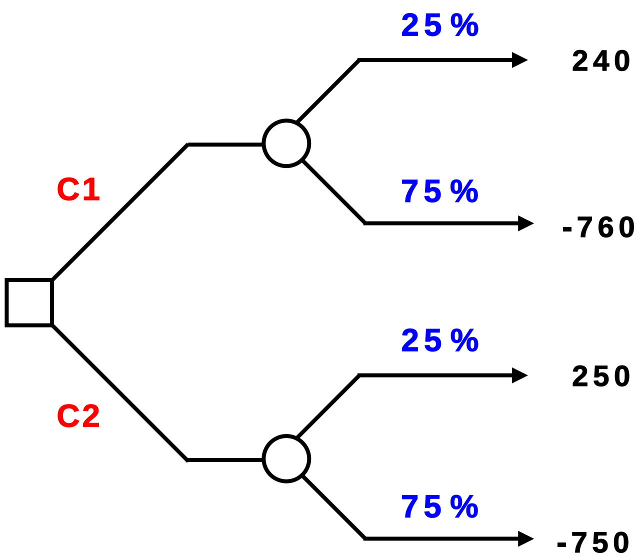
Better A1 or A2 ?

better ...

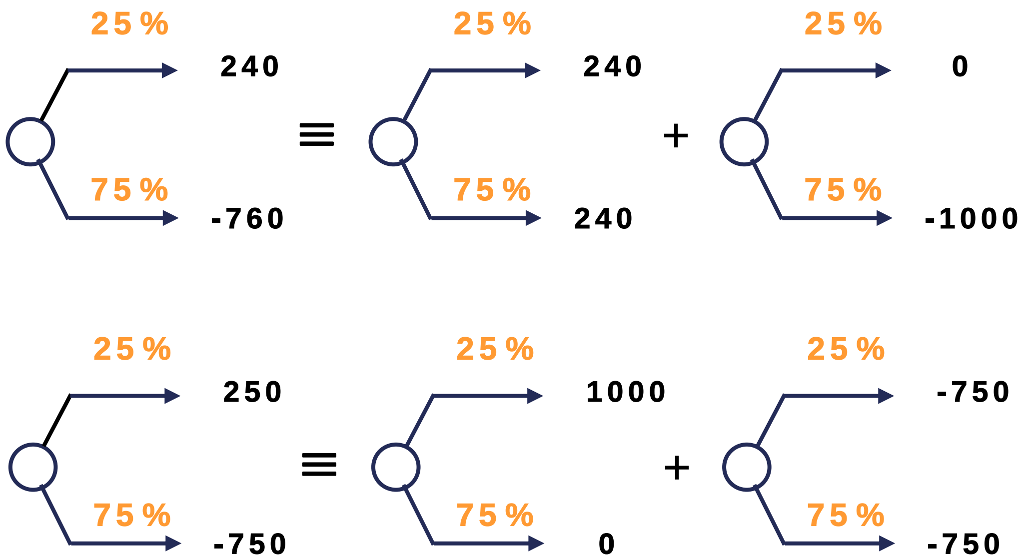
Better B1 or B2 ?

better ...

Lotteries (case C)



But notice that



Better C1 or C2 ?

better …

C1 → lin. comb. of A1 and B2

C2 → lin. comb. of A2 and B1

Ellsberg

A	B
50 (b)	α (b)
50 (n)	**100-** α (n)

White ball win

↓

Better to take from A or B ?

↓

better ...

ambiguity aversion

Now you have a second chance (after the ball is re-inserted)

A	B

the same ...

Black ball win

↓

Better to take from A or B ?

↓

better ...

ambiguity aversion ?

Cognitive theory: a first principle

C-1° **Principle of NON NEUTRALITY**

The aggregation of (decisional) options
is not a neutral operation !

Given the two preferences on A1 and B2, it is **not guaranteed**
that their aggregation (C1) is the preferred one

- **Caution: do not combine too easily the options**
- **Normally, the ambiguity is avoided, “even if this is not rational "**
  (Ellsberg)

N-2° Principle of DOMINANCE

If the DM prefers A with respect to B in every scenario (or context or state of nature) the choice **must be** A

Examples

- I prefer to be missionaire (with respect to engineer) in peace and prefer to be missionaire (...) in war
- I prefer chicken with respect to beef (when there is nothing else) and I prefer chicken … also when there is fish

so choice … is better then ...

(leaving **...** out of consideration)

Counterexamples (see in next lessons)

- Extraction from an urn filled with 100 balls (Tversky e Kahneman, 1986)
- The possible choices in uncertainty conditions (see "Sindaco di Utopia")

Extraction (in two conditions) / 1

n. of balls	situation A	situation B
90 white	0	0
6 red	45	45
1 green	30	45
1 blue	-15	-10
2 yellow	-15	-15

n. of balls	situat. C	situat. D	n. of balls
90 white	0	0	90 white
6 red	45	45	7 red
1 green	30	-10	1 green
3 yellow	-15	-15	2 yellow

Better A or B ?

better ...

Better C or D ?

better ...

but C ≡ A
and D ≡ B

Extraction (in two conditions) / 2

	w1	w2	w3	w4	w5
Invest	0	45	30	-15	-15
Build	0	45	45	-10	-15
p(w)	.90	.06	.01	.01	.02

Better Invest or Build **?**

	w1	w2	w3	w4
Invest	0	45	30	-15
p(w)	.90	.06	.01	.03
Build	0	45	-10	-15
p(w)	.90	.07	.01	.02

Better Invest or Build **?**

Cognitive theory: three more principles

C-2°

Principle of EVIDENCE

The dominance among options should be **obvious**

C-3°

Principle of ASYMMETRY

The possibility of **losing K** is more important than that to **win K**

C-4°

Principle of COMPACTNESS

An aggregated option (A) has an importance less than the sum of the importances of the single sub-options ($A_1.A_2$)

$$\pi(A) < \pi(A_1) + \pi(A_2)$$

N-3° **Principle of TRANSITIVITY**

If the decision prefers A over B and B over C,
then A **must** be preferred over C

Examples:

- Since V. Rossi is better than Stoner, and Stoner is better than Melandri, ...
- Buying emission units (Kyoto protocol) is better than cutting the production, and cutting the production is better than not respecting the constraints on emissions, so ...

Counterexamples:

- a new car + accessories

standard	10.000€
+air cond.	1.000€
+alloy rims	1.000€
+ ...	

(but finally ...)

	A	B	C	D
ob1	50	55	60	65
ob2	50	55	60	65
ob3	50	55	60	65
ob4	40	30	20	10

B > A
C > B
D > C

D > A ?

or rather
the options are incomparable ?

Cognitive theory: progression vs. crash

C-5°

Principle of CRASH

The decision-maker is (relatively) indifferent to small progressive changes, but at some point become aware of the (large) gap and ...

Cognitive theory: estimation

C-6°

Principle of OVER/UNDER-ESTIMATION

There is an inclination to
- over-estimate events with small probability
- under-estimate events with high probability (except in case of certainty)

Asymmetry in dealing with subjective probability