3°

POLITECNICO DI MILANO





Methods and Models for Decision Making

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God in 7 steps:

- Real decision = choice between alternatives,
  → various complexities and aiding tools
- The importance of the communication
  → perception of the problem by the DM
- Design of … → product / service / process
- Analyzing the elements and the whole

But what are you looking for ?

Index:

Classification

Evaluation = classification

- A set of alternatives (solutions, options)
- Possible partitions (classifications)
- Classes:
  - ordered / not ordered
  - not predefined / predefined

	not pred.	predef.
ordered	RANKING	RATING (sorting)
not ord.	ASSIGNM. (recogn.)	CLUSTER.

- Two problems:

  choice → what you want (and the remaining …)

  or

  rejection → what you don't (and the remaining …)

Examples of classification

- Michelin guide
- Medical diagnosis
- Marketing
- Linneo classification
- Envir. impact assess.
- PhD student selection
- Electoral districts
- Measure of land vulnerability
- Feasibility of projects
- Student eval. in 3 cat. (ok - exam - no)
- Level of alert in civil defence
- Breakdown diagnosis
- Smart electoral districting(*)
- More … (suggestions ?)

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In Massachussets in 1821, Governor Elbridge Gerry enacted an electoral redistricting plan that would enable him to be re-elected with high probability.

Gerrymandering

The unusual salamander shape of one of these districts gave origin to the term gerry-mander
(a contraction of *Gerry-salamander*).

Gerrymandering / 1

(adapted from Dixon, Plischke 1950)

EXAMPLE
Consider the territory represented in the figure as a chessboard divided into 81 “elementary zones” (units) with the same population

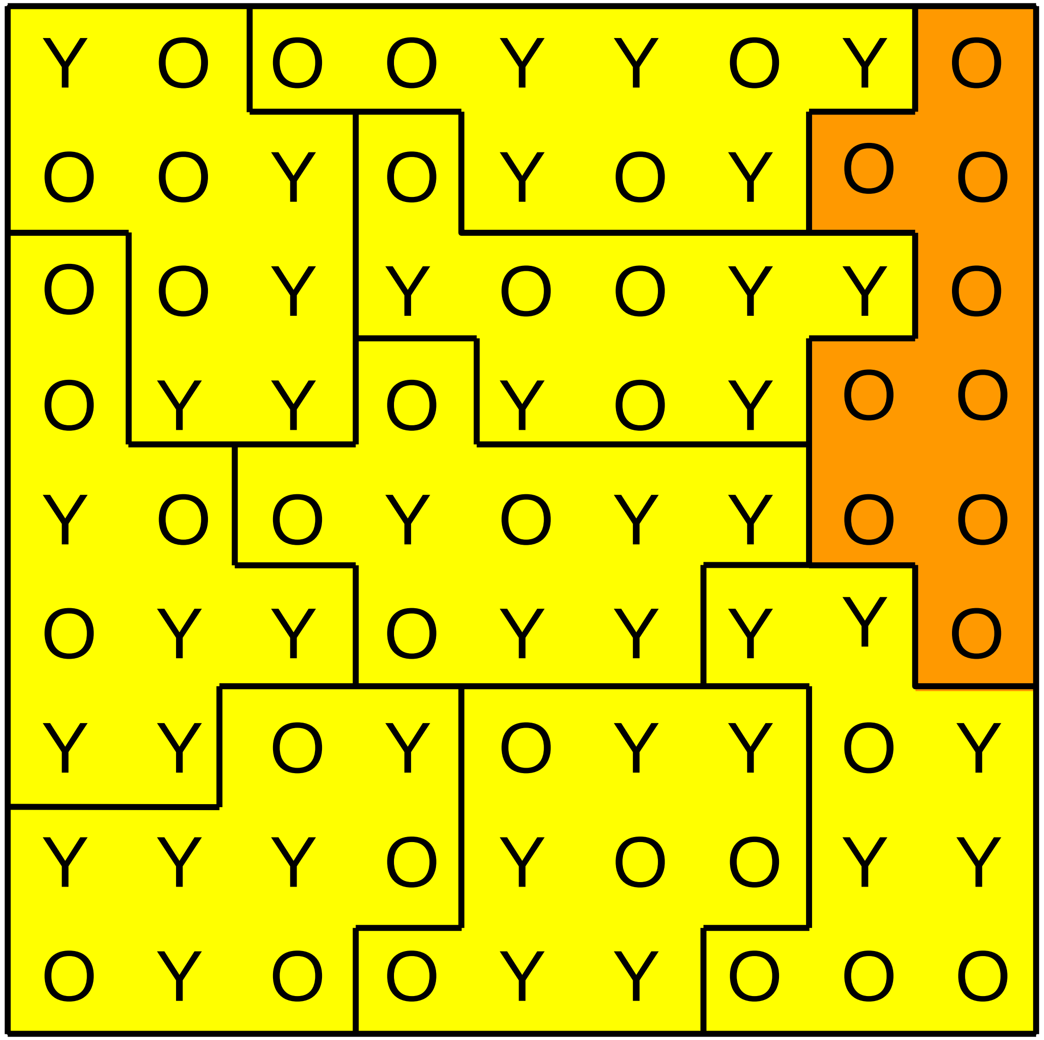
Y	O	O	O	Y	Y	O	Y	O
O	O	Y	O	Y	O	Y	O	O
O	O	Y	Y	O	O	Y	Y	O
O	Y	Y	O	Y	O	Y	O	O
Y	O	O	Y	O	Y	Y	O	O
O	Y	Y	O	Y	Y	Y	Y	O
Y	Y	O	Y	O	Y	Y	O	Y
Y	Y	Y	O	Y	O	O	Y	Y
O	Y	O	O	Y	Y	O	O	O

PROBLEM
Design a map of 9 uninominal districts formed by 9 units each

For simplicity we assume that in each unit the vote is homogeneous: colours yellow (Y) and orange (O) define a possible vote distribution.

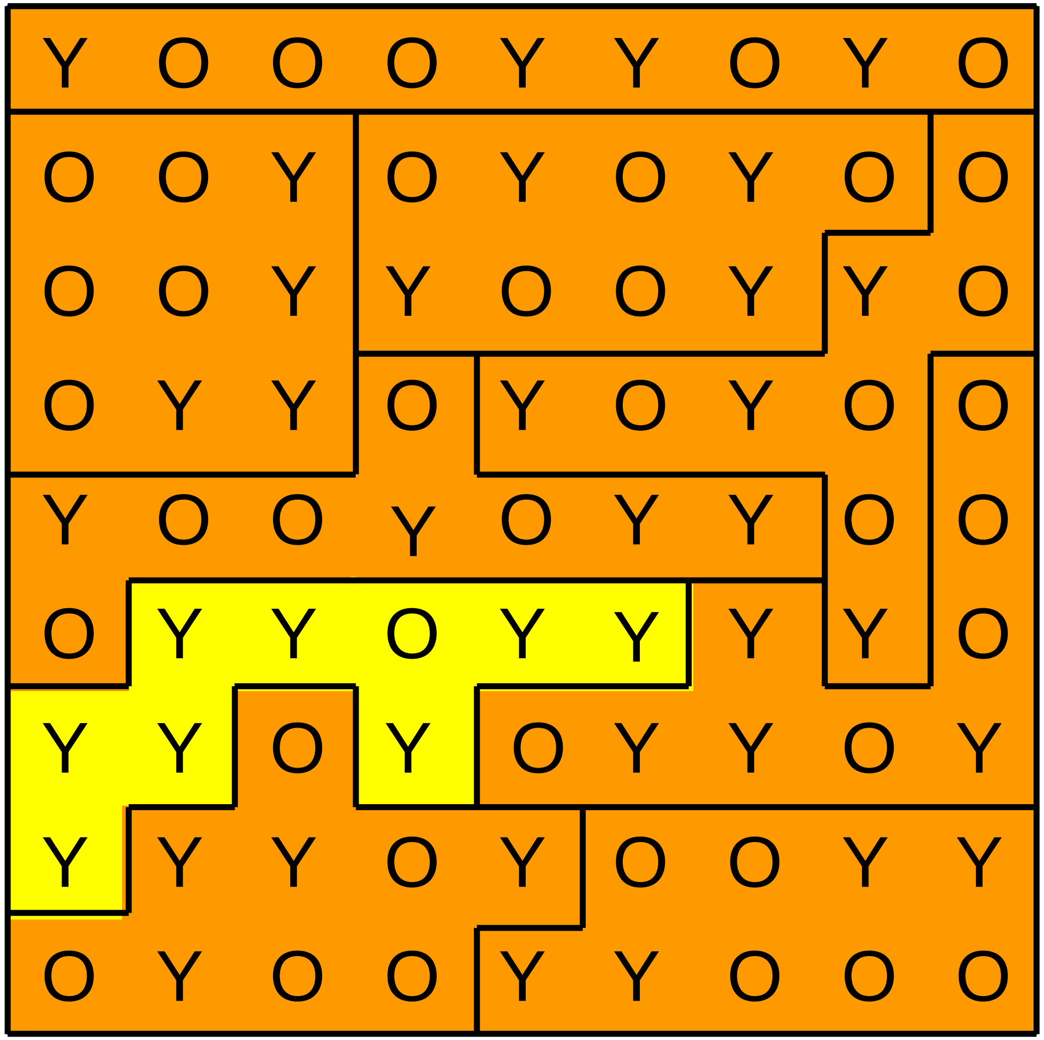
BALANCED VOTE → 41 units Y , 40 units O

Gerrymandering / 2



The **orange party** wins 1 seat

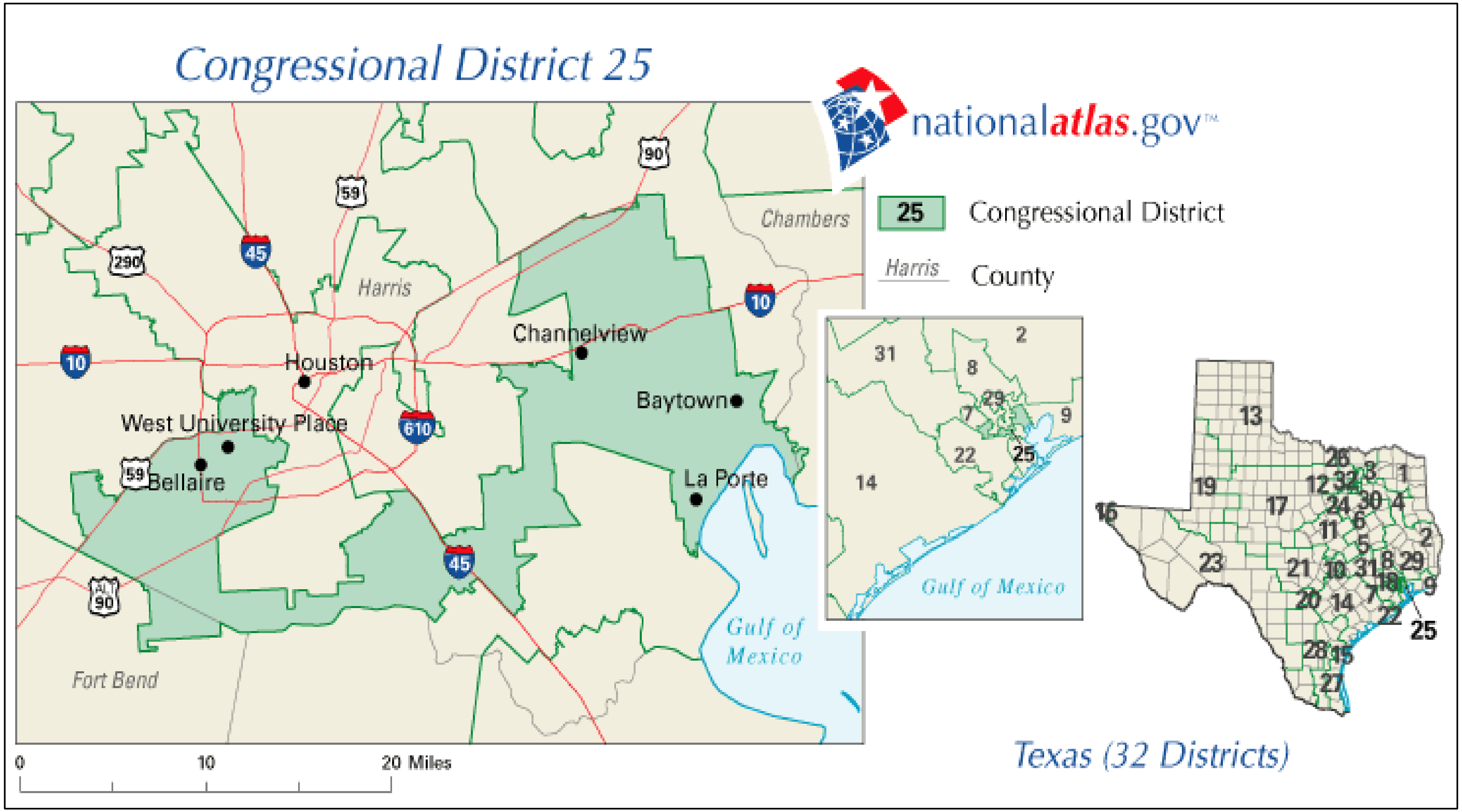
The **yellow party** wins **8 seats**



The **orange party** wins **8 seats**

The **yellow party** wins 1 seat

Presidential Elections – USA 2004



Population equality

District populations must be as balanced as possible.

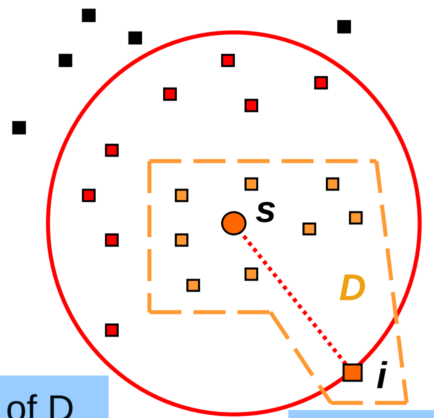
Administrative boundaries

The boundaries of electoral districts and other administrative areas must cross each other as little as possible.

Compactness

The districts must have “regular” geometric shapes: octopus or banana districts must be avoided.

Compactness measure

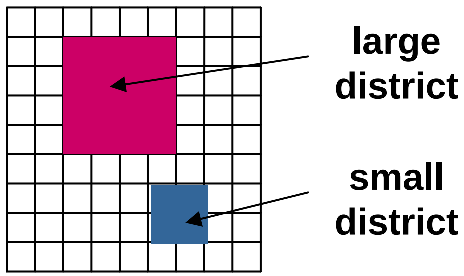


measured by the percentage of units (in the circle) not belonging to *D*

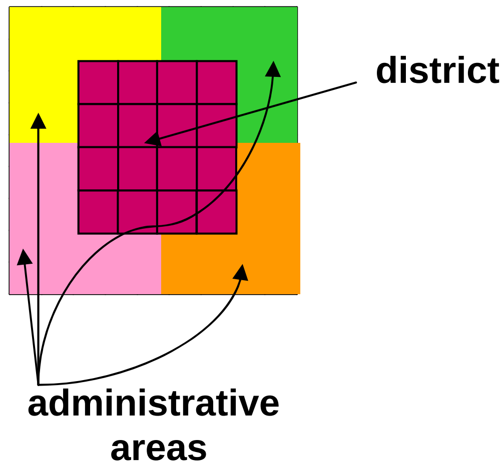
S = center of D

in D unit *i* is the farthest from S

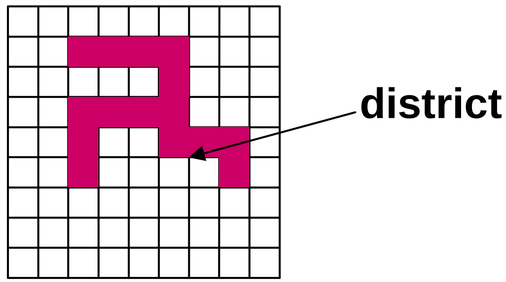
Bad case 1



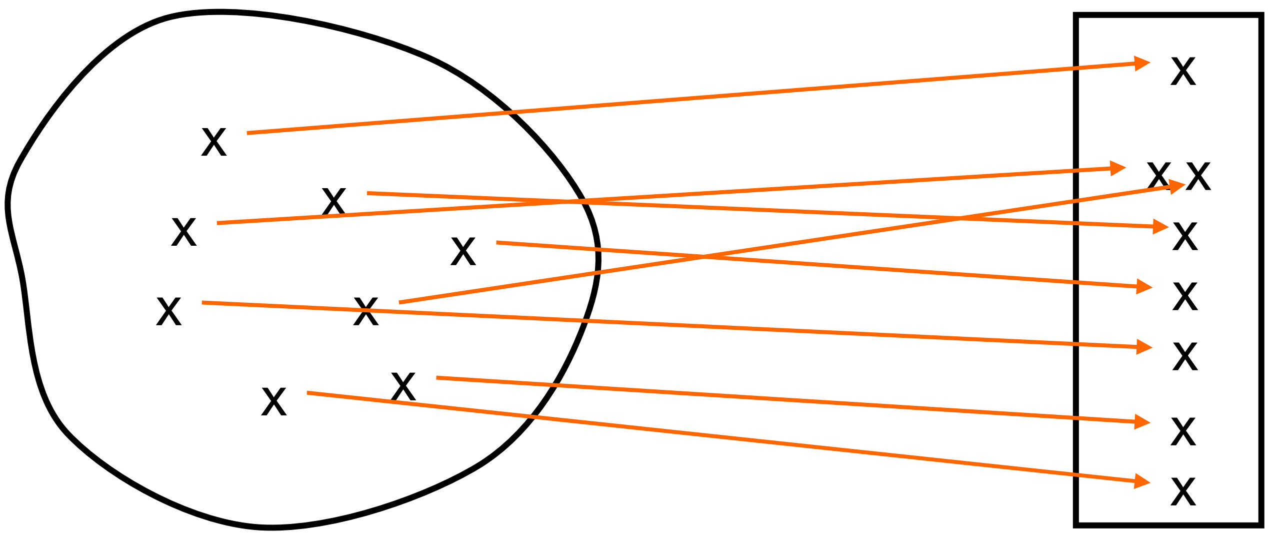
Bad case 2



Bad case 3



Ranking problems



Key-points of an evaluation system

- What relation between A and B ? **a binary one**
  - A better than B → $A > B$
  - A not worse than B → $A \geq B$
  - A indifferent to B → $A \sim B$
  - A not comparable with B → $A ? B$

- Note the difference between
  - $A \sim B$ (I'm able to compare and I say that …)
  - $A ? B$ (I'm not able to compare)

- From a pair (A, B) to a set (A, B, …, Z)
  - ranking
  - rating (sorting)
  - assignement
  - clustering

Ranking problems (the main category)

1. Risk analysis → when the DM has no the complete knowledge of the context (state of nature, exogen variables), **then**

   the choice between the alternatives could depend by the risk attitude of the DM (and also by his/her perception of the problem)

   from values to utilities

2. Multi-criteria analysis → when the DM identifies more than one criterion, **then**

   the choice between alternatives needs the search of a trade-off solution (because usually there is not an alternative better from every point of view)

   from different measures to a common scale

Ranking-1: risk analysis

The mayor of Utopia

An example of:

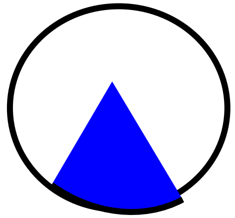
- non-deterministic environment → incomplete data
- making the solution independent of the missing information
- lotteries and risk attitude of the DM
- utility function (difference between value and utility)

An example of …

Utopia → → →

1500 inhabitants, 180 unemployed (12%)

cost c (10% budget), cost 2c (20%)



uncertain results (dependent by the state of nature ω)

A real problem:

the level of employment

Actions

- **action a_1**
  - cost *c*
  - n° of new jobs
    - with ω_1 …
    - with ω_2 …
- **action a_2**
  - cost *c*
  - n° of new jobs
    - with ω_1 …
    - with ω_2 …
- **action a_3**
  - cost *2c*
  - 50 new jobs
    - 50 (certain)
- **action a_4**
  - cost *2c*
  - n° of new jobs
    - with ω_1 …
    - with ω_2 …
- **action a_0**
  - cost 0
  - 0 new jobs

Certain and uncertain data

		a_1	a_2	a_3	a_4	a_0
(certain data) →	cost	c	c	2c	2c	0
(incertain data) →	jobs	?	?	50	?	0

	Jobs	Prob.	Jobs	Prob.	Jobs	Jobs	Prob.	Jobs
ω_1	50	10%	110	5%	50 (certain)	110	50%	0
ω_2	10	90%	10	95%		10	50%	

- How much the mayor is willing to risk (to spend) to create jobs ?
- What action is the best for him ?

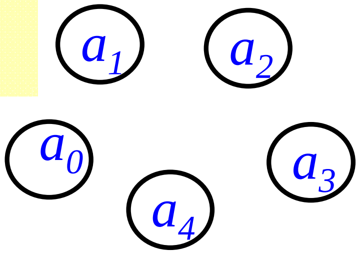
Preliminary questions / 1: preferable solutions

- action $\mathbf{a}_1$
  - cost c
  - n° of new jobs
    - 50 if ω_1 **(10%)**
    - 10 if ω_2 **(90%)**
- action $\mathbf{a}_2$
  - cost c
  - n° of new jobs
    - 110 if ω_1 **(5%)**
    - 10 if ω_2 **(95%)**
- action $\mathbf{a}_3$
  - cost $2c$
  - n° of new jobs
    - 50 certain **(100%)**
- action $\mathbf{a}_4$
  - cost $2c$
  - n° of new jobs
    - 110 if ω_1 **(50%)**
    - 10 if ω_2 **(50%)**
- action $\mathbf{a}_0$ no intervention
  - cost 0
  - 0 certain **(100%)**

Are there actions a priori preferable to others ?

Preliminary questions / 2: preferable solutions without considering costs

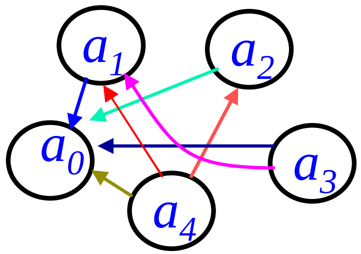
- action $\mathbf{a_1}$	n° of new jobs	50 if ω_1 10 if ω_2
- action $\mathbf{a_2}$	n° of new jobs	110 if ω_1 10 if ω_2
- action $\mathbf{a_3}$	n° of new jobs	50 (certain)
- action $\mathbf{a_4}$	n° of new jobs	110 if ω_1 10 if ω_2
- action $\mathbf{a_0}$	n° of new jobs	0



Considering only the number of jobs (and thus ignoring the costs), are there actions a priori preferable to others ?

Preferable solutions without considering costs

- action $\mathbf{a_1}$ — n° of new jobs — 50 if ω_1 / 10 if ω_2
- action $\mathbf{a_2}$ — n° of new jobs — 110 if ω_1 / 10 if ω_2
- action $\mathbf{a_3}$ — n° of new jobs — 50 (certain)
- action $\mathbf{a_4}$ — n° of new jobs — 110 if ω_1 / 10 if ω_2
- action $\mathbf{a_0}$ — no intervention — 0 jobs (certain)



Preliminary questions / 3: lottery

What is the value of the probability π that makes you believe these two situations are equivalent ?

(i) 110 jobs with prob. π or 10 jobs with prob. $(1- \pi)$

(ii) 50 certain jobs

π = ?

What is the value of the probability π that makes you believe these two situations are equivalent ?

(i) 110 jobs with probability π or 10 jobs with probability $(1- \pi)$

(ii) obtain 50 certain jobs ?

LOTTERY

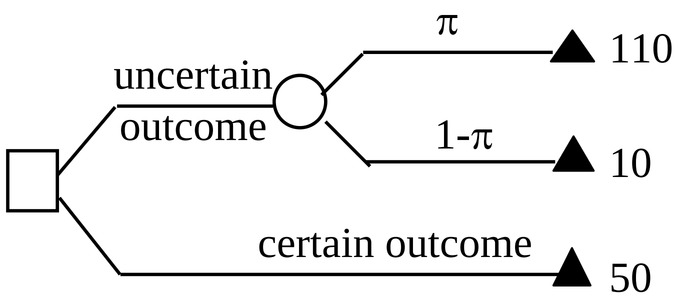
Equivalence between a certain outcome
and a couple of possible outcomes

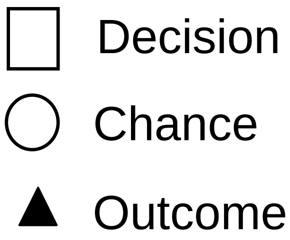
What is the value of the probability π that makes you believe these two situations are equivalent ?

(i) 110 jobs with probability π or 10 jobs with probability (1- π)

(ii) 50 certain jobs

a lottery



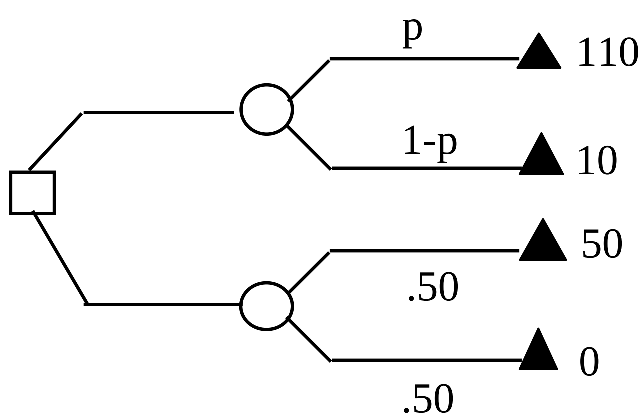


Other questions / 4: certain jobs with cost c

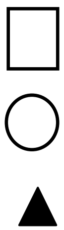
cost *2c* → 50 certain jobs

cost *c* → how many certain jobs ?

What is the value of the probability p that makes you believe these two (not deterministic) situations are equivalent ?



$p = ?$



Final question / 6: utility for the decision-maker

Does the decision-maker deem more useful to go from 10 to 50 jobs, or from 50 to 110 ?

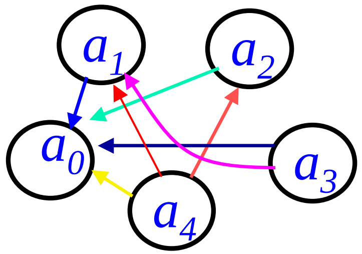
That is→ it is better to have 40 more jobs being in a situation with few employees

or

have 60 more jobs being in a situation that already has a discrete number of employees ?

Possible answers

- Q1: preferable actions **a priori** ? **no**
- Q2: preferable actions evaluating **only the n° of jobs** ? **yes (see figure)**



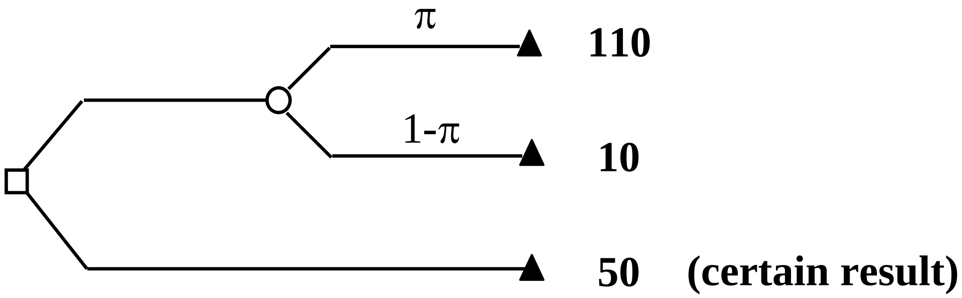
- Q3: probability **π = 0.60**
- Q4: with cost c **20 certain jobs**
- Q5: probability ***p* = 0.25**
- Q6: better to increase the number of jobs **from 10 to 50** (instead than…)

6 questions: 2 for estimating parameters, the others for checking the DM answers

Utility function for the number of jobs

Basic difference:

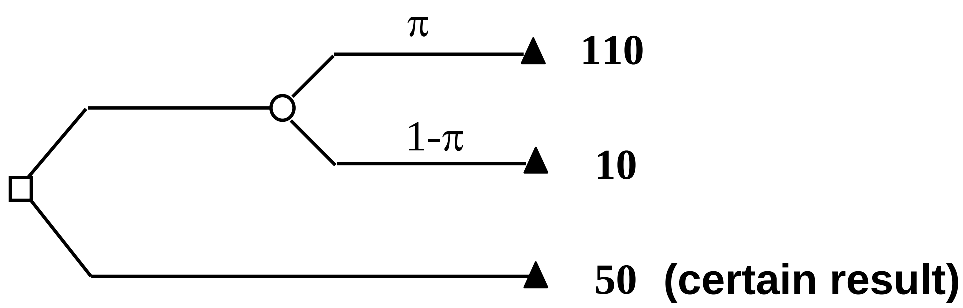
values vs utilities



Utility: $u_{110} = \mathbf{1}$, $u_{50} = \alpha$, $u_{10} = \beta$, $u_0 = \mathbf{0}$

Since utility is measured in a conventional scale (usually between 0 and 1), to the worst outcome (0 jobs) is associated the value 0, while to the best (110 jobs) is associated the value 1.

Utility function: the numerical solution

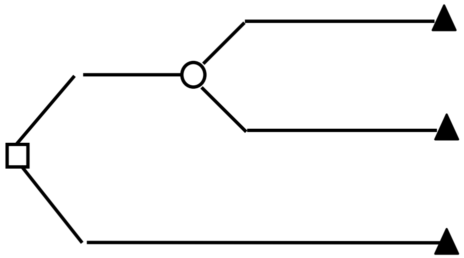


Utility: $u_{110} = 1$, $u_{50} = \alpha$, $u_{10} = \beta$, $u_0 = 0$

$$1 * \pi + \beta * (1- \pi) = \alpha$$

$$1 * p + \beta * (1- p) = \alpha * 0.5 + 0 * 0.5$$

Utility function : the result



$$1 * \pi + \beta * (1- \pi) = \alpha$$

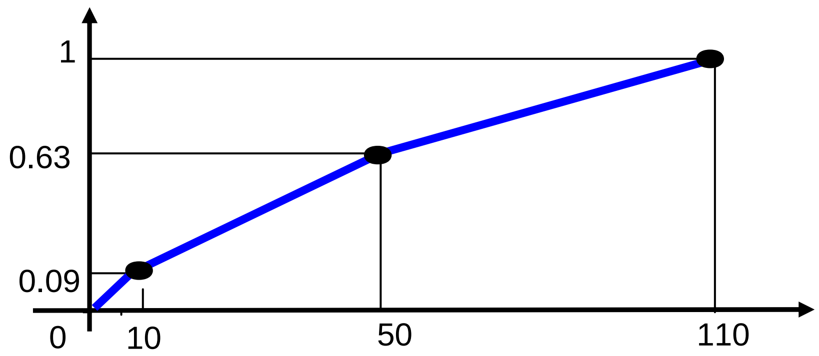
$$1 * p + \beta * (1- p) = \alpha * 0.50 + 0 * 0.50$$

Answers: π **= .60 ,** ***p*** **= .25**

$\alpha = 35/55 \cong 0.63$

$\beta = 5/55 \cong 0.09$

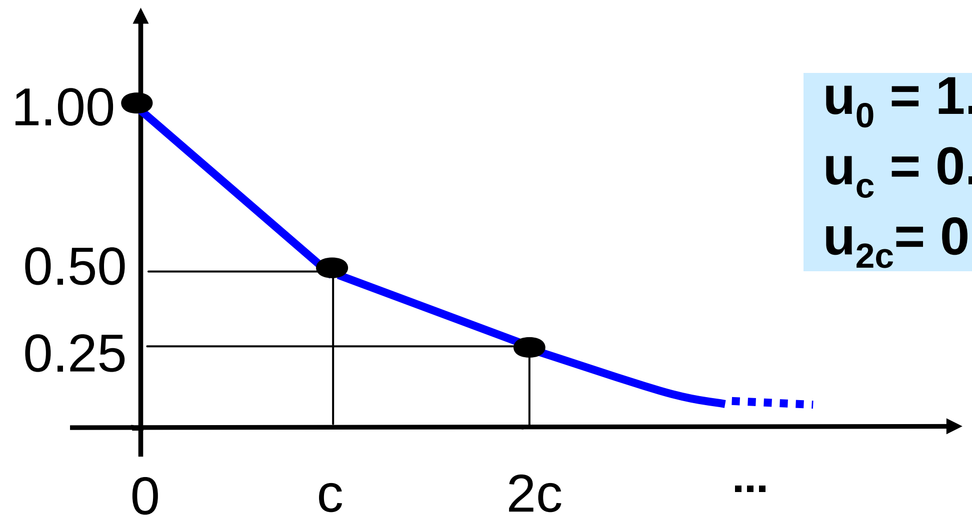
$u_{110} = 1.000$
$u_{50} = 0.630$
$u_{10} = 0.090$
$u_0 = 0.00$



Check: from10 to 50 $\rightarrow$ variation of the utility = (0.63 - 0.09) **= 0.54**

from 50 to 110 $\rightarrow$ variation of the utility = (1.00 - 0.63) **= 0.37**

Utility function for the costs (roughly)



$u_0 = 1.00$
$u_c = 0.50$
$u_{2c} = 0.25$

Of course even for this criterion it is necessary to interview the decision-maker for understanding the shape of his utility function **u(c)** regarding the economical aspects.

Suppose that you have done it and that the result is ...

Evaluation matrix

	a_1	a_2	a_3	a_4	a_0
u_{cost}	0.500	0.500	0.250	0.250	1.000
u_{jobs}	**0.144**	0.135	0.630	0.545	0.000

- How do you get the value **0.144** ?
- It is the **expected utility of a_1** as regards the employment criterion →
  → 0.63 * 0.10 + 0.09 * 0.90 = 0.063 + 0.081 = 0.144

- The problem has two “dominated solutions” (a_4 and a_2)
- The choice between the others has to be done:
  what are the preferences of the decision-maker ?

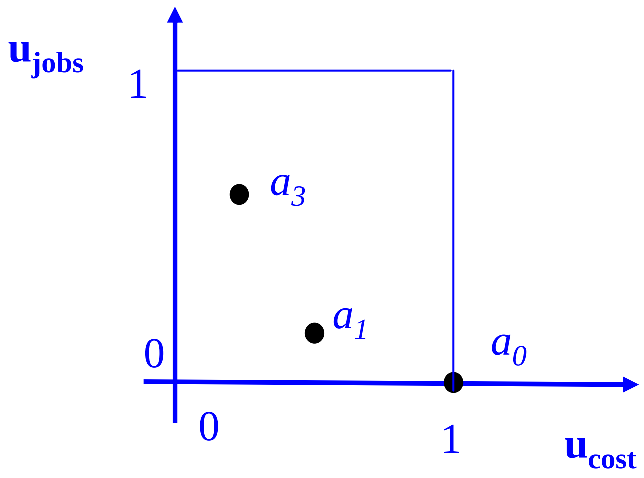
Evaluation matrix

	a_1	a_2	a_3	a_4	a_0
u_{cost}	0.500	0.500	0.250	0.250	1.000
u_{Jobs}	0.144	0.135	0.630	0.545	0.000

Preferences → vector of weights for the criteria → (0.4, 0.6)

The final choice

In the space (in this case a plane) of the criteria



only 3 efficient solutions

Global utility and ranking (using the criteria weights 0.4 and 0.6)

	a_1	a_2	a_3	a_4	a_0
utility	.286	.282	.440	.407	.400
ranking	4°	5°	1°	2°	3°

final choice → a_3

Synthesis

PROBLEM

Risk attitude of the DM

Utility of jobs

Utility of costs

Utility functions

SOLUTION

(depends on the preferences of the decision-maker)

Test-1

In a decision problem under conditions of uncertainty, a utility function is a relationship – expressed in an appropriate scale, usually [0, 1] – between outcome values and utilities perceived by the DM

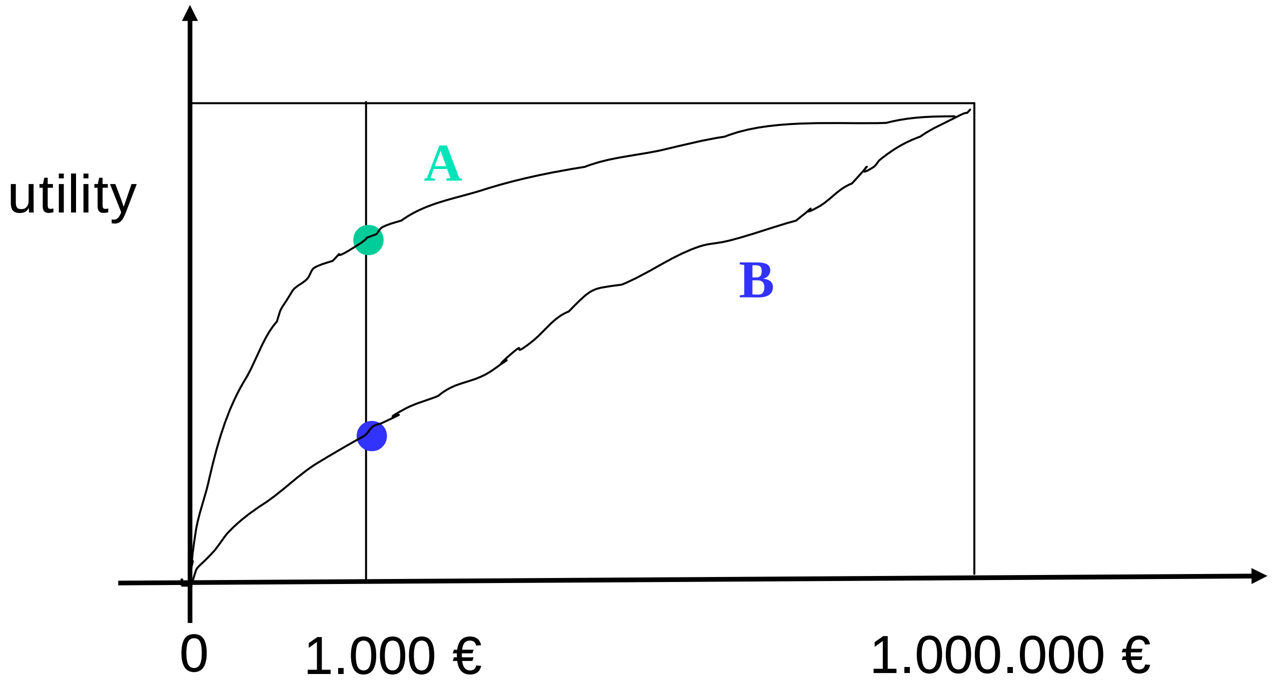
(a) True **(b) False**

Test-2

The following graph shows the utility functions for a worker and an entrepreneur; the utility function of the worker is represented by curve B, while curve A represents the perceived utility by the entrepreneur

(a) True

(b) False



Test-3

As regards the number of jobs, $\mathbf{a_2}$ is preferred with respect to $\mathbf{a_4}$

(a) True **(b) False**

	Jobs	Prob.	Jobs	Prob.	Jobs	Jobs	Prob.	Jobs
ω_1	**50**	**10%**	**110**	**5%**	**50** (certain)	**110**	**50%**	**0**
ω_2	**10**	**90%**	**10**	**95%**		**10**	**50%**	