POLITECNICO DI MILANO

5°





Methods and Models for Decision Making

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God in 7 steps:

- Variable space (decision) & objective space (results)
- Three phases in MCDM (or MODM) problems
- Phase 1 → from indicators to utilities
- Phase 2 → elimination of dominated solutions
- Phase 3 → DM preferences & final choice
- The second is the only "objective" phase
- Utility functions & preferences of *this* DM
- Sensitivity analysis with respect to the weight vector

Index:

A tentative case:
the “Colorni award”

(the best italian newspaper on the web)

The best italian newspaper on the web
(and you are in the jury …)

- What are the alternatives ?
- What are the attributes ?
- What are the utility functions ?
- What are the weights ?
- What is the ranking ? (the personal one and the collective one)

Alternatives

- Repubblica → www.repubblica.it
- Corriere Sera → www.corriere.it
- Sole24Ore → www.sole24.it
- Ansa online → www.ansa.it
- RaiNews24 → www.rainews24.it
- Foglio → www.ilfoglio.it

- Gazzetta d. Sport ? NO, because it is too specific (only sports)
- Novella 2000 ? NO, because it is a magazine weekly

The alternatives must be "similar" (but the concept of similarity is subjective)

- What are the rules ? (if there are rules …)

Attributes

- A lot of (tentative) attributes; note that the attributes must generate a set of indicators that could be measurables

- Four main indicators:
  - arguments in homepage
  - upgrades during the day
  - daily visitors (declaration)
  - quality in a scale [1, 10]

- A search (made by the students) to obtain the data set

Utility functions

- One utility function for each attribute/indicator
- Three steps for each utility function (easy version)
- The results → link
- The evaluation matrix (in a common [1, 10] scale)
- Phase 2 → dominated alternatives ?

Weights (the preference structure)

- We are 30 (approximately) DMs
- What is the way to obtain (shared) weights ?
- *Discussion …*
- A common vector of weights ?
- Total utility of each candidate → final ranking

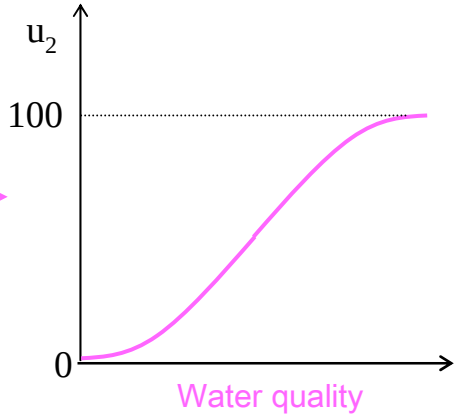
A lot of rankings (the personal paths)

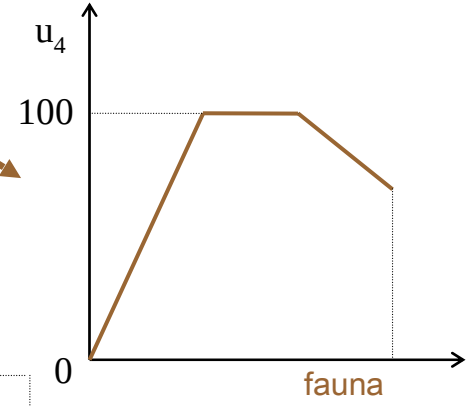
- A set of (~ 30) individual rankings
- What can we do ?
- It is a group decision → see lesson 6
- The final result is the “less conflicting” (or more shared) ranking
- Two main procedures → Distillation or Maastricht

Deeper investigations

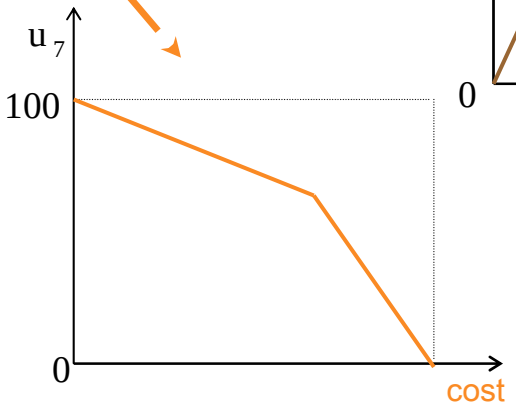
DI-1 → Utility functions

	alt 1	alt 2	alt 3
air quality	0.3	0.8	0.7
water quality	25	47	12
flora	1	6	5
fauna	250	710	940
landscape	7	6	4
employment	53	71	98
cost	409	524	912





	alt 1	alt 2	alt 3
air quality	80	60	70
water quality	70	50	80
flora	40	80	70
fauna	50	70	90
landscape	70	60	40
employment	50	70	80
cost	90	70	40



Pareto dominance (after utility functions)

(attributes)	alt 1	alt 2	alt 3
air quality	0.3	0.7	0.5
water quality	25	47	12
flora	1	5	5
fauna	250	940	710
landscape	7	3	4
employment	53	71	83
cost	409	912	912

Performance matrix

utility

(criteria)	alt 1	alt 2	alt 3
air quality	80	60	70
water quality	70	50	80
flora	40	70	70
fauna	50	70	90
landscape	70	30	40
employment	50	70	90
cost	90	40	40

The Pareto criteria has to be verified only after the application of the utility functions

alt2 dominated by alt3

DI-2 → Mutual preferential independence

The preferential rank (and the structure) between 2 values of an ttribute doesn't depend on the value of the other attribute

Example
2 attributes

the purchase of a radio
- price ↓
- ratio signal/noise ↑

Counterexample1
2 attributes

choice in a menu
- food (fish, meat)
- beverage (white wine, red wine)

Counterexample2
2 attributes

the chemical reaction
- reagent A
- reagent B

keep the ratio 1:1

(4,5) > (3,5)
but (4,2) < (3,2)
complementary goods, synergic effects

More formally

Mutual preferential indipendence

separability $U(x,y)= f(u_x(x) , u_y(y))$

additivity $U(x,y)= f_1 [u_x(x)] + f_2 [u_y(y)]$

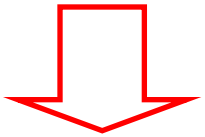
2 steps:

- determine $u_x(x)$ e $u_y(y)$
- determine the functions f_1 e f_2

The mutual preferential indipendence is a necessary condition (but not sufficient) for the additivity

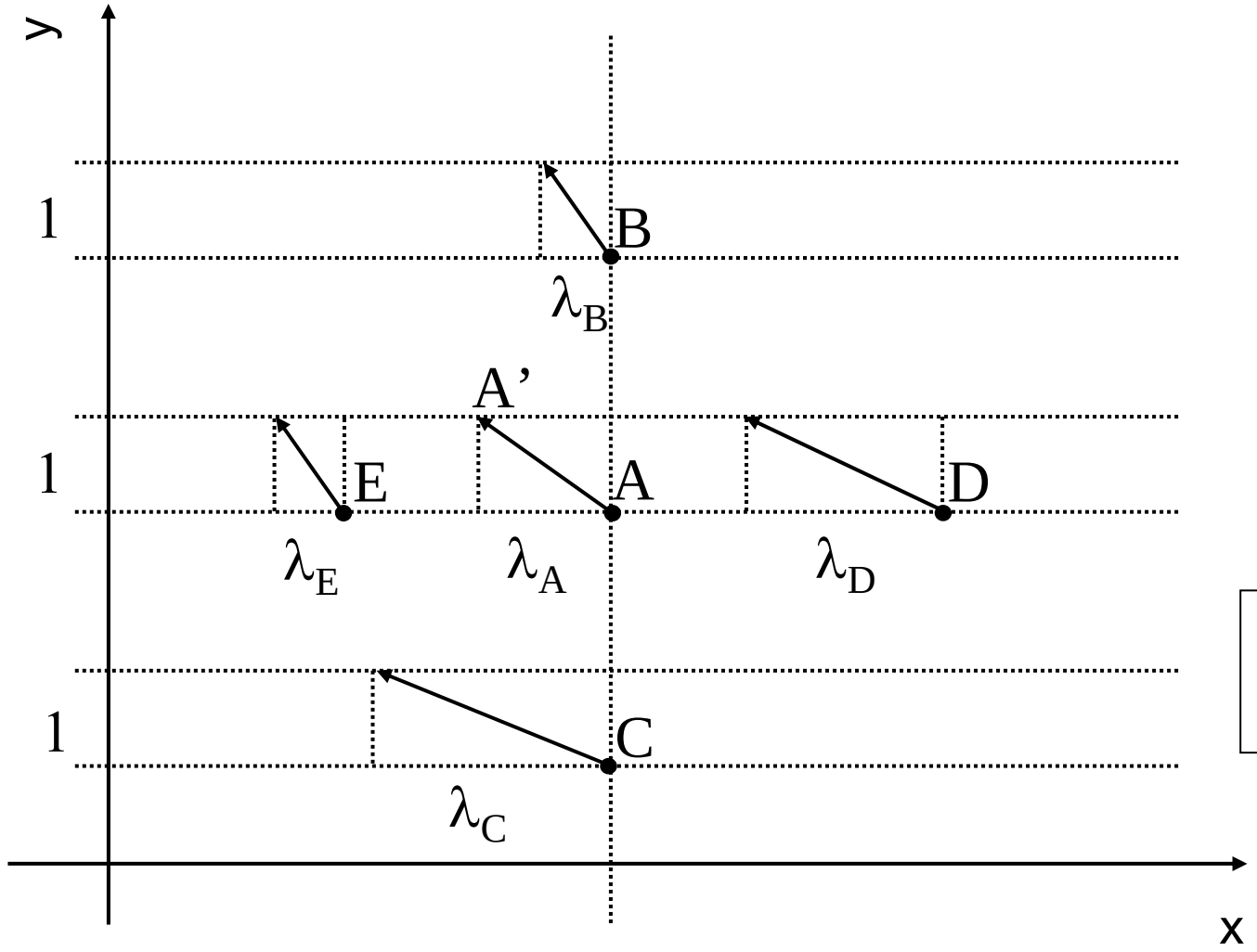
DI-3 → Rate of substitution

The DM is (always) willing to a compensation between a worsening in one attribute and a suitable improvement in another attribute.



Marginal rate of substitution

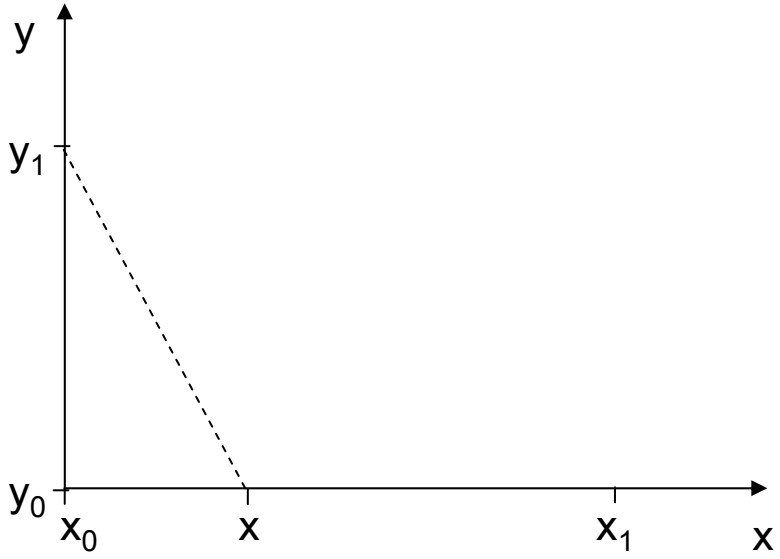
Marginal rate



$\lambda_B < \lambda_A < \lambda_C$

$\lambda_E < \lambda_A < \lambda_D$

DI-4 → Weight assignment



Two questions:

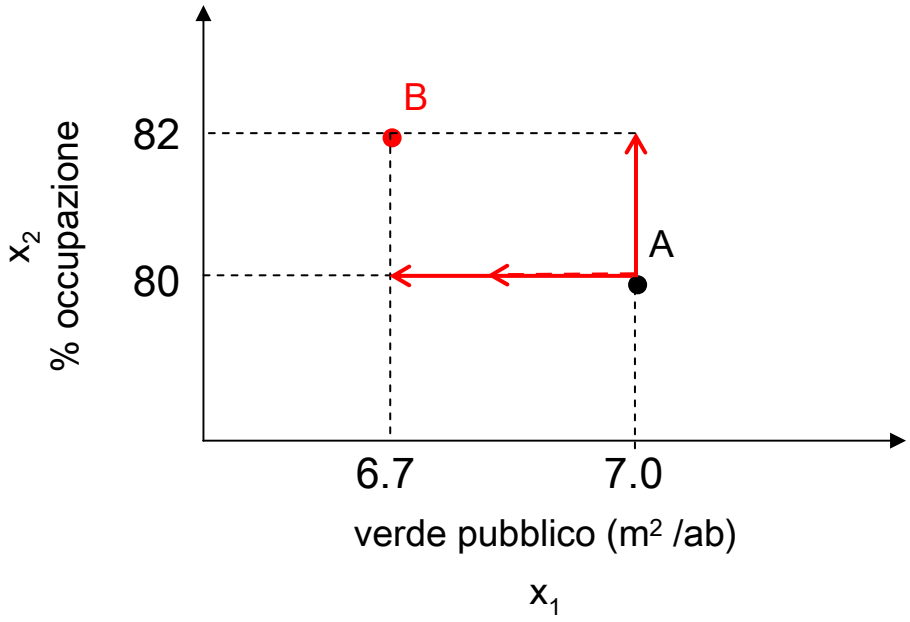
1. is it better (x_0, y_1) or (x_1, y_0) ?

   if the answer is $(x_1, y_0) \Rightarrow w_1 > w_2$

2. suppose to stay in (x_0, y_1), what is the value of x for which (x_0, y_1) is like (x, y_0) ?

$$\begin{cases} w_1\, u_x(x_0) + w_2\, u_y(y_1) = w_1\, u_x(x) + w_2\, u_y(y_0) \\ w_1 + w_2 = 1 \end{cases}$$

Weight assignment: pair comparison



- compensation
- substitution

$\frac{w_2}{w_1}$ is the ratio between the utility differences

n weights $\Rightarrow$ n-1 pairwise comparisons

$\Rightarrow$ n-1 ratios $\frac{w_i}{w_j}$ & $\sum_i w_i = 1$

DI-5 → The quality of life

- The quality of life in the Italian cities
- Report of *Sole24ore* (29 dec. 2008)
- Analyzed 6 sectors, with 36 indicators
- Utility functions, weights, ranking
- Comparison between different rankings based on indicators and perceptions of a sample (approx. 700 people for province)

Test examples

Example-1: free text concerning an argument

(no more than 300 words or 2000 characters)

- Describe how the Sudoku can be seen as a decision (or operational research) problem

or

- What is the Pareto frontier ? Describe it and show an example

or

- Define the concept of lottery. Elaborate with two numerical examples

or

- Describe the main ideas behind the “C-K theory”

or

- …

Example-2

In a problem characterized by two attributes (*x* and *y),* you know the utility functions of such attributes:

$$u_x(x) = 1 - x$$

$$u_y(y) = y^2$$

Moreover, you know that the decision-maker is indifferent to two following situation (A and B)

$$A\begin{cases} x = 1 \\ y = \frac{1}{2} \end{cases} \qquad B\begin{cases} x = \frac{1}{2} \\ y = 0 \end{cases}$$

Please discuss if the situation $K\begin{cases} x = \frac{1}{2} \\ y = \frac{1}{4} \end{cases}$ is preferable to $H\begin{cases} x = \frac{3}{4} \\ y = \frac{1}{2} \end{cases}$

Example-3

A multicriteria decision problem (6 alternatives, 3 criteria = utilities) is showed in this matrix, with its weight vector.

	a_1	a_2	a_3	a_4	a_5	a_6
c_1	60	40	20	70	100	80
c_2	40	40	35	35	35	40
c_3	20	30	60	40	50	50

w_1	0.20
w_2	0.40
w_3	0.40

1. Are there dominated alternatives ?

2. What is the ranking and the final choice ?

3. Is the result changing if w_2 increase ? Is there a rank reversal ?

Explain (briefly) all the answers.