



## Methods and Models for Decision Making

**Alberto Colorni** – Dipartimento INDACO, Politecnico di Milano

**Alessandro Lué** – Consorzio Poliedra, Politecnico di Milano

### God in 7 steps:

- MCDM: a logical path
- Definition of the alternatives
- Choice of the attributes
- Determination of the utility functions
- Preference structure and weights
- Ranking + sensitivity analysis

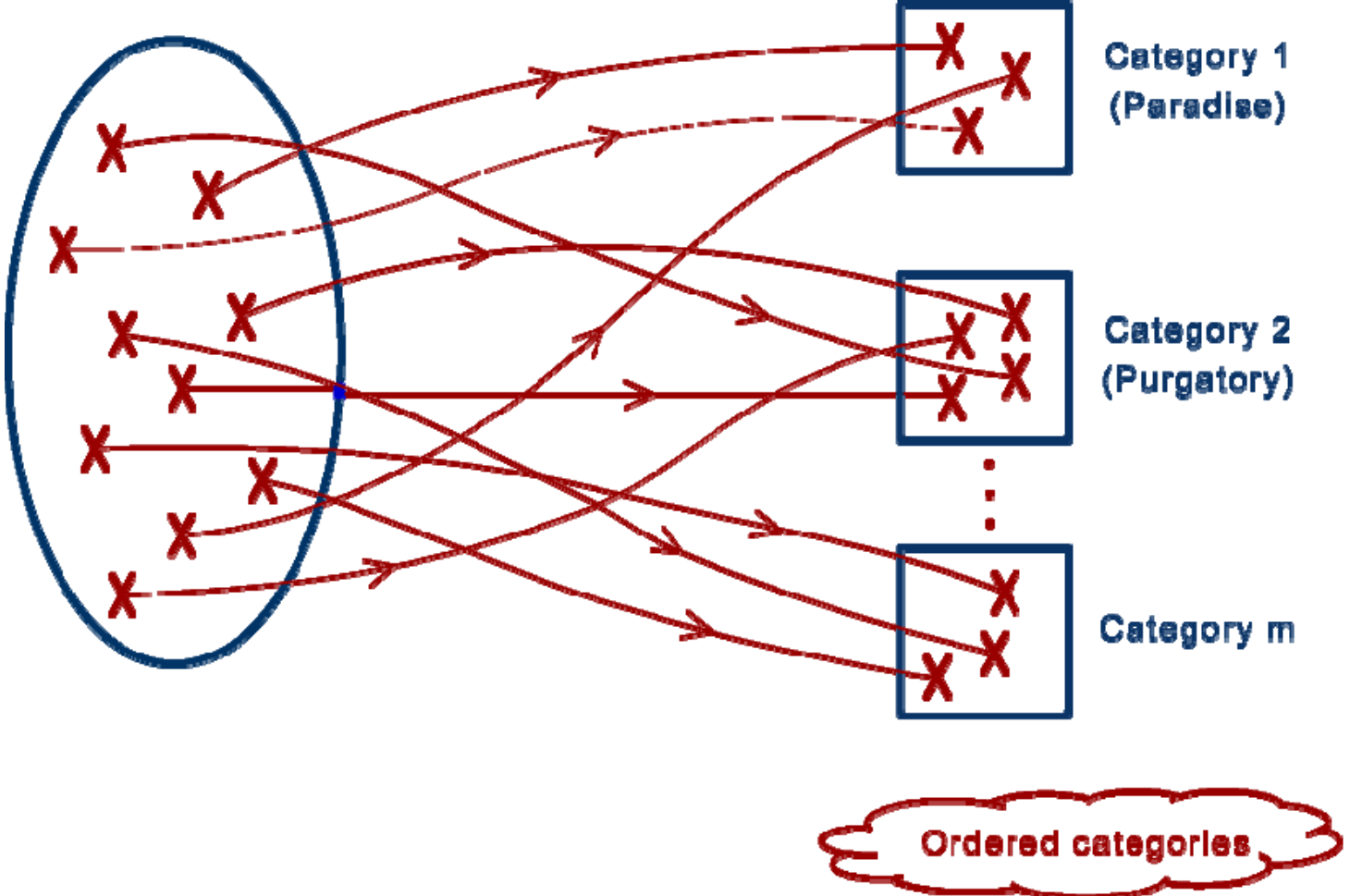
### Index:

- (1) Introduction
- (2) Tools & frame
- (3) Mental models
- (4) Design & decision
- (5) Classification
- (6) Ranking-1, risk analysis
- (7) Ranking-2, multicriteria
- (8) A tentative case
- **(9) Rating problems**
- **(10) Seminar M. Henig**
- *(11) Group decision*
- *(12) Research topics*
- *(13) Conclusions*

# Summary

1. Rating (sorting)
2. An example
3. Definition of...
4. Comparison between objects and profiles
5. When  $K \geq \sum_{j \in S} P_{ij}$
6. Threshold  $\alpha$  (and winning coalitions)
7. Students (A, B, C, D)
8. Rating revised
9. About the method
10. More concerning winning coalitions
11. Test and conclusions

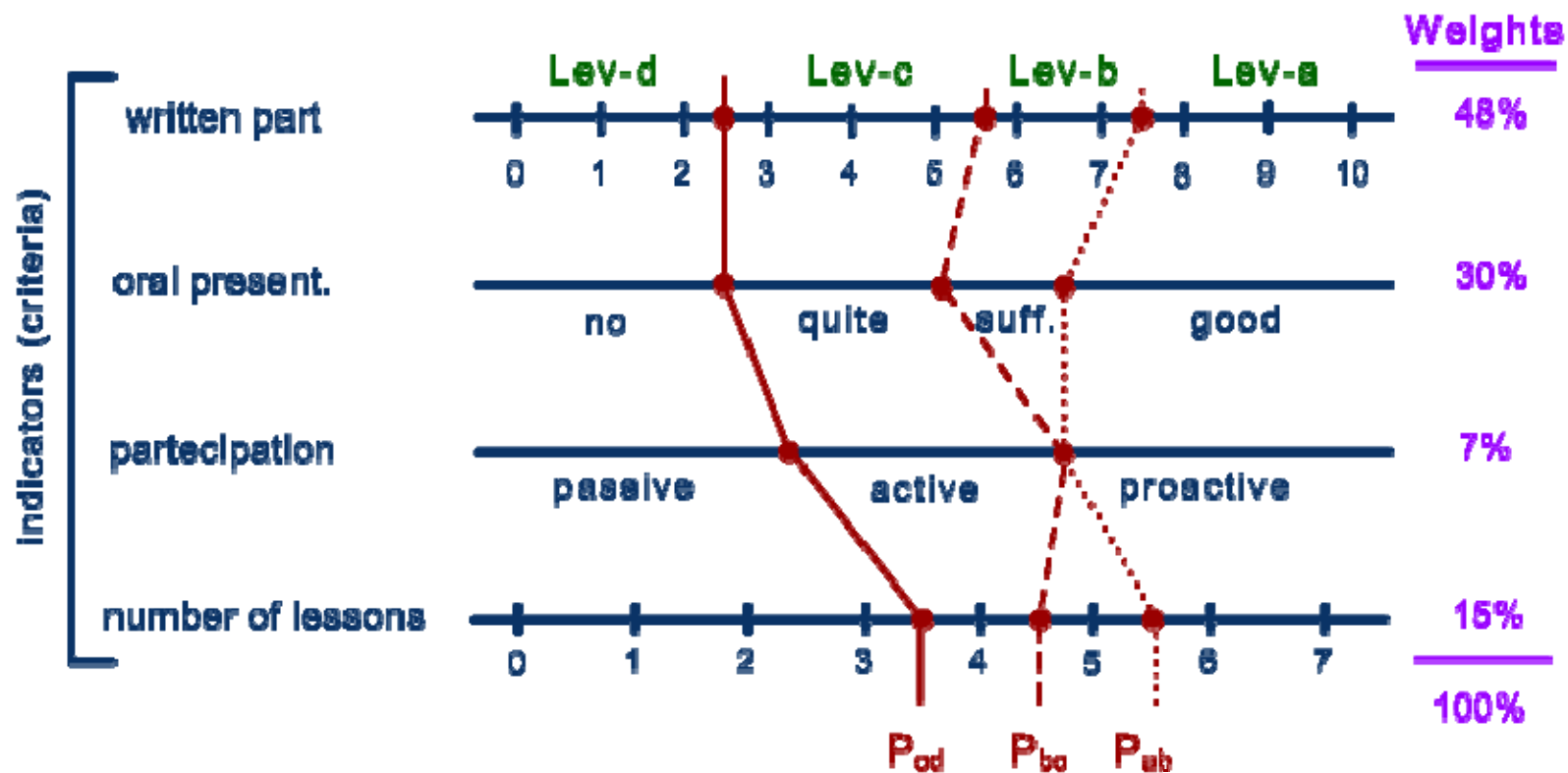
# Rating (sorting)



## An example

- Objects to be rated → the PhD students of MMDM
- Categories (levels) →  
Lev-a = excellent  
Lev-b = good  
Lev-c = sufficient  
Lev-d = insufficient
- What procedure ? → the logical (& subjective) steps

# Definition of...



- ... of
- indicators (criteria)
  - weights
  - levels (categories)
  - profiles (= levels - 1)

$P_{ij}$  = profile = watershed between 2 categories

# Comparison between objects and profiles

Student K  
vs  
profile  $P_{ij}$



$K \succ P_{ij}$



$P_{ij} \succ K$

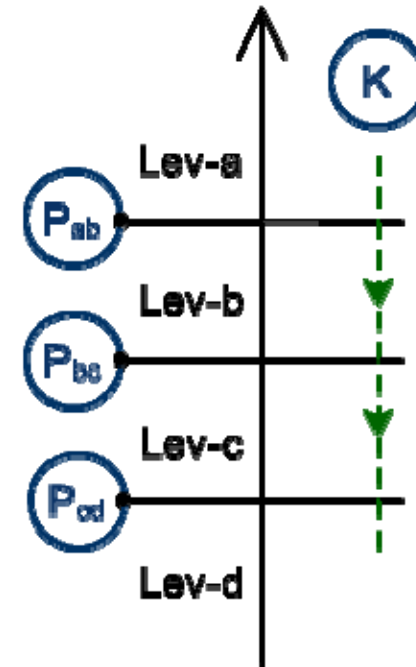


$K \sim P_{ij}$



$K ? P_{ij}$

$\succ$  outrank  
 $\sim$  indifferent  
 $?$  Incompar.



Procedure:

$K$  is in Lev-l

iff  $\left[ \begin{array}{l} K \succ P_{ij} \\ \text{or} \\ K \sim P_{ij} \end{array} \right.$

## When K S Pij

- i. Reasons in favor (concordance) → HIGH ( $\geq \alpha$ )
- ii. Reasons against (discordance) → LOW ( $\leq \beta$ )
- iii. Strong opposition (veto) → NOT PRESENT



(in our case)

- i.  $\Sigma$  weights in favor of K  $\geq \alpha$  (threshold to be fixed)
- ii. (not defined in this case)
- iii. If number of lessons  $< 4 \rightarrow$  veto K S P<sub>cd</sub> (so K in Lev-d)



# Threshold $\alpha$ (and winning coalitions)

|    |       |    |
|----|-------|----|
| C1 | _____ | 48 |
| C2 | _____ | 30 |
| C3 | _____ | 7  |
| C4 | _____ | 15 |

(Weights)

Let we fix  
 $\alpha = 70$

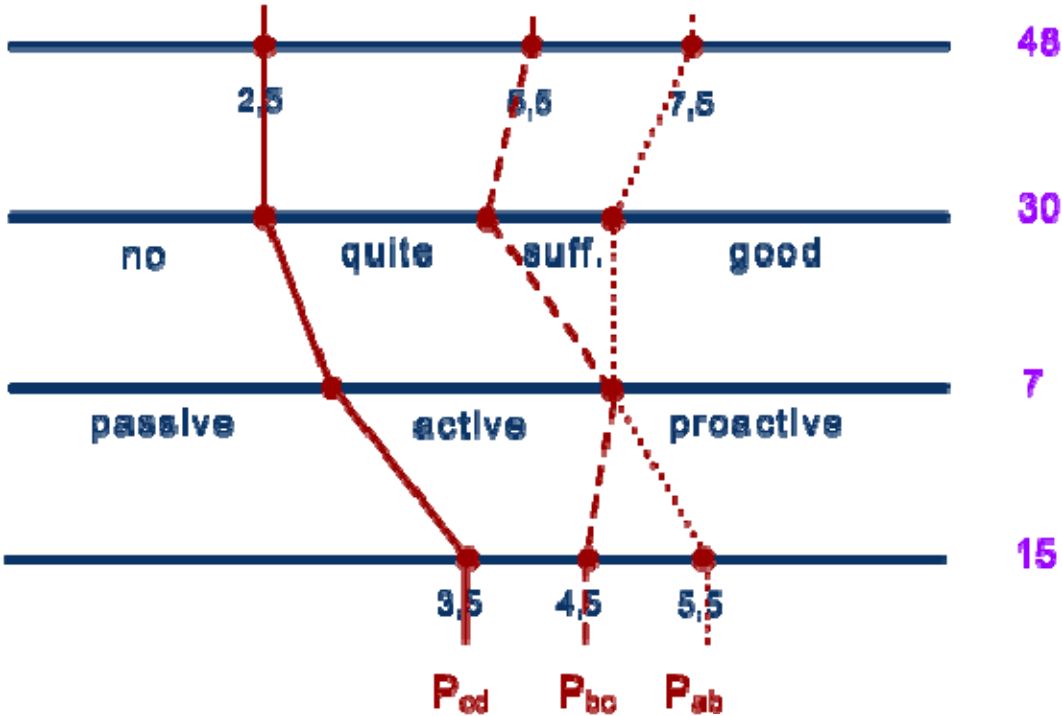
Winning coalitions: C1 + C3 + C4  
C1 + C2 + ...



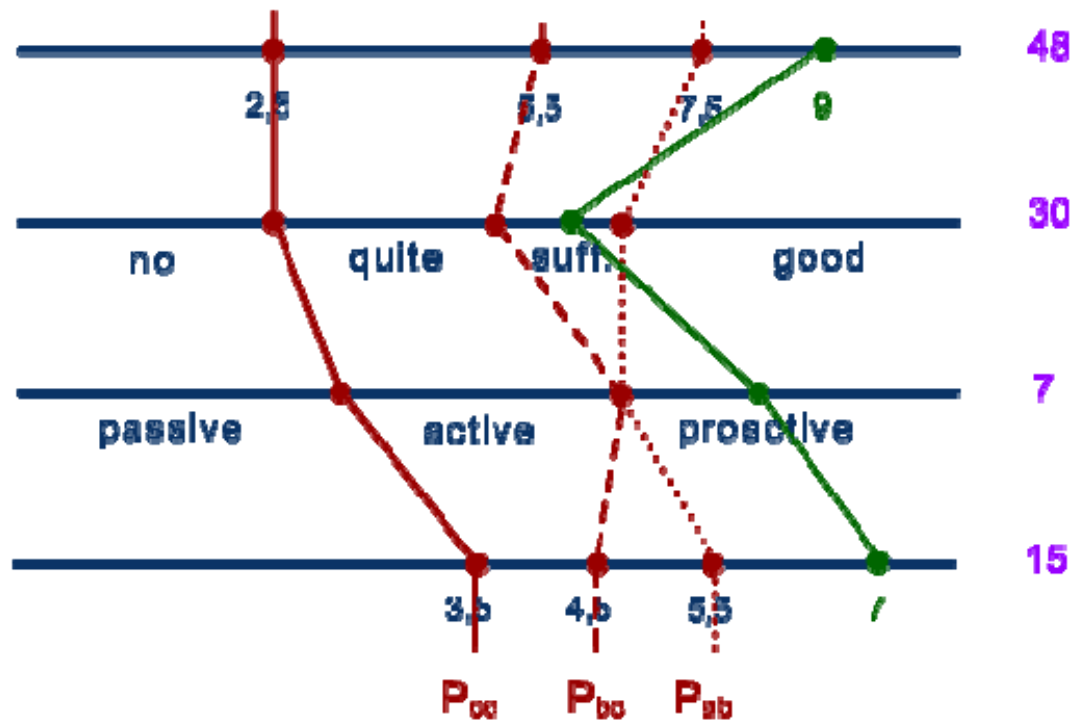
Without C1 no coalition  
with  $\sum w_i \geq \alpha$   
(\* = winning coalition)

| C1 | C2 | C3 | C4 | $\Sigma w_i$ |
|----|----|----|----|--------------|
| 1  | 0  | 0  | 0  | 48           |
| 1  | 0  | 0  | 1  | 63           |
| 1  | 0  | 1  | 0  | 55           |
| 1  | 0  | 1  | 1  | 70*          |
| 1  | 1  | 0  | 0  | 78*          |
| 1  | 1  | 0  | 1  | 93*          |
| 1  | 1  | 1  | 0  | 85*          |
| 1  | 1  | 1  | 1  | 100*         |

# Student

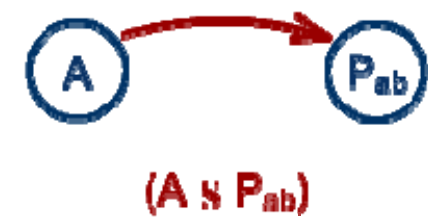


# Student A

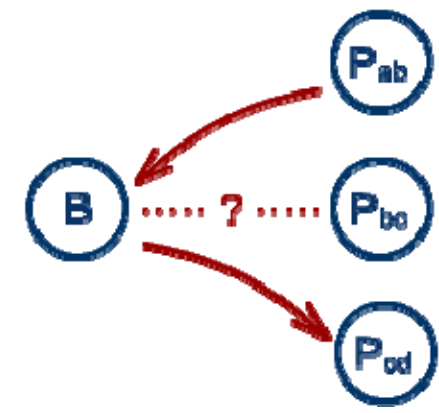
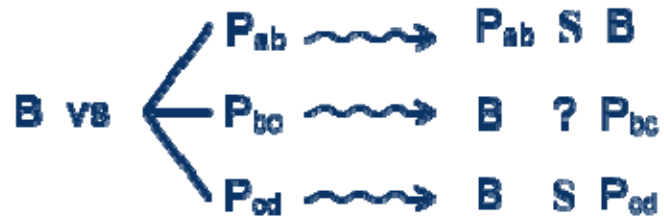
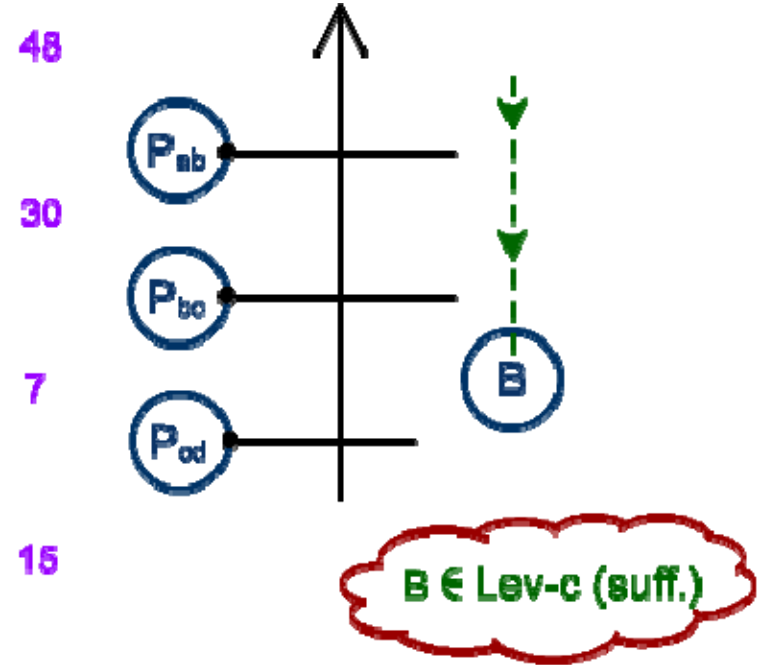
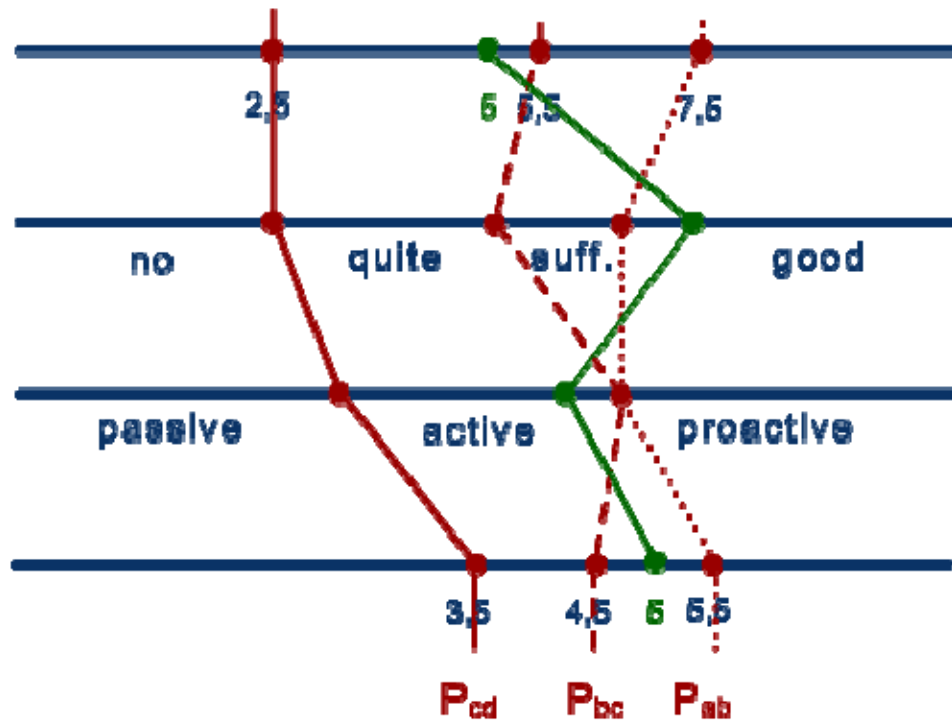


A ∈ Lev-a (excellent)

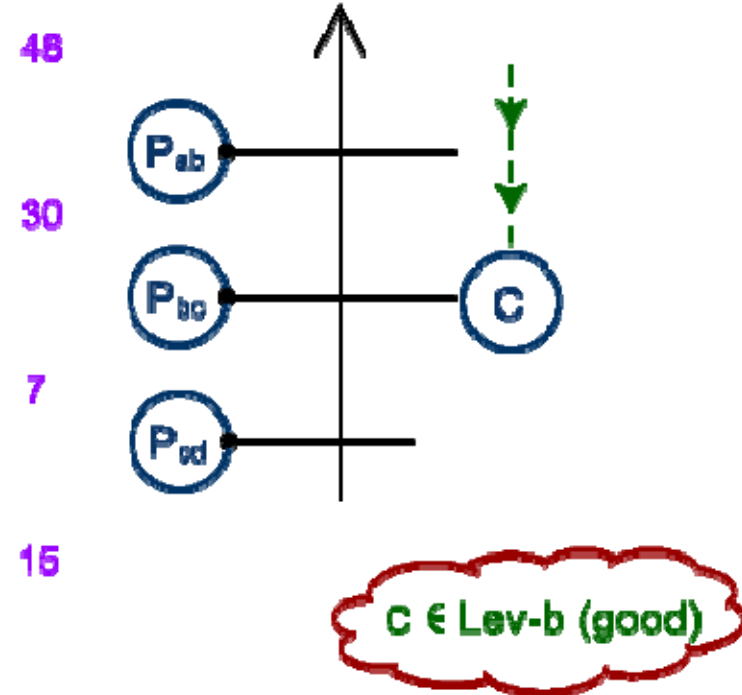
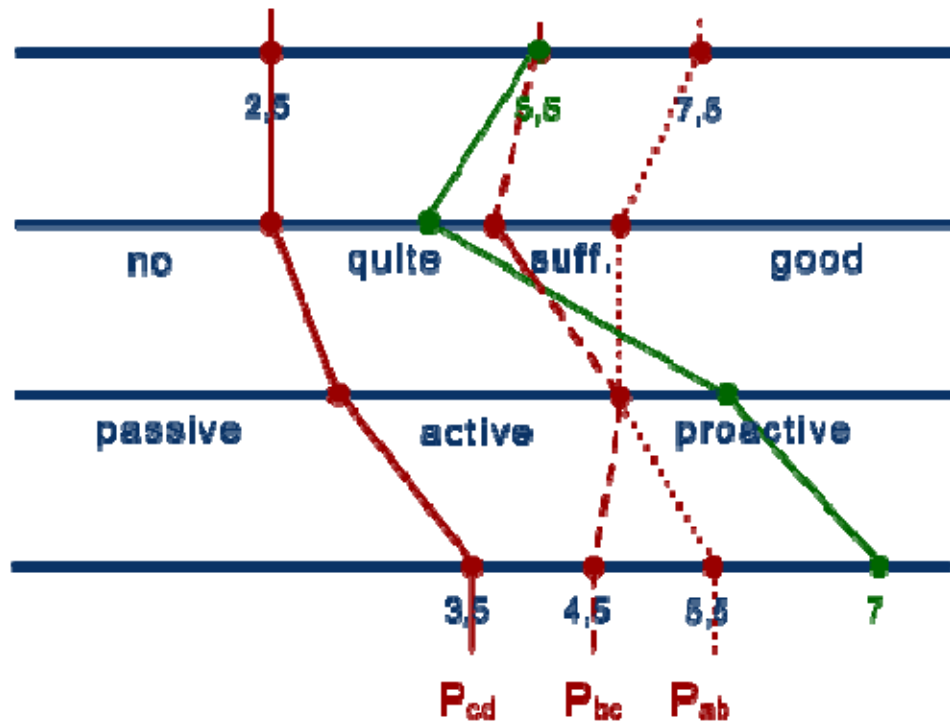
A vs P<sub>ab</sub> →  $\left\{ \begin{array}{l} \text{concordance} = 70 \\ \text{veto} > \text{NO} \end{array} \right.$



# Student B

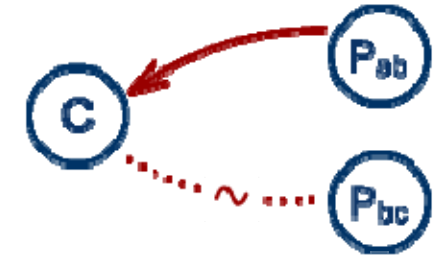


# Student C

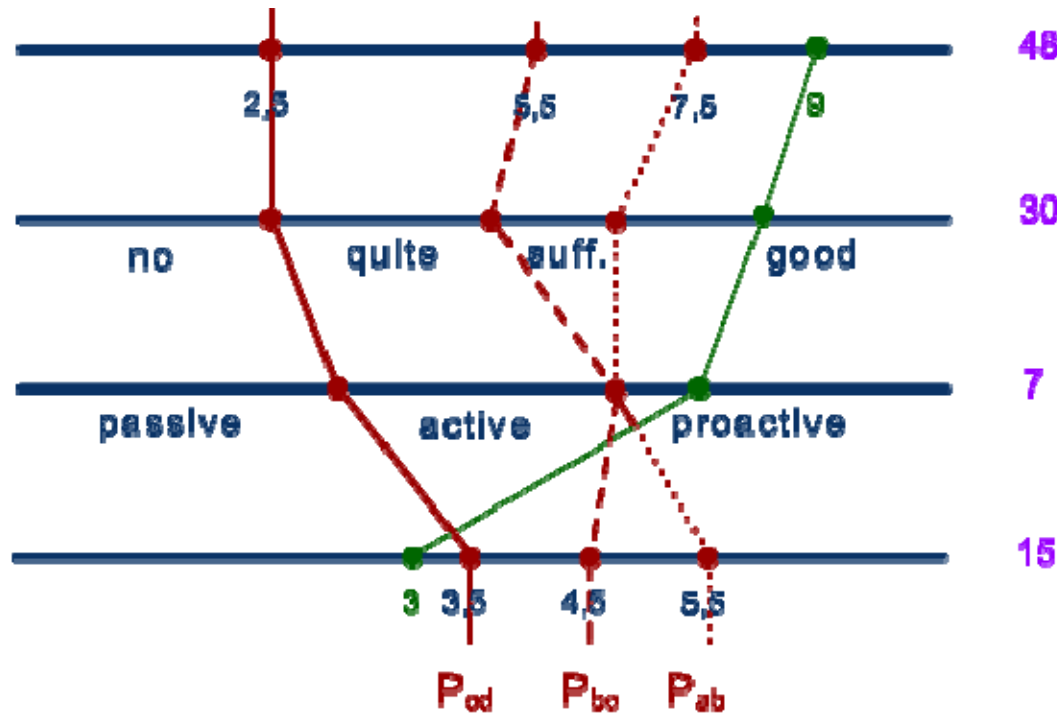


$C$  vs  $P_{ab}$   $\rightsquigarrow$   $P_{ab} \leq C$

$C$  vs  $P_{bc}$   $\rightsquigarrow$  it is  $\begin{cases} C \leq P_{bc} \\ P_{bc} \leq C \end{cases}$



# Student D



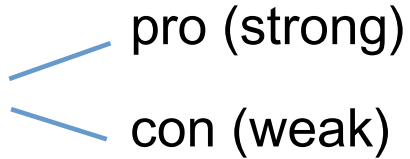
D ∈ Lev-d

D vs P<sub>||</sub> → veto (C4)



0. Data → the categories-levels (4) and the objects (40)
1. Choice of indicators-criteria (4)
2. Choice of weights (48, 30, 7, 15)
3. Definition of profiles (profiles = levels -1)
4. Definition of relations between  $K$  and  $P_{ij}$  (threshold, veto, ...)
5. Performances of students (A, B, C, D, ...)
6. Comparison between  $K$  and  $P_{ij}$  → rating of  $K$

## About the method

- The French school → Electre methods
- The main idea → outranking → reasons 
- Concordance, discordance, veto  
(high)                      (low)                      (no)
- Four cases:  $A \rightarrow B$ ,  $A \leftarrow B$ ,  $A \rightleftarrows B$ ,  $A ? B$
- The importance of incomparability
- Threshold ( $\alpha$ ,  $\beta$ , veto) and sensitivity → what happens if  $\alpha$  decreases ?
- Subjectivity (where ?)



## Winning coalition (more...)

- Three parties →
 

|        |     |
|--------|-----|
| Left   | 48% |
| Center | 3%  |
| Right  | 49% |

- Threshold =  $50\% + \varepsilon$
- What is the power of each party ?

- Coalitions:

| L | C | R | %   |
|---|---|---|-----|
| 0 | 0 | 0 | 0   |
| 0 | 0 | 1 | 49  |
| 0 | 1 | 0 | 3   |
| 0 | 1 | 1 | 52* |
| 1 | 0 | 0 | 48  |
| 1 | 0 | 1 | 97* |
| 1 | 1 | 0 | 51* |
| 1 | 1 | 1 | 100 |

\* = minimal coalition

$P_L = P_C = P_R = 1/3$

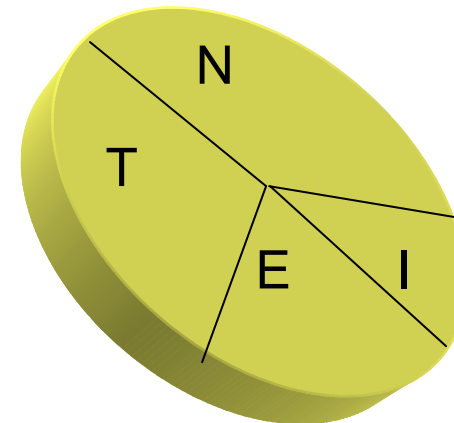


the same power



## The groups:

- North (N) 39%
- Irish (I) 10%
- Editors (E) 21%
- Telefonica (T) 30%

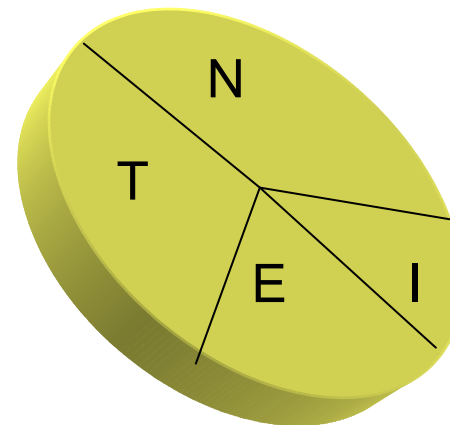


# Coalitions...



**Telecom**

... of who with who ?



## Coalitions

- **Winning coalition (WC)**: a coalition between some DM that permits the governance (that means the coalition overcomes the fixed threshold, usually  $50\% + \varepsilon$ )
- **Critical WC (CWC)**: a WC in which the defection of some DM – but only some – doesn't permit the governance
- **Swing vote (SW)**: in a CWC a SW is a vote that, if modified, determine the failure of the coalition (that means the impossibility of governance).

See also:

**Shapley index** (1953)

**Banzhaf index** (1965)

**Holler index** (public goods, 1982)

N → 39%, I → 10%, E → 21%, T → 30%

Threshold = 50% + ε

| N         | I         | E         | T         | %  | Coa<br>liz. |  |  | N         | I         | E         | T         | %   | Coa<br>liz. |
|-----------|-----------|-----------|-----------|----|-------------|--|--|-----------|-----------|-----------|-----------|-----|-------------|
| <u>39</u> | <u>10</u> | <u>21</u> | <u>30</u> |    |             |  |  | <u>39</u> | <u>10</u> | <u>21</u> | <u>30</u> |     |             |
| 0         | 0         | 0         | 0         | 0  | --          |  |  | 1         | 0         | 0         | 0         | 39  | --          |
| 0         | 0         | 0         | 1         | 30 | --          |  |  | 1         | 0         | 0         | 1         | 69  | CVC         |
| 0         | 0         | 1         | 0         | 21 | --          |  |  | 1         | 0         | 1         | 0         | 60  | CVC         |
| 0         | 0         | 1         | 1         | 51 | CVC         |  |  | 1         | 0         | 1         | 1         | 90  | (CV)        |
| 0         | 1         | 0         | 0         | 10 | --          |  |  | 1         | 1         | 0         | 0         | 49  | --          |
| 0         | 1         | 0         | 1         | 40 | --          |  |  | 1         | 1         | 0         | 1         | 79  | CVC         |
| 0         | 1         | 1         | 0         | 31 | --          |  |  | 1         | 1         | 1         | 0         | 70  | CVC         |
| 0         | 1         | 1         | 1         | 61 | CVC         |  |  | 1         | 1         | 1         | 1         | 100 | (CV)        |

N → ..., I → ..., E → ..., T → ...

N → 40%, I → 10%, E → 20%, T → 30%

Threshold = 50% + ε

| N         | I         | E         | T         | %  | Coa<br>liz. |  |  | N         | I         | E         | T         | %   | Coa<br>liz. |
|-----------|-----------|-----------|-----------|----|-------------|--|--|-----------|-----------|-----------|-----------|-----|-------------|
| <u>40</u> | <u>10</u> | <u>20</u> | <u>30</u> |    |             |  |  | <u>40</u> | <u>10</u> | <u>20</u> | <u>30</u> |     |             |
| 0         | 0         | 0         | 0         | 0  | --          |  |  | 1         | 0         | 0         | 0         | 40  | --          |
| 0         | 0         | 0         | 1         | 30 | --          |  |  | 1         | 0         | 0         | 1         | 70  | CVC         |
| 0         | 0         | 1         | 0         | 20 | --          |  |  | 1         | 0         | 1         | 0         | 60  | CVC         |
| 0         | 0         | 1         | 1         | 50 | --          |  |  | 1         | 0         | 1         | 1         | 90  | (CV)        |
| 0         | 1         | 0         | 0         | 10 | --          |  |  | 1         | 1         | 0         | 0         | 50  | --          |
| 0         | 1         | 0         | 1         | 40 | --          |  |  | 1         | 1         | 0         | 1         | 80  | CVC         |
| 0         | 1         | 1         | 0         | 30 | --          |  |  | 1         | 1         | 1         | 0         | 70  | CVC         |
| 0         | 1         | 1         | 1         | 60 | CVC         |  |  | 1         | 1         | 1         | 1         | 100 | (CV)        |

N → ..., I → ..., E → ..., T → ...