Preference elicitation for MCDA
Behavioral issues

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Introduction

- It is essential to have a thorough analysis of of how DMs actually behave to:
  - have a relevant practice of preference elicitation (as analyst)
  - design efficient preference elicitation procedures (as methodologist).
- One should analyze how DMs actually behave and answer to question that might intervene in elicitation procedures.

Introduction

- Study and analysis of how DMs actually make decisions.

Labels

- Behavioral Decision Research,
- Judgment and Decision Making,
- Psychology of preference and decision,
- Decision et rationality,
- Cognitive illusions,
- Heuristiques and biais.

Plan

- **Introduction**
- Bounded rationality
- Decision strategies
- Contingent decisions
- Beliefs on uncertainty
- Conclusion

Introduction

- Two theoretical approaches:
  - **Normatives theories** : hypothesize a rationality, i.e., an axiom system
  - **Descriptive theories (cognitive)** : express in a model actual mechanisms of real decision making (by real DMs).
- A fundamental question concerns the study of the complex relation between these two approaches,
- **Fundamental observation** : DMs behavior (either in real world or laboratory setting) do not respect axioms of normative theories, ...
- ... but their preferences, choices and decisions "*are neither rational, nor capricious*" Kahneman, Tversky, deviations from axioms are systematic and reproducible.

Introduction

- Normative theories are not (not always) the asymptote towards which real decision processes “converge” in ideal conditions (long reflection, exhaustive information, computations with paper and pen, computer, etc.),
- Often, spontaneous cognitive decision processes are in contradiction with standards in terms of rational decision.

Introduction

- However normative theories are essential to understand in detail phenomena observed in real life or laboratory setting experiments,
- These theories represent the standards for decision making cognitive theories ...

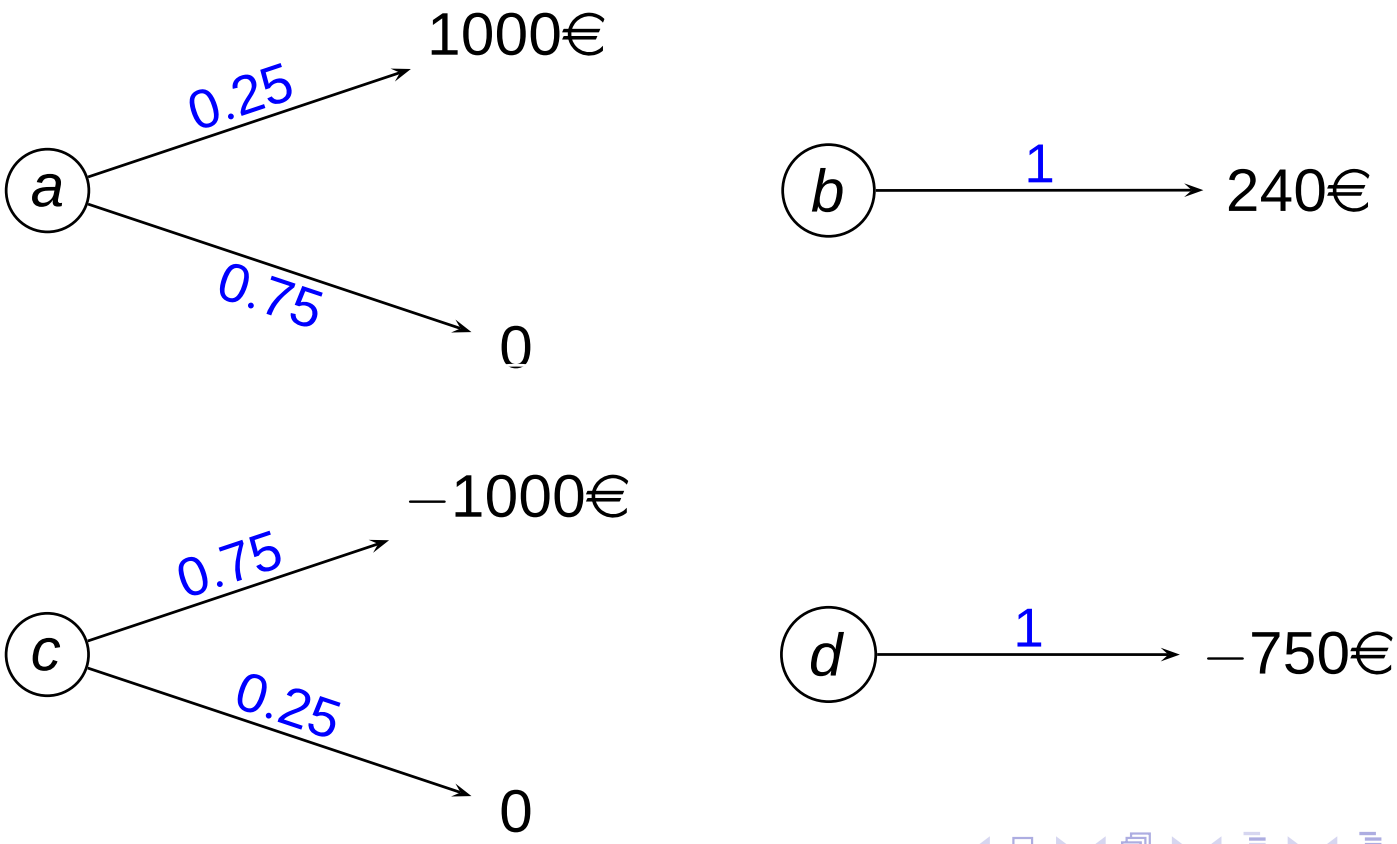
  ... and a legitimate idealization of the cognitive processes associated with decision making,
- Decision making processes can not be reduced to the rational norms but can not be analyzed independently from these norms,
- The comparison between normative principles and the actual decision behavior is fruitful.

Introduction

- Clear decision situations, controllable and reproducible, in which a small number of well defined options are proposed to an experimental subject,
- The subject is invited to make a choice.
- This choice is made by the subject according to a “reasoning” (conscient or not), after a short deliberation period,
- It is possible to propose rationality criteria, with which the subject choices can be compared,
- One observes that subjects contradict rationality criteria,
- but their decision behavior can be explained.

A first example

Subjects should compare a to b then c to d :



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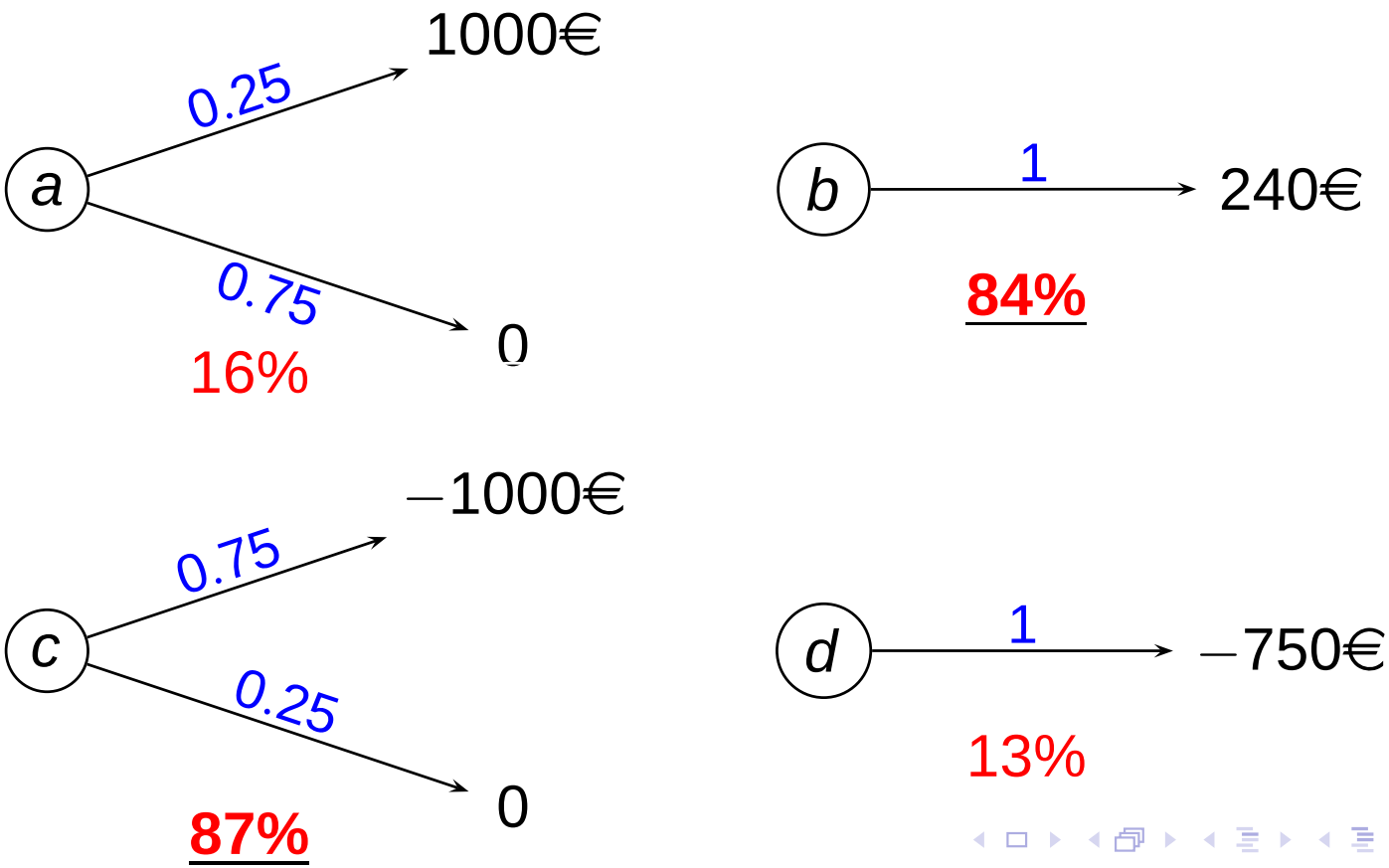
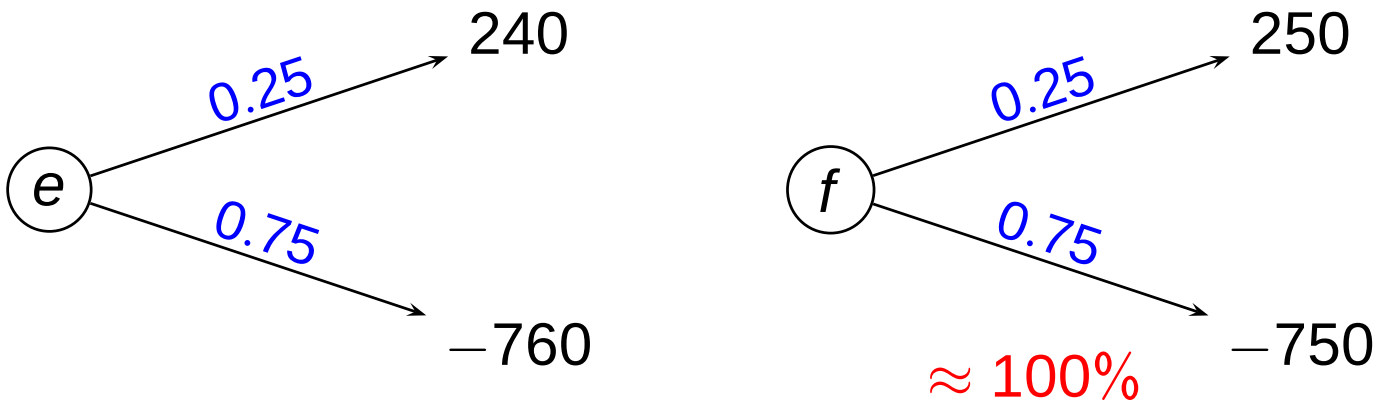


A first example

- These same subjects should also compare e to f.



- but $f = a \oplus d$ and $e = b \oplus c$
- The "aggregation" of two options is not neutral from a cognitive point of view.
- Question : what are the cognitive processes that lead a majority of subjects to prefer b to a and c to d whereas $a \oplus d$ dominates $b \oplus c$?

Another example [Tversky, Kahneman 1986]

Stochastic dominance principle : if A is preferred to B in each state of nature, then A should be chosen.

- Subjects are to choose between 2 loteries (we draw a ball in an urn that contains 100 balls), the number of balls of each color is known.

A 90 black (0), 6 red (45), 1 green (30), 1 blue (-15), 2 yellow (-15)

B 90 black (0), 6 red (45), 1 green (30), 1 blue (-10), 2 yellow (-15)

- All subjects choose B : dominance is obvious.

Another example [Tversky, Kahneman 1986]

Same situation with the urns :

- **C** 90 black (0), 6 red (45), 1 green (30), 3 yellow (-15).
- **D** 90 black (0), 7 red (45), 1 green (-10), 2 yellow (-15).
- 58% of subjects choose D.
- Yet urn D dominates urn C.
- C is attractive, as there is only one case of loss, whereas there exist two for D.

$\Rightarrow$ The dominance effect seems “efficient” only if dominance appear clearly, without any “mental transformation” of options.

Another example [Tversky, Kahneman 1986]

- **A**:90 black (0), 6 red (45), 1 green (30), 1 blue + 2 yellow (-15)
- **C** : 90 black (0), 6 red (45), 1 green (30), 3 yellow (-15).

- **B**: 90 black (0), 6 red + 1 green (30), 1 blue (-10), 2 yellow (-15)
- **D** : 90 black (0), 7 red (45), 1 green (-10), 2 yellow (-15).

- A is strictly equivalent to C !
  and B is strictly equivalent to D !
- but 58% of subjects state : APB et CPD,
- Question : what are the underlying cognitive processes that yield such preference reversal?

Another example [Tversky, Kahneman 1986]

Interpretation

- "Transparence" principle: dominating alternatives are chosen only if dominance appears clearly,
- Compacity effet: an option of disjunctive nature, considered as a whole, has a lower "decision weight" than the sum of the weights associated with each of its components: of $e = (e_1 \vee e_2 \vee e_3)$ then $\pi(e) < \pi(e_1) + \pi(e_3) + \pi(e_3)$
- Asymmetry between gains and losses: a potential loss of x is considered overcomes a possible gain of x.

A third example [Tversky 1969]

Transitivity principe for P (and I): $(aPb \wedge bPc) \Rightarrow aPc$

- A subject is to choose between:
  - a: a gain of 5\$ with a probability of $\frac{7}{24}$
  - b: a gain of 4.75\$ with a probability of $\frac{8}{24}$

$\longrightarrow$ a majority of subjects prefer a to b.

- a second choice involves:
  - b: a gain of 4.75\$ with a probability of $\frac{8}{24}$
  - c: a gain of 4.5\$ with a probability of $\frac{9}{24}$

$\longrightarrow$ a majority of subjects prefer b to c.

... the sequence continues with choices in which outcomes decrease and probabilities increase.

A third example [Tversky 1969]

	Probability	Gain	Expected gain
a	$\frac{7}{24}$	5$	1.46 $
b	$\frac{8}{24}$	4.75$	1.58 $
c	$\frac{9}{24}$	4.5$	1.69 $
d	$\frac{10}{24}$	4.25$	1.77 $
e	$\frac{11}{24}$	4$	1.83 $

- A majority of subjects express preferences such that aPb, bPc, cPd and dPe
  ... but when comparing a to e, ePa.
- Subjects, even after an explicit verification, are reluctant to admit that they have intransitive preferences.

Example 4

Cancelation principle

- A subject is to choose between:
  - a : a certain gain of 10€.
  - b : a gain of 15€ with a probability of 0.8

$\rightarrow$ 78% of subjects prefer a.

- the second choice deals with:
  - c : a gain de 10€ with a probability of $\frac{1}{4}$
  - d : a gain de 15€ with a probability of $\frac{1}{5}$

$\rightarrow$ 58 % of subjects prefer d to c.

- The choice between c and d corresponds to the choice between a and b in which probabilities are divided by 4.

Example 5

- Two therapies are available to treat a type of cancer : one is “radical”, with risks but also possibility of major success; the other one “local”, without risk, but without any hope of great success.
- We consider a 40 years old patient who (without treatment) has life expectancy of about 3 months.
- A group of doctors answered the following questions: which of the following treatment would you choose, knowing that...

Example 5

Choice 1

- Treatment A : 20% chance of immediate death (as a consequence of surgery) and 80% chance of a normal existence (life expectancy : 30 years)
- Treatment B : Certainty of a normal existence, with a life expectancy of 18 ans

- 65% of subjects prefer treatment B
  $\rightarrow$ “Certainty effet”: a certain result, compared to a (only) possible result, has a increased subjective value.

Example 5

Choice 2

- Treatment C : 80% chance of immediate death and 20% chance of a normal existence, with a life expectancy of 30 years.
- Treatment D : 75% chance of immediate death and 25% chance of a normal existence, with life expectancy of 18 years.
- Result : 68% of subjects prefer C
  As in the monetary problem, probabilities have been divided by A as compared to A and B.
  $\rightarrow$ The transgression of the cancelation principle is clear.

Example 5

With this type of cancer, only one patient over 4 reacts positively to treatments. If the cancer reveals not treatable, the life expectancy is about 3 months. If it appears to be treatable, you can choose two treatments: Choice 3

- Treatment E : 20% chance of immediate death and 80% chance of a normal existence, with a life expectancy of 30 years.
- Treatment F : Certainty of a normal existence, with a life expectancy of 18 years.

- Result : 68% of subjects prefer F
- Preferences are the same as for A and B, but final probabilities are identical to the choice between C and D... and not to the choice between A and B.

Example 5

- Mentally subjects suppose that cancer is treatable, and decide accordingly.
- Subjects do not consider what is “upstream” in their choice.
- Subjects tend to boil down the situation to the choice between A and B, effective probabilities (final ones) do not count.
- On “erase” common composants, but not according to the normative principle. There is a form of “mental rewriting”.

Mental Editing

- Subjects are to consider situations of Mr Dupont and Mr Laffont.
- Mr Dupont and Laffont end up with the same economic situation, but subjects consider they are not equally happy/unhappy.
- Mr Dupont wins 20€ in a lottery at his office in the morning, and 80€ in a lottery in his neighborhood in the evening.
- Mr Laffont wins 100€ in a city lottery.
- Which one is the most happy?
- A majority of subjects judges that Mr Dupont is more happy than Mr Laffont.
  $\longrightarrow$ subjects operate a dissociation of gains

Example 5

- Mr Dupont should pay 200€ as a local tax and 800€ as a federal tax.
- Mr Laffont should pay 1000€ as a federal tax.
- Which one is the most unhappy?
- A majority of subjects judges that Mr Laffont is more unhappy than Mr Dupont
  $\rightarrow$ subjects amalgamate losses.

[Thaler, Shafir 1999]

- You bought, some years ago, a case of 12 bottles of a good Bordeaux wine which you payed 25€ each bottle.
- Today, a bottle costs 75€.
- You decide to open one of these bottles to have a nice dinner with some friends.
- What is, according to you, the monetary value of this consumption?

[Thaler, Shafir 1999]

1. Nothing : I had this bottle in my cellar.
2. 25€, its initial price.
3. 25€ plus interests and inflation since you bought it.
4. 75€, its current market price.
5. A gain of 50€ (the difference between its initial and current price)

Answers for (american) subjects:

1	2	3	4	5
30%	18%	7%	20%	25%

[Thaler, Shafir 1999]

- Subjects have a strong opinion and think that all autres answers are nonsense,
- However the distribution is almost uniform...
- It is difficult to “evaluate” economically this consumption.
- More than 50% of subjects consider that this consumption does’nt cost anything, or constitues a gain!

Introduction

- In this research field, the scientific approach is empirical,
- Subjects are placed in “controlled” experimental conditions,
- Analyze observed behavior according to “canons rational standards”,
- Explain possible divergences,
- Contributes to a better understanding of real decision making processes,
- Important to design relevant preference elicitation techniques.

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Hommo economicus

- How should a DM react to a decision task?
- An answer consists in considering that DMs react rationally to decisions of increasing complexity,
- *Hommo economicus* is supposed to have the "*knowledge of the relevant aspects of the environment which, if not absolutely complete, is at least impressively clear and voluminous. He is assumed also to have a well-organized and stable system of preferences and a skill in computation that enables him to calculate, for the alternative courses of action available to him, which of these will permit him to reach the highest attainable point on his preference scale*",
- Many economists consider such behavior as a normative definition of rationality, some consider it also as a descriptive model of observed choices.

Bounded rationality

- "*Human rational behavior is shaped by a scissors whose two blades are the structure of the task environments and the computational capabilities of the actor*", H. Simon,
- For Simon (1955), the study of actual decision behavior requires to analyse how perceptive, cognitive and learning aspects lead decision behavior to differ from the one predicted by the model of "*hommo economicus*",
- The limited computation abilities of DMs interact with the structure of the decision task to produce a "*bounded rationality*".

Bounded rationality

- As consequence of bounded rationality, Simon suggests that decision behaviors do not even correspond to an approximation of those predicted by normative models.,
- The choice among a finite set of alternatives results from the identification of an "*sufficiently good*" alternative,
- Rather than choosing the alternative that maximizes some utility function, DMs select the first alternative that is sufficiently good (satisficing).

Bounded rationality

- Limitations concerning information manipulation and computation can explain that preferences on objects are often constructed and not only revealed during a choice or judgment.
- The concept of constructed preferences is that DMs do not have pre-defined values on most alternatives under consideration, ...
- On the contrary, they construct their preferences on the spot when necessary (i.e., when they have to evaluate/compare alternatives),
- This view sheds doubts on the idea that observed preferences result from a “master list” present in memory,
- Moreover, this implies that expressed judgments and choices do not result from the application of the application of an invariant algorithm (such as expected utility computation).

Bounded rationality

- The constructive nature of preferences induce that observed choices are highly contingent to factors related to the decision task and context,
- Factors related to the decision task are the characteristics of the decision problem (response mode, information format, ...)
- Factors related to the context (e.g., similarities of alternatives) are linked to the value of alternatives under consideration,
- Due to bounded rationality, DMs are insensitive to factors to which they should be normatively, and are sensitive to normatively irrelevant factors.

Bounded rationality

- Factors related to the decision task and context make some aspects of the problem more salient and can induce different way to use information,
- Seemingly uninformative aspects of a decision problem can, at least partly determine observed preferences.

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Weighted additive value

- Consists in solving les conflits entre multicriteria alternatives considering substitution rate among criteria,
- $v(x) = \sum_{i=1}^{n_{crit}} w_i x_i$, with x_i the value of x on criterion i and w_i the "weight" of criterion i,
- Consistent with normative choice procedures [Keeney, Raiffa 76],
- This procedure uses all available information, solves explicitly solves conflict through the identification of substitution rates and selects the alternative x^* which maximizes $v(x^*)$,
- The additive form "*so completely dominates the literature that is has no competitors*" (Edwards, Tversky 67).

Weighted additive value

- How do DMs understand the weights w_i in this context?
- Sometimes DMs confer to w_i a semantic ("*local*") linked to the range of the scale for the alternatives in consideration (Goldstein 1990) ...
- ... sometimes a semantic ("*global*") independent from the scale (Beattie, Baron 1991).

Probabilistic choice models

- Some procedures propose to analyze choice behavior as a stochastic process,
- When comparing a to b, $p(a, \{a, b\})$ defines the probability to choose a in the set $\{a, b\}$,
- The probability $p(a, \{a, b\})$ "evaluates" the de degree of preference of a over b,
- The Logit Multinomial model defines the probabilities to choose a_i in $\{a_1, a_2, \ldots, a_n\}$ by: $p(a, \{a_1, \ldots, a_n\}) = \frac{e^{v(a_i)}}{\sum_{j=1}^{n} e^{v(a_j)}}$
- where $v(a_i) = \sum_{k=1}^{n_{crit}} w_k g_k(a_i) + b_i$ où $g_k(a_i)$ is the value of a_i on criterion k, w_k a scaling constant, and b_i a constant taking into account aspects of a_i absent from the criteria

$\Rightarrow$ Not independent vis-à-vis a third alternative.

The satisficing heuristic

- One of the first heuristics described by Simon 1955,
- Alternatives are considered one after another, in a random way,
- The value on each criterion of the current alternative is compared to a predefined level (aspiration level),
- The alternative is rejected if its value on a criterion does not meet the aspiration level,
- The first alternative which passes this test is chosen,
- If no alternative can be chosen, the levels are revised and a new iteration starts.

The satisficing heuristic

- The chosen alternative depends on the order by which alternatives are considered ,
- no relative comparison of a and b is required,
- If a and b meet the levels on all criteria, the first alternative observed is chosen.

The majority heuristic

- Majority of confirming dimensions heuristic [Russo, Dosher 83],
- This heuristic compares two alternatives on each attribute and selects the one which is better on a majority of criteria (possibly weighted),
- The preferred alternative is then compared to the following one, ...
- The final choice depends on the order by which alternatives are considered,
- The pairwise comparison graph is not necessarily transitive (Condorcet).

The elimination by aspect heuristic

- A minimum required level is defined on the most important criterion,
- Alternatives that do not reach the required level on this criterion (on this aspect) are removed,
- In [Tversky 72] the importance order was'nt explicitly considered, the choice of the criterion was probabilistic (probability related to the importance),
- The process goes on with the second important criterion, until only one alternative remains,

The elimination by aspect heuristic

- This heuristique violates the normative principe according to which all the relevant information should be taken into account in the decision,
- In this heuristic, a limited part of the information is used,

→ A process grounded in a “bounded rationality” is typical is choice heuristics.

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Procedure invariance

- Procedure invariance is a fundamental principe of rational decision making,
- "*strategically equivalent ways of eliciting a DM's preference should result in the same revealed preferences*",
- Different response modes:
  - *Choice* : which alternative do you prefer among (x_1, x_2) and (y_1, y_2)?
  - *Matching* : determine the missing value so that$(x_1, ?)I(y_1, y_2)$.
  - *Willingness to pay* : What is the maximum amount you are willing to pay to obtain (x_1, x_2) ?
  - *Willingness to sell* : What is the minimum amount for which you would accept to give up (x_1, x_2) ?
  - *Rating* : Rate the attractivity of (x_1, x_2) on a [0,100] scale.
  - ...

Procedure invariance

- [Tversky, Sattath, Slovic 88] observed systematic preference reversals between choice and matching,
- Example : Consider security programs for roads (casualties, cost in M€),
- Choice task : Which programm do you prefer?
  a = (570, 12€), b = (500, 55M€)
- A majority of subjects choose b
  $\Rightarrow$ saving 70 lives is more important than 43M€.

Procedure invariance

- Matching task : Determine x so that programs a and b are indifferent.

  a = (570, 12M€), b = (500, xM€)

- A majority of answers are such that $x < 55$M€
  $\Rightarrow$ a difference in cost $<$43M€ is equivalent to saving 70 lives
- The tradeoff"*lives saved/cost*" (weighting of criteria) varies according to the response mode,
- Proeminence effect: the most important criterion is "over-weighted" when preferences are elicited through choices by matching questions: choice anwers are closer to lexicography than matching.

Procedure invariance

- the two tasks implicitly encourage different heurstics/reasoning,
- Choices leads to qualitative reasoning of ordinal nature (lexicography without tradeoffs),
- Matching implies a cardinal reasoning focussing on tradeoffs,
- More generally, [Tversky, Sattath, Slovic 88] talk about "*strategy compatibility*" to state that subjects adopt a strategy compatible with the response mode,
- Choice implies an ordinale answer and refers ordinal arguments based on attribute order.

Procedure invariance

- [Fischer, Hawkins 93] explain proeminence effect by a notion of (*scale compatibility*),
- A stimulus is more taken into account when the answer is given on a scale similar to the stimulus,
- Ex: The cost criterion is more salient in a matching task than in a choice task.

Descriptive invariance

- Different representations of a same choice problem should lead to the expression of equivalent preferences,
- Many experiments showed that DMs fail to respect such principle expressing varying preferences according to the way the the decision problem is posed,
- Two types of descriptive (in)variance effets have been studied:
  - $\rightarrow$ Framing effects,
  - $\rightarrow$ Information presentation effect.

Descriptive invariance

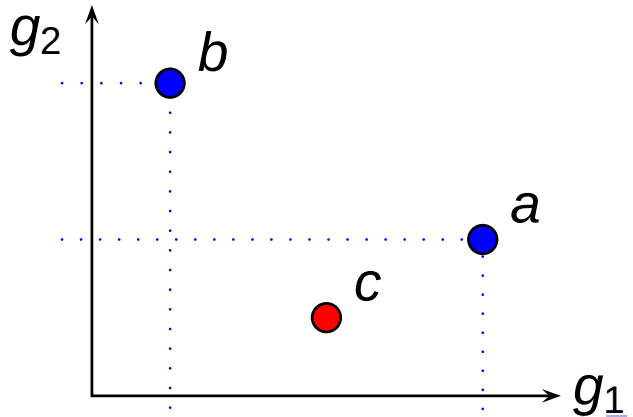
- Preferences can change according to the way the decision problem is frames,
- [Kahneman, Tversky 1981] showed that a choice among cancer treatments strongly depends on the framing (possibility of death or survival),
- Differences appears whether the framing induce DMs to encode outcomes as gains or losses,
- Gains and losses are subjectively perceived differently (a difference perceived as a gain is weighted more than the same difference in terms of loss),
- Mental accounting effect (Theatre ticket example)

Descriptive invariance

- [Slovic 1972] suggests that "*decision makers tend to use information in the form it is displayed, without transforming it, as a way to conserve cognitive effort*",
- [Russo 77] showed that prices are more considered in a store when products are sorted by price than when they are not,
- [MacGregor, Slovic 86]showed that a criterion is more taken into account in a binary choice when its visual presentation is "saillant".

Asymmetric dominance effects

- Response mode and framing effects are related to the decision task,
- The decision context can also have a influence (nature of the set of alternatives),
- A standard effect is the "*asymmetric dominance effect*",
- Consider a ternary choice task, 3 alternatives a, b et c, s.t. $a \Delta c$ and $\neg b \Delta c$



Asymmetric dominance effects

- Considering 3 alternatives a, b and c where $a \Delta c$ and $\neg b \Delta c$, preference reversals are observed between the choice in $\{a, b, c\}$, and in $\{a, b\}$,
- Frequent statements: choice($\{a, b\}$)=b and choice($\{a, b, c\}$)=a,
- The presence of c provides a comparative advantage to a when compared to b, and influences the relative value of a and b,
- Change in reference point (reference dependent model).

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Reasoning about probability

- $p(e)$ denotes the probability for e to become true,
- Axioms on probabilities are:
  1. $p(e) \in [0,1]$, $p(e) = 0 \Leftrightarrow$ "e is impossible", $p(e) = 1 \Leftrightarrow$ "e is certain".
  2. $p(\neg e) = 1 - p(e)$,or if e and $\neg e$ is all what can happen $p(e) + p(\neg e) = 1$.
  3. for two distincts events e_1 et e_2, the probability of the disjunction is $p(e_1 \vee e_2) = p(e_1) + p(e_2)$, in particular, if $e_1, e_2, \ldots, e_n$ corresponds to all states $\sum_{i=1}^{n} p(e_i) = 1$.
  4. if e_1 and e_2 2 are independent :$p(e_1 \wedge e_2) = p(e_1) \times p(e_2)$.
  5. $p(e_1 \vee e_2) = p(e_1) + p(e_2) - p(e_1 \wedge e_2)$.
  6. $p(e_1/e_2) = \frac{p(e_1 \wedge e_2)}{p(e_2)}$.

Reasoning about probability

- Each of the axiom, considered independently is “unquestionable”,
- However, in many situations, subjects choose in contradiction with these axioms,
- Often, subjects remain convinced that their choice is correct, and that nothing is incorrect in their reasoning,
- many works studied how intuitive judgments of probability deviates from predictions of the normative laws of statistics.

Reasoning about probability

- [Kahneman, Tversky 73] hypothesized that DMs use heuristics to make probability judgments.,
- Availability: evaluating the probability of an event is grounded on the ease by which an instance of this event can come to mind,
- Representativeness: the probability of an event is evaluated by the appreciation of the degree to which this event corresponds to a mental model of this class of events (probability of success of a new product $\approx$ similarity to success products).

Reasoning about probability

- Anchoring/Adjustment : in the evaluation process a starting data/answer plays the role of starting value.An adjustment process (using additional information) revises the initial value/answer. Adjustments are usually insufficient.

- A manager is to forecast sales for next year on the basis of the sales of the current year. Insufficient adjustment means that the sales forecast can refer more to this year sales than to variation anticipation.

Reasoning on probabilities

Russian roulette (Zeckhauser)

- You have a six-shooter revolver and you are to play a cruel and fatal game: shoot yourself,
- You can pay to remove a bullet from the cylinder,
- How much are you willing to pay to reduce the number of bullets:
  1. from 6 to 5
  2. from 4 to 3
  3. from 1 to zero
- The average amount in case **2** is the lowest (by far),
- However, each situation increases the your probability to survive by $\frac{1}{6}$.

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Lottery portfolio

- Samuelson (1963), Redelmeier & Tversky (1992), Kahneman & Lovallo (1993)
- Theory imposes that, if a lottery has an expected value of v, then n repetitions of this lottery have an expected value nv (if time and cash limitation are excluded, ...),
- It is not rational *rationnel* to refuse a lottery and to accept n repetitions of this lottery (or the reverse).
- Such situation is frequently observed: expressed preferences do not match normative principles.

Lottery portfolio

A portfolio is more attractive than each component lottery:

- A majority of subjects refuse the lottery (+200€, 0.5, -100€) mais ...
- ... but accept a portfolio of 6 or 10 repetitions of this lottery, when the distribution of possible gains and losses is explicit,
- for $n = 3$, the portfolio :
(600€, 0.125; 300€, 0.5; 0€, 0.25; -300€, 0.125) ,
- ... is perceived as attractive.
- When n increases (up to 6 - 10 repetitions), the portfolio is increasingly attractive.

Lottery portfolio

6 is not 5+1 (Shafir & Tversky 1993)

- Subjects who did accept a portfolio of 6 repetitions of the lottery (25$, 0.5, -10$),
- ... do not accept a 6th trial, when they are invited to imagine that the 5 first trial have been played.

Other example : a normatively attractive lottery

- A standard subject is indifferent to the following proposal: (250€, 0.5, -100€) expected value of 75€, but subjective utility null, as the subject is indifferent,
- The portfolio of 2 repetitions is (500€, 0.25; 150€, 0.5; -200€, 0.25), which is perceived as attractive,
- For a “large” portfolio (6 repetitions), experimentally, the value of each trial after the third one is $\sim$45€.
- Six times zero equals $\sim$270€.

Sub-additivity (compact-uncompact)

Groupe A :
Mr Dupont just bought a new car. We do not know anything about Mr Dupont. Grounded on your knowledge of the proportions of cars brands, evaluate the following probabilities: Probability that Mr Dupond bought a:

Brand	Probability
Renault	.....
Citroën	.....
Fiat	.....
Honda	.....
other	.....

Sub-additivity (compact-uncompact)

Groupe B : same situation, same question.

Brand	Probability
Renault	.....
Citroën	.....
Fiat	.....
Honda	.....
Toyota	.....
Volkswagen	.....

Brand	Probability
Saab	.....
Volvo	.....
Lancia	.....
Mercedes	.....
Ford	.....
other	.....

Sub-additivity (compact-uncompact)

- Group A assigns to “other” a probability much lower than the sum of probabilities assign to the corresponding brand by group B.
- Could we conclude that subjects do not have in mind the list of all possible car brands ? yes, ...
- But there is some more. The same result is obtained with car professionals (dealers, journalists etc.).

Sub-additivity of expert mecanics

This morning, Mr Dupont's car could not start. He had to call a mechanic. What are the probabilities of the breakdown causes:

group A	
Cause	Probability
No gas	.....
Low battery	.....
Carburation defect	.....
Other causes	.....

group B	
Cause	Probability
No gas	.....
Low battery	.....
Carburation defect	.....
Starter broken	.....
Electrical problem	.....
sparking plug out of order	.....
Alternator out of order	.....
Other causes	.....

sub-additive probabilities expressed by doctors

- Experiment described in [Redelmeier *et al.*, 1996].
- Doctors from the Toronto University Hospital are invited to evaluate the probability of an outcome for a patient described by a real clinical file.
- Each doctor is to evaluate the probability of one among the 4 possible outcomes (randomly selected).
  1. the patient will die during the current hospitalization,
  2. (s)he will survive, but will die within 5 years,
  3. (s)he will survive 5 years, but will die within 10 years,
  4. (s)he will survive at least 10 years,
- These four cases describe all possible event; The sum of probabilities should not diverge too much from 1.

sub-additive probabilities expressed by doctors

- An over-estimation of the probability of each of the possible outcome is observed (when compared to epidemiologic standards representing the patient characteristics).
- The sum of mean values of probability estimations is 1.64.
- Doctors, when informed *a posteriori*, are reluctant to believe in such result.
- How to explain such over-estimation ?
- Each outcome is equivalent to the negation of all other three (probability of (1) is equivalent to the probability of $(\neg 2 \wedge \neg 3 \wedge \neg 4)$.
- Being implicit and compact, the disjonctive decomposition is not mentally salient, does not spontaneously come to the mind.
- Only the outcome (1) is explicit and consciously, it is hence over-estimated.

Plan

- Introduction
- Bounded rationality
- Decision strategies
- Contingent decisions
- Beliefs on uncertainty
- **Conclusion**

Implications in terms of preference elicitation

- DMs do not behave as normative theories expect them to do,
- When making preference judgements, DMs consider normatively irrelevant information and ignore normatively relevant information,
- Experiments strongly supports the idea that preferences are (at least partly) constructed while expressing these preferences,
- ... rather than revealed from a stable master list in the DM's mind,
- All "irrational" observed behaviors constitutes a "*stone in the shoe*" of the analyst,