Preference elicitation for MCDA
Robust elicitation of ranking model

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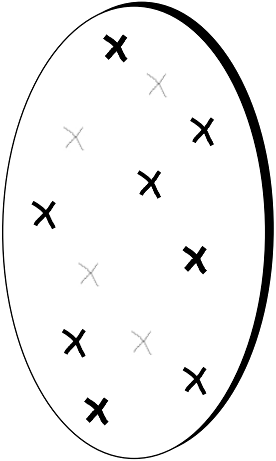
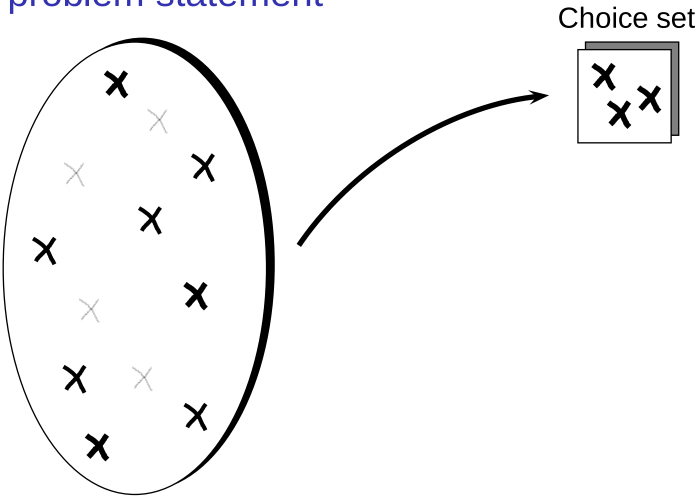
Problem statements

- Choosing, from a set of potential alternatives, the best alternative or a small sub set of the best alternatives
- Sorting alternatives to pre-defined and (ordered) categories
- Ranking the alternatives from the best to the worst (the ranking can be complete or not)

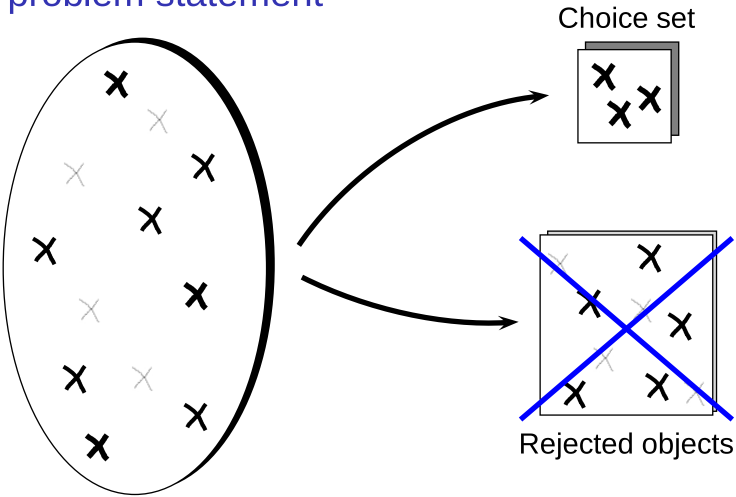
Choice problem statement



Choice problem statement



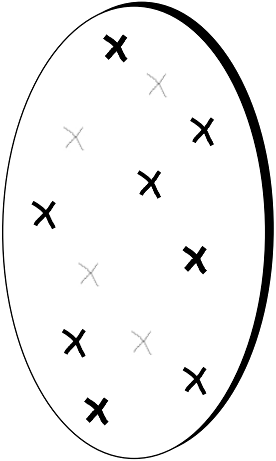
Choice problem statement



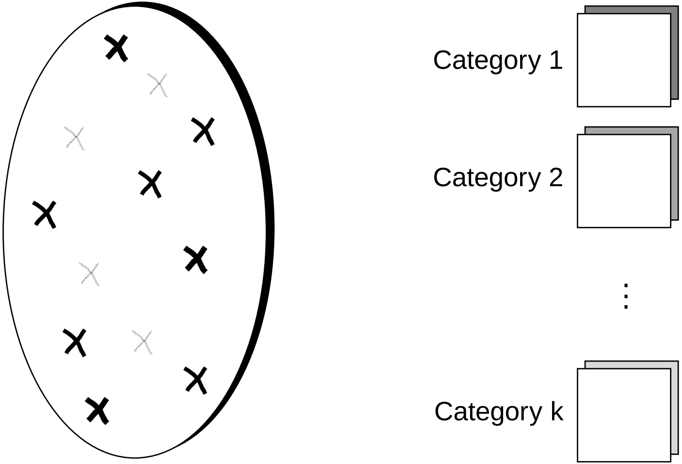
Problem statements

- Assigning alternatives to pre-defined and order categories

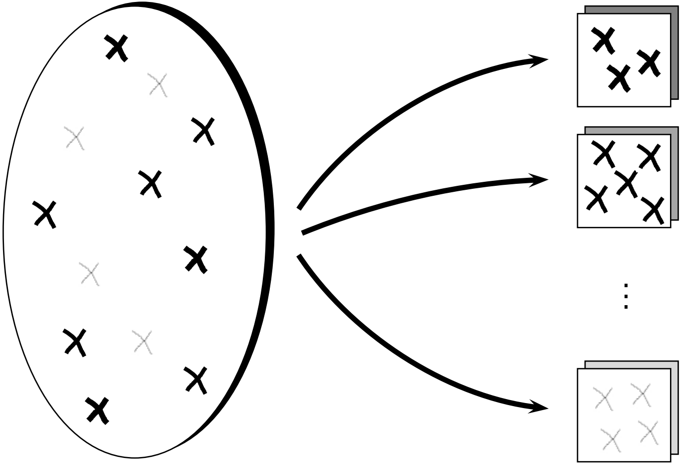
Sorting problem statement



Sorting problem statement



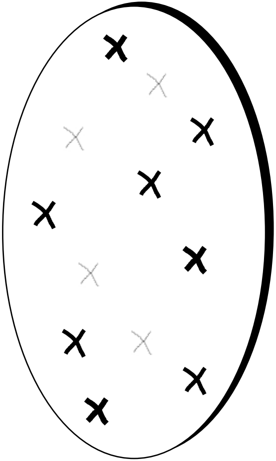
Sorting problem statement



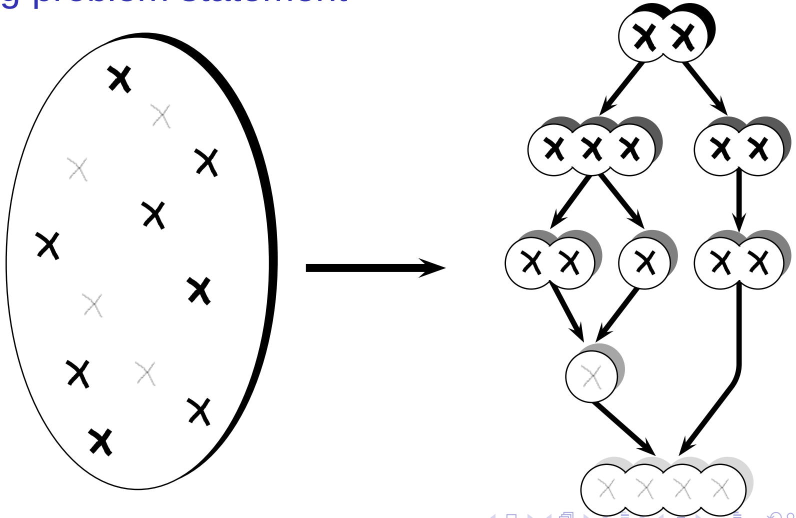
Problem statements

- Ranking the alternatives from the best to the worst (the ranking can be complete or not)

Ranking problem statement



Ranking problem statement



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Ordinal regression paradigm

- Traditional aggregation paradigm: The criteria aggregation model is first constructed and then applied on set A to get information about the comprehensive preference
- Disaggregation-aggregation (or ordinal regression) paradigm: Comprehensive preferences on a subset $A^R \subset A$ is known a priori, and a consistent criteria aggregation model is inferred from this information to be applied on set A.

Ordinal regression paradigm

- In UTA^{GMS}, the preference model is a set of additive value functions compatible with a non-complete set of pairwise comparisons of reference alternatives and information about comprehensive and partial intensities of preference
- We focus on the ranking problem statement (but the ideas can be extended to choice and sorting)

Contents

UTA-GMS/GRIP

- Robust elicitation of a ranking model,
- Preference model = set of monotone additive value functions,
- Preference information = pairwise comparisons of alternatives/evaluation vectors and information about intensities of preference.

Contents

Elementary notation

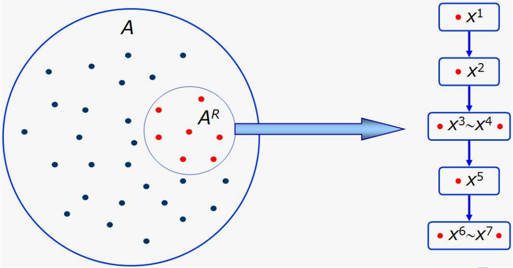
- $A = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$ is finite set of alternatives
- $g_1, g_2, \ldots, g_j, \ldots, g_n$ n criterion functions, F is the set of criteria indices
- $g_j(a_i)$ is the evaluation of the alternative a_i on criterion g_j
- G_j - domain of criterion g_j,
- $\succsim$ - weak preference (outranking) relation on G: for each $x, y \in G$
  - $x \succsim y \ \Leftrightarrow$ "x is at least as good as y"
  - $x \succ y \ \Leftrightarrow [x \succsim y$ and not$(y \succsim x)]$ "x is preferred to y"
  - $x \sim y \ \Leftrightarrow [x \succsim y$ and $y \succsim x]$ "x is indifferent to y"

Reminder on UTA

- For each g_j, $G_j = [\alpha_j, \beta_j]$ is the criterion evaluation scale, $\alpha_j \leq \beta_j$,
- U is an additive value function on G: for each $x \in G$, $U(x) = \sum_{j \in F} u_j[g_j(x)]$,
- u_j are non-decreasing marginal value functions, $u_j : G_j \mapsto \mathbb{R}, \forall j \in F$

Reminder on UTA

- The preference information is given in the form of a complete pre-order on a subset of reference alternatives $A^R \subseteq A$, called reference pre-order.
- $A^R = \{a_1, a_2, ..., a_{m1}\}$ is rearranged such that $a_k \succsim a_{k+1}, \; k = 1, ..., m_1 - 1$, where $m_1 = |A^R|$.



Reminder on UTA

- The inferred value of each $a \in A^R$ is :

$$U(a) + \sigma^+(a) - \sigma^-(a),$$

- In UTA , the marginal value functions u_i are assumed to be piecewise linear, so that the intervals $[\alpha_i, \beta_i]$ are divided into $\gamma_i \geq 1$ equal sub-intervals

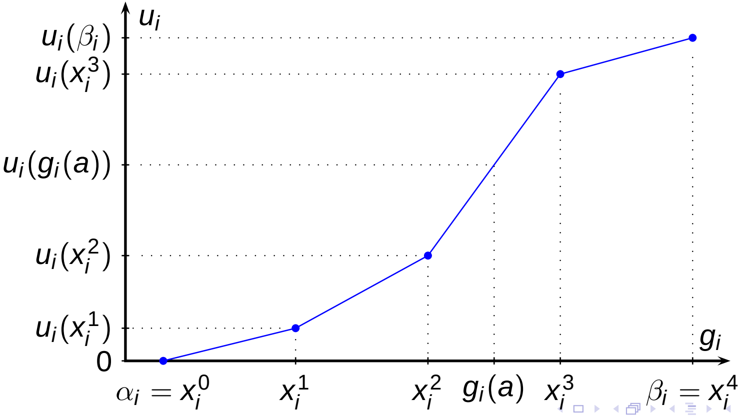
$$[x_i^0, x_i^1], [x_i^1, x_i^2], \ldots, [x_i^{\gamma_i - 1}, x_i^{\gamma_i}],$$

where,

$$x_i^j = \alpha_i + \frac{j(\beta_i - \alpha_i)}{\gamma_i}, j = 0, \ldots, \gamma_i, i = 1, \ldots, n.$$

Reminder on UTA

The piecewise linear value model is defined by the marginal values at break points: $u_i(x_i^0) = u_i(\alpha_i),\ u_i(x_i^1), u_i(x_i^2), \ldots, u_i(x_i^{\gamma_i}) = u_i(\beta_i)$



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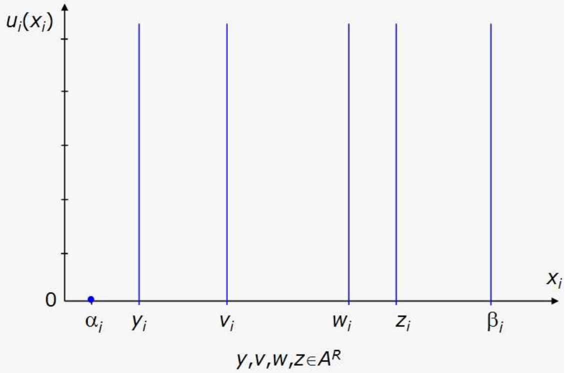
The UTA^{GMS} method: Main features

UTA^{GMS} method generalizes the UTA method in three aspects:

- The preference information is a partial preorder (not necessary complete): $\mathcal{I} = B^R \subset A^R \times A^R$
- It takes into account all additive value functions compatible with indirect preference information, while UTA is using only one such function.
- The marginal value functions are general monotone non-decreasing functions, and not piecewise linear only.

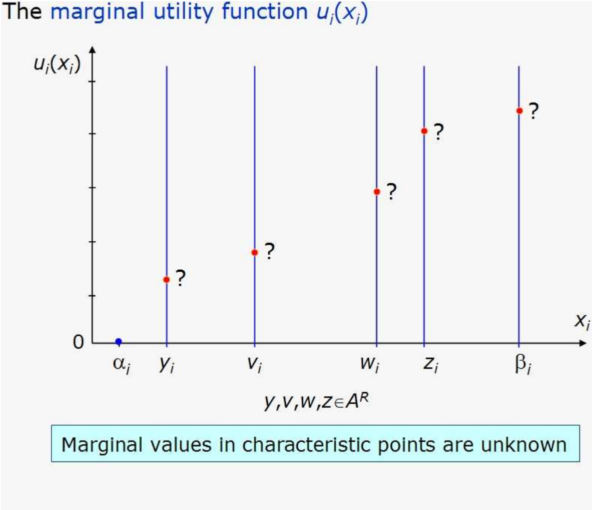
General monotone non-decreasing value functions

The marginal utility function $u_i(x_i)$



Characteristic points of marginal utility functions are fixed on actual evaluations of actions from A

General monotone non-decreasing value functions



UTA^{GMS}

Consider $x, y \in A \setminus A^R$

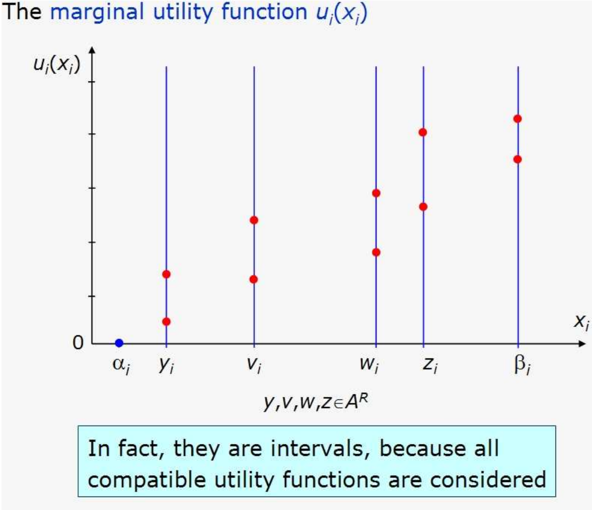
- Let π_i be a permutation on the set of alternatives $A^R \cup \{x, y\}$ that reorders them according to increasing evaluation on criterion g_i:

$$g_j(a_{[\pi_i(1)]}) \leq g_j(a_{[\pi_i(1)]}) \leq ... \leq g_j(a_{[\pi_i(\omega-1)]}) \leq g_j(a_{[\pi_i(\omega)]})$$

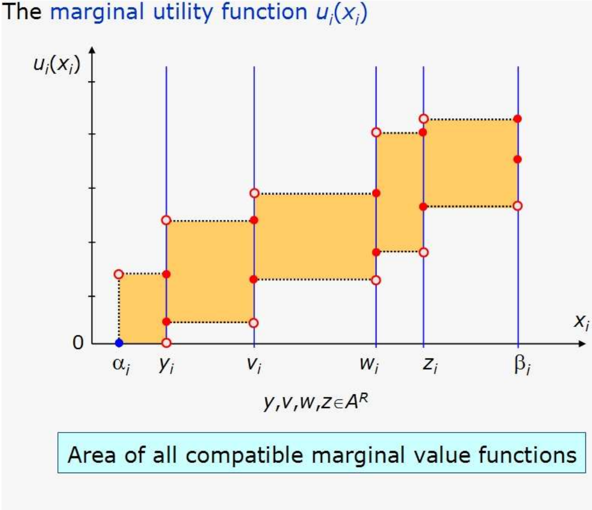
where $\omega = |A^R| + 2$, $|A^R| + 1$ or $|A^R|$ depending on $g_i(x)$ and $g_i(y)$,

- The characteristic points of $u_i(x_j)$, $i = 1, ..., m$, are then fixed according to this reordering

General monotone non-decreasing value functions

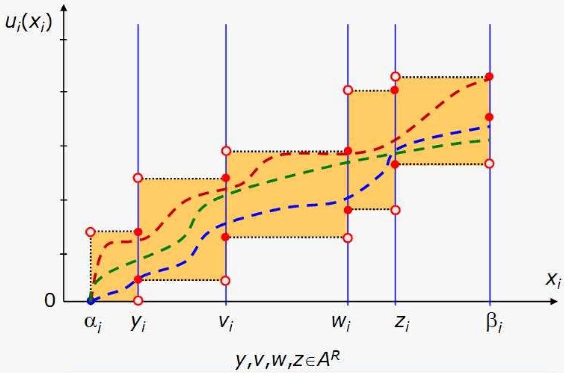


General monotone non-decreasing value functions



General monotone non-decreasing value functions

The marginal utility function $u_i(x_i)$



In the area the marginal compatible value functions must be monotone

UTA^{GMS}

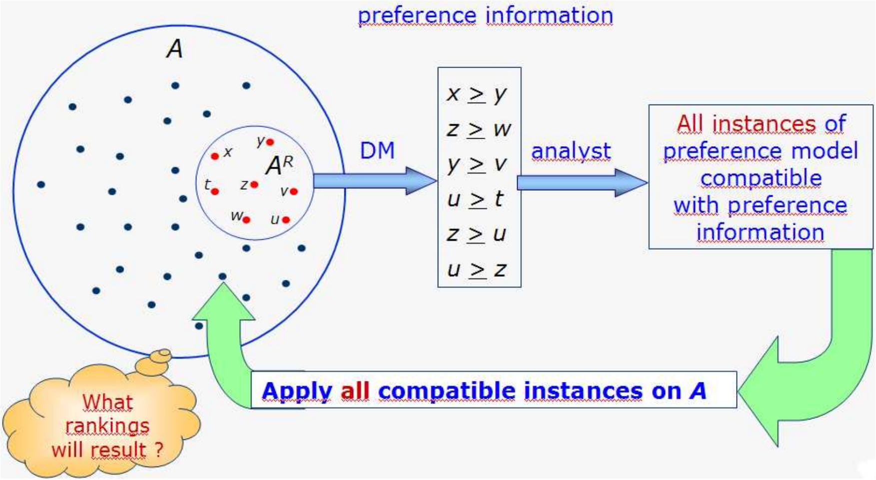
- For any pair of alternatives $(a, b) \in A$, and for available preference information represented by B^R, preference of a over b is determined by compatible utility functions U verifying set $E(a,b)$ of constraints:

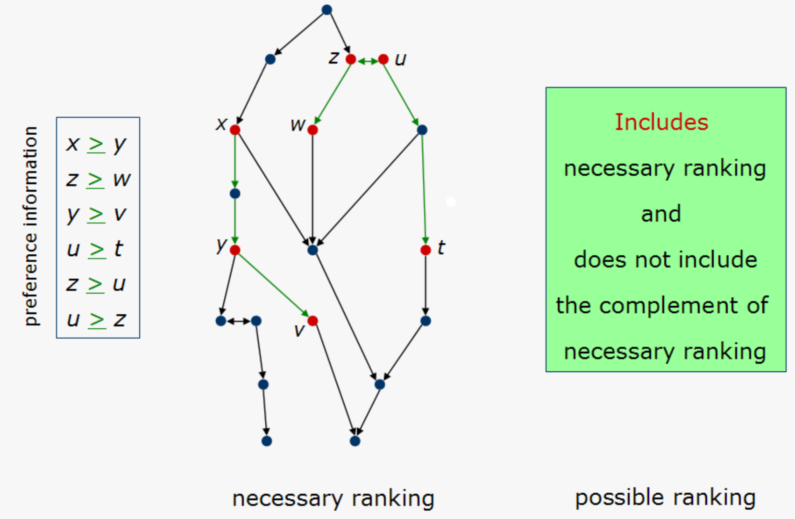
$$\left.\begin{array}{l} \left.\begin{array}{l} U(c) \geq U(d) + \varepsilon \;\Leftrightarrow\; c \succ d \\ U(c) = U(d) \;\Leftrightarrow\; c \sim d \end{array}\right\} \text{for all } (c,d) \in B^R \\ u_i(x_i^j) - u_i(x_i^{j-1}) \geq 0,\; i = 1,\dots,n,\; j = 1,\dots,\omega+1 \\ u_i(g_i^0) = 0,\; i = 1,\dots,n \\ \sum_{i=1}^n u_i(g_i^{\omega+1}) = 1, \end{array}\right\} E(a,b)$$

The UTA^{GMS} method: Main features

The method produces two rankings in the set of alternatives A, such that for any pair of alternatives $a, b \in A$,

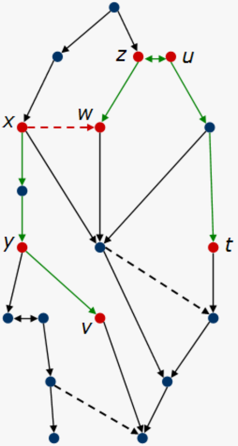
- In the *necessary order*, a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for all value functions compatible with the preference information.
- In the *possible order*, a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for at least one value function compatible with the preference information.


preference information
A
x
y
A^R
t
z
v
w
u
DM
x ≥ y
z ≥ w
y ≥ v
u ≥ t
z ≥ u
u ≥ z
analyst
All instances of preference model compatible with preference information
Apply all compatible instances on A
What rankings will result ?


preference information
x ≥ y
z ≥ w
y ≥ v
u ≥ t
z ≥ u
u ≥ z
z
u
x
w
y
t
v
Includes
necessary ranking
and
does not include
the complement of
necessary ranking
necessary ranking
possible ranking

additional preference information

$x \geq y$
$z \geq w$
$y \geq v$
$u \geq t$
$z \geq u$
$u \geq z$
$x \geq w$



necessary ranking
enriched

Includes
necessary ranking
and
does not include
the complement of
necessary ranking

possible ranking
impoverished

Computing necessary and possible relations: $\succsim^N$, $\succsim^P$

- d(x,y)= $Min_{U \in \mathcal{U}} U(x) - U(y)$ and
  D(x,y)= $Max_{U \in \mathcal{U}} U(x) - U(y)$
  where
  $\mathcal{U} = \{$value fct compatible with the DM's statements$\}$
- $x \succsim^P y \;\Leftrightarrow\; D(x,y) \geq 0$
- $x \succsim^N y \;\Leftrightarrow\; d(x,y) \geq 0$

Elaboration of the final rankings

- for the necessary preference relation being a partial preorder (supported by all compatible value functions)
  - preference: $x \succ^N y$ if $x \succeq^P y$ and $not y \succeq^N x$
  - indifference: $x \sim^N y$ if $x \succeq^N y$ and $y \succeq^N x$
  - incomparability: $x?y$ if not $x \succeq^N y$ and not $y \succeq^N x$
- for the possible preference relation being complete (supported by at least one compatible value function)
  - $x \succ^P y$ if $x \succeq^P y$ and not $y \succeq^P x$
  - $x \sim^P y$ if $x \succeq^P y$ and $y \succeq^P x$

Properties of relations $\succsim^N$, $\succsim^P$

- $d(x,y) = Min\{U(x) - U(y)\} = -Max\{-(U(x) - U(y))\} = -Max\{U(y) - U(x)\} = -D(y,x)$
- $(x,y) \in B^R \Rightarrow x \succsim^N y$,
- $x \succsim^N y \Rightarrow x \succsim^P y$,
- $\succsim^N$ is a partial preorder (reflexive and transitive),
- $\succsim^P$ is strongly complete ($x \succsim^P y$ or $y \succsim^P x$), but not necessarily transitive.
- In absence of preference information: necessary ranking = weak dominance relation, possible ranking is complete,
- For complete pairwise comparisons: necessary ranking = possible ranking

Contents

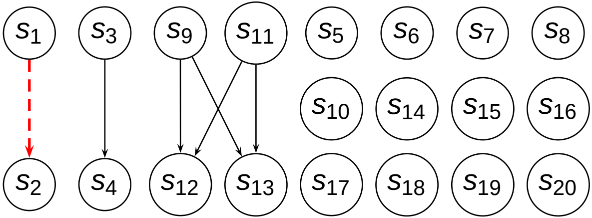
Illustrative example

20 alternatives, 5 criteria (all alternatives are efficient).

$s_1 = (14.5, 147, 4, 1014, 5.25)$	$s_{11} = (15.75, 164.375, 41.5, 311, 6.5)$
$s_2 = (13.25, 199.125, 4, 1014, 4)$	$s_{12} = (13.25, 181.75, 41.5, 311, 4)$
$s_3 = (15.75, 164.375, 16.5, 838.25, 5.25)$	$s_{13} = (12, 199.125, 41.5, 311, 2.75)$
$s_4 = (12, 181.75, 16.5, 838.25, 4)$	$s_{14} = (17, 147, 16.5, 662.5, 5.25)$
$s_5 = (12, 164.375, 54, 838.25, 4)$	$s_{15} = (15.75, 199.125, 16.5, 311, 6.5)$
$s_6 = (13.25, 199.125, 29, 662.5, 5.25)$	$s_{16} = (13.25, 164.375, 54, 311, 4)$
$s_7 = (13.25, 147, 41.5, 662.5, 5.25)$	$s_{17} = (17, 181.75, 16.5, 486.75, 5.25)$
$s_8 = (17, 216.5, 16.5, 486.75, 1.5)$	$s_{18} = (14.5, 164.375, 41.5, 838.25, 4)$
$s_9 = (17, 147, 41.5, 486.75, 5.25)$	$s_{19} = (15.75, 181.75, 41.5, 135.25, 5.25)$
$s_{10} = (15.75, 216.5, 41.5, 662.5, 1.5)$	$s_{20} = (15.75, 181.75, 41.5, 311, 2.75)$

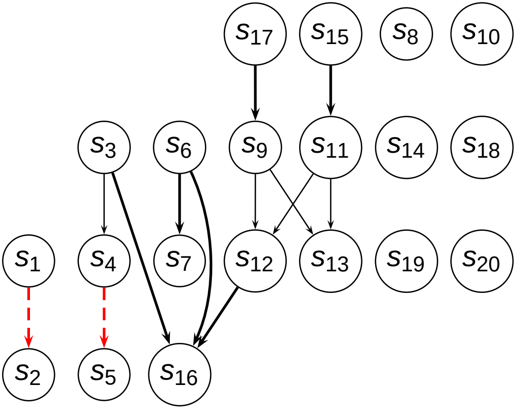
Illustrative example

First information: $s_1 \succ s_2$.



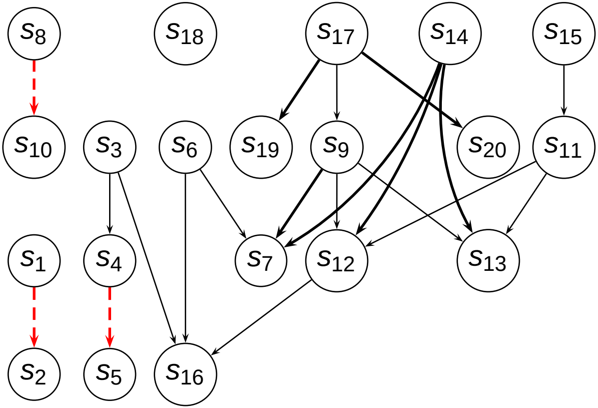
Illustrative example

Second information: $s_4 \succ s_5$.



Illustrative example

Third information: $s_8 \succ s_{10}$.



Accounting for confidence on preference statements

- Preference information = nested partial preorders $B_1^R \subset B_2^R \subset ... \subset B_p^R$ with decreasing credibility,
- From B_i^R to B_{i+1}^R, we add new constraints and reduce the set of compatible value functions $\mathcal{U}(B_{i+1}^R) \subset \mathcal{U}(B_i^R)$,
- If $Min_{U \in \mathcal{U}(B_i^R)}(u(x) - u(y)) > 0$, then $Min_{U \in \mathcal{U}(B_{i+1}^R)}(u(x) - u(y)) > 0$,
- $\succsim^N (B_1^R), \succsim^N (B_2^R), ..., \succsim^N (B_p^R)$ define a set of nested partial preorders.

Contents

Inconsistency management

- When DM's statement are not representable in the additive model
  →inconsistency,
- DM's statements induce linear constraints on the variables (marginal values of alternatives)
- When such inconsistency occurs, we should check how to "solve" inconsistency,
- Which modification of the DM's input will lead to representable preferences ?
- Are they different ways to do so ?
- What is the minimum number of constraints to delete ?

Inconsistency management

- solution of minimal cardinality is not necessarily the most interesting one for the DM,
- The knowledge of the various ways to solve inconsistency is useful for the DM,
- This permits to:
  - help the DM to understand the conflicting aspects of his/her statement,
  - create a context in which the DM car learn about his/her preferences,
  - make the elicitation process more flexible,

Inconsistency resolution via constraints deletion

- m constraints induced by the DM's statements

$$\left\{ \begin{array}{lll} \sum_{j=1}^{n} \alpha_1^j x_j & \geq & \beta_1 \\ & \vdots & \\ \sum_{j=1}^{n} \alpha_{m-1}^j x_j & \geq & \beta_{m-1} \\ \sum_{j=1}^{n} \alpha_m^j x_j & \geq & \beta_m \end{array} \right. \quad [1]$$

- $I = \{1, \ldots, m\}$; subset $S \subset I$ solves [1] iff $I \setminus S \neq \emptyset$
- We search for $S_1, S_2, \ldots, S_p \subset I$ such that :
  - (i) S_i resolves [1], $i \in \{1, 2, ..., p\}$;
  - (ii) $S_i \not\subseteq S_j, i, j \in \{1, ..., p\}, i \neq j$;
  - (iii) $|S_i| \leq |S_j|, i, j \in \{1, 2, ..., p\}, i < j$;
  - (iv) if $\exists$ S that resolves [1] s.t. $S \not\subseteq S_i$, $\forall i = 1, 2, \ldots, p$, then $|S| > |S_p|$.

Inconsistency management

- Let y_i ($\in \{0,1\}$, $i \in I$), s.t. :

$$
\begin{array}{rcl}
y_i & = & 1 \text{ if constraint } i \text{ is removed} \\
 & = & 0 \text{ otherwise}
\end{array}
$$

$$
P_1 \left\{
\begin{array}{ll}
Min & \sum_{i \in I} y_i \\
\text{s.t.} & \sum_{j=1}^{n} \alpha_{ij} x_j + M y_i \geq \beta_i, \quad \forall i \in I \\
 & x_j \geq 0, \quad j = 1, \ldots, n \\
 & y_i \in \{0,1\}, \quad \forall i \in I
\end{array}
\right.
$$

- $S_1 = \{i \in I : y_i^* = 1\}$ corresponds to (one of the) subset(s) of constraints resolving [1] of smallest cardinality,
- We define P_2 adding to P_1 the constraint $\sum_{i \in S_1} y_i \leq |S_1| - 1$

Inconsistency management

- P_{k+1} is defined adding to P_k the constraint $\sum_{i \in S_k} y_i \leq |S_k| - 1$
- We compute $S_1, S_2, \ldots, S_k$, and stop when $|S_{k+1}| > \Omega$,

```
Begin
     k  ⟵ 1
     moresol  ⟵ true
     While moresol
       Solve PM_k
       If (PM_k  has no solution) or (PM_k  has an optimal value > Ω)
         Then moresol  ⟵ false
         Else
           - S_k ⟵ {i ∈ I : y_i^* = 1}
           - Add constraint ∑_{i∈S_k} y_i ≤ |S_k| − 1 to PM_k  so as to define PM_k
           - k  ⟵k+1
       End if
     End while
End
```

Inconsistency management

- Each S_i corresponds to a set of DM's preference statements (presented to the DM),
- Sets S_i represent (for the DM) "incompatible" comparisons, each one specifies a way to solve inconsistency,
- Deleting/modifying the smallest set of preference assertions might not be the best idea
- Consider the confidence statements to ranking inconsistency resolutions.

Contents

The GRIP method: Main features

GRIP extends UTA^{GMS}: additional preference information in form of comparisons of intensities of preference between some pairs of reference alternatives.

1) Comprehensive, on all criteria, "x is preferred to y at least as much as w is preferred to z".
$\rightarrow (x,y) \succsim^* (w,z) \Leftrightarrow U(x) - U(y) \geq U(w) - U(z)$

2) Partial, on each criterion, "x is preferred to y at least as much as w is preferred to z, on criterion $g_i \in F$".
$\rightarrow (x,y) \succsim_i^* (w,z) \Leftrightarrow U_i(x) - U_i(y) \geq U_i(w) - U_i(z)$

The GRIP method: Preference Information

DM is expected to provide the following preference information,

- A partial pre-order $\succsim$ on A^R,
  $x \succsim y \Leftrightarrow x$ is at least as good as y.
  $$\rightarrow x \succsim (y) \;\Leftrightarrow\; U(x) \geq U(y)$$

- A partial pre-order $\succsim^*$ on $A^R \times A^R$,
  $(x, y) \succsim^* (w, z) \Leftrightarrow$
  x is preferred to y at least as much as w is preferred to z
  $$\rightarrow (x, y) \succsim^* (w, z) \;\Leftrightarrow\; U(x) - U(y) \geq U(w) - U(z)$$

- A partial pre-order $\succsim_i^*$ on $A^R \times A^R$,
  $(x, y) \succsim_i^* (w, z) \Leftrightarrow$ x is preferred to y at least as much as w is preferred to z on criterion g_i.
  $$\rightarrow (x, y) \succsim_i^* (w, z) \;\Leftrightarrow\; U_i(x) - U_i(y) \geq U_i(w) - U_i(z)$$

The GRIP method: Output

If the set of compatible value functions is not empty, we compute:

- $\succsim^N$: $x \succsim^N y \Leftrightarrow min_{U \in \mathcal{U}}(U(x) - U(y)) \geq 0$
- $\succsim^P$: $x \succsim^P y \Leftrightarrow max_{U \in \mathcal{U}}(U(x) - U(y)) \geq 0$
- $\succsim^{*N}$: $(x, y) \succsim^{*N} (w, z) \Leftrightarrow min_{U \in \mathcal{U}}((U(x) - U(y)) - (U(w) - U(z))) \geq 0$
- $\succsim^{*P}$: $(x, y) \succsim^{*N} (w, z) \Leftrightarrow max_{U \in \mathcal{U}}((U(x) - U(y))(U(w) - U(z))) \geq 0$

Contents

Ordinal regression for group ranking: UTA^{GMS}-Group

- Set of DMs: $\mathcal{D} = \{d_1, ..., d_p\}$
- Preference information provided by d_h, $h = 1, ..., p$: $B^R(d_h)$ a partial preorder on a set of reference actions,

UTA^{GMS}-Group

- We consider the set of value functions for each $d_h \in \mathcal{D}^* \subseteq \mathcal{D}$ stemming from UTA-GMS,
- For each $d_h \in \mathcal{D}^*$, 4 situations are interesting for $(x, y) \in A$:
  - $x \succsim^{N,N} (\mathcal{D}^*) y$: $x \succsim^N y$ for all $d_h \in \mathcal{D}^*$,
  - $x \succsim^{N,P} (\mathcal{D}^*) y$: $x \succsim^N y$ for at least one $d_h \in \mathcal{D}^D*$,
  - $x \succsim^{P,N} (\mathcal{D}^*) y$: $x \succsim^P y$ for all $d_h \in \mathcal{D}^*$,
  - $x \succsim^{P,P} (\mathcal{D}^*) y$: $x \succsim^P y$ for at least one $d_h \in \mathcal{D}^*$,

UTA^{GMS}-Group

Properties

- $\succeq^{N,N}(\mathcal{D}^*)$ is a partial preorder
- $\succeq^{N,P}(\mathcal{D}^*)$ is not necessarily transitive
- $\succeq^{P,P}(\mathcal{D}^*)$ is strongly complete
- $x \succeq^{N,N}(\mathcal{D}^*)y \Rightarrow x \succeq^{N,P}(\mathcal{D}^*)y$
- $x \succeq^{N,P}(\mathcal{D}^*)y \Rightarrow x \succeq^{P,P}(\mathcal{D}^*)y$

When $\mathcal{D}^* \subset \mathcal{D}^{**}$, it holds

- $x \succeq^{N,N}(\mathcal{D}^{**})y \Rightarrow x \succeq^{N,N}(\mathcal{D}^*)y$
- $x \succeq^{N,P}(\mathcal{D}^{**})y \Rightarrow x \succeq^{N,P}(\mathcal{D}^*)y$
- $x \succeq^{P,N}(\mathcal{D}^{**})y \Rightarrow x \succeq^{P,N}(\mathcal{D}^*)y$
- $x \succeq^{P,P}(\mathcal{D}^{**})y \Rightarrow x \succeq^{P,P}(\mathcal{D}^*)y$

UTA^{GMS}-Group

- Given $\mathcal{D}^* \subseteq \mathcal{D}$, a value function U is compatible with $\mathcal{D}^*$ if:

$$\mathcal{U}_{\mathcal{D}^*}\begin{cases} \left.\begin{array}{l} U(c) > U(d) \;\Leftrightarrow\; c \succ d \\ U(c) = U(d) \;\Leftrightarrow\; c \sim d \end{array}\right\} \text{for all } c,d \in B^R(\mathcal{D}^*) \\ u_i(g_i(a_{\tau_i(j)})) - u_i(g_i(a_{\tau_i(j-1)})) \geq 0,\; i = 1,...,n,\; j = 2,...,m \\ u_i(g_i(a_{\tau_i(1)})) \geq 0, u_i(g_i(a_{\tau_i(m)})) \leq u_i(\beta_i),\; i = 1,...,n, \\ u_i(\alpha_i) = 0,\; i = 1,...,n \\ \sum_{i=1}^n u_i(\beta_i) = 1, \end{cases}$$

where τ_i is the permutation that reorders alternatives according to their increasing evaluation on g_i.

UTA^{GMS}-Group

- If $\mathcal{U}_{\mathcal{D}^*} \neq \emptyset$, one obtains two rankings such that for any pair of actions $(x, y) \in A$:
  - $x \succsim^N (D*) y$: x is ranked at least as good as y iff $U^{D^*}(x) \geq U^{D^*}(y)$ for all U compatible with the preference information ($\succsim^N$ being a partial preorder)
  - $x \succsim^P (D*) y$: x is ranked at least as good as y iff $U^{D^*}(x) \geq U^{D^*}(y)$ for at least one U compatible with the preference information ($\succsim^P$ being a strongly complete and negatively transitive binary relation)
- However, the set $\mathcal{U}_{\mathcal{D}^*}$ of compatible value function can be empty...

UTA^{GMS}- Group

Suppose $\mathcal{U}_{\mathcal{D}^*} = \emptyset$

- $\mathcal{U}_{\mathcal{D}^*}$ = intersection of sets of compatible value functions for all $d_h \in \mathcal{D}^*$ (each one being non-empty).
- Pairwise comparisons of two (or more) DMs are conflicting.
- Identifying which are these conflicting comparisons amounts at solving inconsistency
- This leads to know which comparison to remove to obtain a consistent collective model.
- Performing these computations $\forall \mathcal{D}^* \subseteq \mathcal{D}$ allows to identify coalitions of convergent DMs.

Contents

UTA^{GMS}- Group: Illustrative example

- Group ranking with 3 DMs d_1, d_2 and d_3,

	$g_1(a_i)$	$g_2(a_i)$	$g_3(a_i)$	$g_4(a_i)$	$g_5(a_i)$
a_1	2	0	0	5	3
a_2	1	3	0	5	2
a_3	3	1	1	4	3
a_4	0	2	1	4	2
a_5	1	1	4	3	2
a_6	3	3	2	3	3
a_7	0	0	3	3	3
a_8	4	4	1	2	0
a_9	3	0	3	2	3
a_{10}	3	4	3	3	0
a_{11}	3	1	3	1	4
a_{12}	3	2	3	1	2
a_{13}	3	3	3	1	1
a_{14}	1	0	1	3	3
a_{15}	1	3	1	1	4
a_{16}	4	1	4	1	2
a_{17}	1	2	1	2	3
a_{18}	3	1	3	4	2
a_{19}	3	2	3	0	3
a_{20}	3	2	3	1	1

- empty dominance relation.

UTA^{GMS}- Group: Illustrative example

- Statements of DMs:
  - d_1: $a_1 \succ a_2$, $a_6 \succ a_7$ and $a_{17} \succ a_{20}$
  - d_2: $a_9 \succ a_{13}$, $a_4 \succ a_5$ and $a_{14} \succ a_7$
  - d_3: $a_4 \succ a_3$, $a_{15} \succ a_{11}$ and $a_8 \succ a_{10}$
- $\mathcal{U}_{\{d1,d3\}} = \mathcal{U}_{\{d1,d2,d3\}} = \emptyset$, i.e., d_1 and d_3 statements contradict:
  - d_1: $a_1 \succ a_2 \Rightarrow a_3 \succ a_4$
  - d_3: $a_4 \succ a_3 \Rightarrow a_2 \succ a_1$
- If (d_1 removes $a_1 \succ a_2$) or (d_3 removes $a_4 \succ a_3$) then $\mathcal{U}_{\{d1,d2,d3\}} \neq \emptyset$.

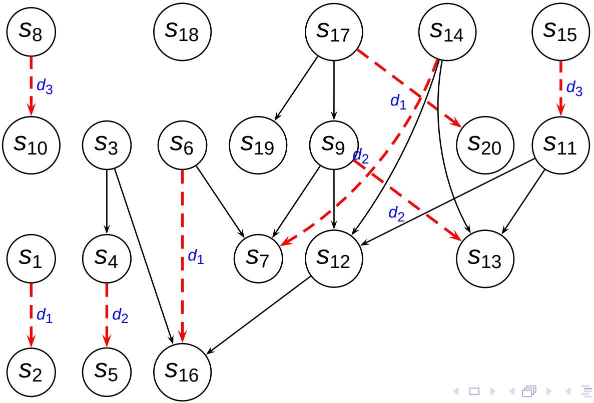
UTA^{GMS}- Group: Illustrative example

Although $\mathcal{U}_{\{d1,d2,d3\}} = \emptyset$, the following relations are not empty:

- $\succsim^{N,N}(\{d_1,d_2,d_3\}) = \{(a_6,a_7)\}$, $a_6 \succsim^{N} a_7$ for all d_h
- $\succsim^{N,N}(\{d_1,d_2\}) = \{(a_6,a_7),(a_9,a_{13})\}$,
- $\succsim^{N,N}(\{d_1,d_3\}) = \{(a_6,a_7),(a_{17},a_{20})\}$,
- $\succsim^{N,N}(\{d_2,d_3\}) = \{(a_6,a_7),(a_{11},a_{15})\}$,
- $x \succsim^{N,P}(\{d_1,d_2,d_3\})(d1,d2,d3)y$: $x \succsim^{N} y$ for at least one d_h
- $x \succsim^{P,N}(\{d_1,d_2,d_3\})(d1,d2,d3)y$: $x \succsim^{P} y$ for all d_h
- $x \succsim^{P,P}(\{d_1,d_2,d_3\})(d1,d2,d3)y$: $x \succsim^{P} y$ for at least one d_h

UTA^{GMS}- Group: Illustrative example

- If d_3 removes $a_4 \succ a_3$ then $\mathcal{U}_{\{d1,d2,d3\}} \neq \emptyset$ leading to the collective necessary ranking:



Contents

Software demonstration

Software demonstration: Visual-UTA 2.0

- AGRITEC is a medium size firm (350 persons approx.) producing some tools for agriculture,
- The C.E.O., M^{r} Becault, intends to double the production and multiply exports by 4 within 5 years.
- He wants to hire a new international sales manager.
- A recruitment agency has interviewed 17 potential candidates which have been evaluated on 3 criteria (sales management experience, international experience, human qualities) evaluated on a [0,100] scale.

Software demonstration

	Crit 1	Crit 2	Crit 3
Alexievich	4	16	63
Bassama	28	18	28
Calvet	26	40	44
Dubois	2	2	68
El Mrabat	18	17	14
Ferret	35	62	25
Fleichman	7	55	12
Fourny	25	30	12
Frechet	9	62	88
Martin	0	24	73
Petron	6	15	100
Psorgos	16	9	0
Smith	26	17	17
Varlot	62	43	0
Yu	1	32	64

Software demonstration

- it0 without preference information,
- it1 Ferret $\sim$ Frechet $\succ$ Fourny $\succ$ Fleichman,
- it2 Ferret $\sim$ Frechet $\succ$ Martin $\succ$ Fourny $\sim$ El Mrabat $\succ$ Fleichman,
  $\rightarrow$inconsistency: Ferret $\sim$ Frechet vs Fourny $\sim$ El Mrabat
- it3 Ferret $\sim$ Frechet $\succ$ Martin $\succ$ Fourny $\succ$ Fleichman,

Contents

Conclusion

- UTA-GMS/GRIP:
  - General additive value function,
  - Intuitive information required from the DM,
  - Robust elicitation of a ranking model,
  - Necessary and Possible rankings,
  - Inconsistency management.

Conclusion

Insufficient attention is devoted in MCDA to develop elicitation tools an methodologies which should contribute to the definition of a doctrine for MCDA practitioners.

More research is needed to :

- develop methodologies/tools to organize the interaction with DMs in a given MCAP,
- test the operational validity of the developed tools.