

Chapter 8: Symmetry Breaking (Balanced Incomplete Block Designs)

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ECLIPSe ELearning [Overview](#)



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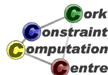
Outline

- 1 Problem
- 2 Program
- 3 Symmetry Breaking



What we want to introduce

- BIBD - Balanced Incomplete Block Designs
- Using lex constraints to remove symmetries
- Finding all solutions to a problem
- Using timeout to limit search



Outline

- 1 Problem
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Problem Definition

BIBD (Balanced Incomplete Block Design)

A BIBD is defined as an arrangement of v distinct objects into b blocks such that each block contains exactly k distinct objects, each object occurs in exactly r different blocks, and every two distinct objects occur together in exactly λ blocks. A BIBD is therefore specified by its parameters (v, b, r, k, λ) .



Motivation: Test Planning

Consider a new release of some software with v new features. You want to regression test the software against combinations of the new features. Testing each subset of features is too expensive, so you want to run b tests, each using k features. Each feature should be used r times in the tests. Each pair of features should be tested together exactly λ times. How do you arrange the tests?



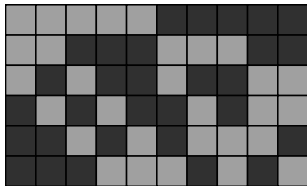
Model

Another way of defining a BIBD is in terms of its incidence matrix, which is a binary matrix with v rows, b columns, r ones per row, k ones per column, and scalar product λ between any pair of distinct rows.



Model

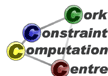
Another way of defining a BIBD is in terms of its incidence matrix, which is a binary matrix with v rows, b columns, r ones per row, k ones per column, and scalar product λ between any pair of distinct rows.



A (6,10,5,3,2) BIBD

Outline

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Model

- A binary $v \times b$ matrix. Entry V_{ij} states if item i is in block j .
- Sum constraints over rows, each sum equal r
- Sum constraints over columns, each sum equal k
- Scalar product between any pair of rows, the product value is λ .



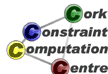
Top Level Program

```
:-module(bibd) .  
:-export(top/0) .  
:-lib(ic) .  
:-lib(ic_global) .  
  
top:-  
    bibd(6,10,5,3,2,Matrix),writeln(Matrix) .  
  
bibd(V,B,R,K,L,Matrix):-  
    model(V,B,R,K,L,Matrix),  
    extract_array(row,Matrix,List),  
    search(L,0,input_order,indomain,  
          complete,[]).
```



Top Level Program

```
:-module(bibd) .  
:-export(top/0) .  
:-lib(ic) .  
:-lib(ic_global) .  
  
top:-  
    bibd(6,10,5,3,2,Matrix),writeln(Matrix) .  
  
bibd(V,B,R,K,L,Matrix):-  
    model(V,B,R,K,L,Matrix), ⇨ Set up model  
    extract_array(row,Matrix,List),  
    search(L,0,input_order,indomain,  
           complete,[]).
```



Top Level Program

```
:-module(bibd) .
:-export(top/0) .
:-lib(ic) .
:-lib(ic_global) .
```

```
top:-
    bibd(6,10,5,3,2,Matrix),writeln(Matrix) .
```

```
bibd(V,B,R,K,L,Matrix):-
    model(V,B,R,K,L,Matrix),
    extract_array(row,Matrix,List), ⇒ Get list
    search(L,0,input_order,indomain,
           complete,[]) .
```



Top Level Program

```
:-module(bibd) .  
:-export(top/0) .  
:-lib(ic) .  
:-lib(ic_global) .
```

```
top:-  
    bibd(6,10,5,3,2,Matrix),writeln(Matrix) .
```

```
bibd(V,B,R,K,L,Matrix):-  
    model(V,B,R,K,L,Matrix),  
    extract_array(row,Matrix,List),  
    search(L,0,input_order,indomain,  
           complete,[]) => Search
```



Constraint Model

```

model (V,B,R,K,L,Matrix,Method):-
    dim(Matrix,[V,B]),
    Matrix[1..V,1..B] :: 0..1,
    (for(I,1,V), param(Matrix,B,R) do
        sumlist(Matrix[I,1..B],R)
    ),
    (for(J,1,B), param(Matrix,V,K) do
        sumlist(Matrix[1..V,J],K)
    ),
    (for(I,1,V-1), param(Matrix,V,B,L) do
        (for(I1,I+1,V), param(Matrix,I,B,L) do
            scalar_product(Matrix[I,1..B],
                Matrix[I1,1..B],L)
        )
    )
).

```



Constraint Model

```

model (V,B,R,K,L,Matrix,Method):-
    dim(Matrix,[V,B]), ⇨ Define Binary Matrix
    Matrix[1..V,1..B] :: 0..1,
    (for(I,1,V), param(Matrix,B,R) do
        sumlist(Matrix[I,1..B],R)
    ),
    (for(J,1,B), param(Matrix,V,K) do
        sumlist(Matrix[1..V,J],K)
    ),
    (for(I,1,V-1), param(Matrix,V,B,L) do
        (for(I1,I+1,V), param(Matrix,I,B,L) do
            scalar_product(Matrix[I,1..B],
                Matrix[I1,1..B],L)
        )
    )
).

```



Constraint Model

```

model (V,B,R,K,L,Matrix,Method):-
    dim(Matrix,[V,B]),
    Matrix[1..V,1..B] :: 0..1,
    (for(I,1,V), param(Matrix,B,R) do
        sumlist(Matrix[I,1..B],R)
    ), => Row Sum = R
    (for(J,1,B), param(Matrix,V,K) do
        sumlist(Matrix[1..V,J],K)
    ),
    (for(I,1,V-1), param(Matrix,V,B,L) do
        (for(I1,I+1,V), param(Matrix,I,B,L) do
            scalar_product(Matrix[I,1..B],
                Matrix[I1,1..B],L)
        )
    )
).

```



Constraint Model

```

model (V,B,R,K,L,Matrix,Method):-
    dim(Matrix,[V,B]),
    Matrix[1..V,1..B] :: 0..1,
    (for(I,1,V), param(Matrix,B,R) do
        sumlist(Matrix[I,1..B],R)
    ),
    (for(J,1,B), param(Matrix,V,K) do
        sumlist(Matrix[1..V,J],K)
    ), ⇨ Column Sum = K
    (for(I,1,V-1), param(Matrix,V,B,L) do
        (for(I1,I+1,V), param(Matrix,I,B,L) do
            scalar_product(Matrix[I,1..B],
                Matrix[I1,1..B],L)
        )
    )
).

```



Constraint Model

```

model (V,B,R,K,L,Matrix,Method):-
    dim(Matrix,[V,B]),
    Matrix[1..V,1..B] :: 0..1,
    (for(I,1,V), param(Matrix,B,R) do
        sumlist(Matrix[I,1..B],R)
    ),
    (for(J,1,B), param(Matrix,V,K) do
        sumlist(Matrix[1..V,J],K)
    ),
    (for(I,1,V-1), param(Matrix,V,B,L) do
        (for(I1,I+1,V), param(Matrix,I,B,L) do
            scalar_product(Matrix[I,1..B],
                Matrix[I1,1..B],L)
        )
    )
).

```

⇒ Scalar product between all rows

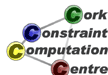


scalar_product

```

scalar_product (XVector, YVector, V) :-
  collection_to_list (XVector, XList),
  collection_to_list (YVector, YList),
  (foreach (X, XList),
   foreach (Y, YList),
   fromto (0, A, A1, Term) do
     A1 = A + X * Y
  ),
  eval (Term) #= V.

```

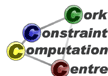


scalar_product

```

scalar_product (XVector, YVector, V) :-
    collection_to_list (XVector, XList),
    collection_to_list (YVector, YList),  $\Rightarrow$  Get lists
    (foreach (X, XList),
     foreach (Y, YList),
     fromto (0, A, A1, Term) do
         A1 = A + X * Y
    ),
    eval (Term) #= V.

```

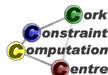


scalar_product

```

scalar_product (XVector, YVector, V) :-
    collection_to_list (XVector, XList),
    collection_to_list (YVector, YList),
    (foreach (X, XList), ⇨ Iterate over lists
     foreach (Y, YList), ⇨ ...in parallel
     fromto (0, A, A1, Term) do
         A1 = A + X * Y
    ),
    eval (Term) #= V.

```



scalar_product

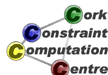
```
scalar_product (XVector, YVector, V) :-  
    collection_to_list (XVector, XList),  
    collection_to_list (YVector, YList),  
    (foreach (X, XList),  
     foreach (Y, YList),  
      fromto (0, A, A1, Term) do  $\Rightarrow$  Build term  
        A1 = A+X*Y  
    ),  
    eval (Term) #= V.
```



scalar_product

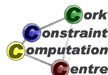
```

scalar_product (XVector, YVector, V) :-
  collection_to_list (XVector, XList),
  collection_to_list (YVector, YList),
  (foreach (X, XList),
   foreach (Y, YList),
   fromto (0, A, A1, Term) do
     A1 = A+X*Y  $\Rightarrow$  Construct term
  ),
  eval (Term) #= V.
  
```



scalar_product

```
scalar_product (XVector, YVector, V) :-  
    collection_to_list (XVector, XList),  
    collection_to_list (YVector, YList),  
    (foreach (X, XList),  
     foreach (Y, YList),  
     fromto (0, A, A1, Term) do  
         A1 = A + X * Y  
     ),  
    eval (Term) #= V. ⇨ State Constraint
```



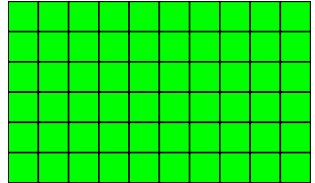
Search Routine

- Static variable order
- First fail does not work for binary variables
- Enumerate variables by row
- Use utility predicate `extract_array/3`
- Assign with `indomain`, try value 0, then value 1
- Use simple `search` call



Basic Model - First Solution

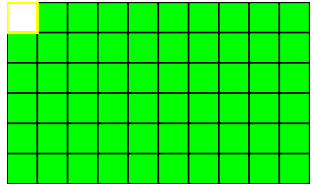
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Basic Model - First Solution

1
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2



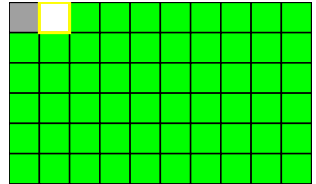
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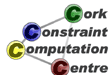
Basic Model - First Solution

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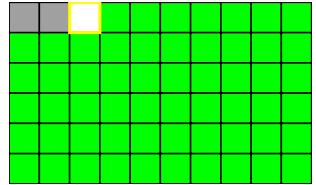
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Basic Model - First Solution

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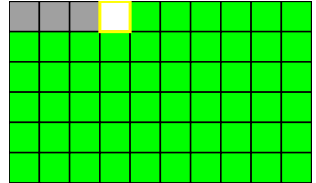
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Basic Model - First Solution

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5



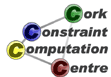
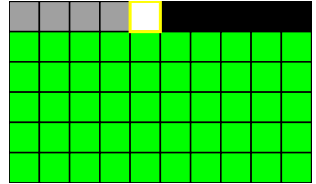
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Basic Model - First Solution

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11



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Basic Model - First Solution

1
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12

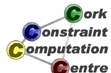
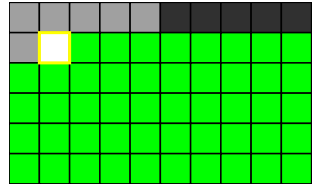


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Basic Model - First Solution

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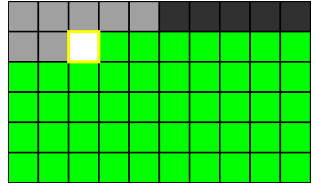


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Basic Model - First Solution

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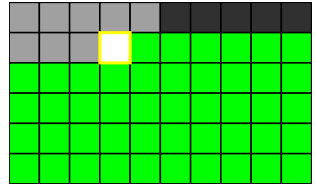


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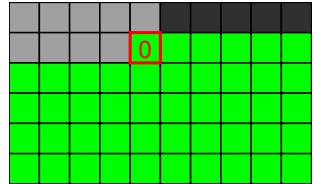


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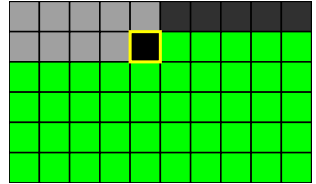
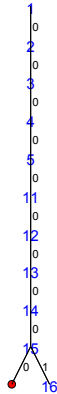
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Basic Model - First Solution



Basic Model - First Solution

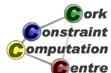
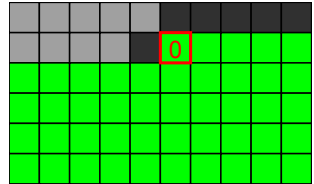


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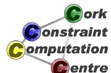
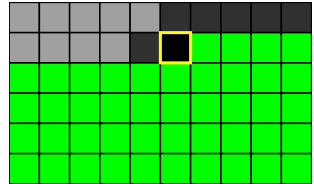
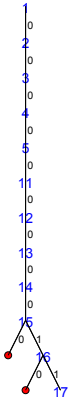


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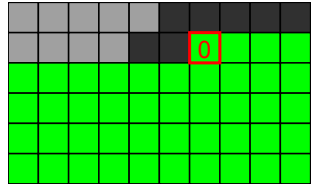
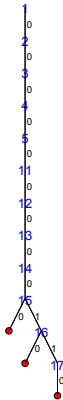


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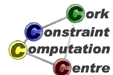
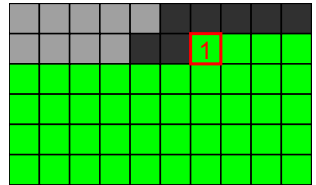
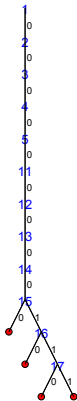


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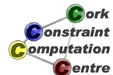
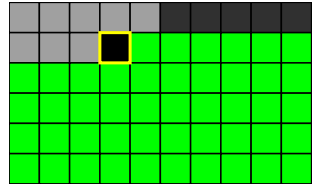
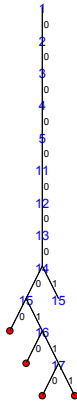


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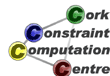
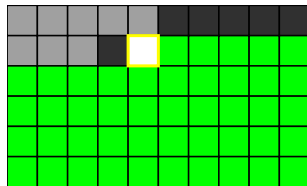
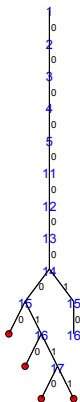
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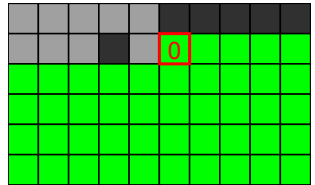
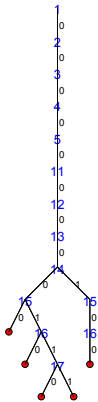


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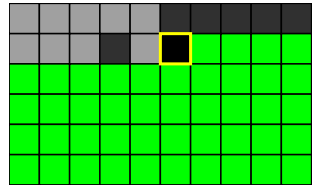
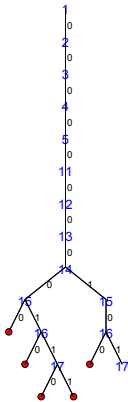


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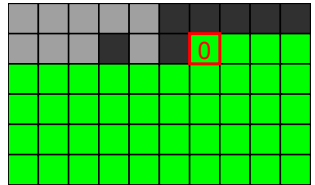
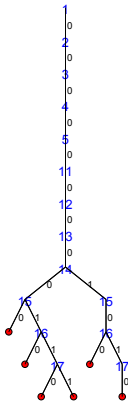
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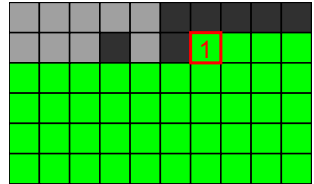
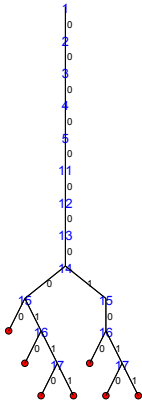
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◀ Back to Start ▶ Skip Animation



Basic Model - First Solution

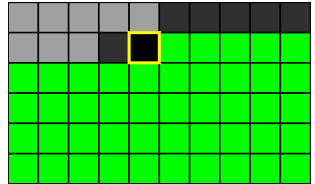
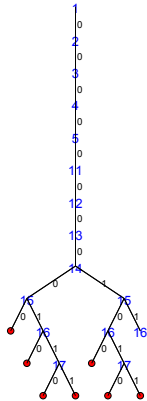


◀ Back to Start

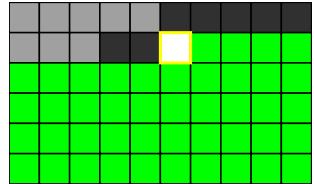
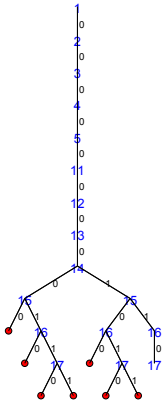
▶▶ Skip Animation



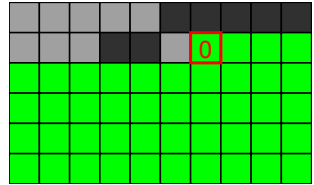
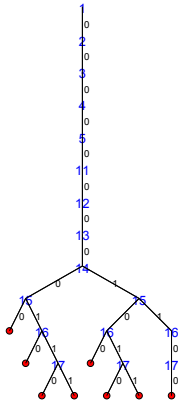
Basic Model - First Solution



Basic Model - First Solution



Basic Model - First Solution

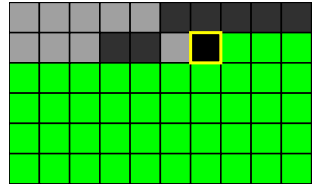
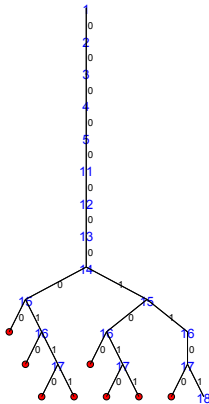


◀ Back to Start

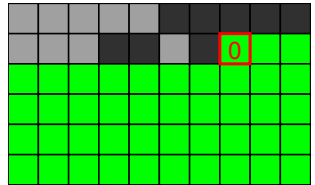
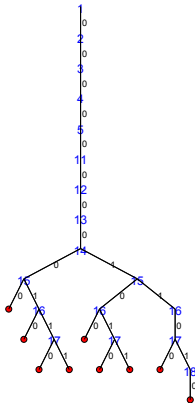
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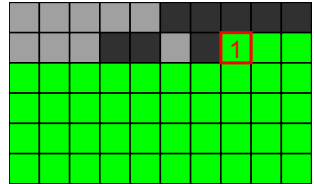
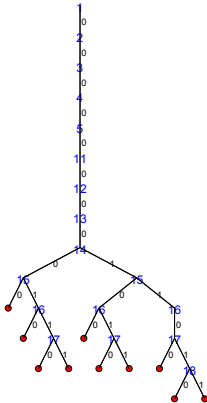
Basic Model - First Solution



Basic Model - First Solution



Basic Model - First Solution

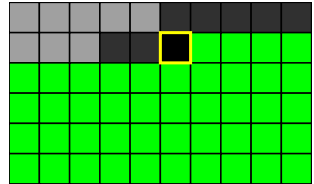
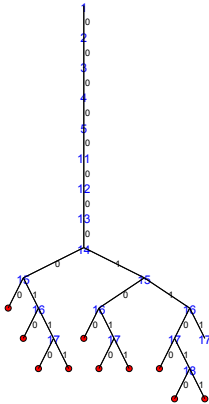


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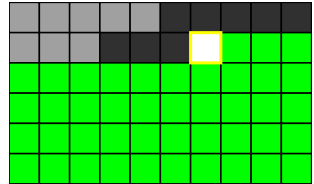
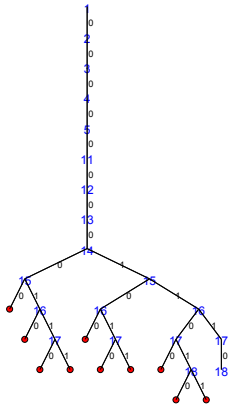
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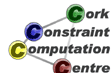
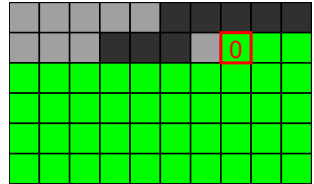
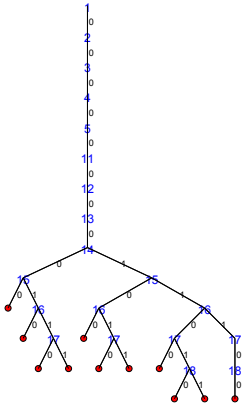
Basic Model - First Solution



Basic Model - First Solution



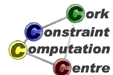
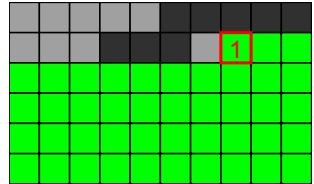
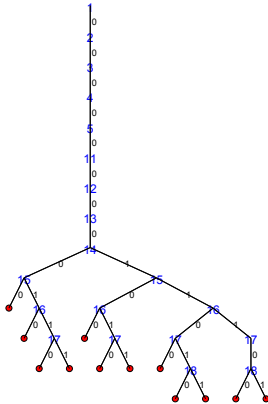
Basic Model - First Solution



◀ Back to Start ▶ Skip Animation



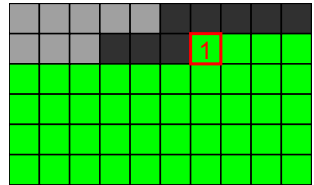
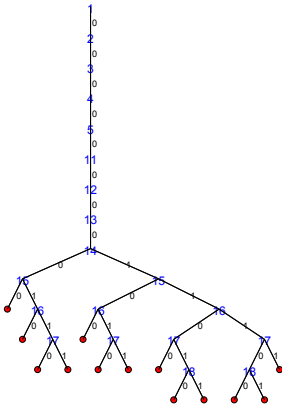
Basic Model - First Solution



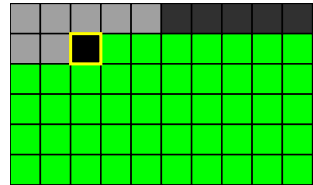
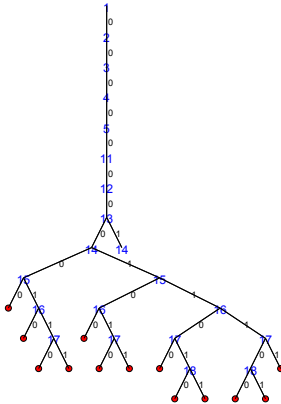
◀ Back to Start ▶ Skip Animation



Basic Model - First Solution



Basic Model - First Solution

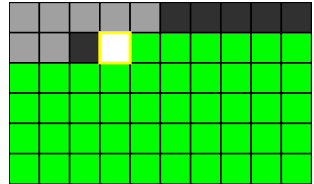
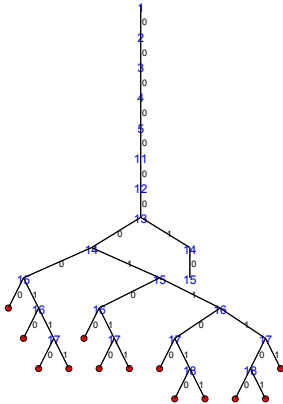


◀ Back to Start

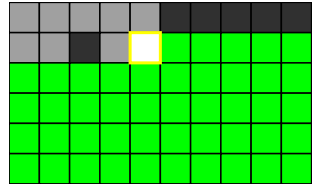
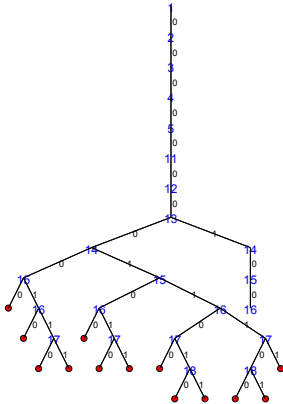
▶▶ Skip Animation



Basic Model - First Solution



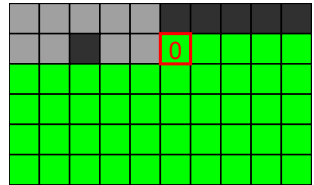
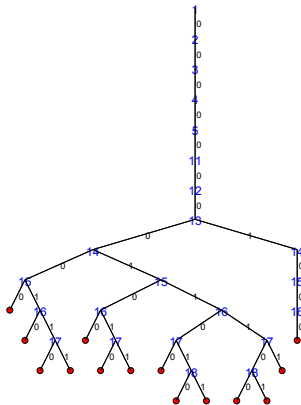
Basic Model - First Solution



◀ Back to Start

▶▶ Skip Animation

Basic Model - First Solution

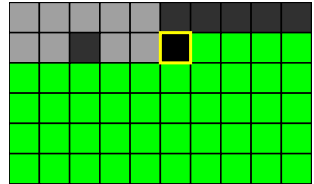
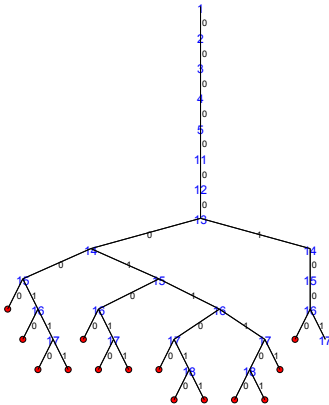


◀ Back to Start

▶▶ Skip Animation



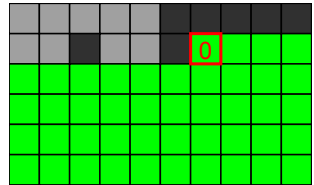
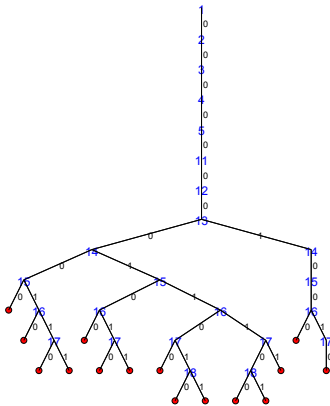
Basic Model - First Solution



◀ Back to Start

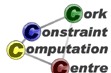
▶▶ Skip Animation

Basic Model - First Solution

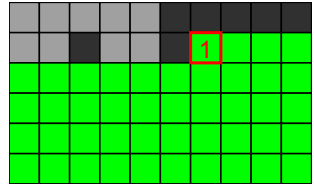
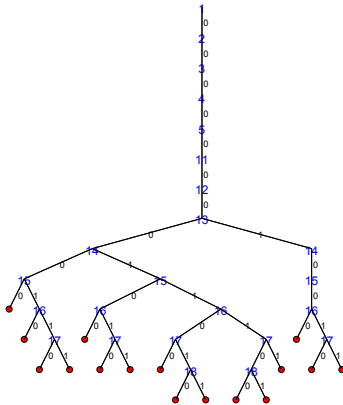


◀ Back to Start

▶▶ Skip Animation

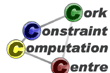


Basic Model - First Solution

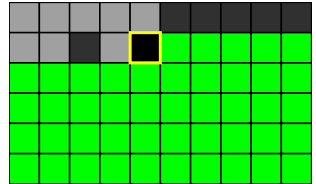
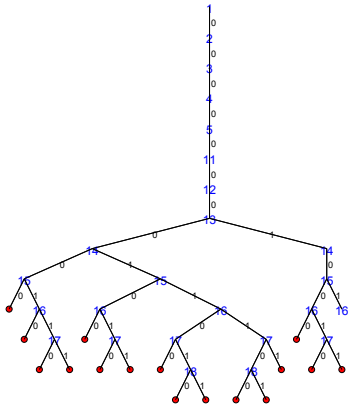


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

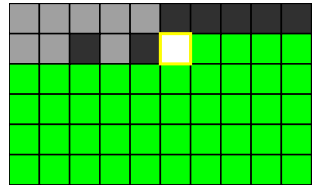
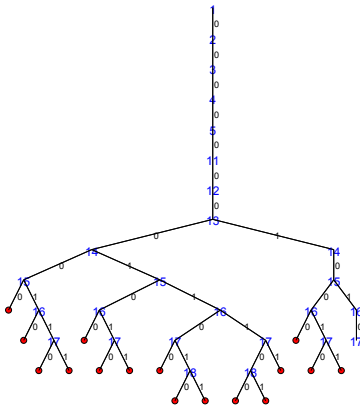


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

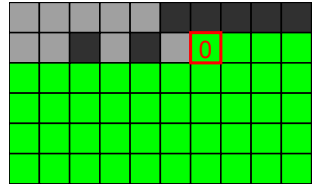
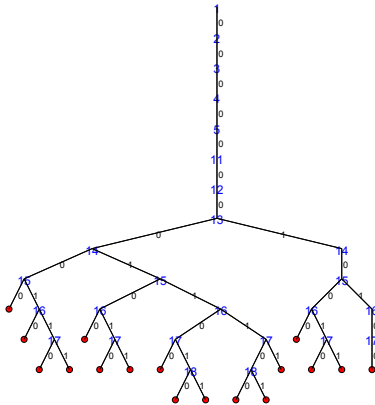


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

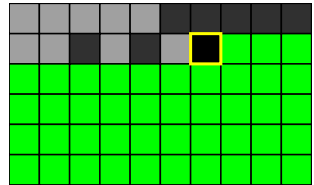
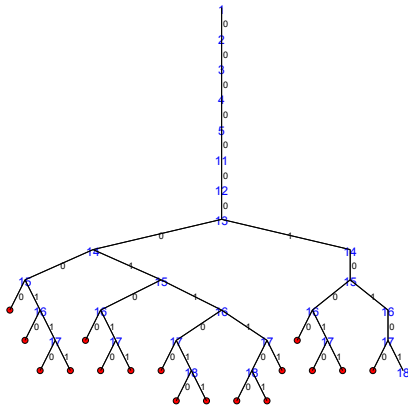


◀ Back to Start

▶▶ Skip Animation



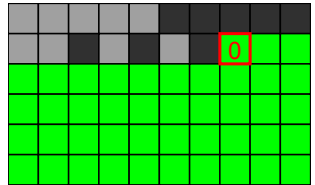
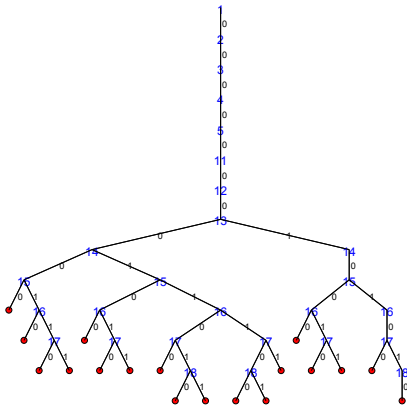
Basic Model - First Solution



◀ Back to Start

▶▶ Skip Animation

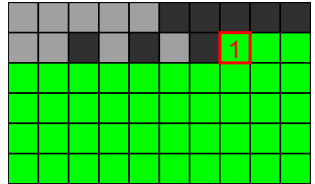
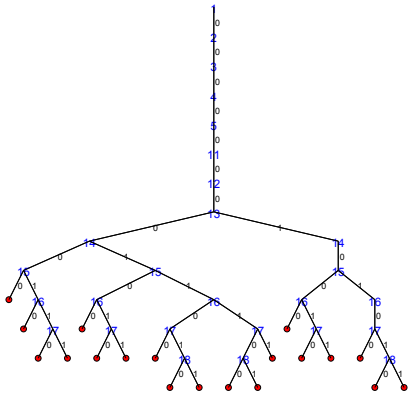
Basic Model - First Solution



◀ Back to Start

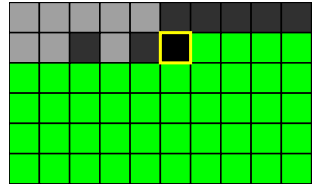
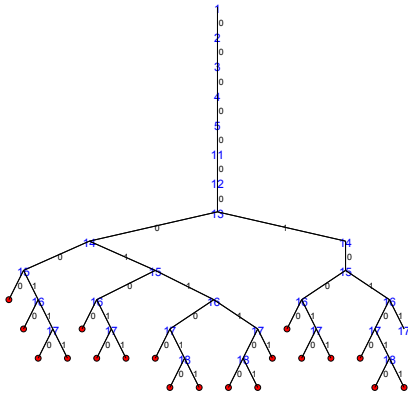
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Basic Model - First Solution



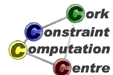
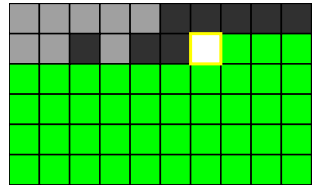
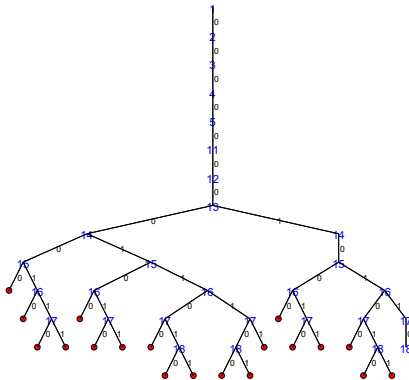
◀ Back to Start ▶▶ Skip Animation

Basic Model - First Solution



◀ Back to Start ▶ Skip Animation

Basic Model - First Solution

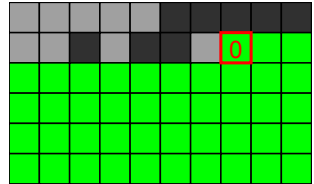
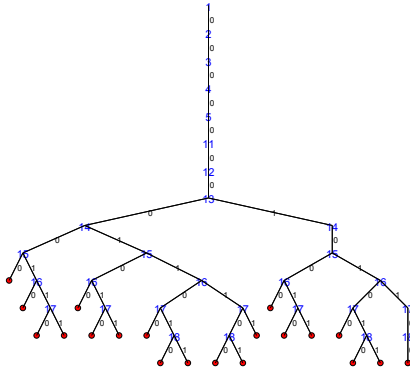


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

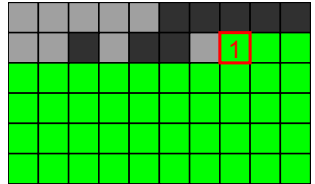
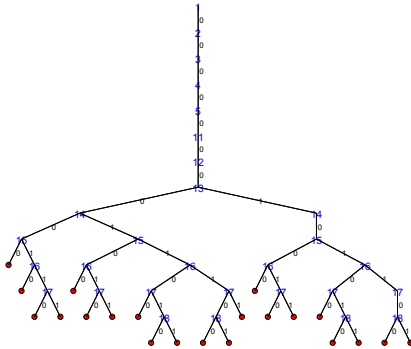


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

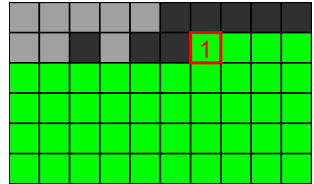
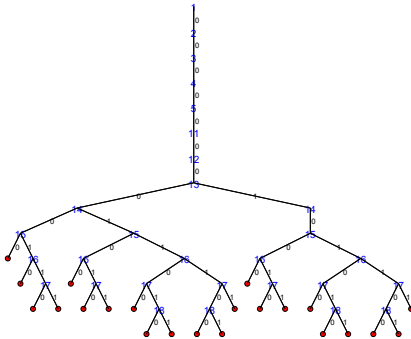


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

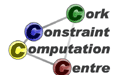
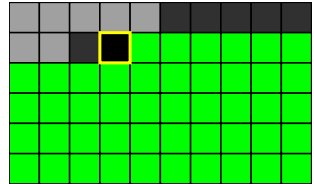
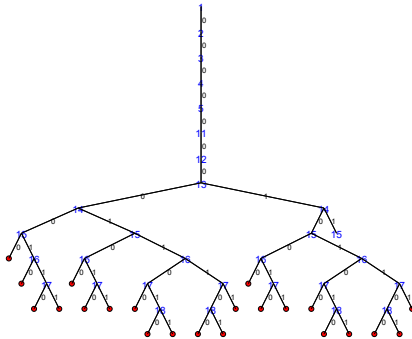


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

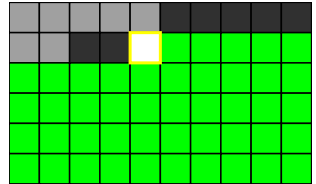
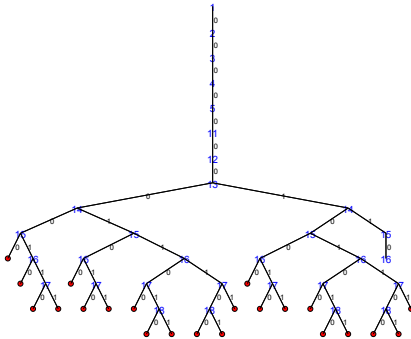


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

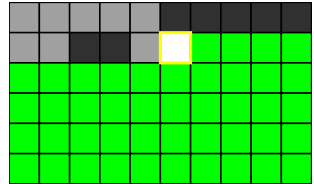
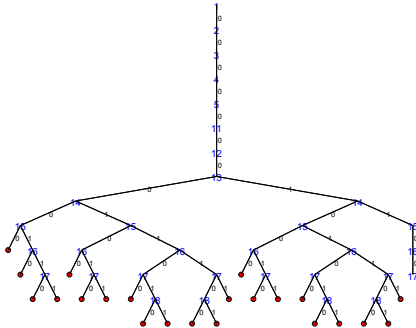


◀ Back to Start

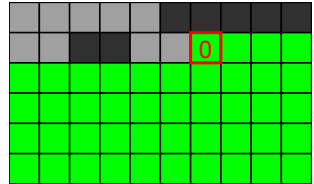
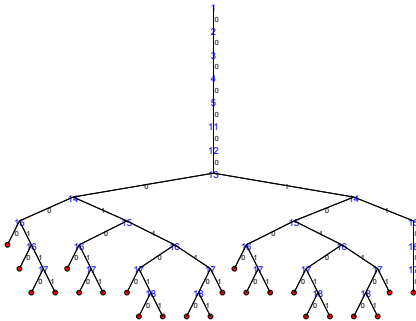
▶▶ Skip Animation



Basic Model - First Solution

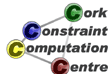


Basic Model - First Solution

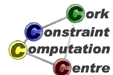
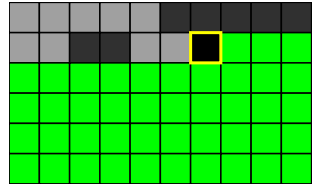
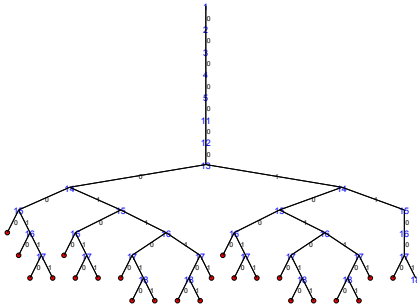


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

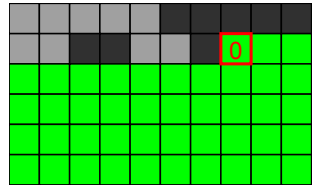
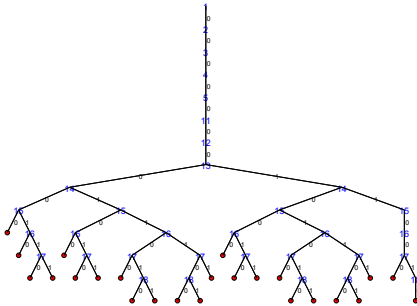


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

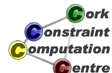
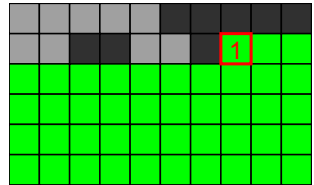
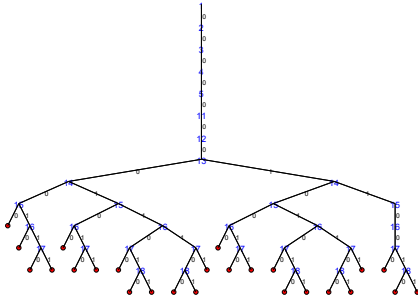


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

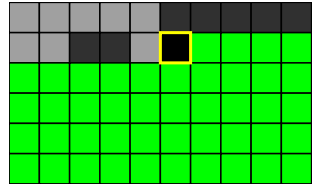
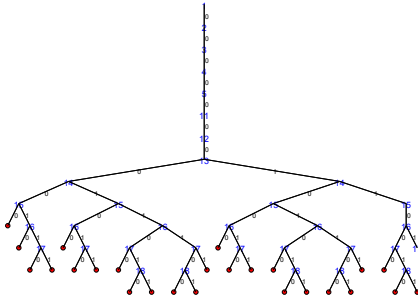


◀ Back to Start

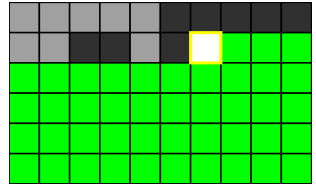
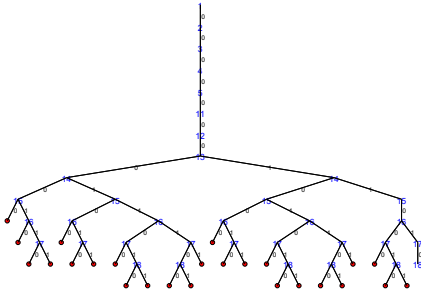
▶▶ Skip Animation



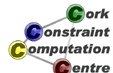
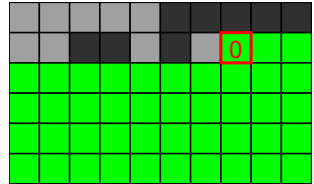
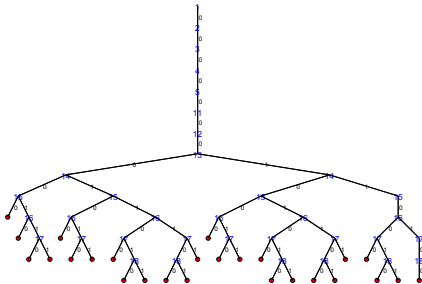
Basic Model - First Solution



Basic Model - First Solution



Basic Model - First Solution

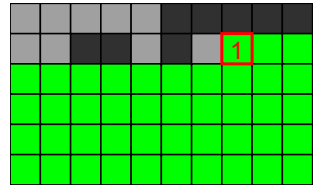
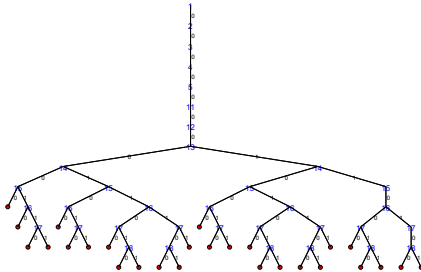


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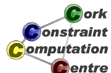
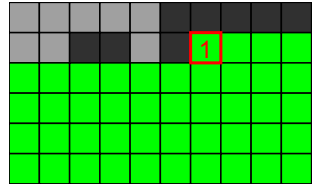
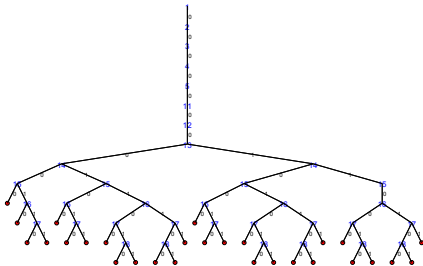
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Basic Model - First Solution



Basic Model - First Solution

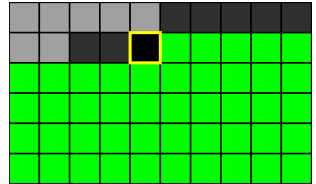
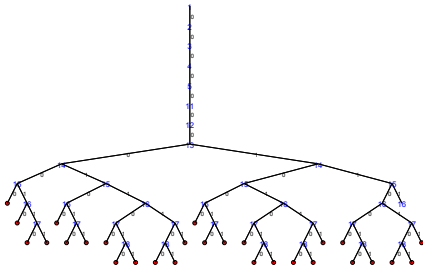


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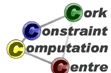


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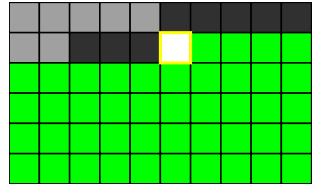
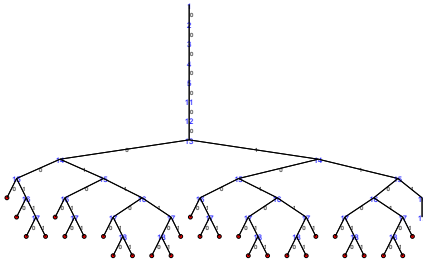


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▶▶ Skip Animation

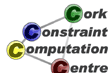


Basic Model - First Solution

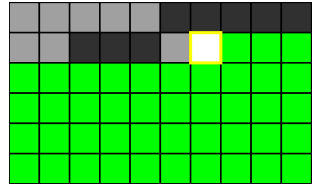
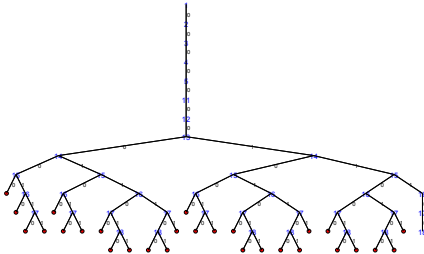


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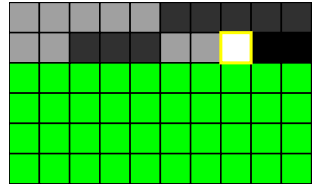
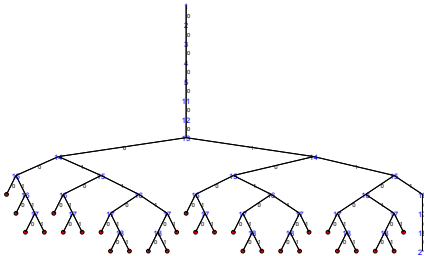
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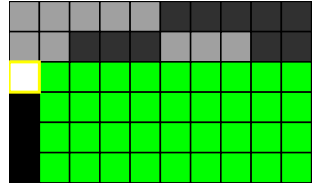
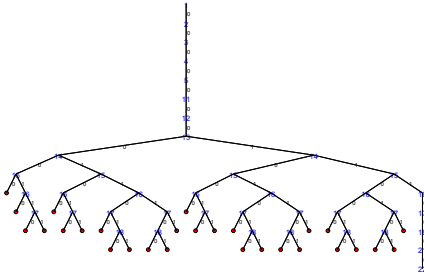
Basic Model - First Solution



Basic Model - First Solution



Basic Model - First Solution

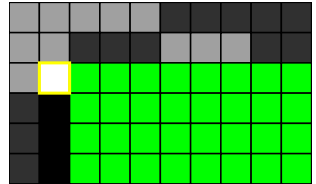
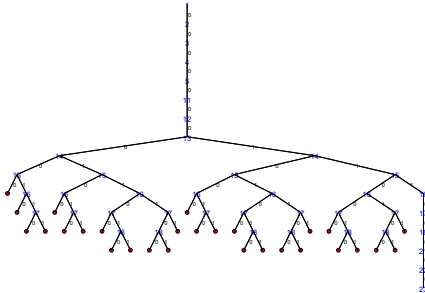


◀ Back to Start

▶▶ Skip Animation



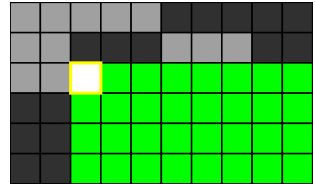
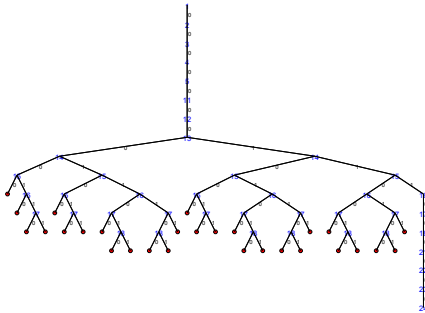
Basic Model - First Solution



◀ Back to Start

▶▶ Skip Animation

Basic Model - First Solution

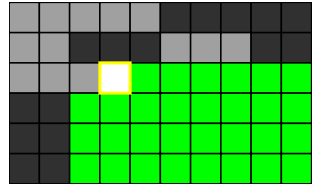
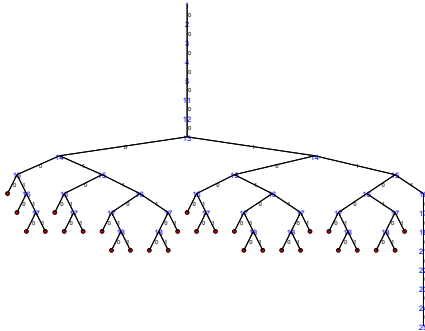


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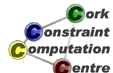
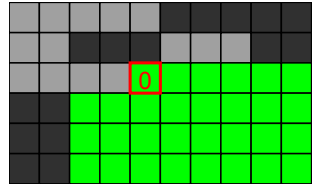
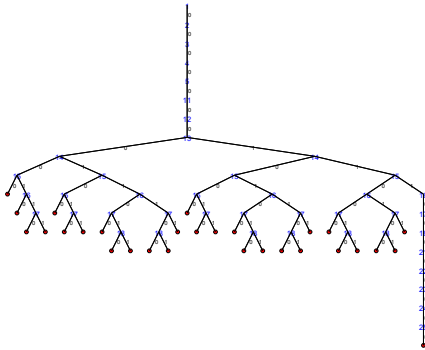
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Basic Model - First Solution



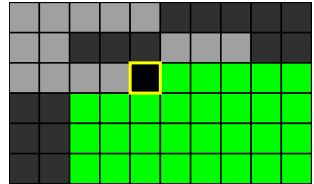
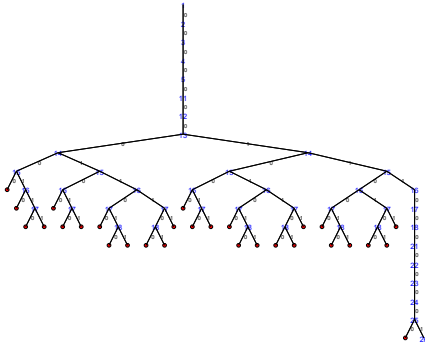
Basic Model - First Solution



◀ Back to Start ▶ Skip Animation



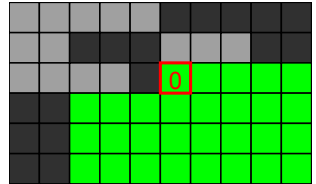
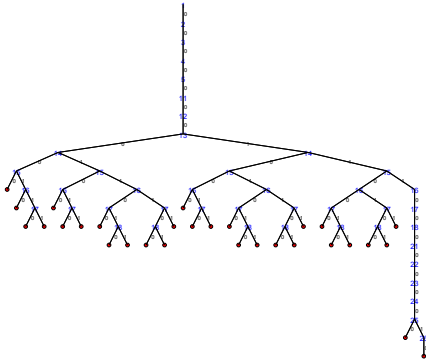
Basic Model - First Solution



◀ Back to Start

▶▶ Skip Animation

Basic Model - First Solution

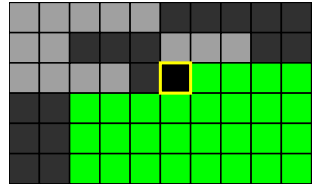
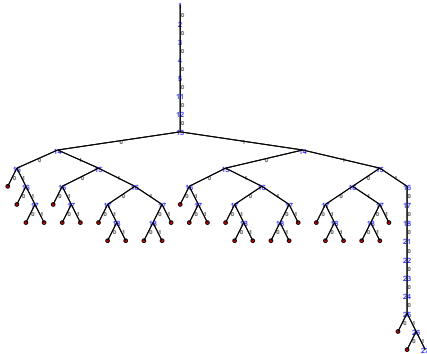


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

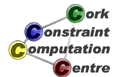
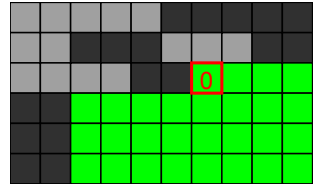
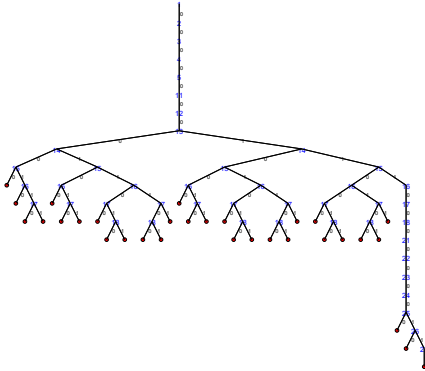


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

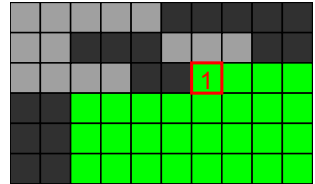
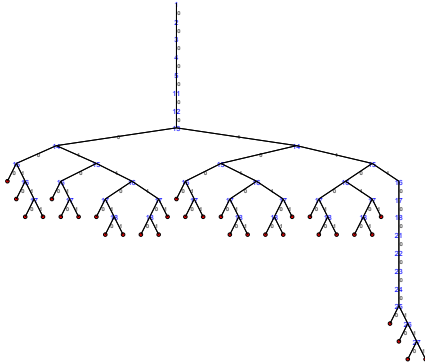


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

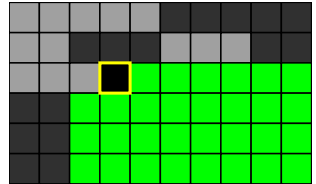
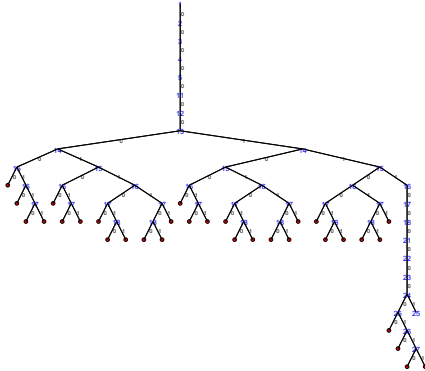


◀ Back to Start

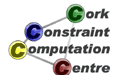
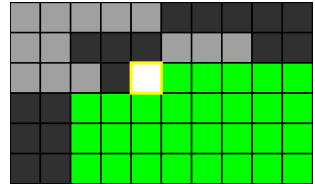
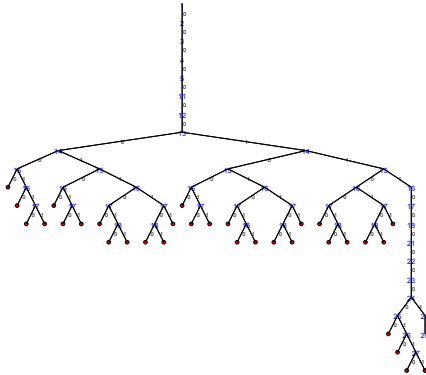
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Basic Model - First Solution



Basic Model - First Solution

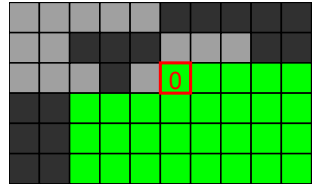
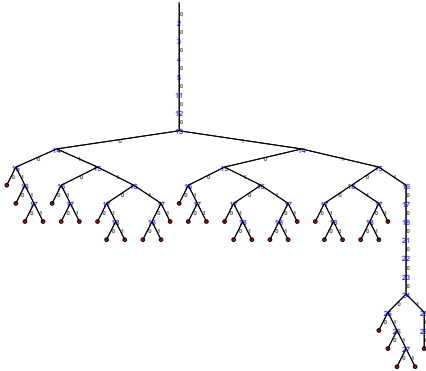


◀ Back to Start

▶▶ Skip Animation

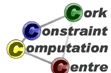


Basic Model - First Solution

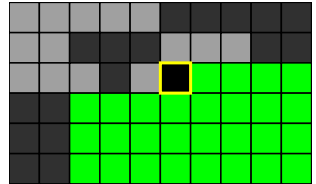
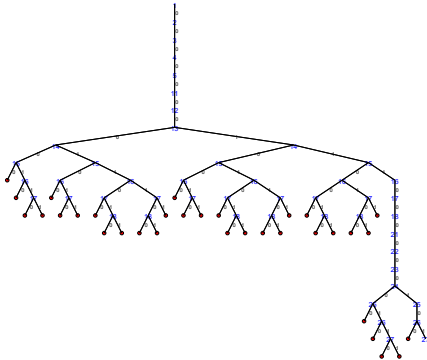


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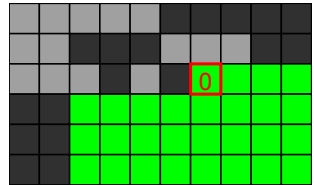
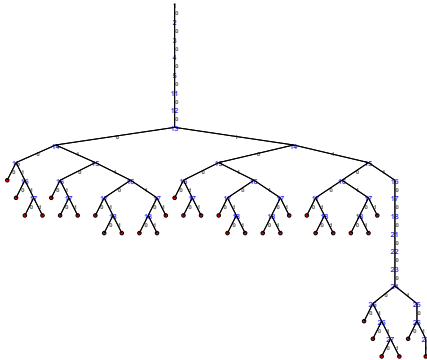
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Basic Model - First Solution



Basic Model - First Solution

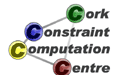
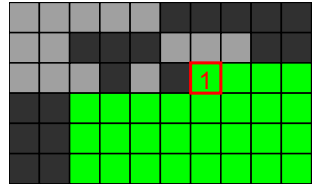
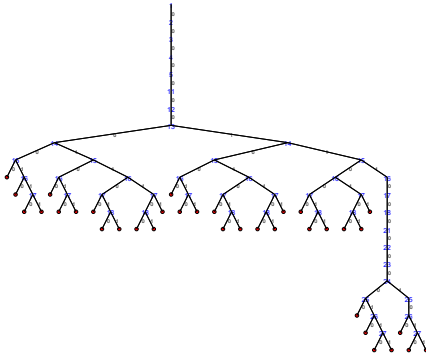


◀ Back to Start

▶▶ Skip Animation



Basic Model - First Solution

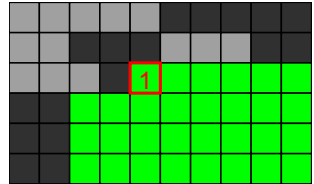
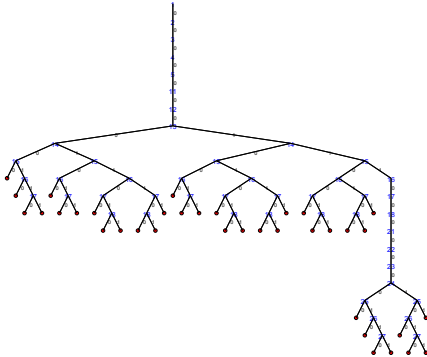


◀ Back to Start

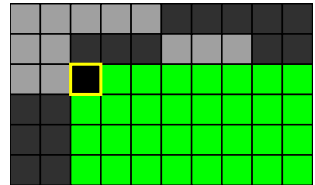
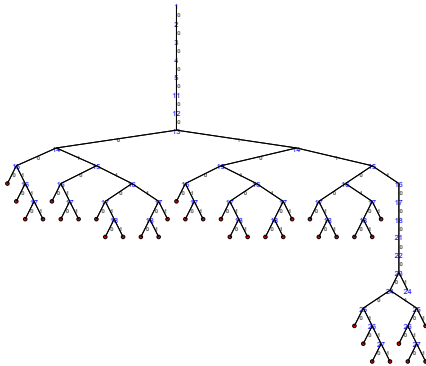
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Basic Model - First Solution

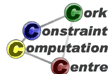


Basic Model - First Solution

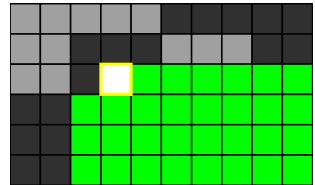
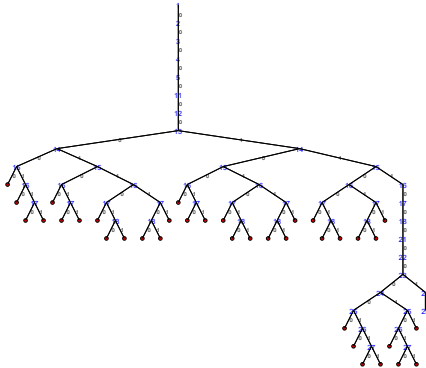


◀ Back to Start

▶ Skip Animation



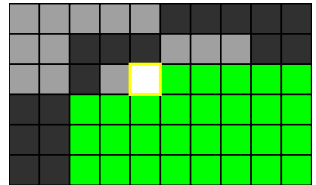
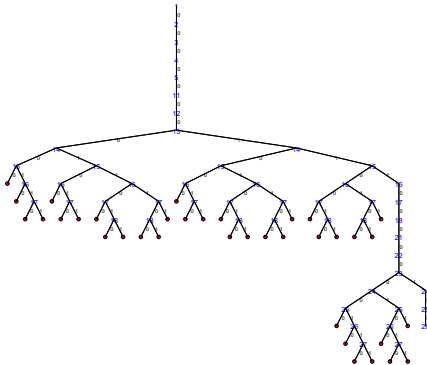
Basic Model - First Solution



◀ Back to Start ▶ Skip Animation



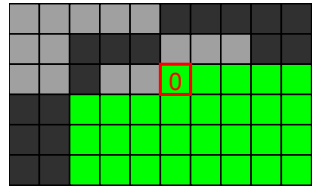
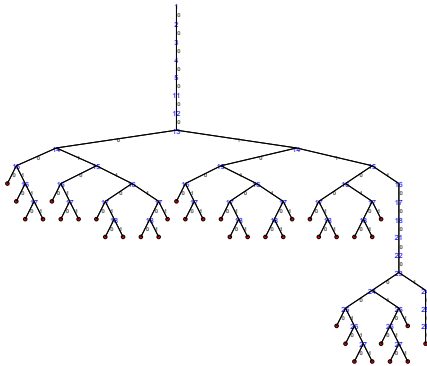
Basic Model - First Solution



◀ Back to Start ▶ Skip Animation



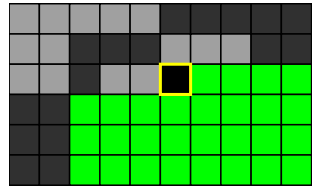
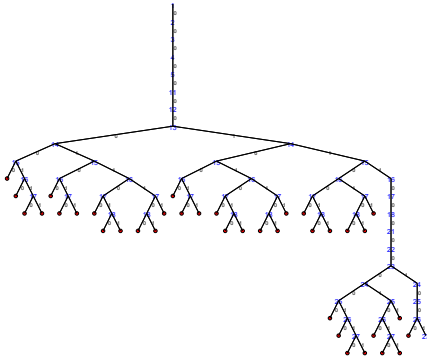
Basic Model - First Solution



◀ Back to Start

▶ Skip Animation

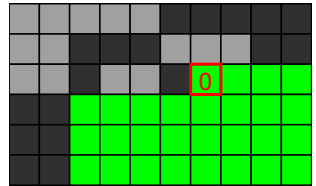
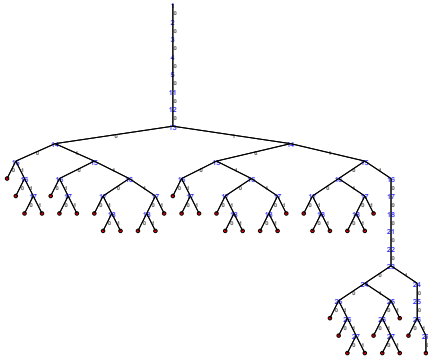
Basic Model - First Solution



◀ Back to Start ▶ Skip Animation



Basic Model - First Solution

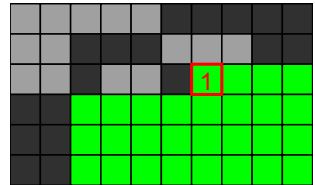
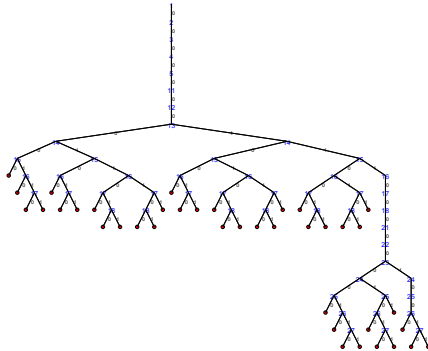


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

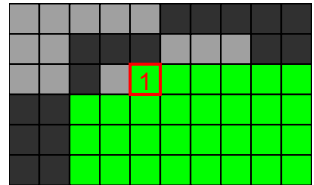
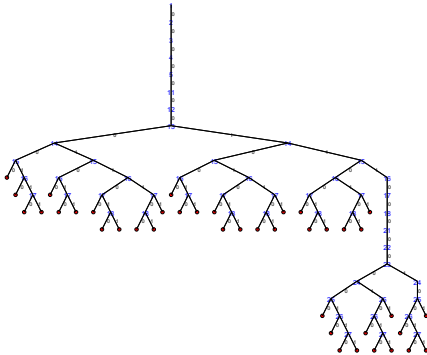


◀ Back to Start

▶ Skip Animation



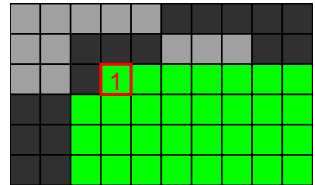
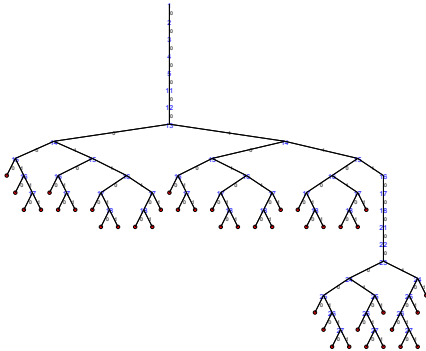
Basic Model - First Solution



◀ Back to Start ▶ Skip Animation



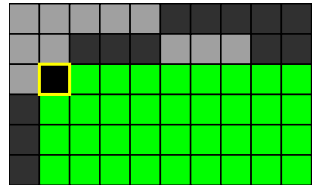
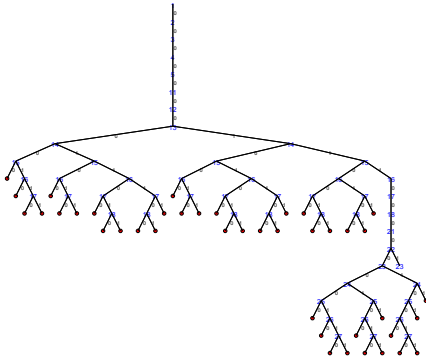
Basic Model - First Solution



◀ Back to Start

▶ Skip Animation

Basic Model - First Solution

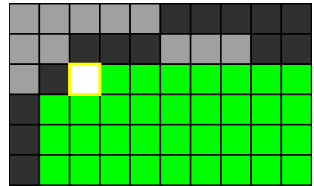
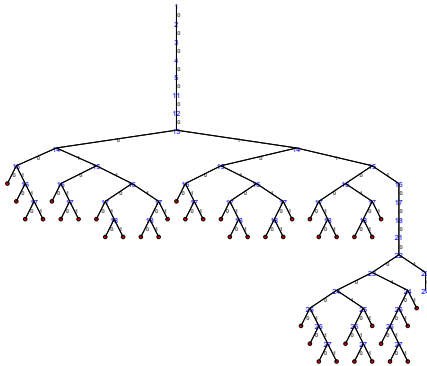


◀ Back to Start

▶ Skip Animation



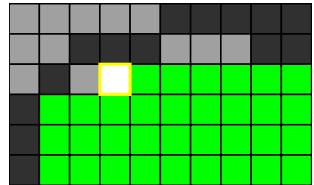
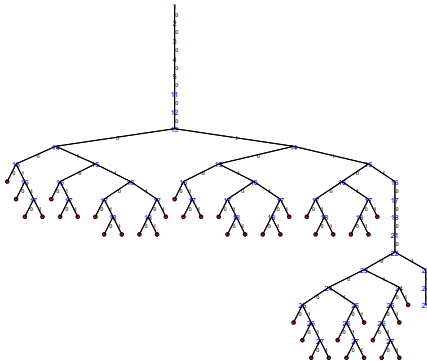
Basic Model - First Solution



◀ Back to Start ▶ Skip Animation



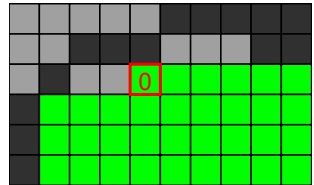
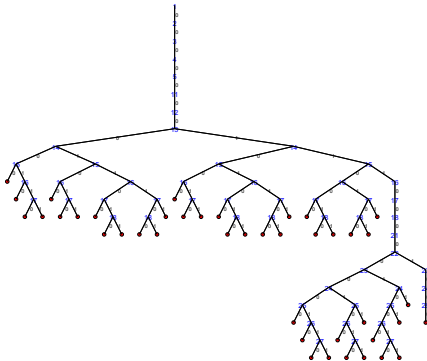
Basic Model - First Solution



◀ Back to Start ▶ Skip Animation



Basic Model - First Solution

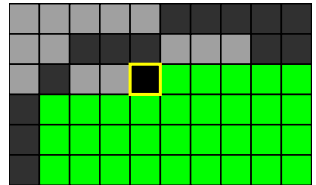
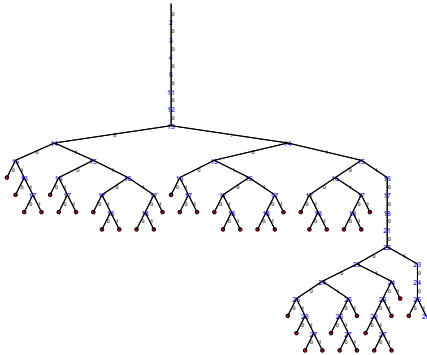


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

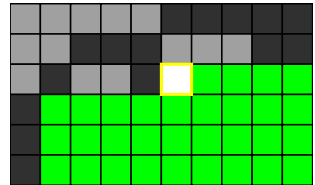
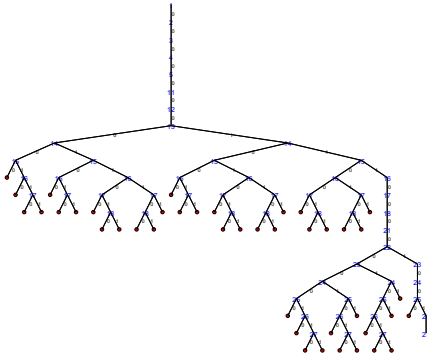


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

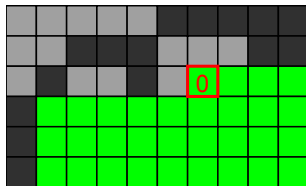
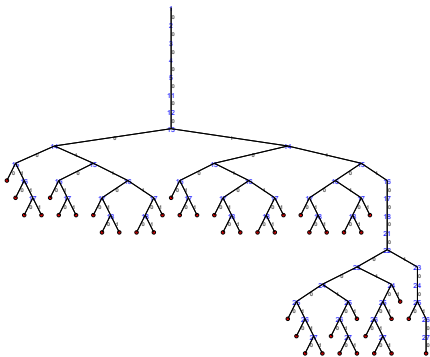


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

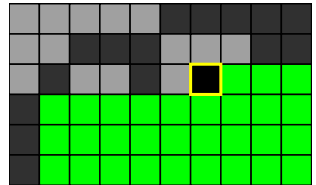
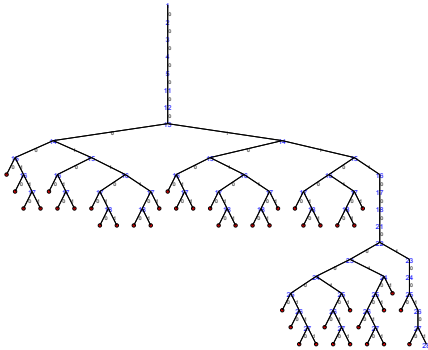


◀ Back to Start

▶ Skip Animation

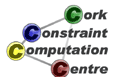


Basic Model - First Solution

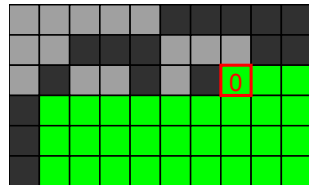
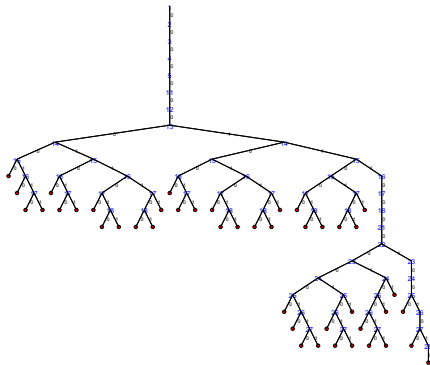


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

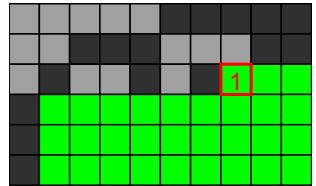
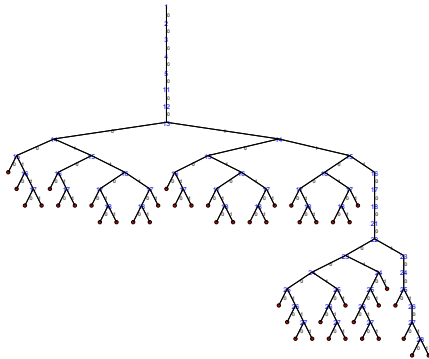


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

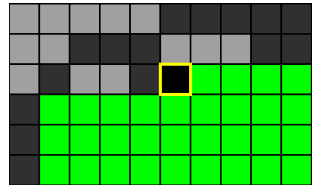
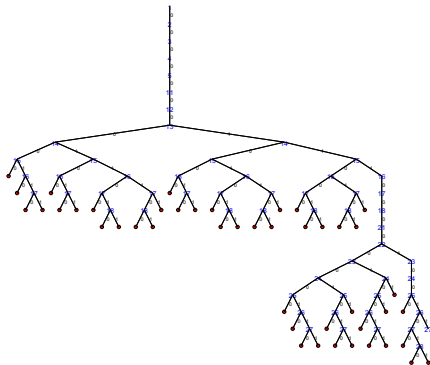


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

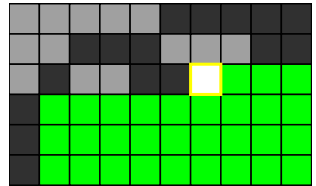
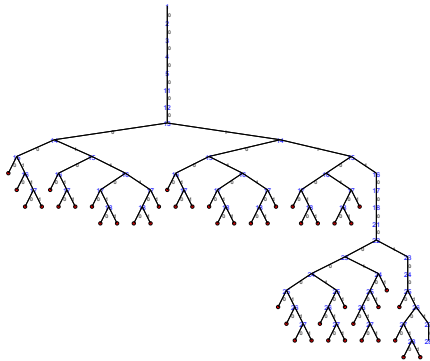


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

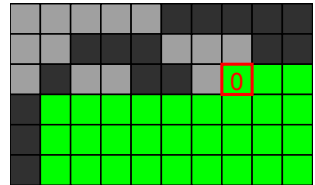
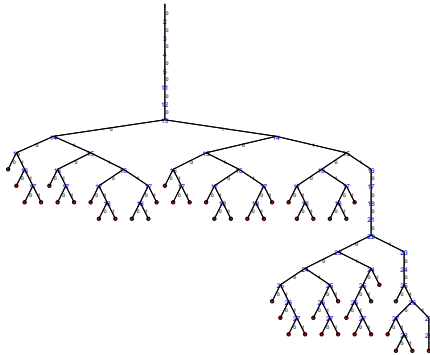


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

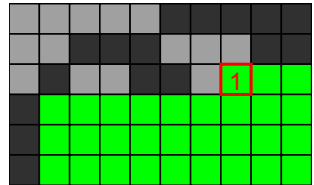
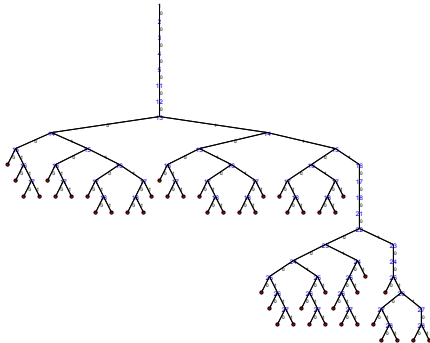


◀ Back to Start

▶ Skip Animation



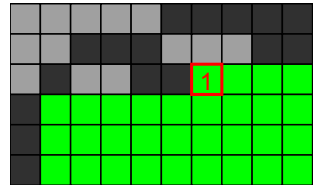
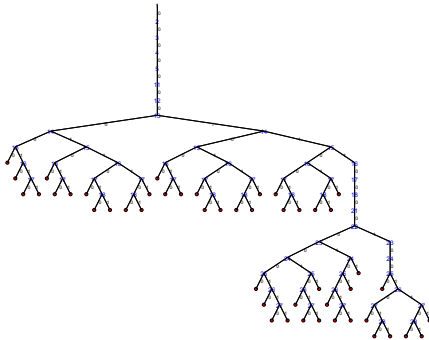
Basic Model - First Solution



◀ Back to Start

▶ Skip Animation

Basic Model - First Solution

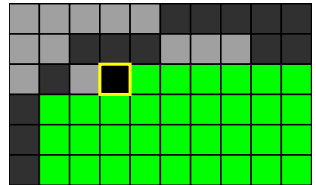
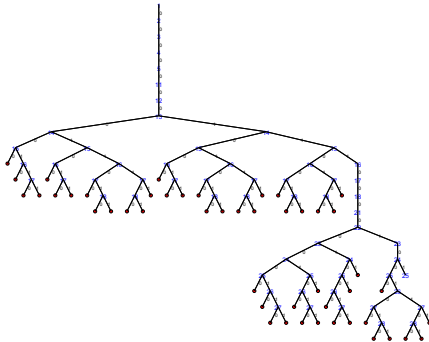


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

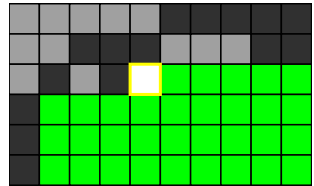
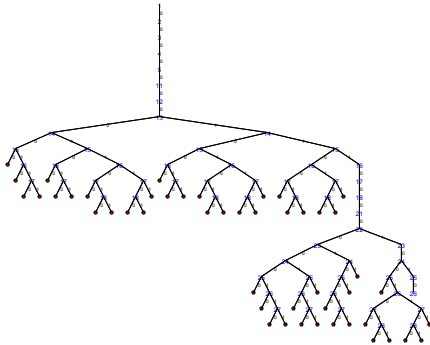


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

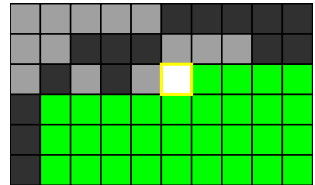
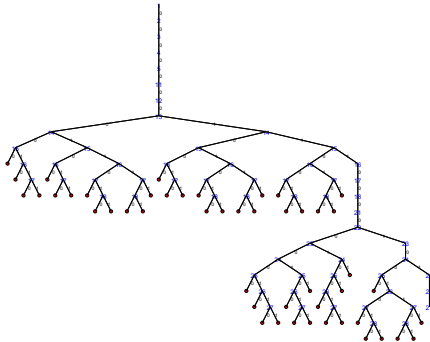


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

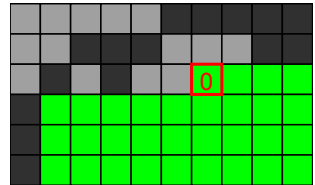
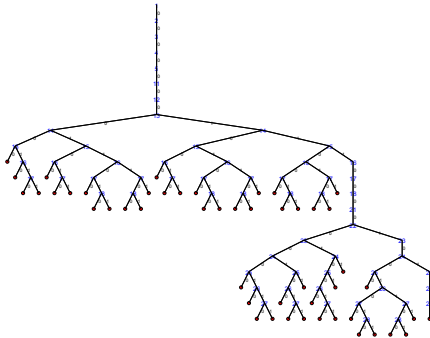


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

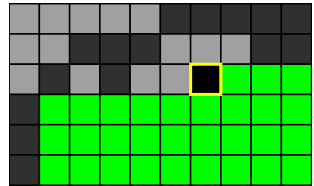
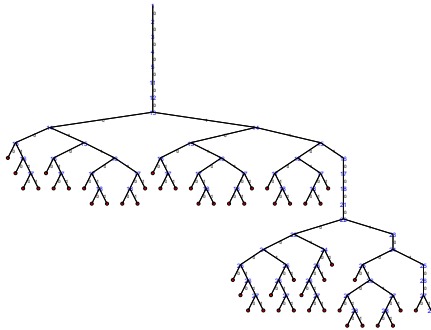


◀ Back to Start

▶ Skip Animation

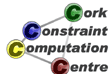


Basic Model - First Solution

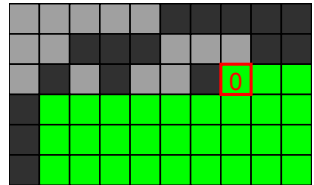
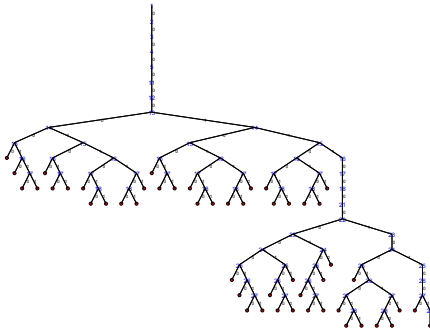


◀ Back to Start

▶ Skip Animation



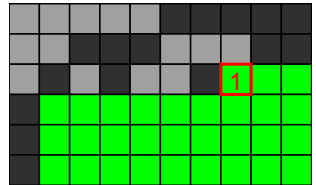
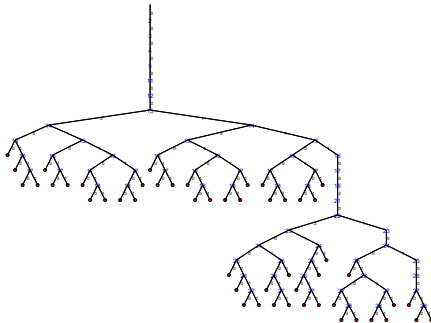
Basic Model - First Solution



◀ Back to Start

▶ Skip Animation

Basic Model - First Solution

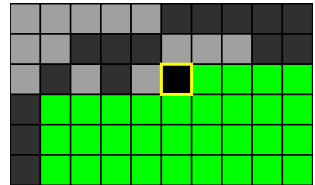
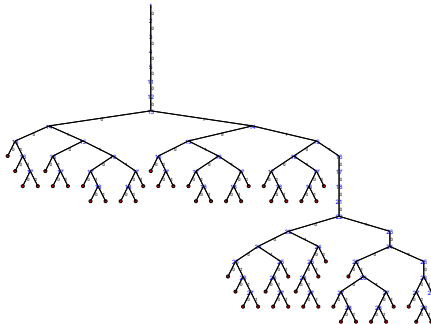


◀ Back to Start

▶ Skip Animation



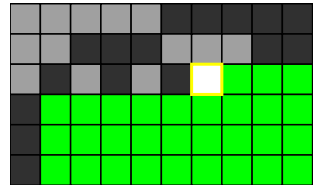
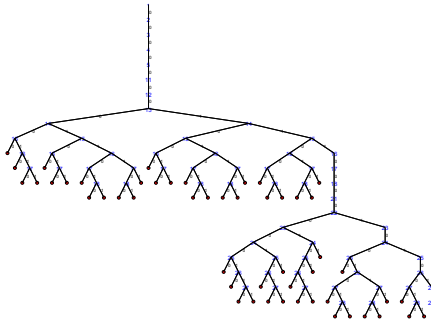
Basic Model - First Solution



◀ Back to Start

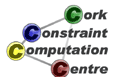
▶ Skip Animation

Basic Model - First Solution

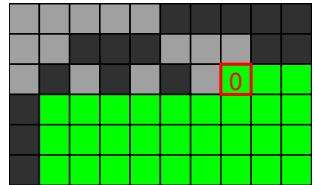
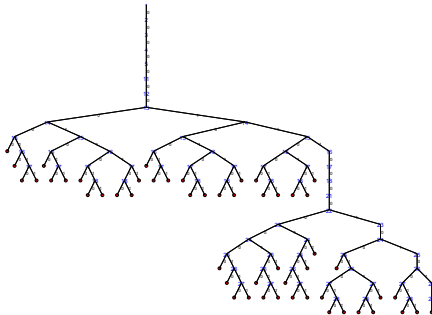


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

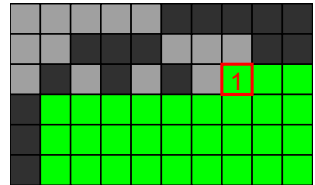
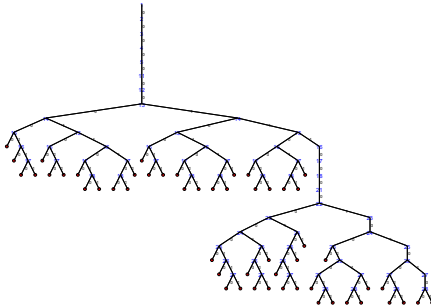


◀ Back to Start

▶ Skip Animation

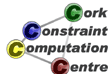


Basic Model - First Solution

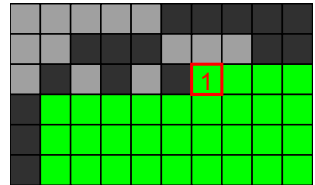
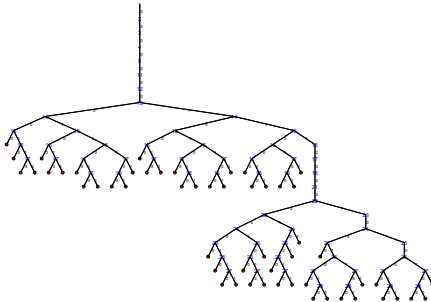


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

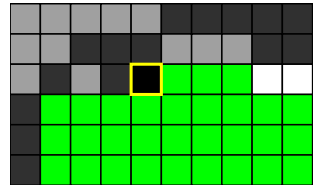
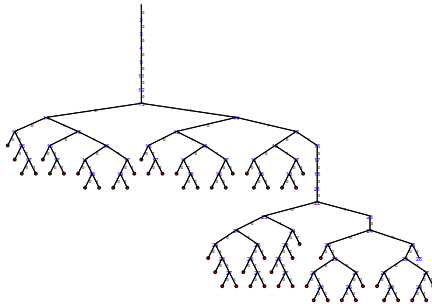


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

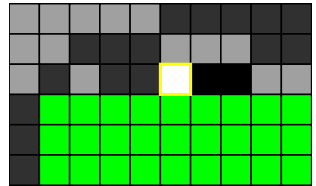
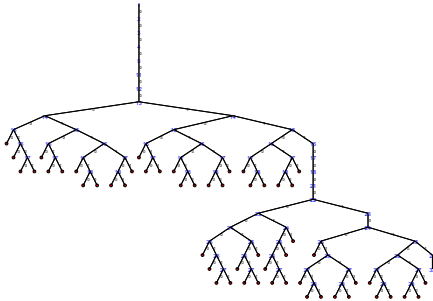


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

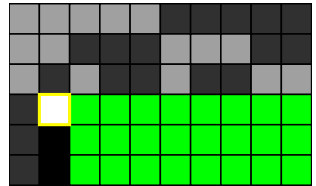
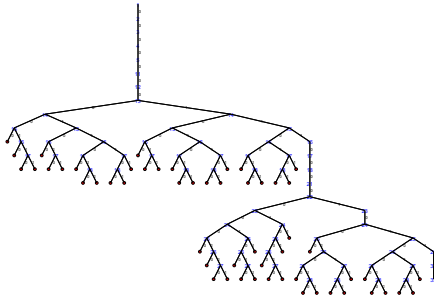


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

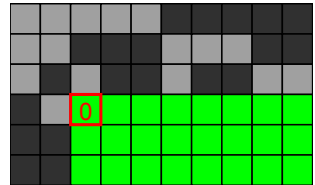
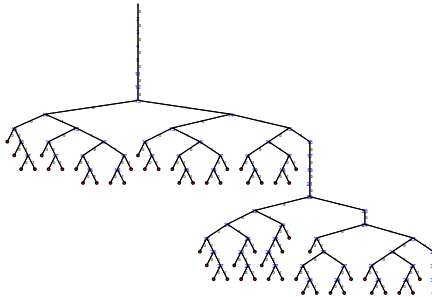


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

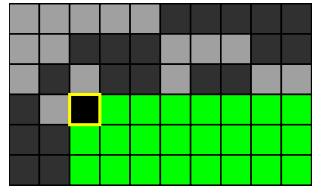
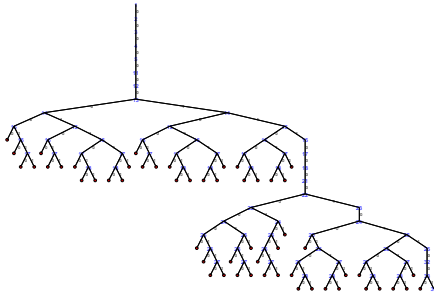


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

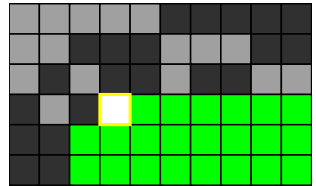
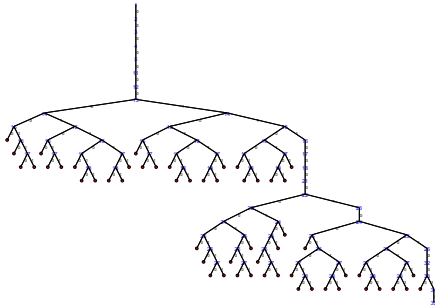


◀ Back to Start

▶ Skip Animation



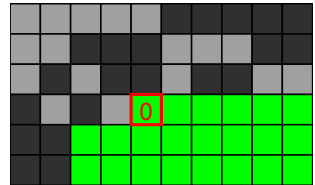
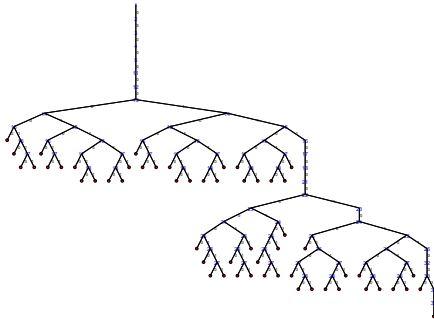
Basic Model - First Solution



◀ Back to Start

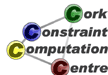
▶ Skip Animation

Basic Model - First Solution

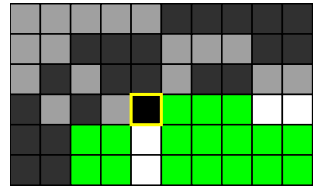
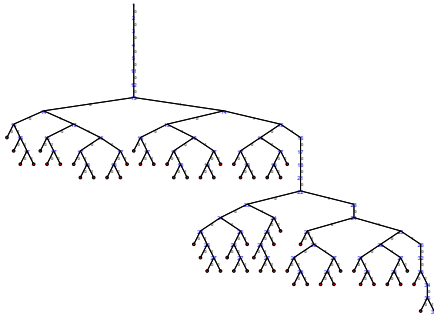


◀ Back to Start

▶ Skip Animation

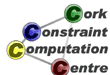


Basic Model - First Solution

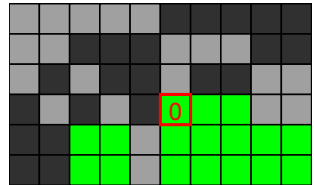
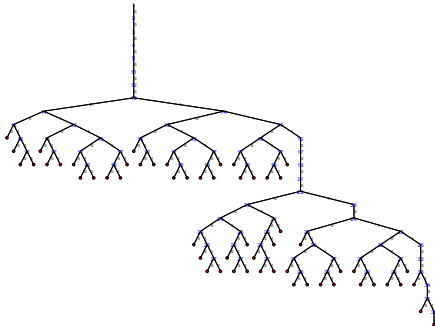


◀ Back to Start

▶ Skip Animation

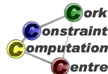


Basic Model - First Solution

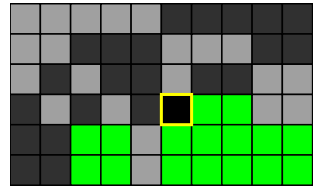
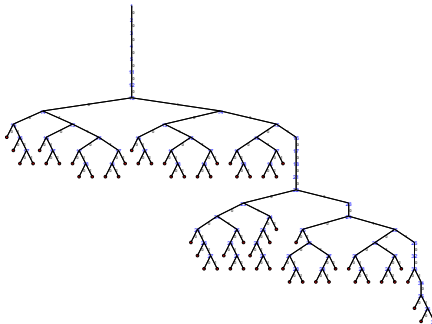


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

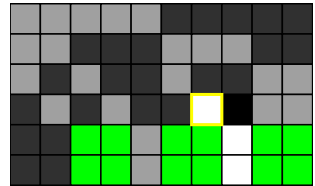
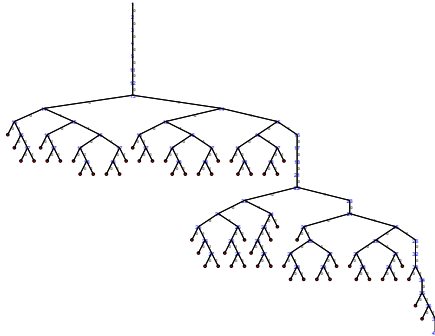


◀ Back to Start

▶ Skip Animation

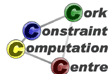


Basic Model - First Solution

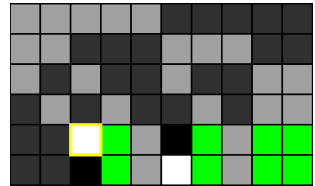
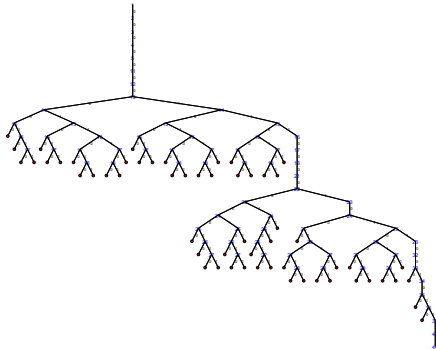


◀ Back to Start

▶ Skip Animation



Basic Model - First Solution

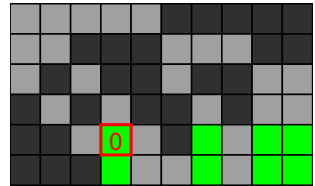
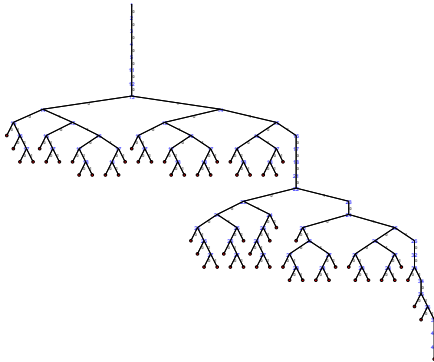


◀ Back to Start

▶ Skip Animation

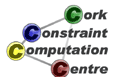


Basic Model - First Solution

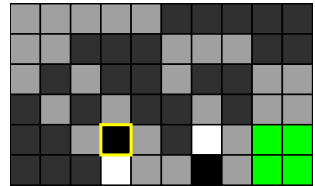
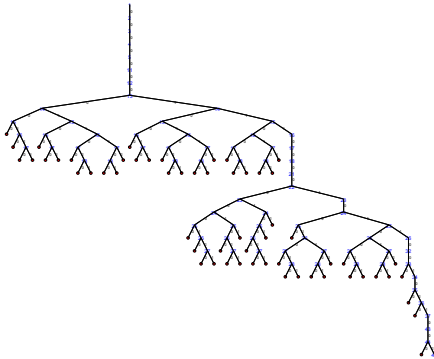


◀ Back to Start

▶ Skip Animation

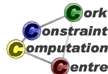


Basic Model - First Solution

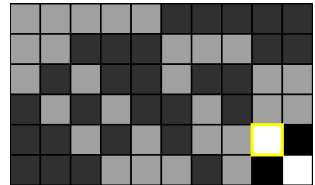
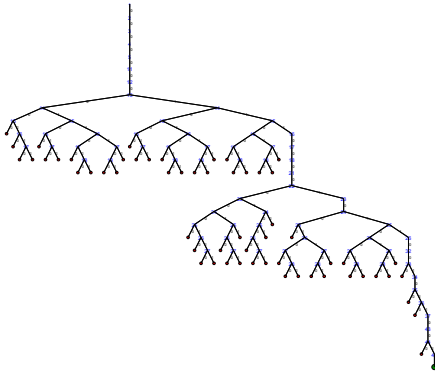


◀ Back to Start

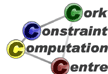
▶ Skip Animation



Basic Model - First Solution



◀ Back to Start



Finding all solutions - Hack!

```
:-module(bibd) .
:-export(top/0) .
:-lib(ic) .
:-lib(ic_global) .
```

top:-

```
    bibd(6,10,5,3,2,Matrix),writeln(Matrix),
    fail.
```

bibd(V,B,R,K,L,Matrix):-

```
    model(V,B,R,K,L,Matrix),
    extract_array(row,Matrix,List),
    search(L,0,input_order,indomain,
           complete,[]).
```



Finding all solutions - Hack!

```
:-module(bibd) .
:-export(top/0) .
:-lib(ic) .
:-lib(ic_global) .
```

top:-

```
    bibd(6,10,5,3,2,Matrix),writeln(Matrix),
    fail. ⇨ Force Backtracking
```

bibd(V,B,R,K,L,Matrix):-

```
    model(V,B,R,K,L,Matrix),
    extract_array(row,Matrix,List),
    search(L,0,input_order,indomain,
           complete,[]).
```



Finding all solutions - Proper

```
:-module(bibd) .
:-export(top/0) .
:-lib(ic) .
:-lib(ic_global) .
```

top:-

```
    findall(Matrix,bibd(6,10,5,3,2,Matrix),Sols),
    writeln(Sols) .
```

bibd(V,B,R,K,L,Matrix):-

```
    model(V,B,R,K,L,Matrix),
    extract_array(row,Matrix,List),
    search(L,0,input_order,indomain,
           complete,[]).
```



Finding all solutions - Proper

```
:-module(bibd) .
:-export(top/0) .
:-lib(ic) .
:-lib(ic_global) .
```

top:-

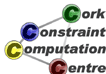
```
    findall(Matrix,bibd(6,10,5,3,2,Matrix),Sols) ,
    writeln(Sols) .
```

```
bibd(V,B,R,K,L,Matrix):-
    model(V,B,R,K,L,Matrix) ,
    extract_array(row,Matrix,List) ,
    search(L,0,input_order,indomain,
           complete,[]) .
```



findall predicate

- `findall(Template, Goal, Collection)`
- Finds all solutions to `Goal` and collects them into a list `Collection`
- `Template` is used to extract arguments from `Goal` to store as solution
- Backtracks through all choices in `Goal`
- Solutions are returned in order in which they are found



Problem

- Program now only stops when it has found all solutions
- This takes too long!
- How can we limit the amount of time to wait?
- Use of the `timeout` library



Finding all solutions - Proper

```
:-module(bibd) .  
:-export(top/0) .  
:-lib(ic) .  
:-lib(ic_global) .  
:-lib(timeout) .
```

```
top:-
```

```
    findall(Matrix, timeout(bibd(6, 10, 5, 3, 2, Matrix),  
                            10,  
                            fail), Sols),  
    writeln(Sols) .
```



Finding all solutions - Proper

```
:-module(bibd) .  
:-export(top/0) .  
:-lib(ic) .  
:-lib(ic_global) .  
:-lib(timeout) . ⇨ Load library
```

top:-

```
    findall(Matrix, timeout(bibd(6, 10, 5, 3, 2, Matrix),  
                            10,  
                            fail), Sols),  
    writeln(Sols) .
```



Finding all solutions - Proper

```
:-module(bibd) .
:-export(top/0) .
:-lib(ic) .
:-lib(ic_global) .
:-lib(timeout) .
```

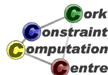
top:-

```
    findall(Matrix, timeout(bibd(6, 10, 5, 3, 2, Matrix),
                            10, ↪ seconds
                            fail), Sols),
    writeln(Sols) .
```

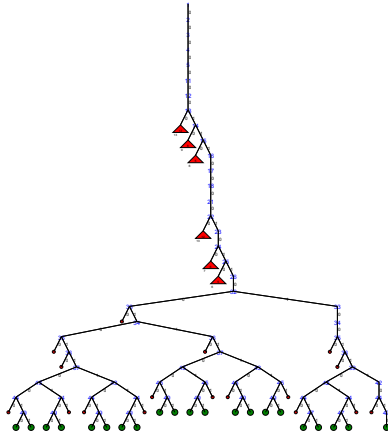


timeout library

- `timeout (Goal, Limit, TimeoutGoal)`
- **Runs Goal for Limit seconds**
- **If Limit is reached, Goal is stopped and TimeoutGoal is run instead**
- **If Limit is not reached, it has no impact**
- **Must load `:-lib(timeout).`**

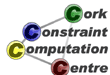


Search Tree 200 Nodes

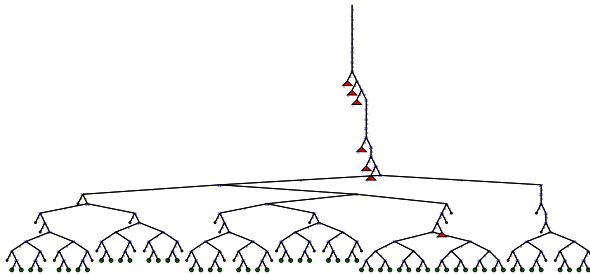


Observation

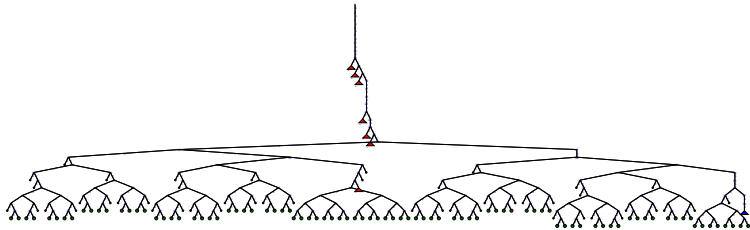
- Surprise! There are many solutions



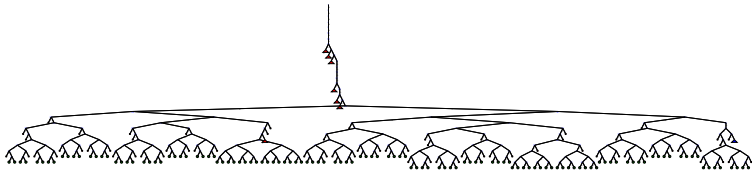
Search Tree 300 Nodes



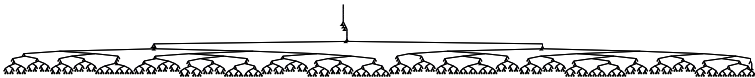
Search Tree 400 Nodes



Search Tree 500 Nodes



Search Tree 1000 Nodes



Search Tree 2000 Nodes



Problem

- There are too many solutions to collect in a reasonable time
- Most of these solutions are very similar
- If you take one solution and
 - exchange two rows
 - and/or exchange two columns
- ... you have another solution
- Can we avoid exploring them all?



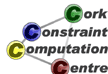
Outline

- 1 Problem
- 2 Program
- 3 Symmetry Breaking
 - Experiment with alternative value order



Symmetry Breaking Techniques

- Remove all symmetries
 - Reduce the search tree as much as possible
 - May be hard to describe all symmetries
 - May be expensive to remove symmetric parts of tree
- Remove some symmetries
 - Search is not reduced as much
 - May be easier to find some symmetries to remove
 - Cost can be low



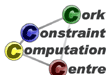
Symmetry Breaking Techniques

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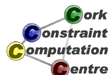
Symmetry Breaking Techniques

- Symmetry removal by forcing partial, initial assignment
 - Easy to understand
 - Rather weak, does not affect search
- Symmetry removal by stating constraints
 - Removing all symmetries may require exponential number of constraints
 - Can conflict with search strategies
- Symmetry removal by controlling search
 - At each node, decide if it needs to be explored
 - Can be expensive to check



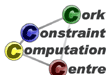
Symmetry Breaking Techniques

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 - Removing all symmetries may require exponential number of constraints
 - Can conflict with search strategies
- Symmetry removal by controlling search
 - At each node, decide if it needs to be explored
 - Can be expensive to check



Solution used here: Double Lex

- Partial symmetry removal by adding lexicographical ordering constraints
- Our problem has full row and column symmetries
- Any permutation of rows and/or columns leads to another solution
- Idea: Order rows lexicographically
- Rows must be different from each other, strict order on rows
- Columns might be identical, non strict order on columns
 - This can be improved in some cases
- Constraints only between adjacent rows(columns)



Added Constraints

```
dim(Matrix, [V, B]),  
(for(I, 1, V-1),  
  param(Matrix, B) do  
    I1 is I+1,  
    lex_less(Matrix[I1, 1..B], Matrix[I, 1..B])  
  ),  
(for(J, 1, B-1),  
  param(Matrix, V) do  
    J1 is J+1,  
    lex_leq(Matrix[1..V, J1], Matrix[1..V, J])  
  ),
```

Added Constraints

```

dim(Matrix, [V, B]),
(for(I, 1, V-1),
  param(Matrix, B) do
    I1 is I+1,
    lex_less(Matrix[I1, 1..B], Matrix[I, 1..B])
), ⇨ Row lex constraints
(for(J, 1, B-1),
  param(Matrix, V) do
    J1 is J+1,
    lex_leq(Matrix[1..V, J1], Matrix[1..V, J])
),

```



Added Constraints

```
dim(Matrix, [V, B]),  
(for(I, 1, V-1),  
  param(Matrix, B) do  
    I1 is I+1,  
    lex_less(Matrix[I1, 1..B], Matrix[I, 1..B])  
  ),  
(for(J, 1, B-1),  
  param(Matrix, V) do  
    J1 is J+1,  
    lex_leq(Matrix[1..V, J1], Matrix[1..V, J])  
  ),  $\Rightarrow$  Column lex constraints
```



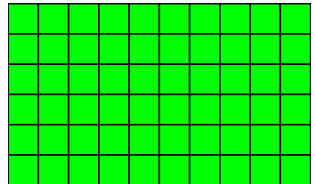
Two new global constraints

- `lex_leq(List1, List2)`
 - `List1` is lexicographical smaller than or equal to `List2`
 - Achieves domain consistency
- `lex_less(List1, List2)`
 - `List1` is lexicographical smaller than `List2`
 - Achieves domain consistency



Complete Search Tree with Double Lex

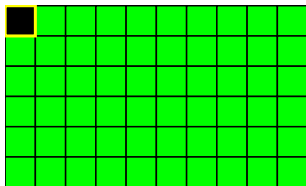
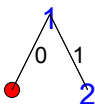
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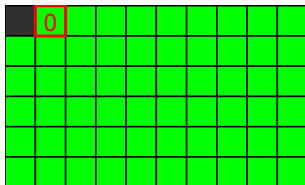
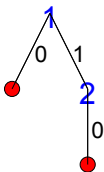
» Skip Animation



Complete Search Tree with Double Lex

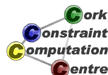
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Complete Search Tree with Double Lex

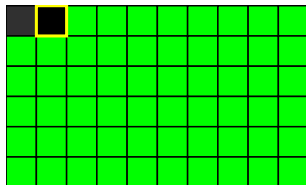
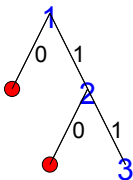


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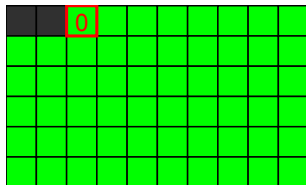
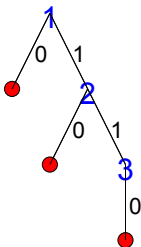


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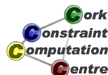


Complete Search Tree with Double Lex

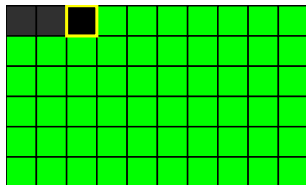
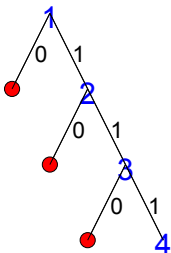


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Complete Search Tree with Double Lex

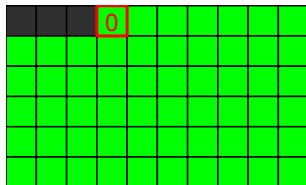
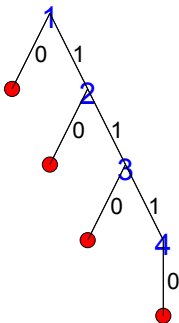


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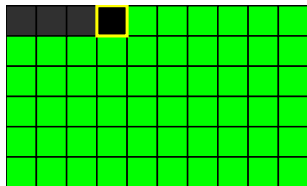
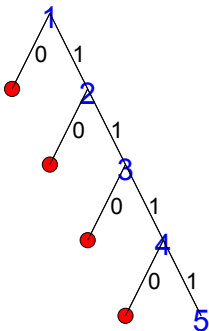


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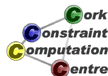


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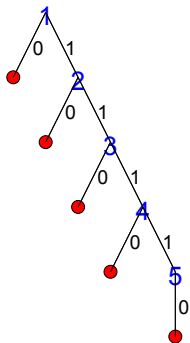


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Complete Search Tree with Double Lex



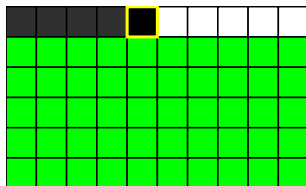
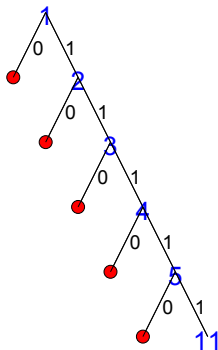
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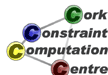


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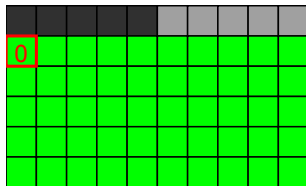
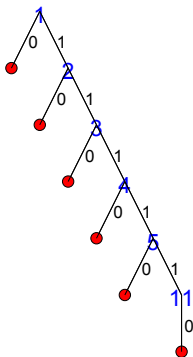


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Complete Search Tree with Double Lex

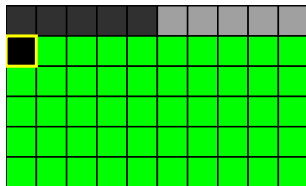
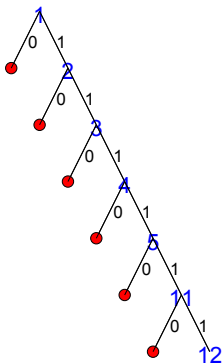


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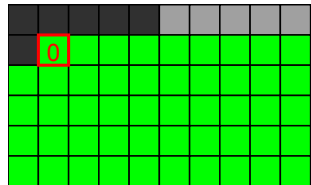
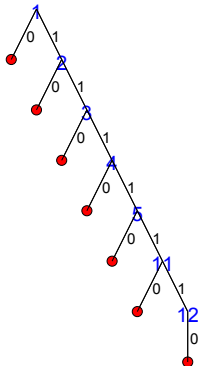


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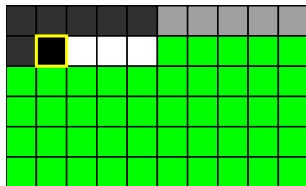
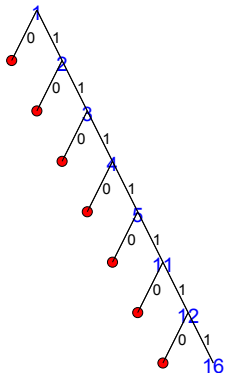


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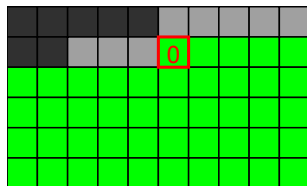
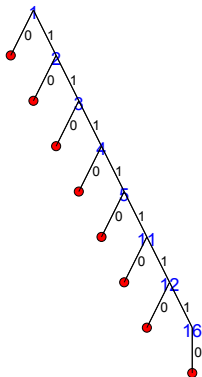


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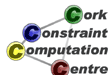


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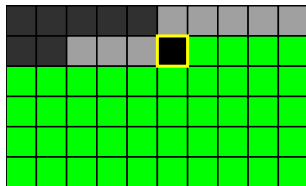
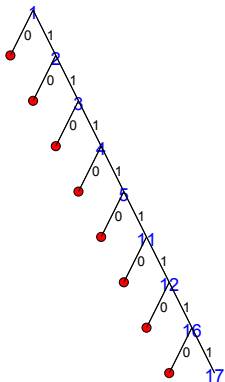


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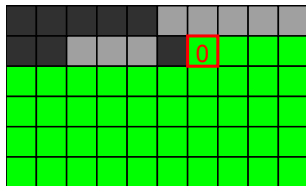
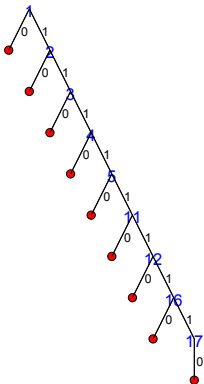


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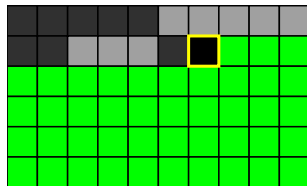
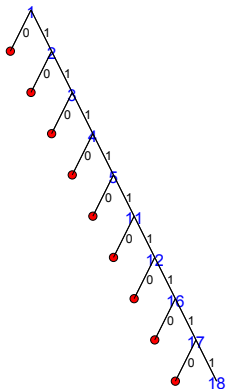


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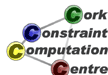


Complete Search Tree with Double Lex

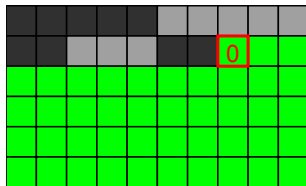
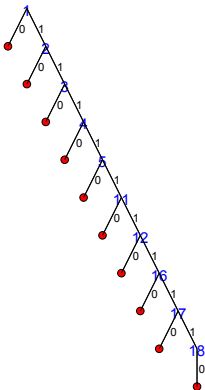


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Complete Search Tree with Double Lex

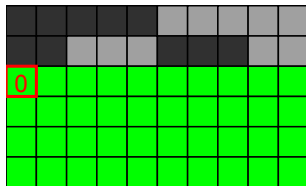
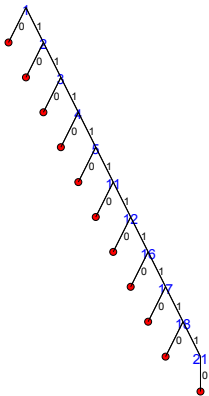


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Complete Search Tree with Double Lex

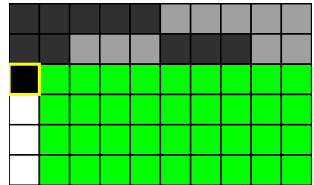
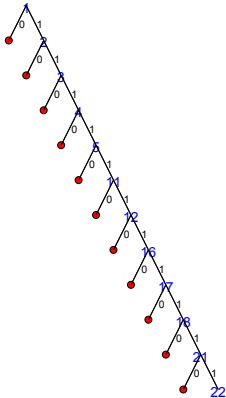


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Complete Search Tree with Double Lex

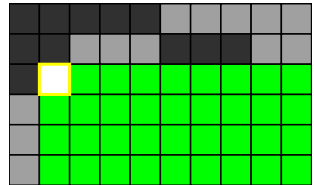
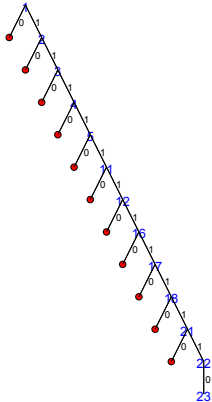


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Complete Search Tree with Double Lex

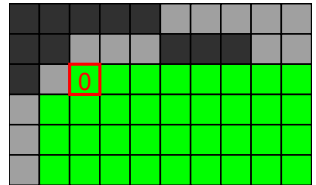
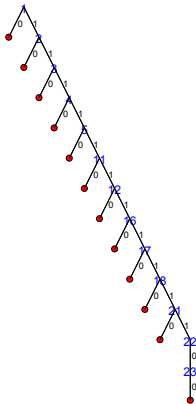


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Complete Search Tree with Double Lex

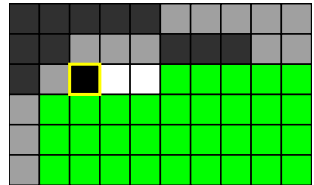
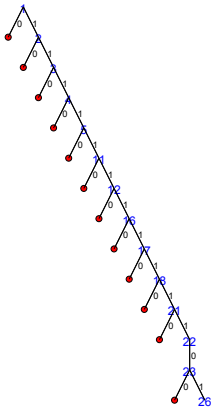


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Complete Search Tree with Double Lex

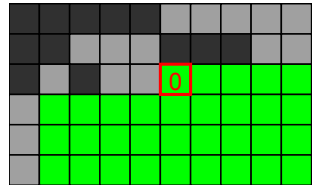
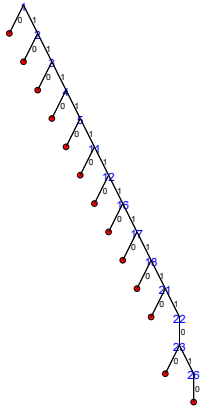


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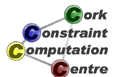


Complete Search Tree with Double Lex

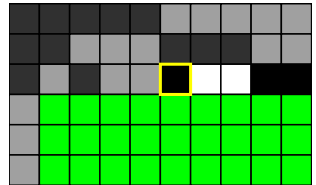
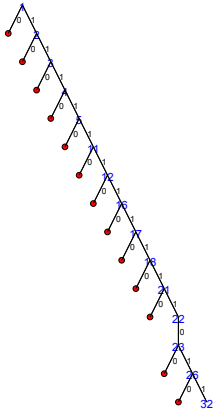


◀ Back to Start

▶ Skip Animation



Complete Search Tree with Double Lex

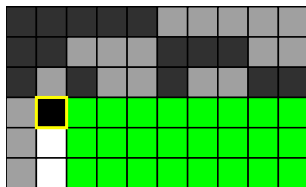
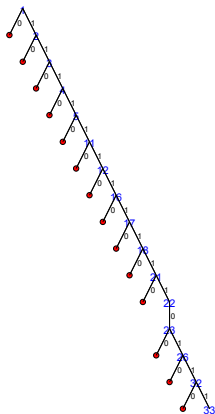


◀ Back to Start

▶ Skip Animation

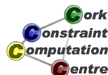


Complete Search Tree with Double Lex

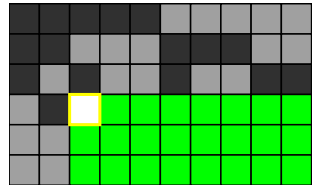
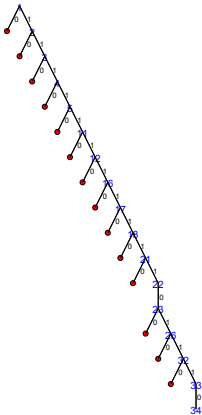


◀ Back to Start

▶ Skip Animation



Complete Search Tree with Double Lex

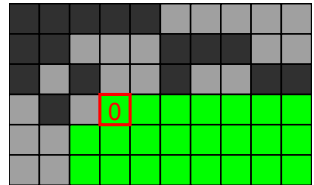
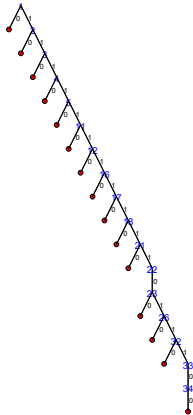


◀ Back to Start

▶ Skip Animation



Complete Search Tree with Double Lex

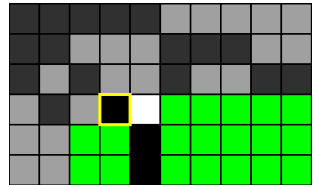
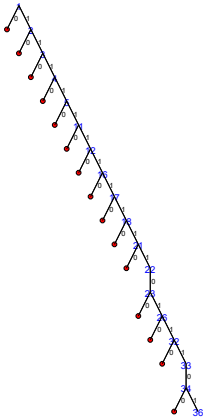


◀ Back to Start

▶ Skip Animation

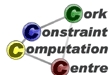


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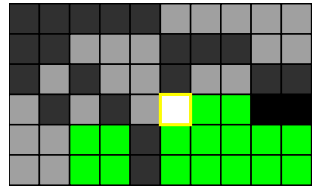
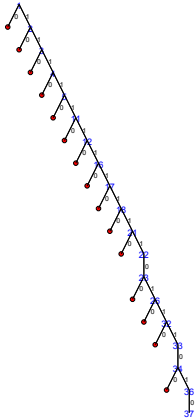


◀ Back to Start

▶ Skip Animation



Complete Search Tree with Double Lex

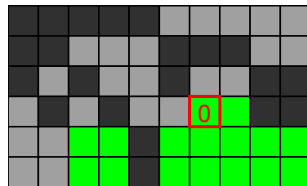
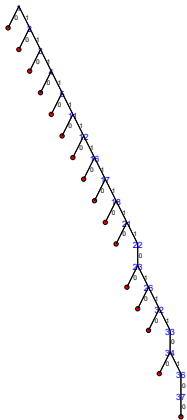


◀ Back to Start

▶ Skip Animation



Complete Search Tree with Double Lex

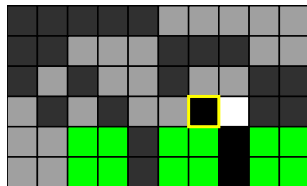
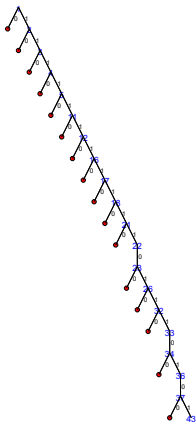


◀ Back to Start

▶ Skip Animation



Complete Search Tree with Double Lex

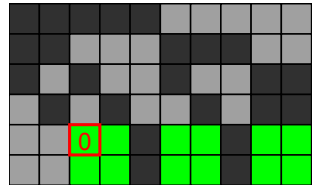
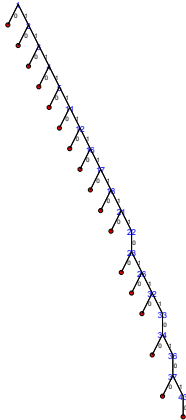


◀ Back to Start

▶ Skip Animation



Complete Search Tree with Double Lex

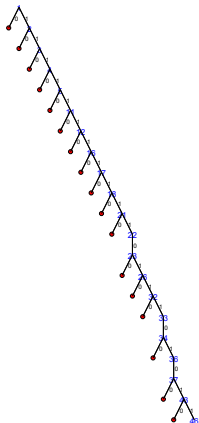


◀ Back to Start

▶ Skip Animation



Complete Search Tree with Double Lex

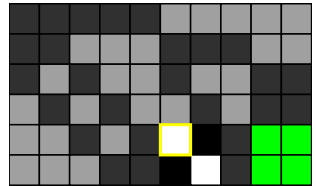
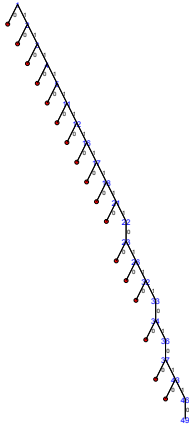


◀ Back to Start

▶▶ Skip Animation



Complete Search Tree with Double Lex

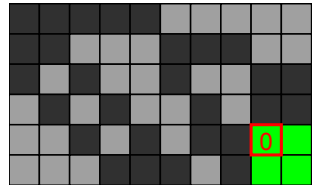
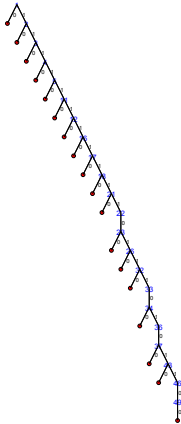


◀ Back to Start

▶▶ Skip Animation

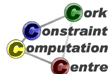


Complete Search Tree with Double Lex

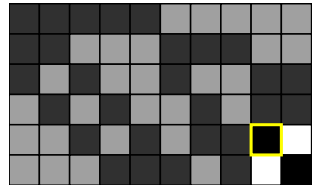
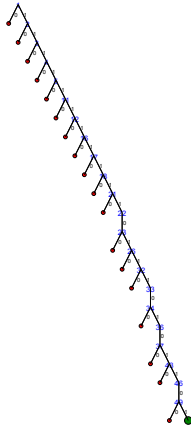


◀ Back to Start

▶ Skip Animation



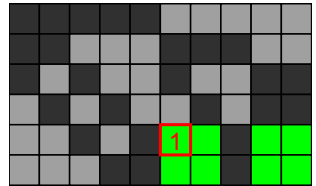
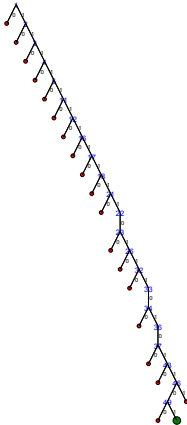
Complete Search Tree with Double Lex



◀ Back to Start ▶ Skip Animation



Complete Search Tree with Double Lex

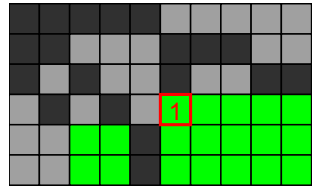
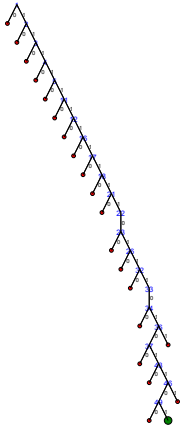


◀ Back to Start

▶▶ Skip Animation

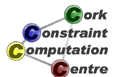


Complete Search Tree with Double Lex

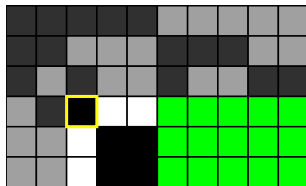
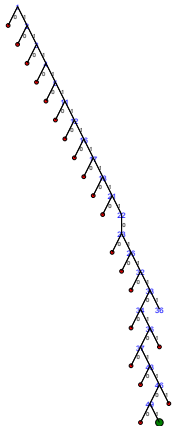


◀ Back to Start

▶ Skip Animation

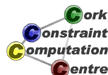


Complete Search Tree with Double Lex

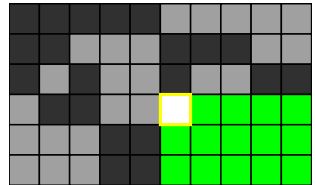
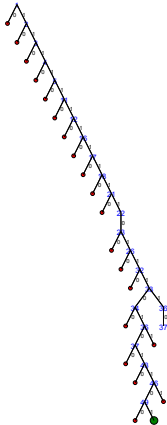


◀ Back to Start

▶▶ Skip Animation



Complete Search Tree with Double Lex

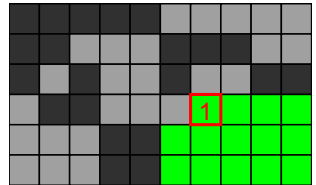
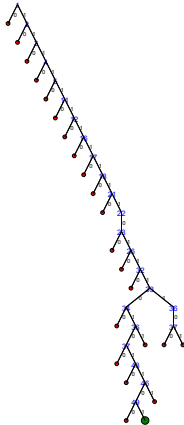


◀ Back to Start

▶▶ Skip Animation



Complete Search Tree with Double Lex

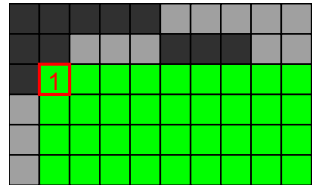
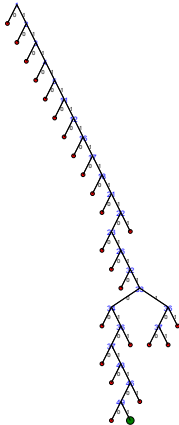


◀ Back to Start

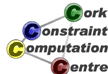
▶ Skip Animation



Complete Search Tree with Double Lex

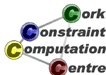


◀ Back to Start



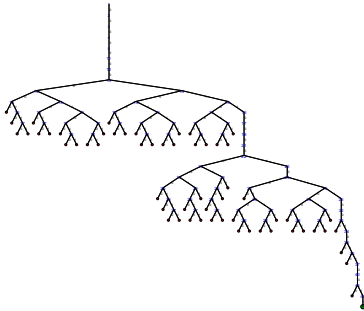
Observation

- Enormous reduction in search space
- We are solving a different problem!
- Not just good for finding all solutions, also for first solution!
- Value choice not optimal for finding first solution
- There is a lot of very shallow backtracking, can we avoid that?

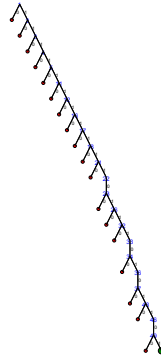


Effort for First Solution

Basic Model



With double Lex



Alternative Value Order

```
:-module(bibd) .  
:-export(top/0) .  
:-lib(ic) .  
:-lib(ic_global) .
```

```
top:-
```

```
    bibd(6,10,5,3,2,Matrix),writeln(Matrix) .
```

```
bibd(V,B,R,K,L,Matrix):-
```

```
    model(V,B,R,K,L,Matrix),  
    extract_array(row,Matrix,List),  
    search(L,0,input_order,  
           indomain_max,  
           complete,[]).
```



Alternative Value Order

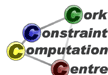
```
:-module(bibd) .  
:-export(top/0) .  
:-lib(ic) .  
:-lib(ic_global) .
```

```
top:-
```

```
    bibd(6,10,5,3,2,Matrix),writeln(Matrix) .
```

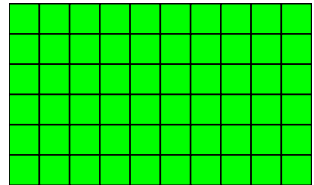
```
bibd(V,B,R,K,L,Matrix):-
```

```
    model(V,B,R,K,L,Matrix),  
    extract_array(row,Matrix,List),  
    search(L,0,input_order,  
           indomain_max, ⇨ Start with 1  
           complete, []).
```



Assigning Value 1 First

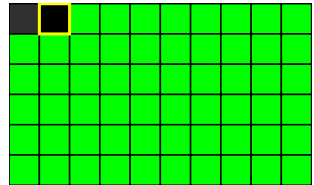
1



» Skip Animation



Assigning Value 1 First

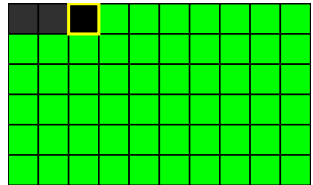
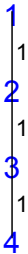


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

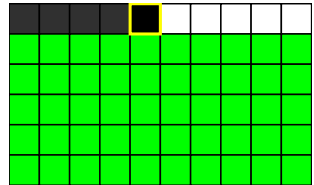


◀ Back to Start

▶ Skip Animation

Assigning Value 1 First

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2
1
3
1
4
1
5
1
11

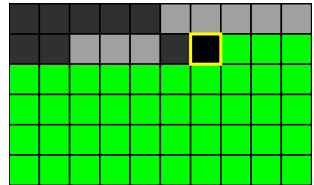


◀ Back to Start

▶ Skip Animation

Assigning Value 1 First

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1
12
1
16
1
17
1
18



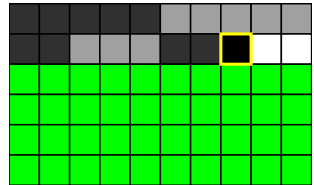
◀ Back to Start

▶ Skip Animation



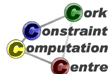
Assigning Value 1 First

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18
1
21

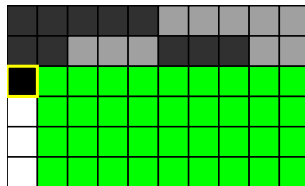
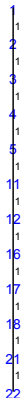


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

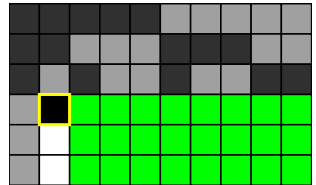


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

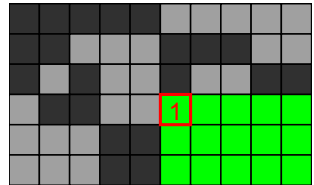


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

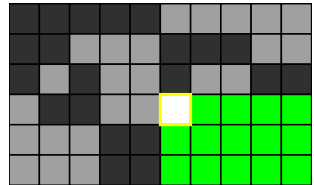


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

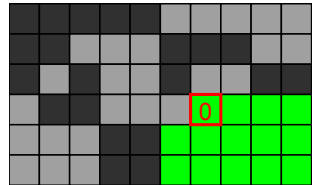


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

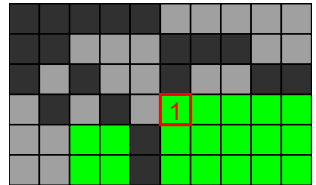
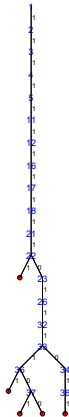


◀ Back to Start

▶ Skip Animation



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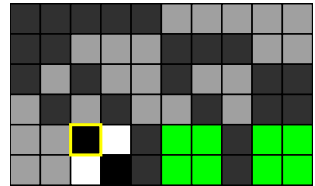
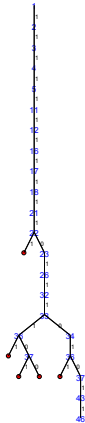


◀ Back to Start

▶ Skip Animation



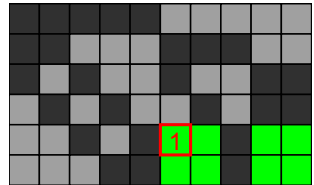
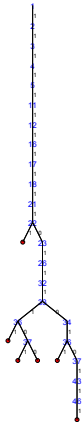
Assigning Value 1 First



◀ Back to Start

▶ Skip Animation

Assigning Value 1 First

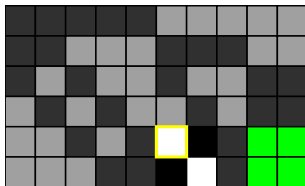
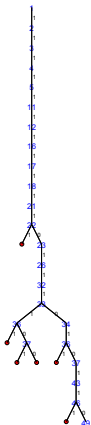


◀ Back to Start

▶ Skip Animation

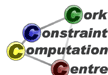


Assigning Value 1 First

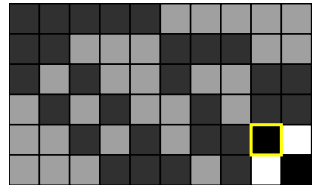
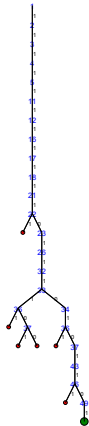


◀ Back to Start

▶ Skip Animation

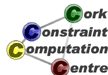


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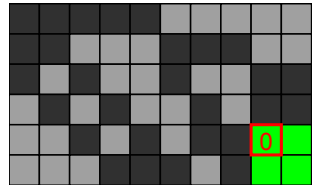
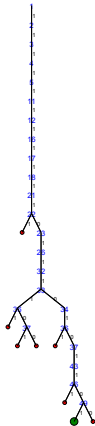


◀ Back to Start

▶ Skip Animation

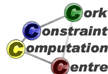


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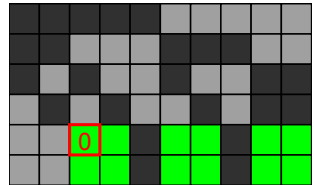
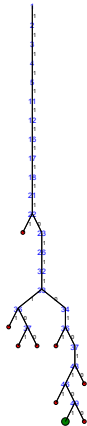


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

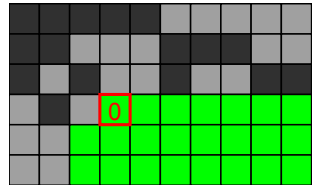
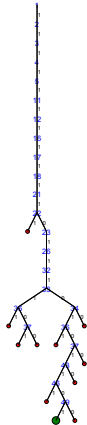


◀ Back to Start

▶ Skip Animation

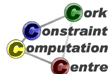


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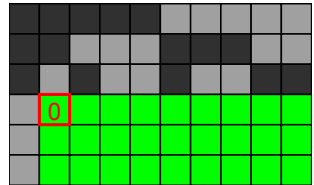
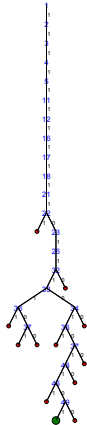


◀ Back to Start

▶ Skip Animation

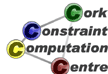


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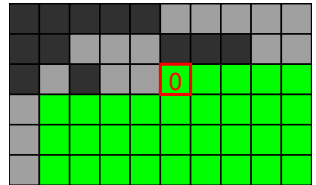
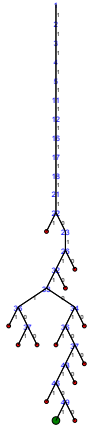


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

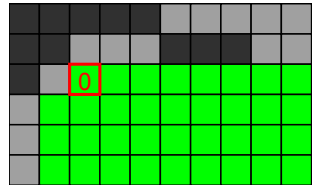


◀ Back to Start

▶▶ Skip Animation



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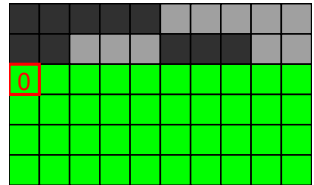


◀ Back to Start

▶▶ Skip Animation



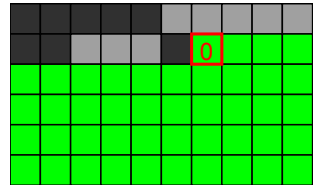
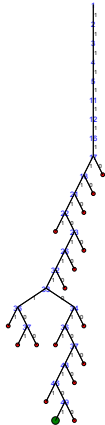
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◀ Back to Start

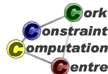
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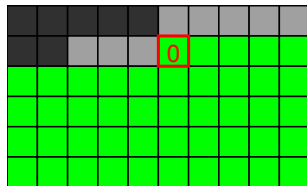
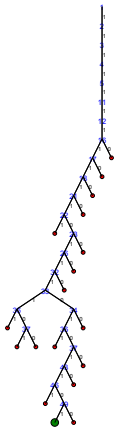


◀ Back to Start

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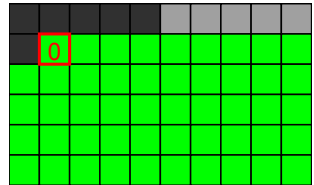
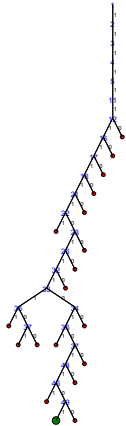


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

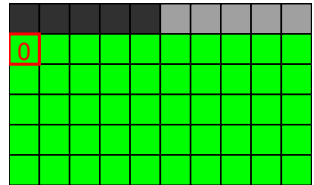
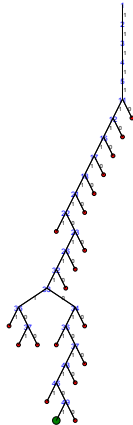


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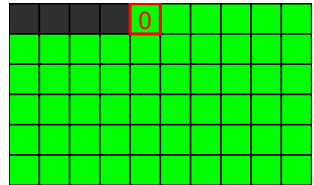
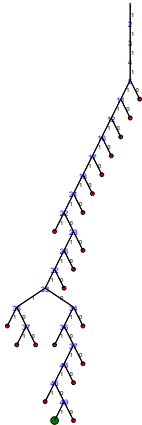


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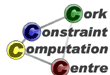


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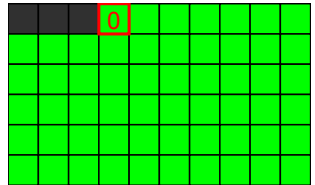
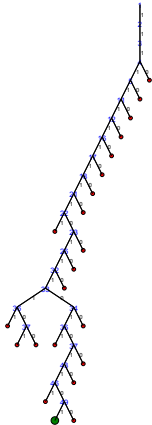


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First

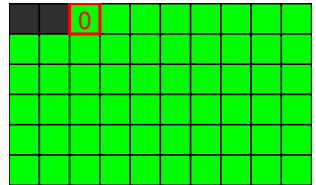
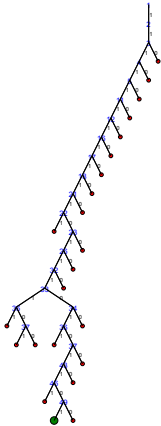


◀ Back to Start

▶ Skip Animation



Assigning Value 1 First



◀ Back to Start

▶ Skip Animation



Observation

- First solution is found more quickly
- Size of tree for all solutions unchanged
- Value order does not really affect search space when exploring all choices!



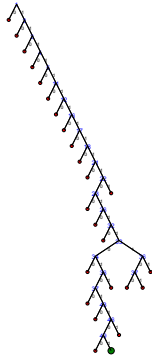
Observation

- First solution is found more quickly
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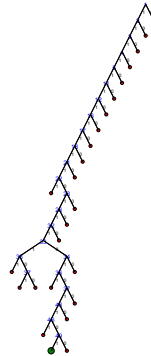


Effort for All Solutions

Assign 0, then 1

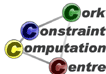


Assign 1, then 0



Conclusions

- Symmetry breaking can have huge impact on model
- Mainly works for pure problems
- Partial symmetry breaking with additional constraints
- Double lex for row/column symmetries
- Only one variant of many symmetry breaking techniques



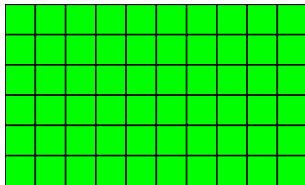
Outline

- 4 Why assign by row?
- 5 Alternative Models
- 6 Exercises



Variable Selection by Column

1

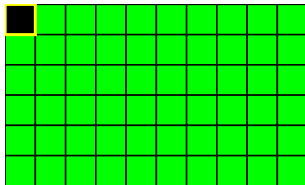


» Skip Animation



Variable Selection by Column

1
|
1
|
2



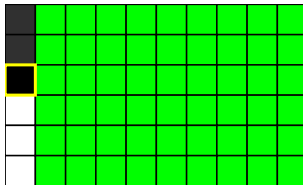
◀ Back to Start

▶▶ Skip Animation

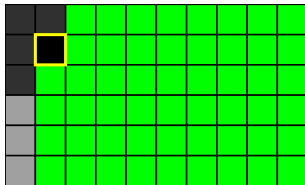
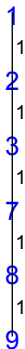


Variable Selection by Column

1
1
2
1
3
1
7



Variable Selection by Column



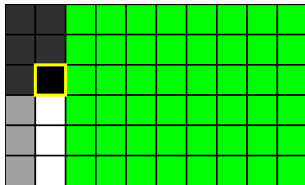
◀ Back to Start

▶▶ Skip Animation



Variable Selection by Column

1
1
2
1
3
1
7
1
8
1
9
1
13

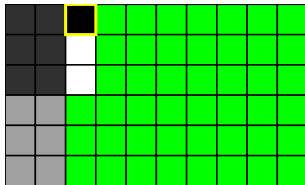


◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column

1
1
2
1
3
1
7
1
8
1
9
1
13
1
16

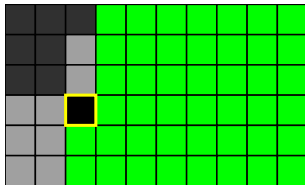


◀ Back to Start

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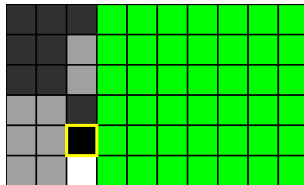
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3
1
7
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8
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9
1
13
1
16
1
17



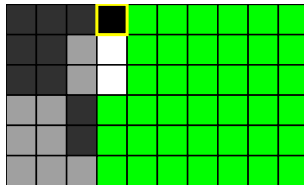
Variable Selection by Column

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7
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8
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9
1
13
1
16
1
17
1
19



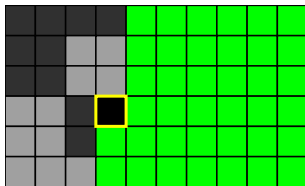
Variable Selection by Column

1
1
2
1
3
1
7
1
8
1
9
1
13
1
16
1
17
1
19
1
22



Variable Selection by Column

1
2
3
7
8
9
13
16
17
19
22
23

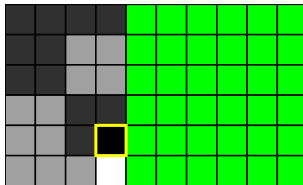


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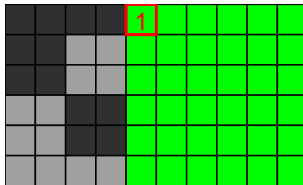
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Variable Selection by Column

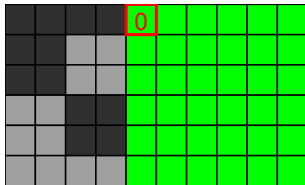
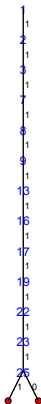
1
1
2
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3
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7
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13
1
16
1
17
1
19
1
22
1
23
1
25



Variable Selection by Column



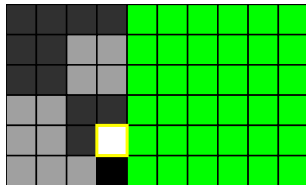
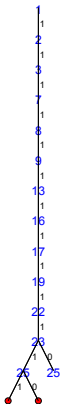
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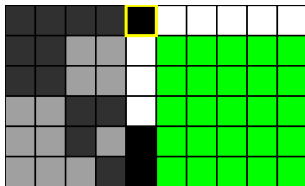
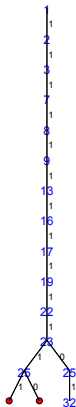
◀ Back to Start

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Variable Selection by Column



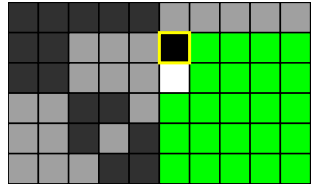
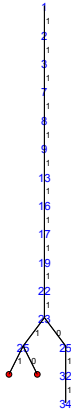
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◀ Back to Start

▶▶ Skip Animation

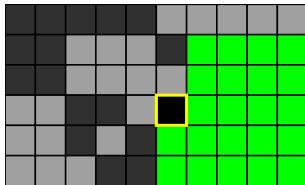
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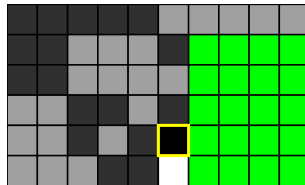
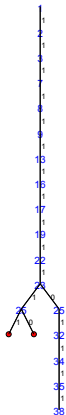
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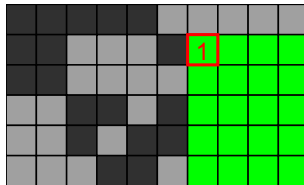
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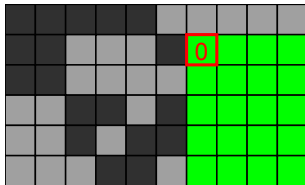
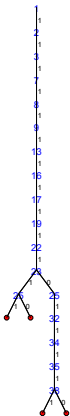
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Variable Selection by Column



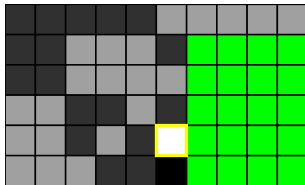
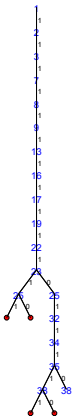
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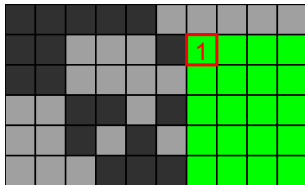
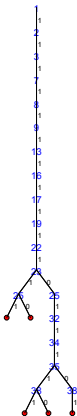
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Variable Selection by Column



Variable Selection by Column

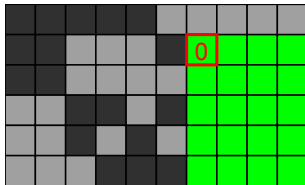
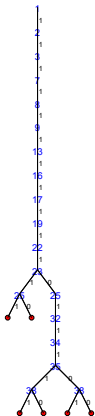


◀ Back to Start

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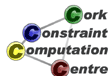


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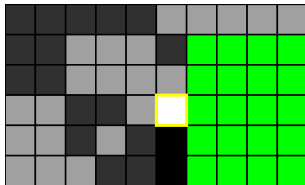
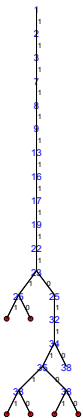


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▶▶ Skip Animation



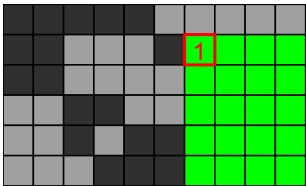
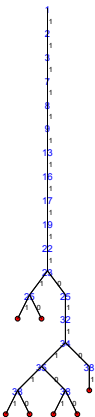
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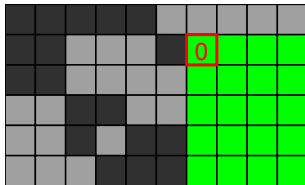
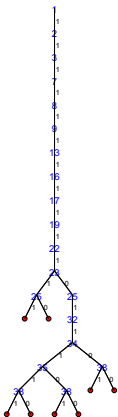
◀ Back to Start

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Variable Selection by Column



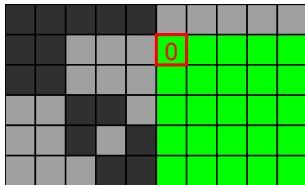
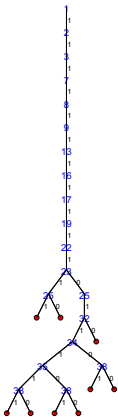
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◀ Back to Start

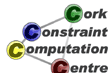
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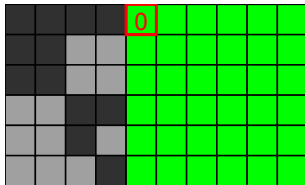
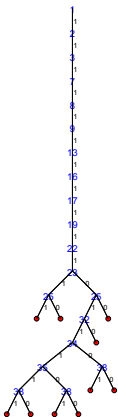


◀ Back to Start

▶▶ Skip Animation



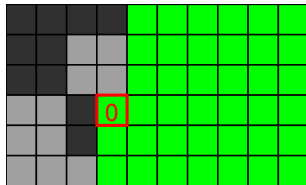
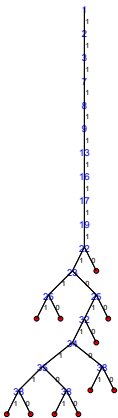
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

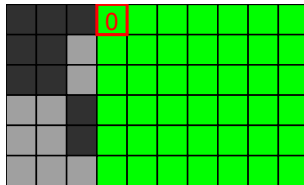
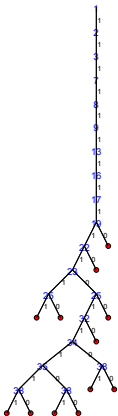
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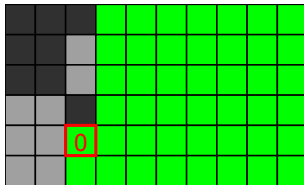
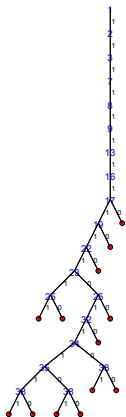
◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column



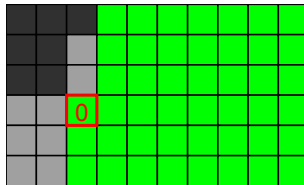
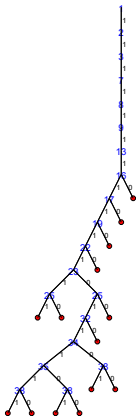
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

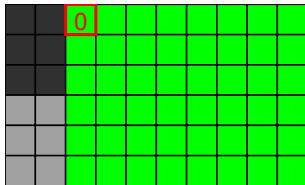
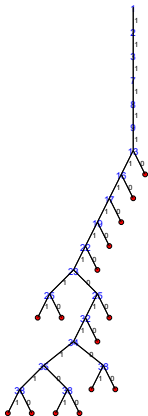
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

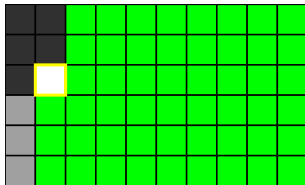
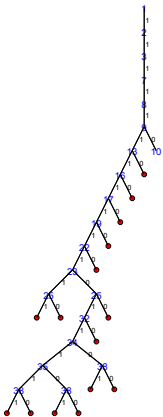
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

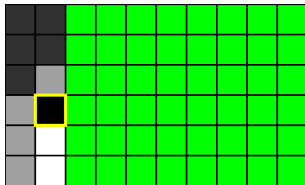
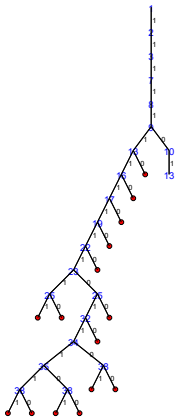
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

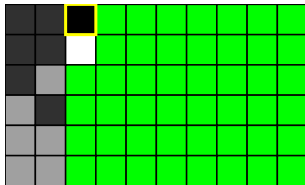
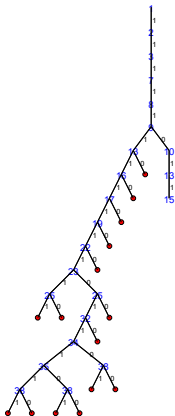
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column

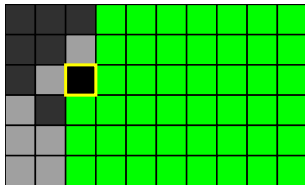
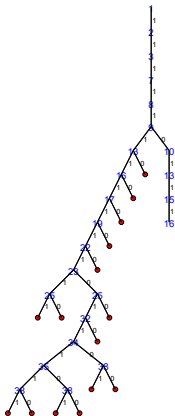


◀ Back to Start

▶▶ Skip Animation



Variable Selection by Column

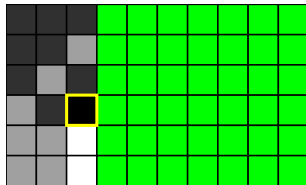
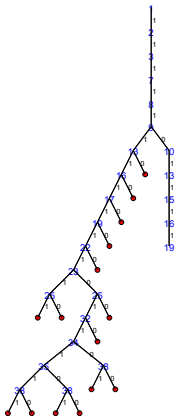


◀ Back to Start

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Variable Selection by Column

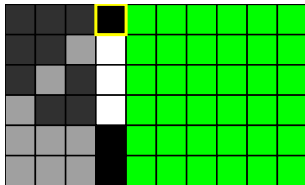
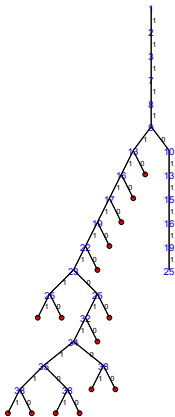


◀ Back to Start

▶▶ Skip Animation



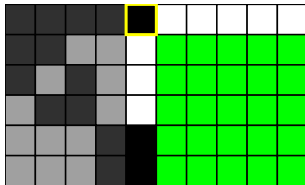
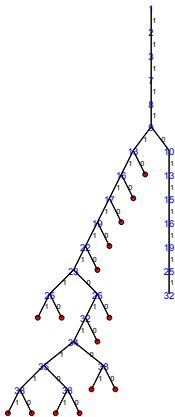
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◀ Back to Start

▶▶ Skip Animation

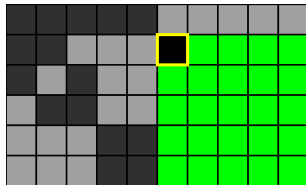
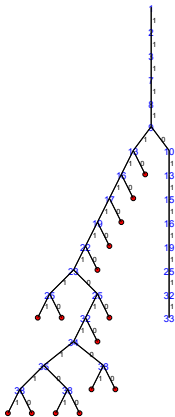
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column

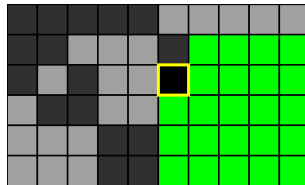
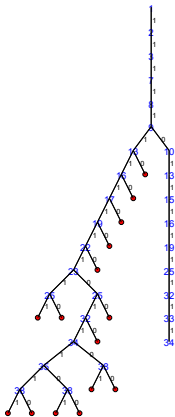


◀ Back to Start

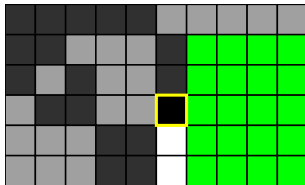
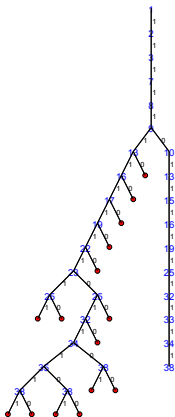
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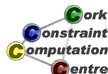


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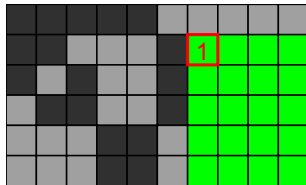
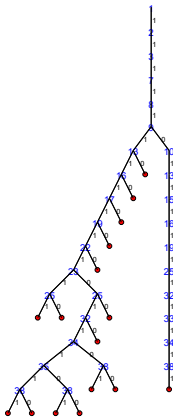


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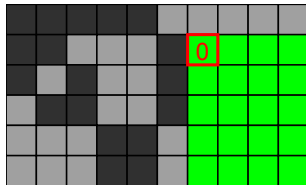
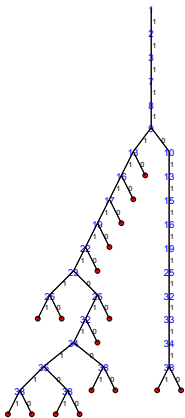
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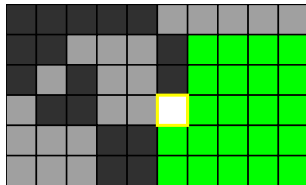
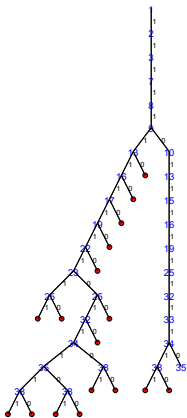
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

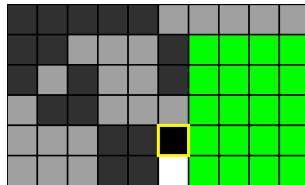
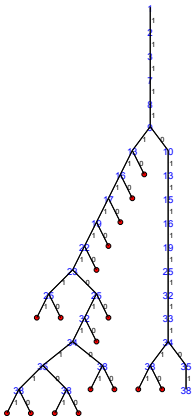
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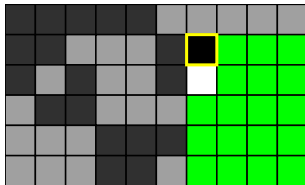
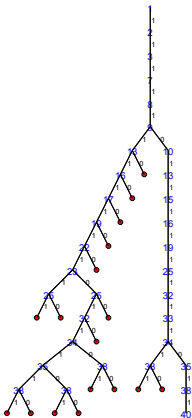
◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column



Variable Selection by Column

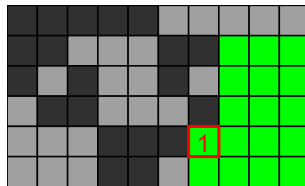
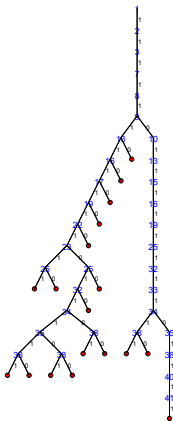


◀ Back to Start

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Variable Selection by Column

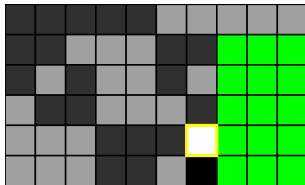
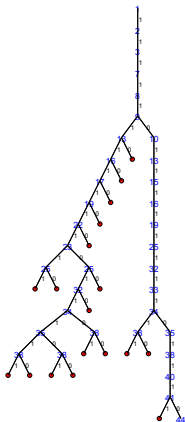


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Variable Selection by Column

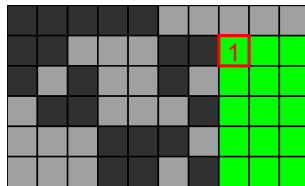
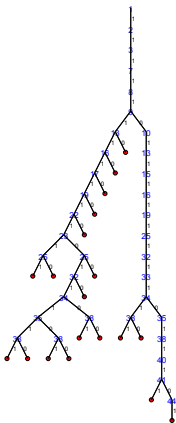


◀ Back to Start

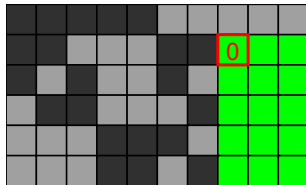
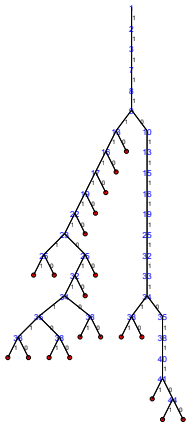
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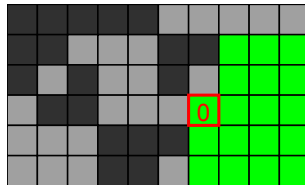
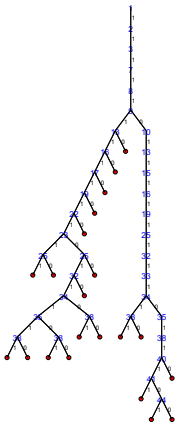
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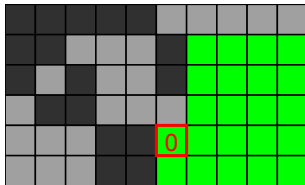
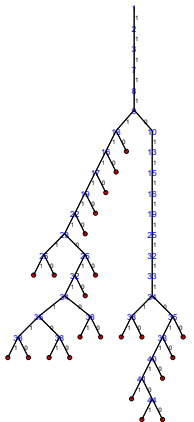
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Variable Selection by Column



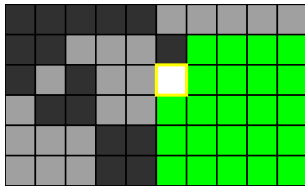
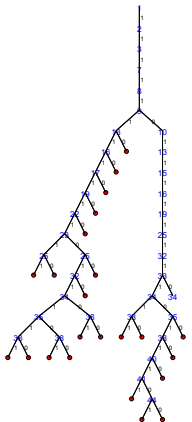
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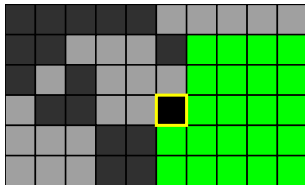
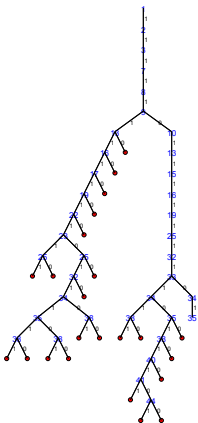
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▶▶ Skip Animation

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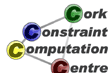


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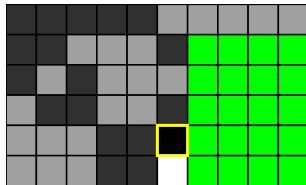
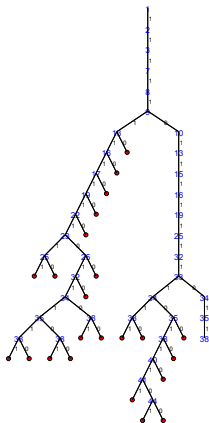


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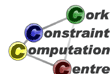
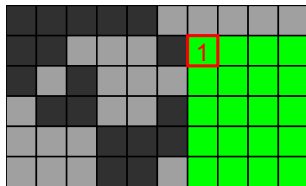
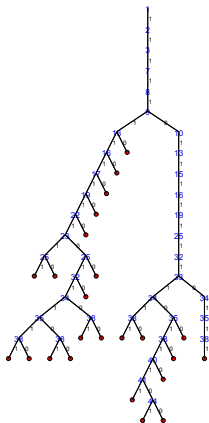
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Variable Selection by Column



Variable Selection by Column

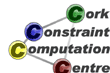
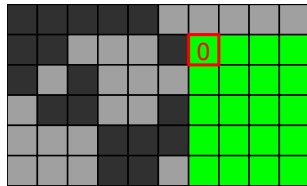
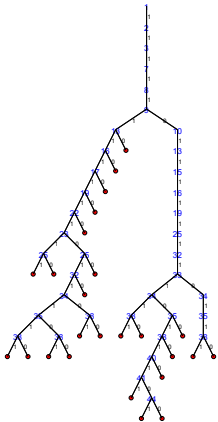


◀ Back to Start

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Variable Selection by Column

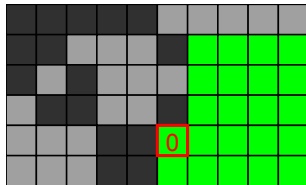
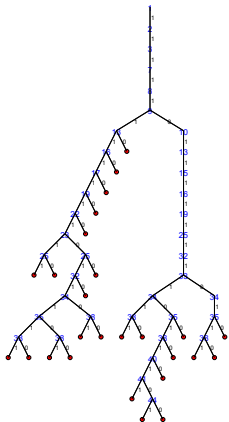


◀ Back to Start

▶▶ Skip Animation

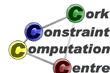


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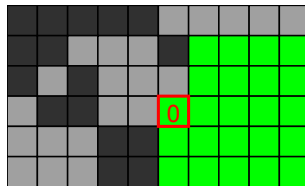
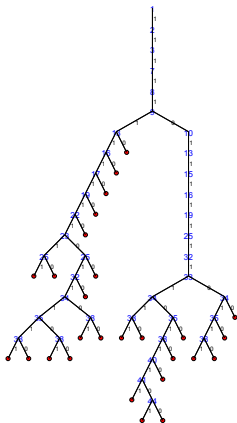


◀ Back to Start

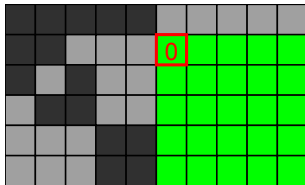
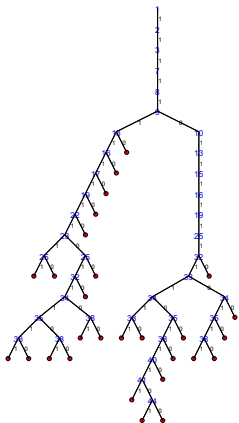
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Variable Selection by Column



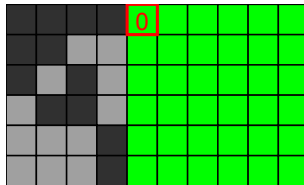
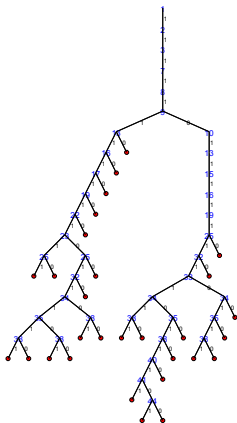
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◀ Back to Start

▶▶ Skip Animation

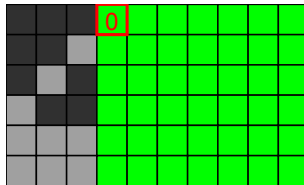
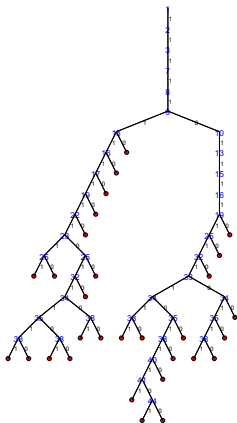
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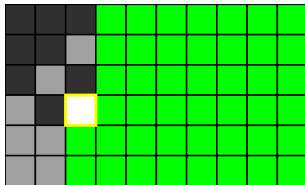
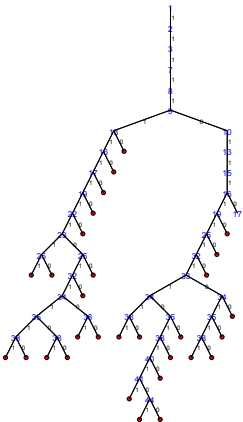
◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column



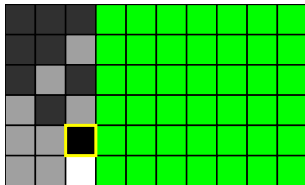
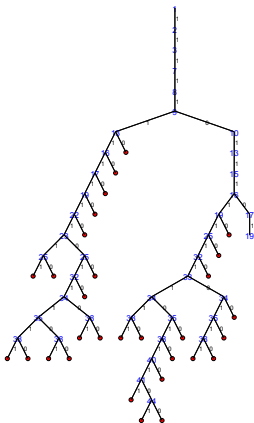
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

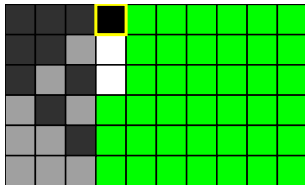
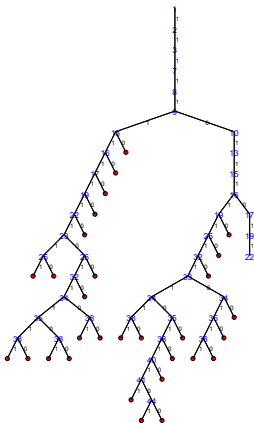
Variable Selection by Column



◀ Back to Start

▶▶ Skip Animation

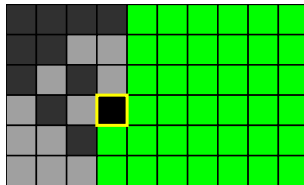
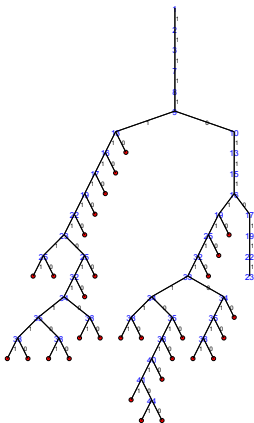
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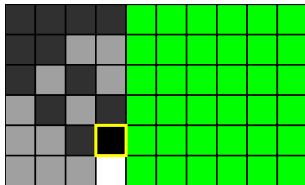
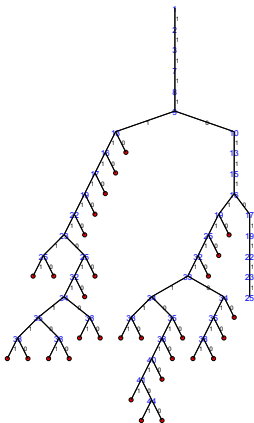
◀ Back to Start

▶▶ Skip Animation

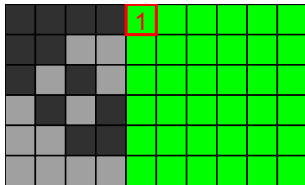
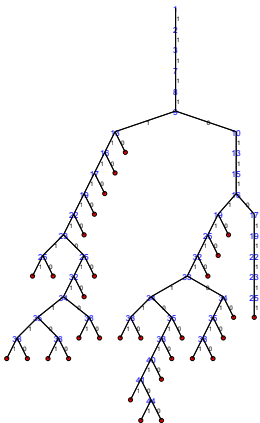
Variable Selection by Column



Variable Selection by Column



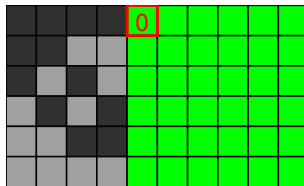
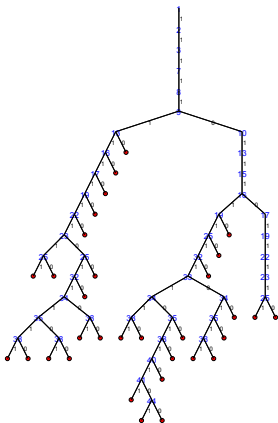
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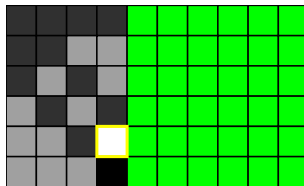
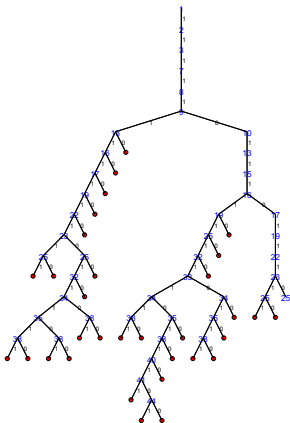
◀ Back to Start

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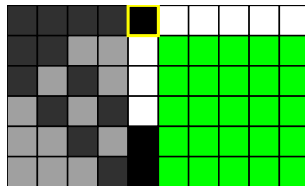
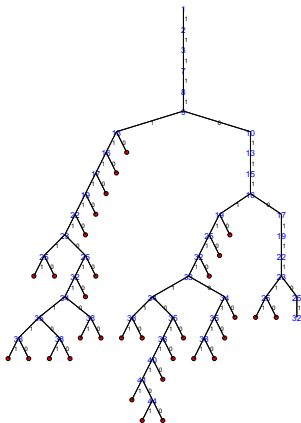
Variable Selection by Column



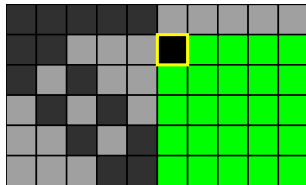
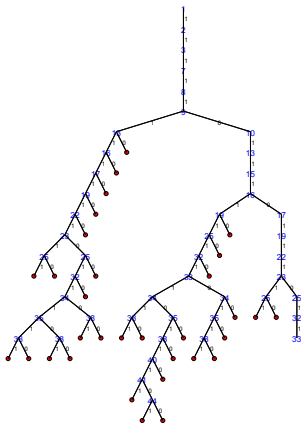
Variable Selection by Column



Variable Selection by Column



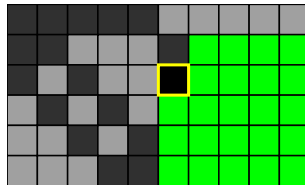
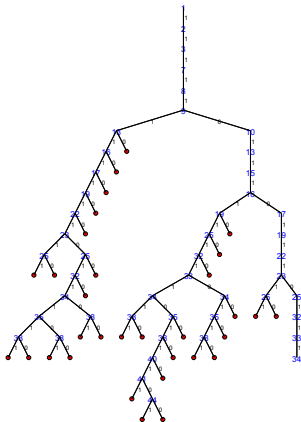
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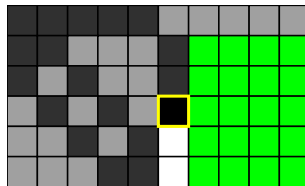
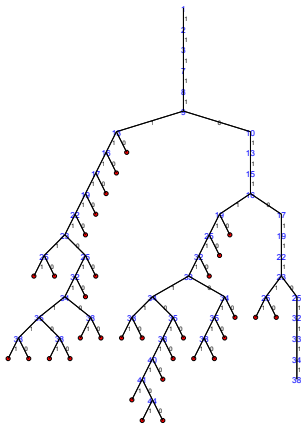
◀ Back to Start

▶▶ Skip Animation

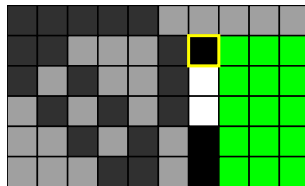
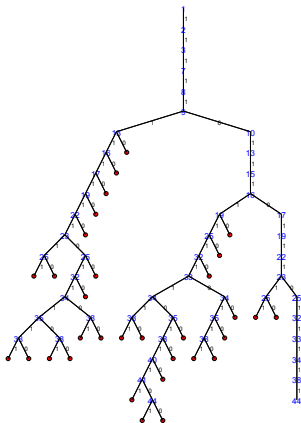
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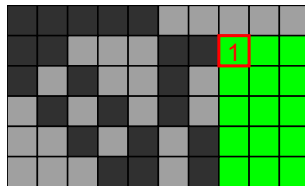
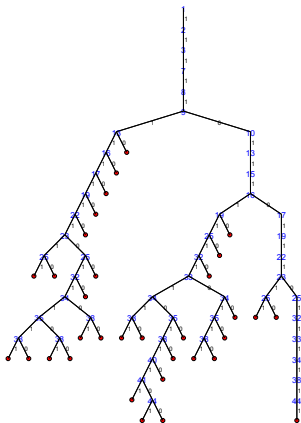
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Variable Selection by Column



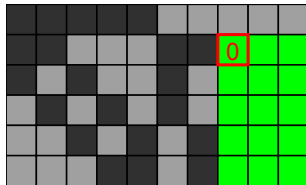
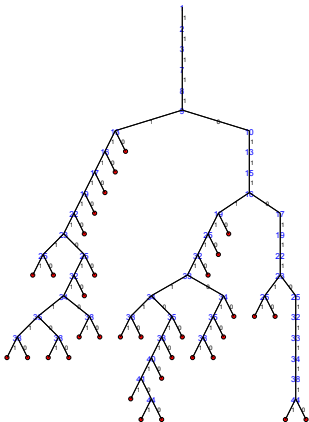
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◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column

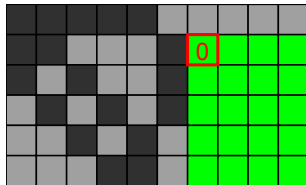
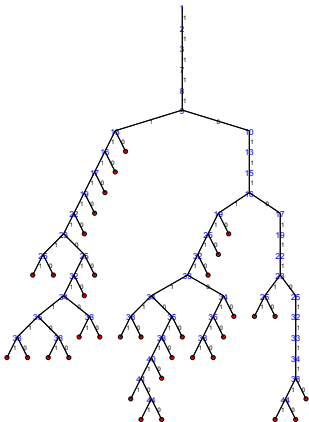


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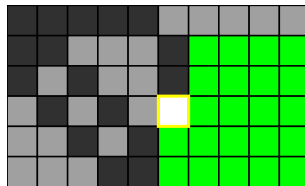
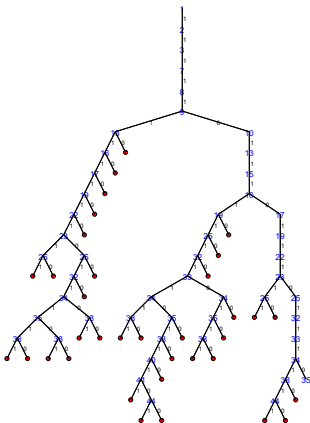
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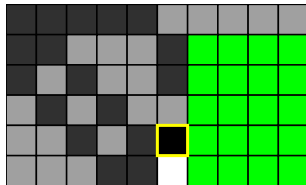
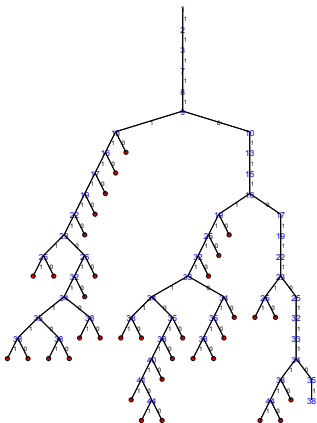
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Variable Selection by Column



Variable Selection by Column

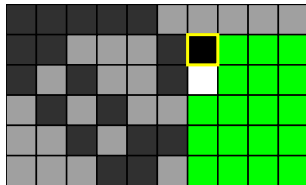
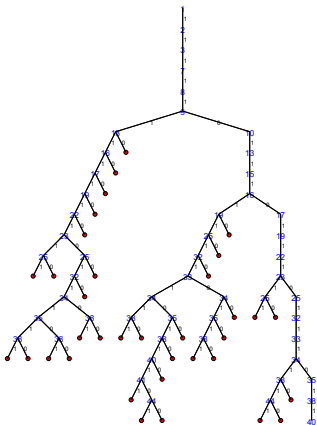


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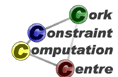
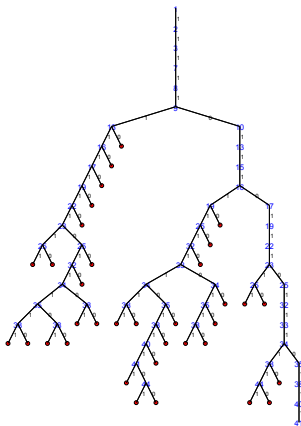
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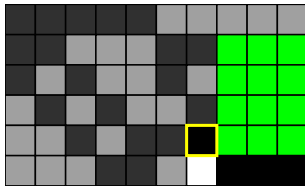
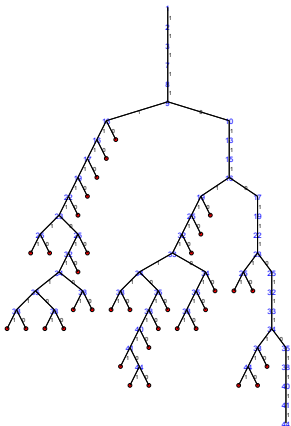
Variable Selection by Column



◀ Back to Start ▶ Skip Animation



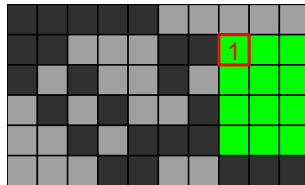
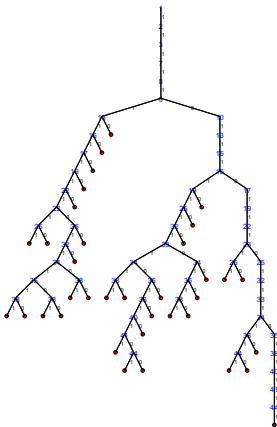
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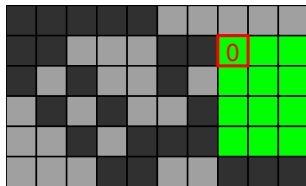
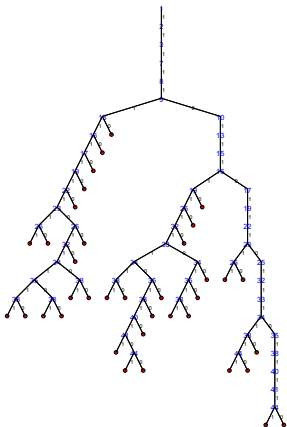
◀ Back to Start

▶▶ Skip Animation

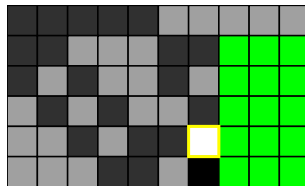
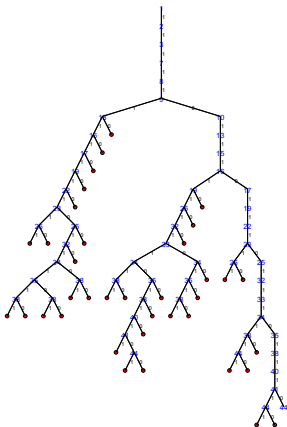
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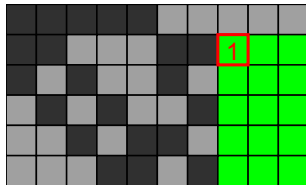
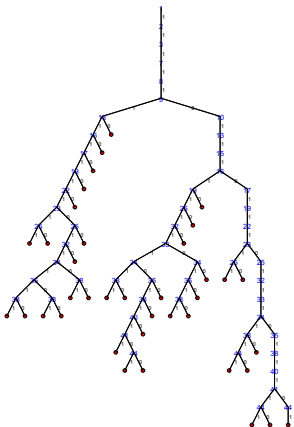
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Variable Selection by Column



Variable Selection by Column

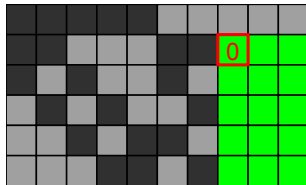
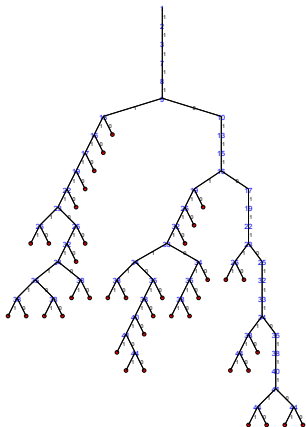


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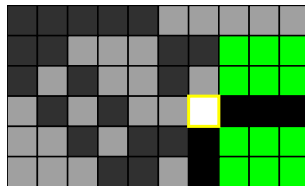
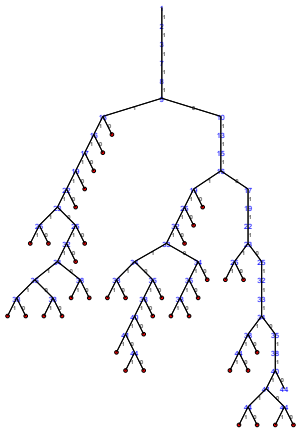
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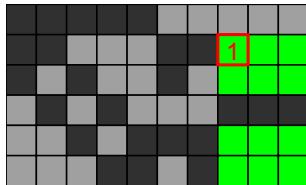
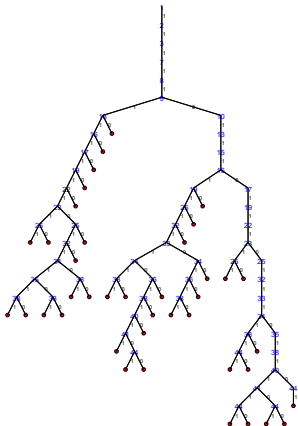
Variable Selection by Column



Variable Selection by Column



Variable Selection by Column

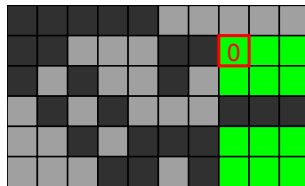
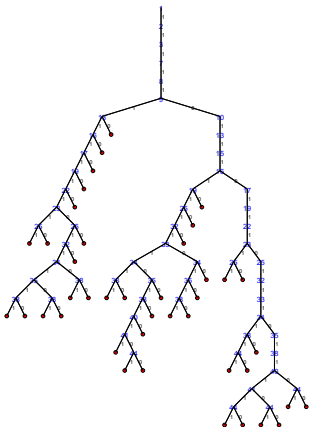


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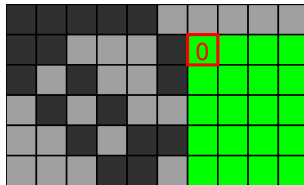
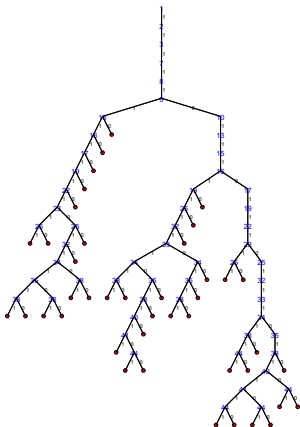
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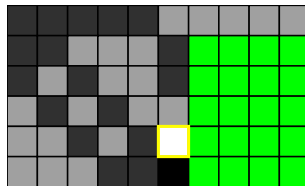
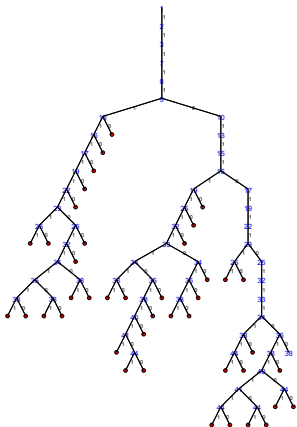
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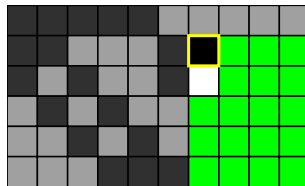
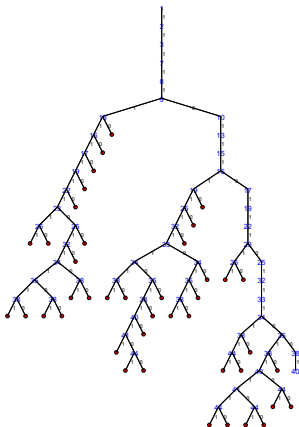
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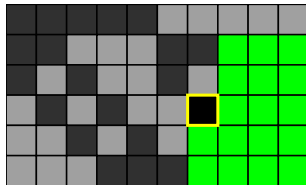
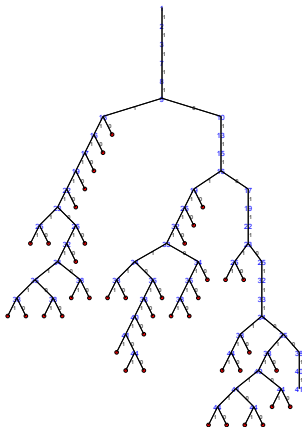
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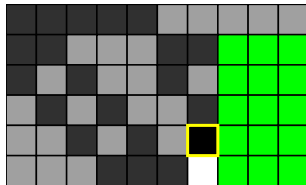
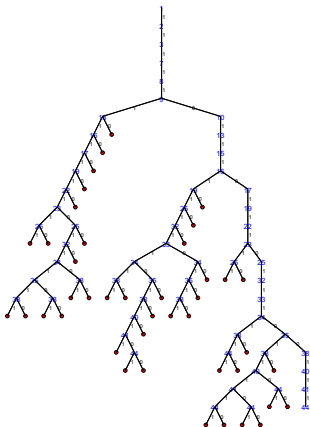
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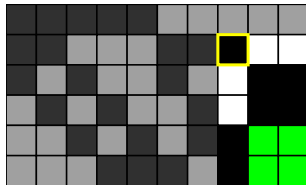
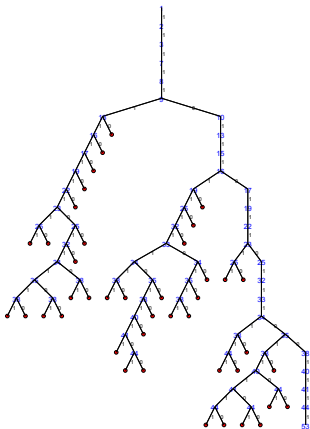
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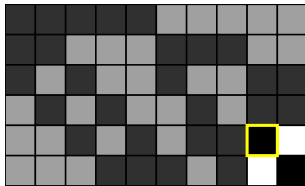
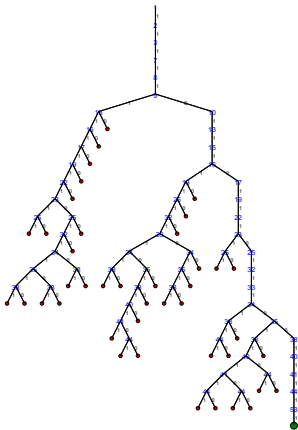
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Variable Selection by Column



Variable Selection by Column

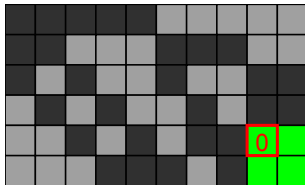
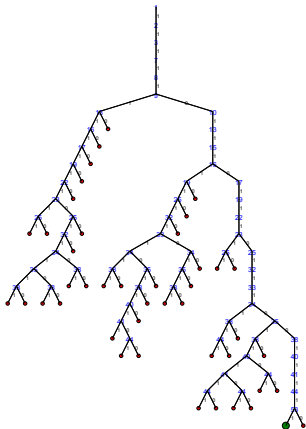


◀ Back to Start

▶▶ Skip Animation

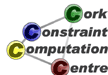


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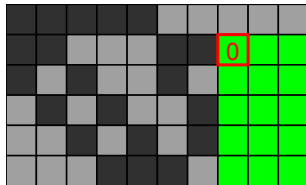
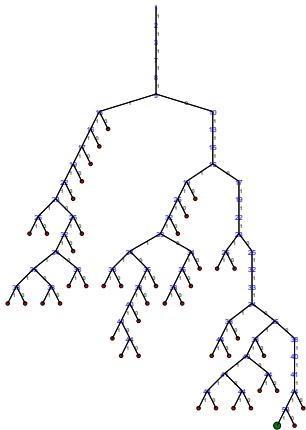


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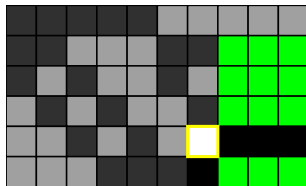
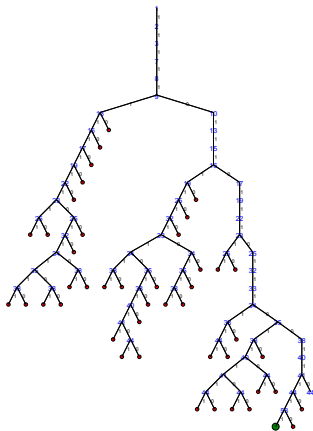
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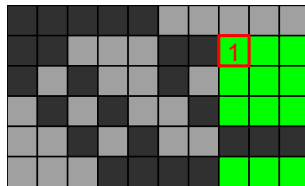
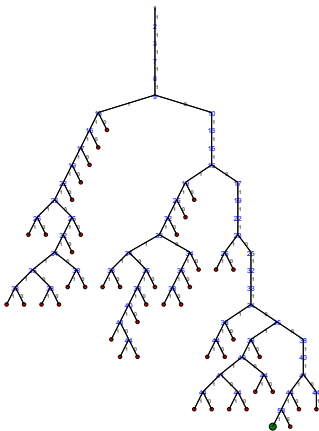
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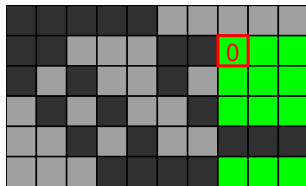
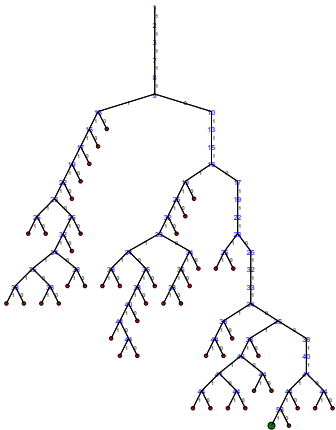
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Variable Selection by Column



Variable Selection by Column

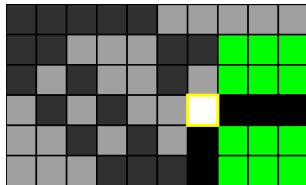
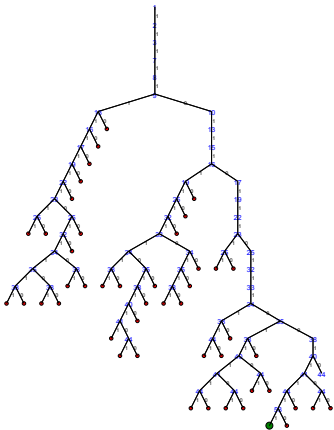


◀ Back to Start

▶▶ Skip Animation



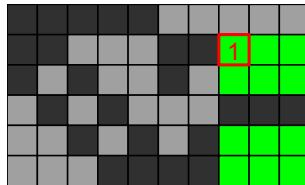
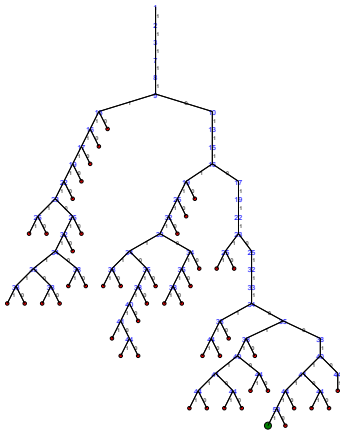
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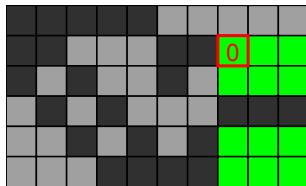
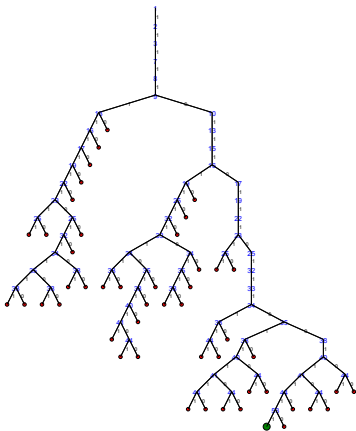
◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column



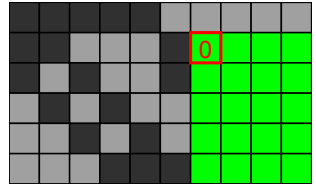
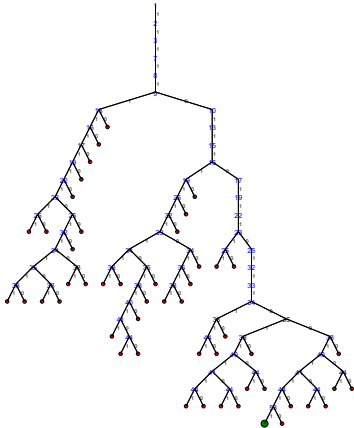
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▶▶ Skip Animation

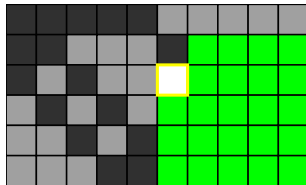
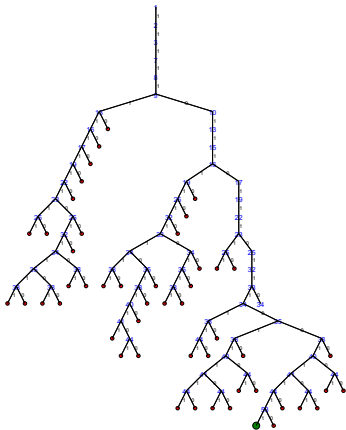
Variable Selection by Column



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▶▶ Skip Animation

Variable Selection by Column

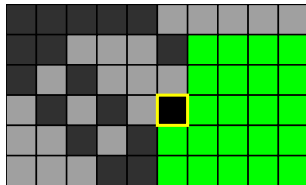
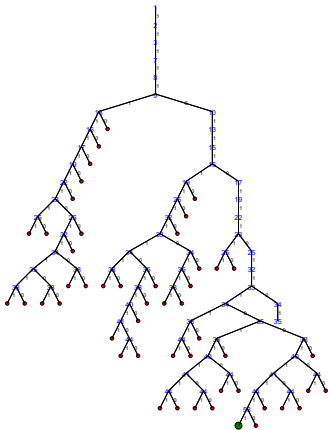


◀ Back to Start

▶▶ Skip Animation



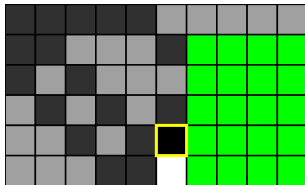
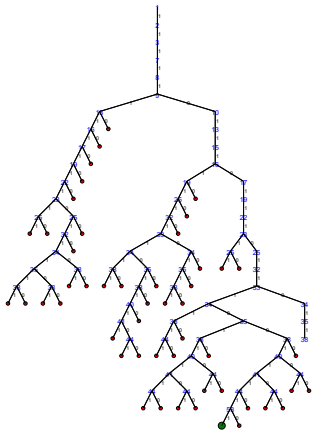
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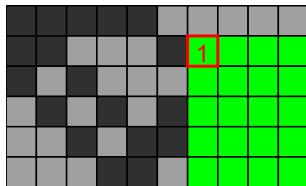
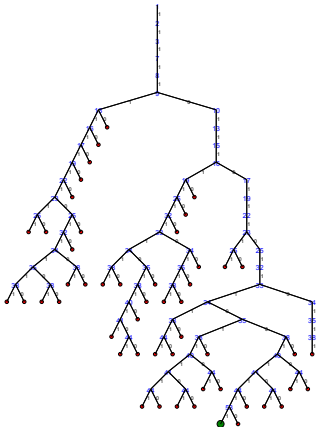
◀ Back to Start

▶▶ Skip Animation

Variable Selection by Column



Variable Selection by Column

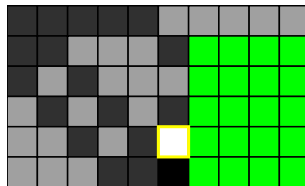
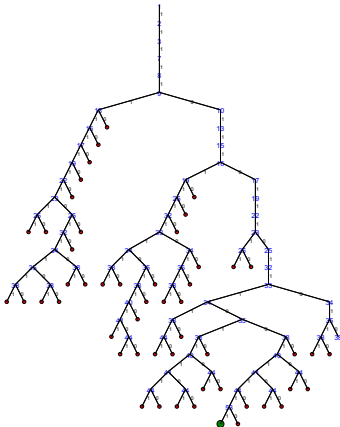


◀ Back to Start

▶ Skip Animation



Variable Selection by Column

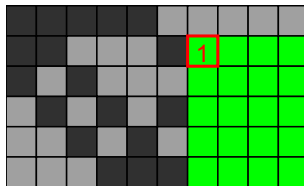
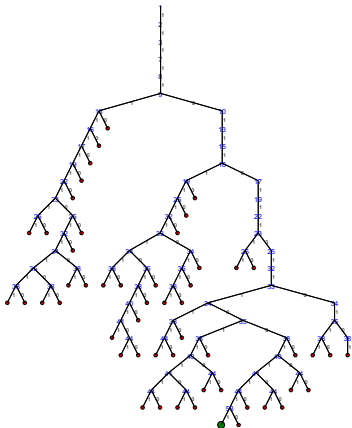


◀ Back to Start

▶ Skip Animation



Variable Selection by Column

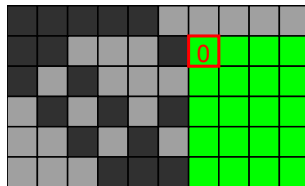
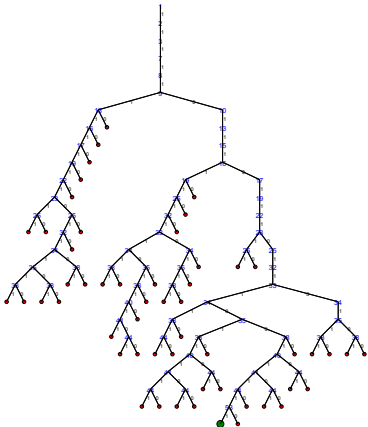


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▶ Skip Animation



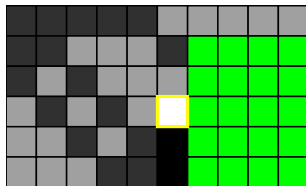
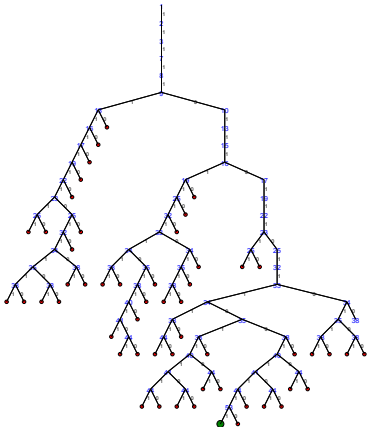
Variable Selection by Column



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▶ Skip Animation

Variable Selection by Column

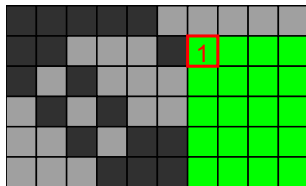
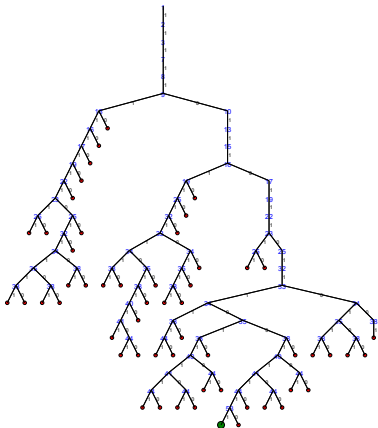


◀ Back to Start

▶ Skip Animation



Variable Selection by Column

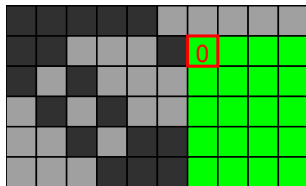
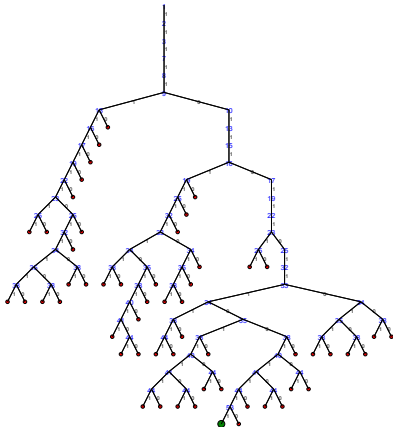


◀ Back to Start

▶ Skip Animation



Variable Selection by Column

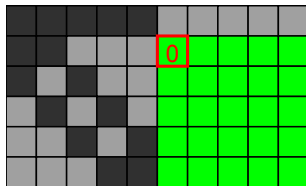
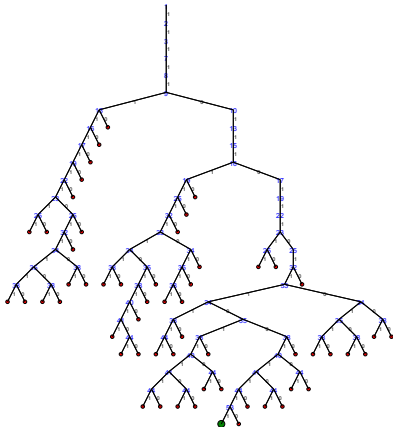


◀ Back to Start

▶ Skip Animation



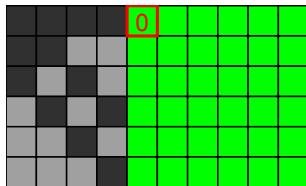
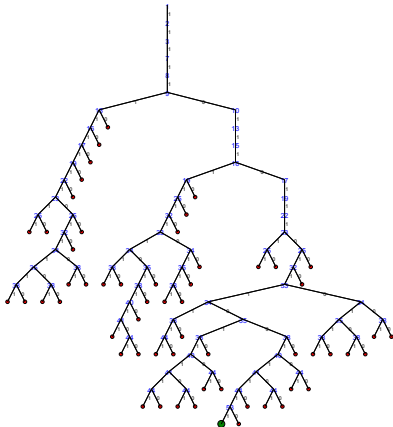
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

Variable Selection by Column

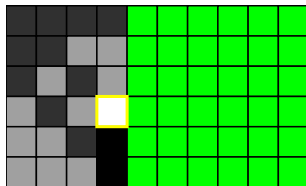
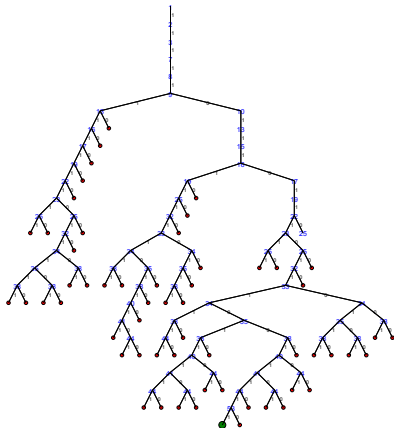


◀ Back to Start

▶ Skip Animation



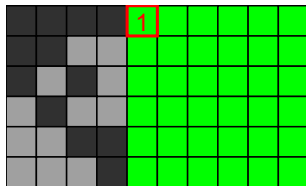
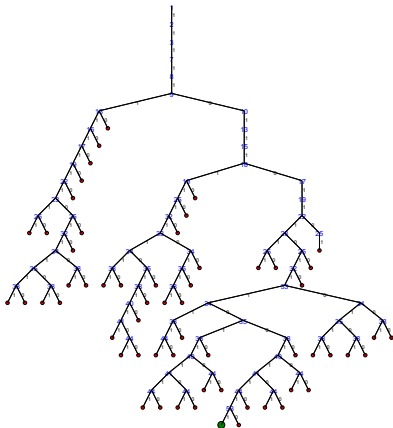
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◀ Back to Start

▶ Skip Animation

Variable Selection by Column

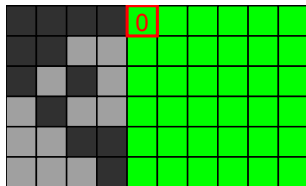
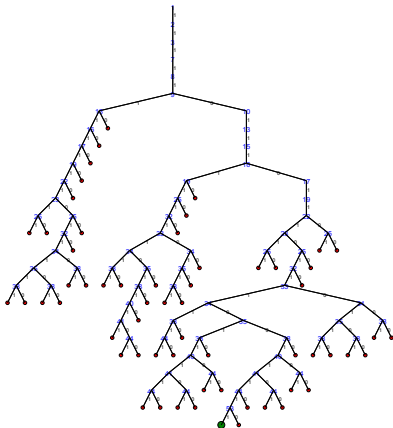


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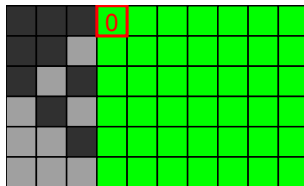
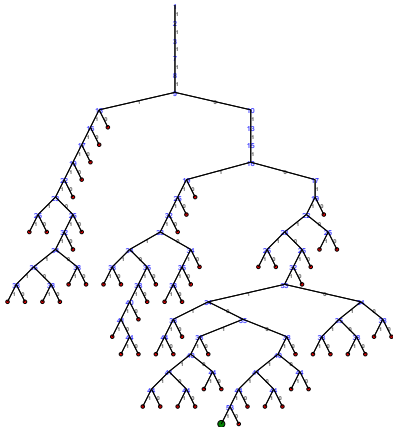
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◀ Back to Start

▶ Skip Animation

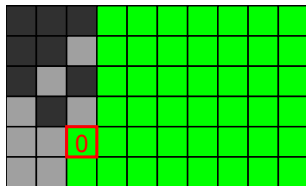
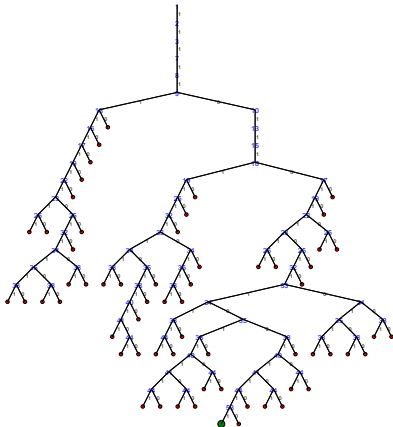
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◀ Back to Start

▶ Skip Animation

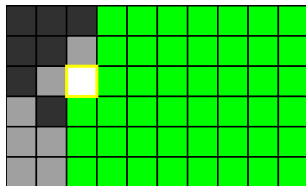
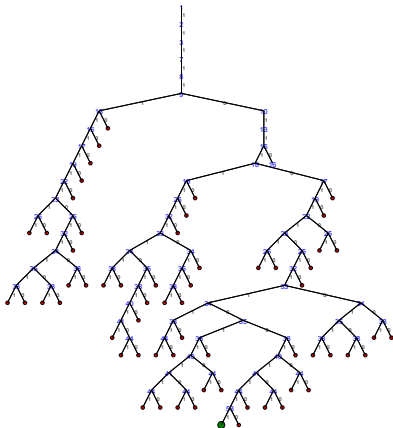
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◀ Back to Start

▶ Skip Animation

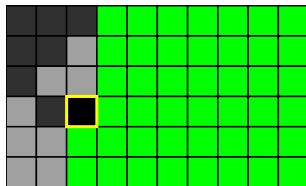
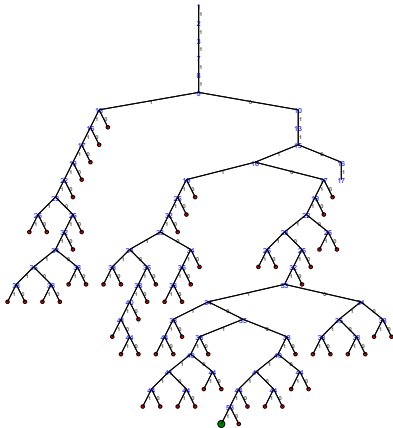
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◀ Back to Start

▶ Skip Animation

Variable Selection by Column

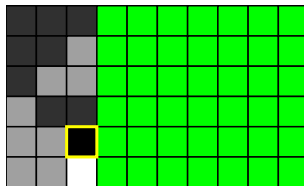
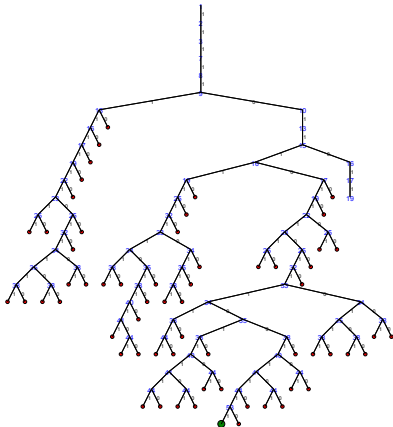


◀ Back to Start

▶ Skip Animation



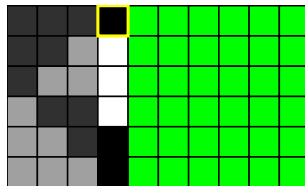
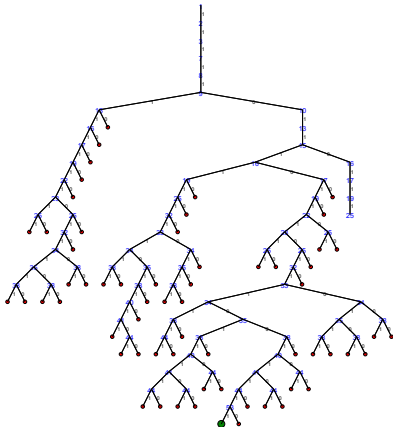
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

Variable Selection by Column

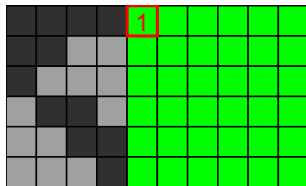
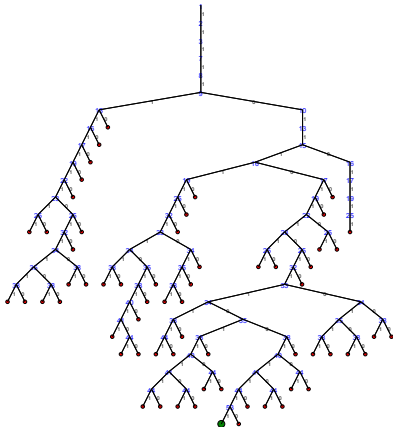


◀ Back to Start

▶ Skip Animation



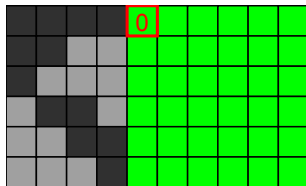
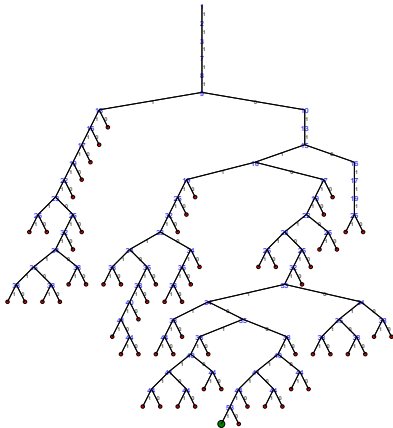
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

Variable Selection by Column

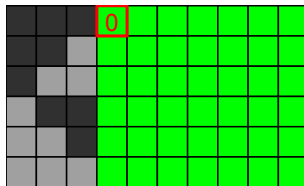
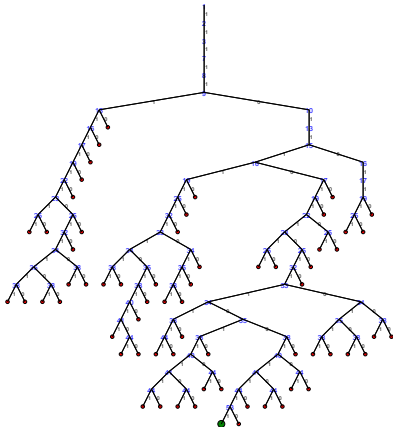


◀ Back to Start

▶ Skip Animation



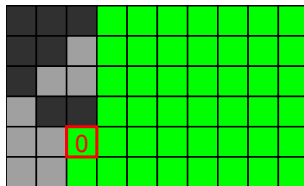
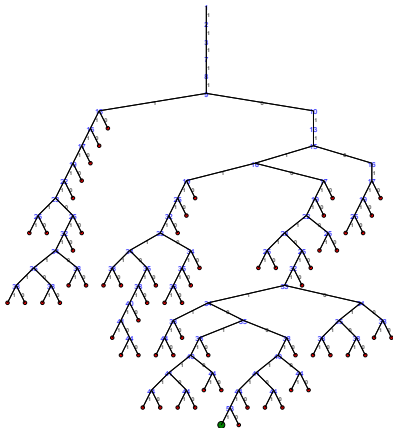
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

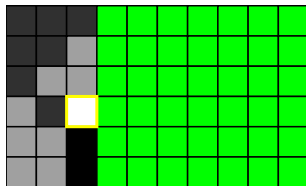
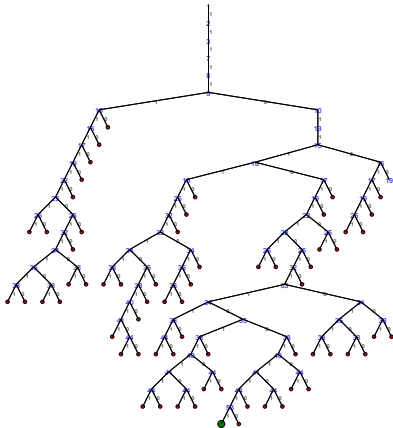
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◀ Back to Start

▶ Skip Animation

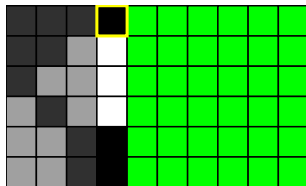
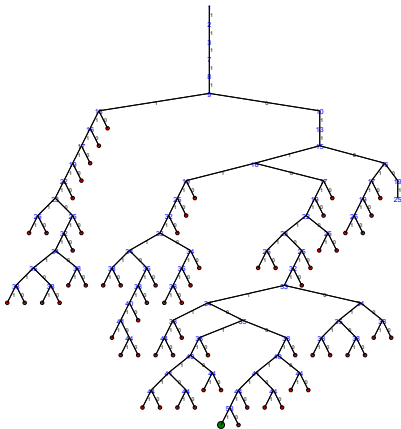
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

Variable Selection by Column

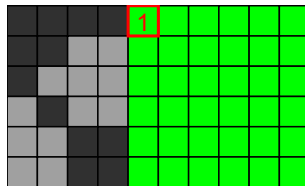
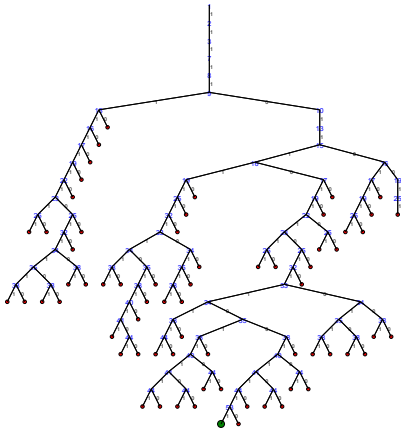


◀ Back to Start

▶ Skip Animation



Variable Selection by Column

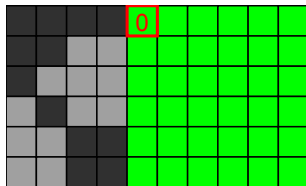
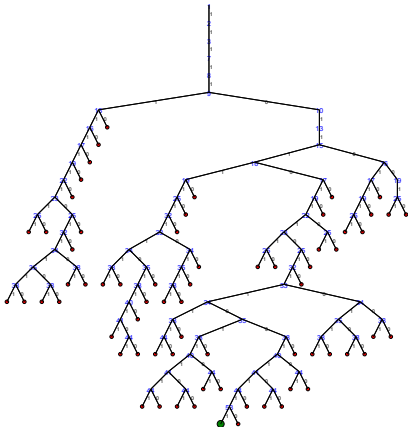


◀ Back to Start

▶ Skip Animation



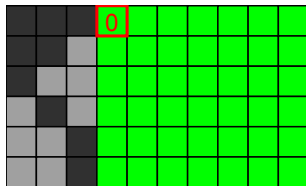
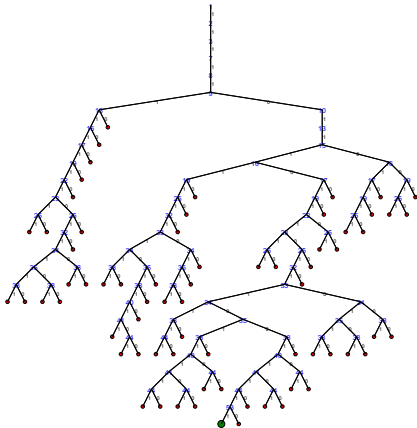
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

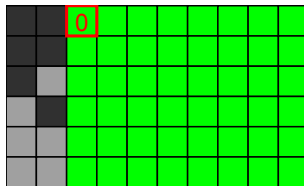
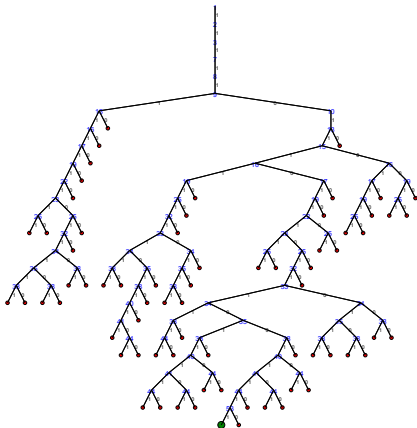
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

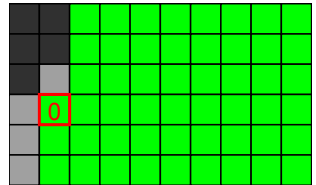
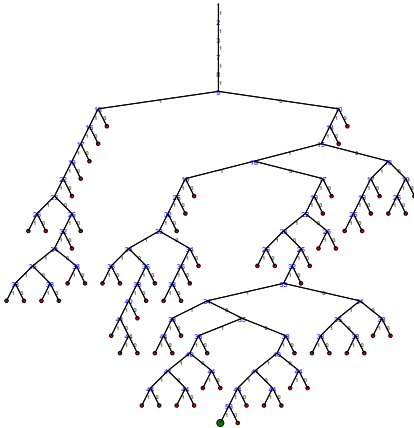
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◀ Back to Start

▶ Skip Animation

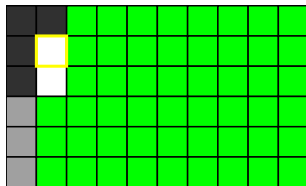
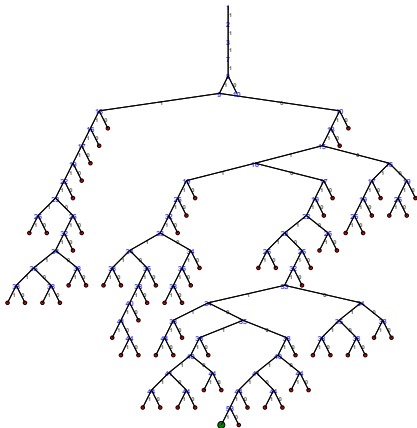
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

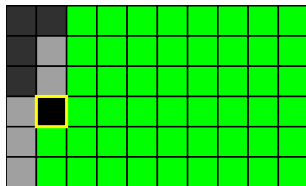
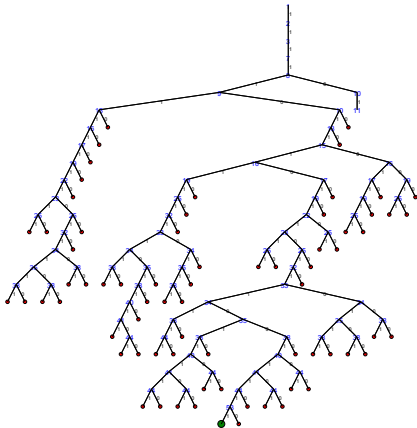
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

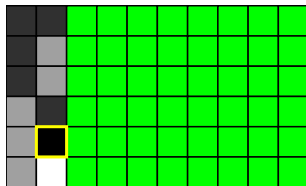
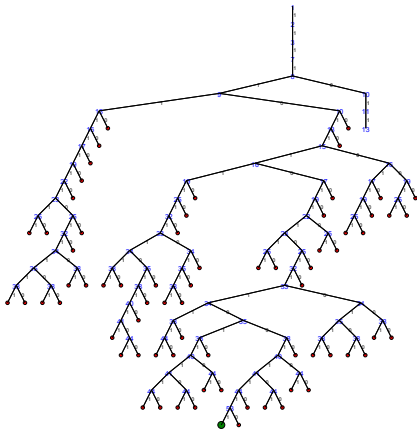
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

Variable Selection by Column

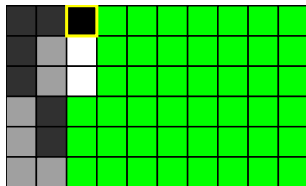
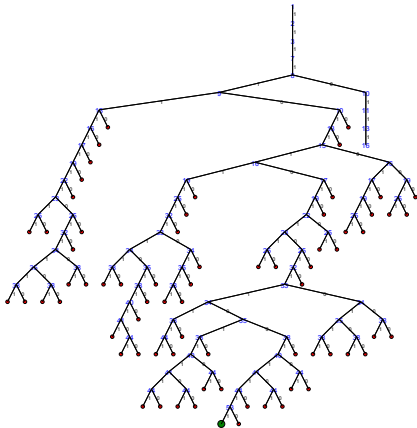


◀ Back to Start

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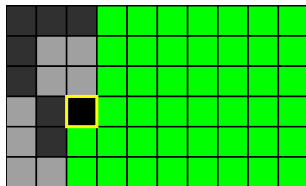
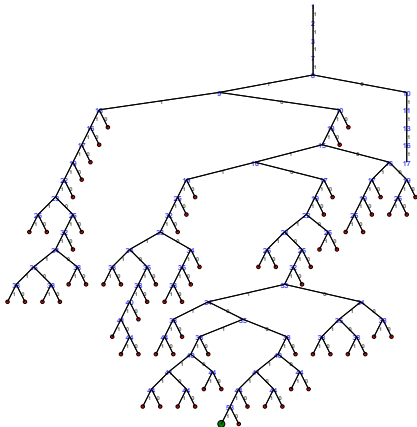
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◀ Back to Start

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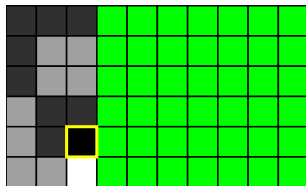
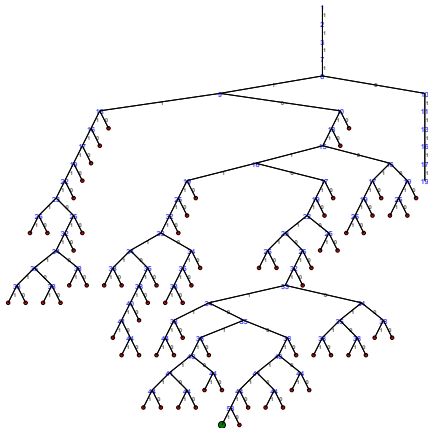
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◀ Back to Start

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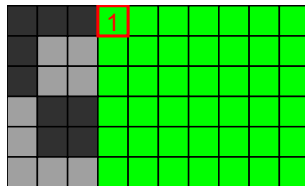
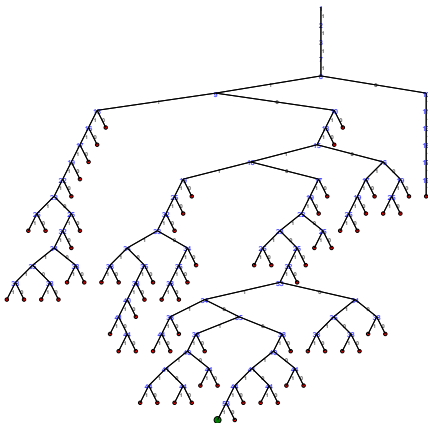
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◀ Back to Start

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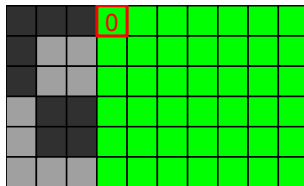
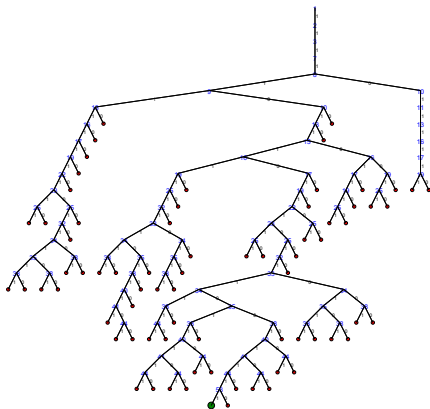
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◀ Back to Start

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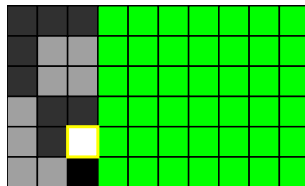
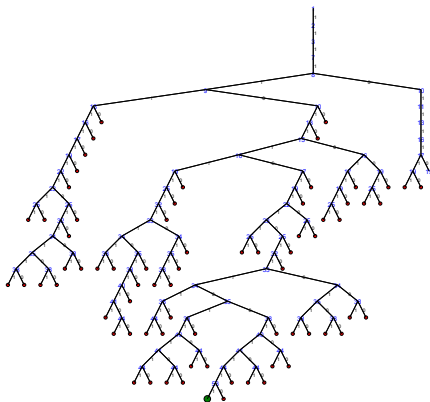
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◀ Back to Start

▶ Skip Animation

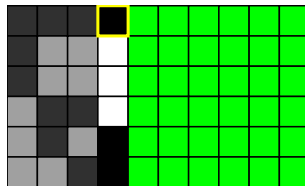
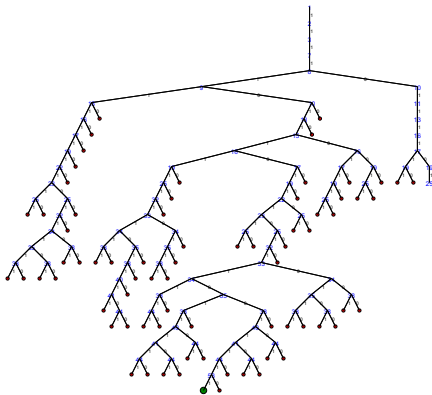
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◀ Back to Start

▶ Skip Animation

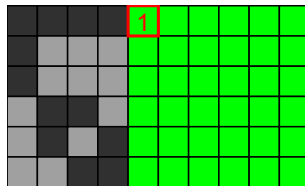
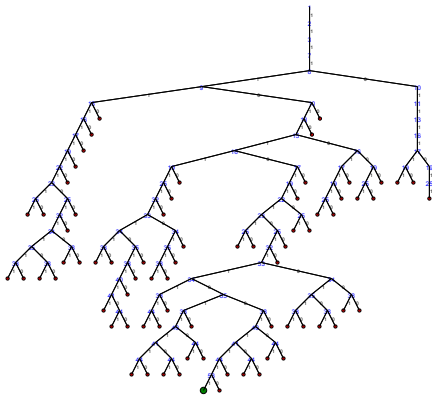
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

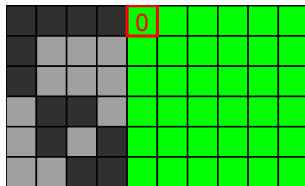
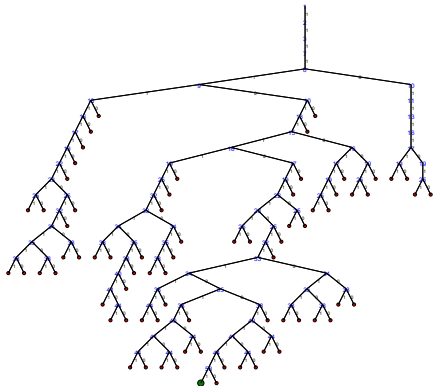
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

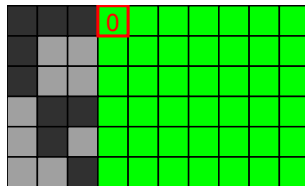
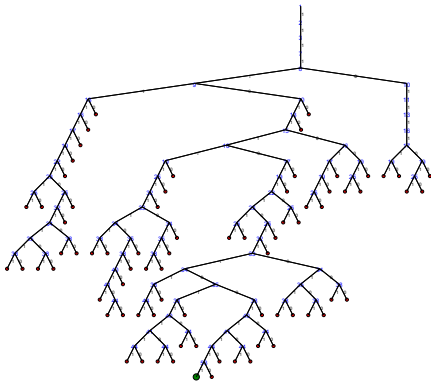
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

Variable Selection by Column

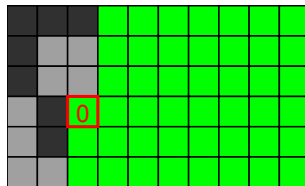
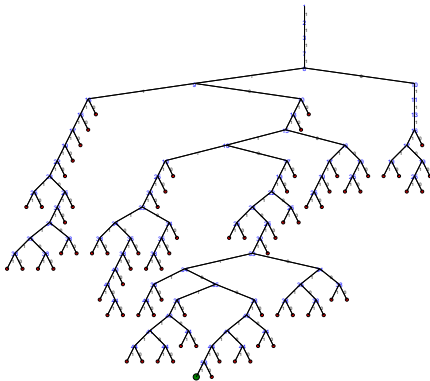


◀ Back to Start

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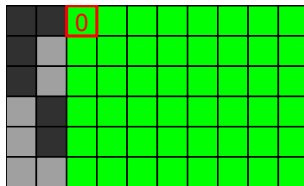
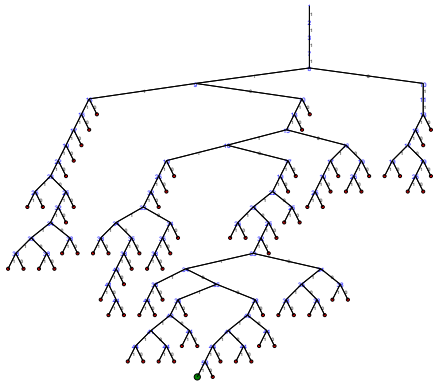
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◀ Back to Start

▶ Skip Animation

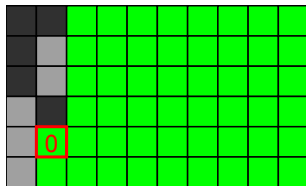
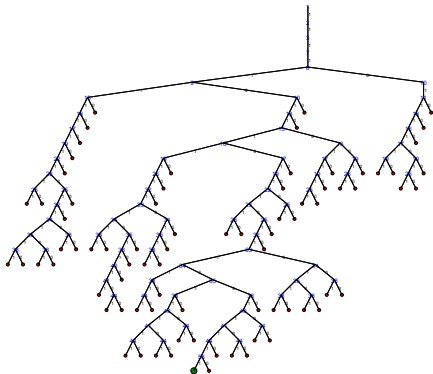
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

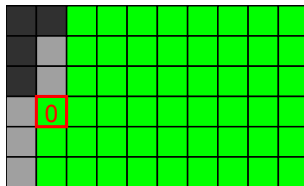
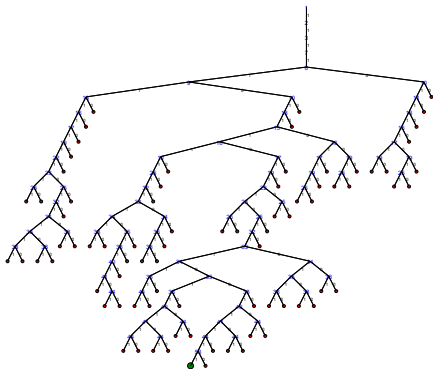
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

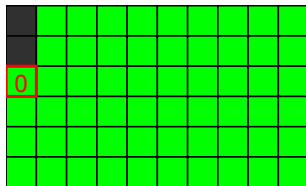
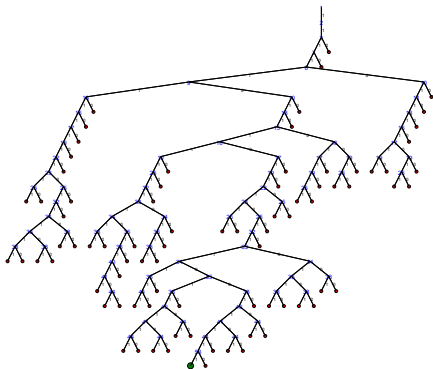
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◀ Back to Start

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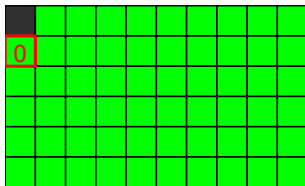
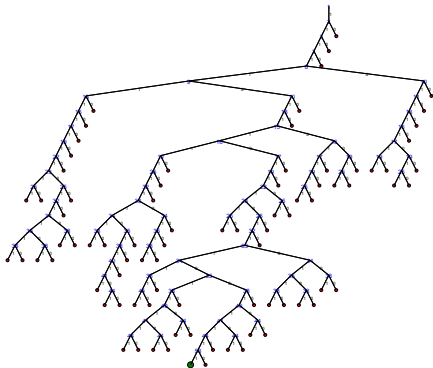
Variable Selection by Column



◀ Back to Start

▶ Skip Animation

Variable Selection by Column

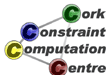


◀ Back to Start

▶ Skip Animation

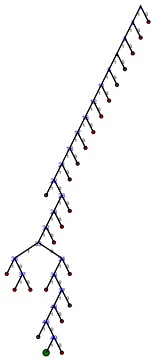
Observation

- Good, but not as good as row order
- Value choice unimportant even for first solution
- Changing the variable selection does affect size of search space, even for all solutions

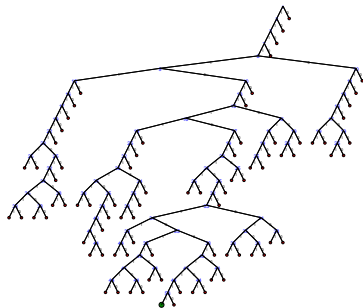


Effort for All Solutions

By Row



By Column



Possible Explanations

- There are fewer rows than columns
- Strict lex constraints on rows, but not on columns
 - More impact of first row
- Needs more testing



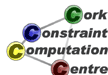
Outline

- 4 Why assign by row?
- 5 **Alternative Models**
- 6 Exercises



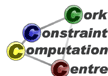
Do we need binary variables?

- Consider a model with finite domain variables
- Each of b blocks consists of k variables ranging over v values
- The values in a block must be all different (ordered)
- Each value can occur r times
- Scalar product more difficult
- Even better expressed with finite set variables



Outline

- 4 Why assign by row?
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Exercises

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