Multiple classifiers

JERZY STEFANOWSKI
Institute of Computing Sciences
Poznań University of Technology

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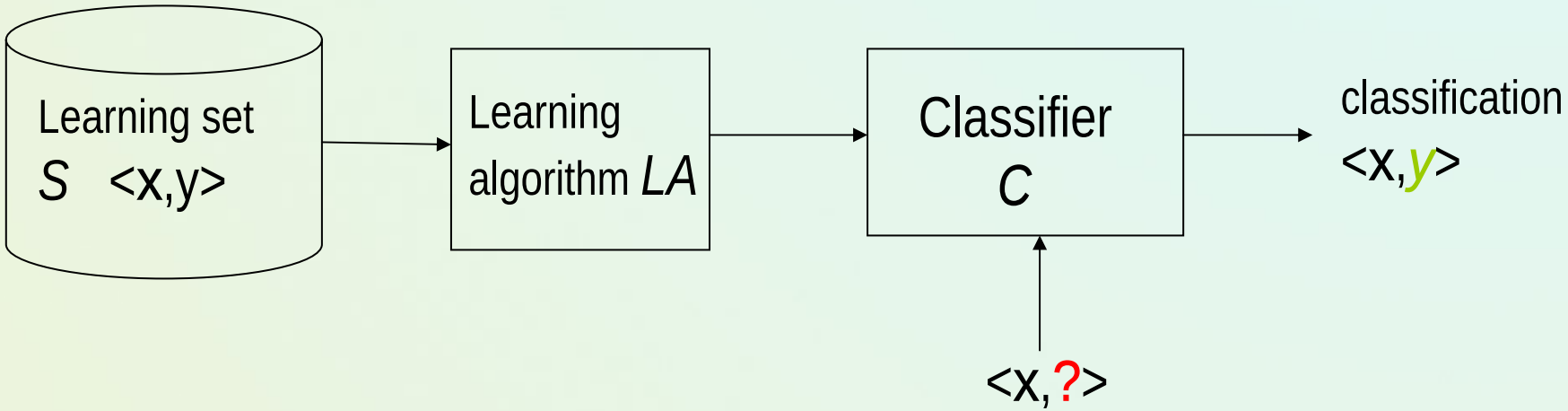
Outline of the presentation

1. Introduction
2. Why do multiple classifiers work?
3. Stacked generalization – combiner.
4. Bagging approach
5. Boosting
6. Feature ensemble
7. n^2 classifier for multi-class problems

Machine Learning and Classification

Classification - assigning a decision class label to a set of objects described by a set of attributes



Set of learning examples $S = \{\langle \mathbf{x}_1, y_1 \rangle, \langle \mathbf{x}_2, y_2 \rangle, \cdots; \langle \mathbf{x}_n, y_n \rangle\}$

for some unknown classification function f: $y = f(\mathbf{x})$

$\mathbf{x}_i = \langle x_{i1}, x_{i2}, \ldots, x_{im} \rangle$ example described by m attributes

y – class label; value drawn from a discrete set of classes $\{Y_1, \ldots, Y_K\}$

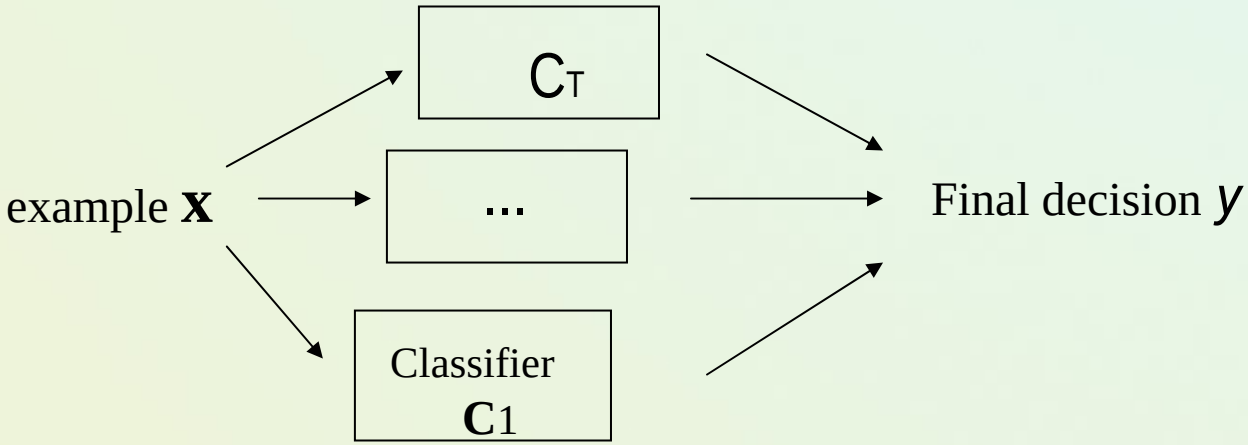
Why could we integrate classifiers?

- Typical research → create and evaluate a single learning algorithm; compare performance of some algorithms.
- Empirical observations or applications → a given algorithm may outperform all others for a specific subset of problems
  - There is no one algorithm achieving the best accuracy for all situations!
- A complex problem can be decomposed into multiple sub-problems that are easier to be solved.
- Growing research interest in combining a set of learning algorithms / classifiers into one system

> *„Multiple learning systems try to exploit the local different behavior of the base learners to enhance the accuracy of the overall learning system"*
>
> - G. Valentini, F. Masulli

Multiple classifiers - definitions

- Multiple classifier – a set of classifiers whose individual predictions are combined in some way to classify new examples.
- Various names: ensemble methods, committee, classifier fusion, combination, aggregation,…
- Integration should improve predictive accuracy.



Multiple classifiers – review studies

- Relatively young research area – since the 90's
- A number of different proposals or application studies
- Some review papers or book:
  - L.Kuncheva, Combining Pattern Classifiers: Methods and Algorithms, 2004 (large review + list of bibliography).
  - T.Dietterich, Ensemble methods in machine learning, 2000.
  - J.Gama, Combining classification algorithms, 1999.
  - G.Valentini, F.Masulli, Ensemble of learning machines, 2001 [exhaustive list of bibliography].
  - J.Kittler et al., On combining classifiers, 1998.
  - J.Kittler et al. (eds), Multiple classifier systems, Proc. of MCS Workshops, 2000, … ,2003.
  - See also many papers by L.Breiman, J.Friedman, Y.Freund, R.Schapire, T.Hastie, R.Tibshirani,

Multiple classifiers – why do they work?

- How to create such systems and when they may perform better than their components used independently?
- Combining identical classifiers is useless!

> A necessary condition for the approach to be useful is that member classifiers should have a substantial level of disagreement, i.e., they make error independently with respect to one another

- Conclusions from some studies (e.g. Hansen&Salamon90, Ali&Pazzani96):
  Member classifiers should **make uncorrelated errors** with respect to one another; each classifier should perform better than a random guess.

Diversification of classifiers - intuition

Two classifiers are diverse, if they make different errors on a new object

Assume a set of three classifiers $\{h_1, h_2, h_3\}$ and a new object **x**

- If all are identical, then when $h_1(\mathbf{x})$ is wrong, $h_2(\mathbf{x})$ and $h_3(\mathbf{x})$ will be also wrong
- If the classifier errors are uncorrelated, then when $h_1(\mathbf{x})$ is wrong, $h_2(\mathbf{x})$ and $h_3(\mathbf{x})$ may be correct $\rightarrow$ a majority vote will correctly classify **x**!

Improving performance with respect to a single classifier

- An example of binary classification (50% each class), classifiers have the same error rate and make errors independently; final classification by uniform voting $\rightarrow$ the expected error of the system should decrease with the number of classifiers

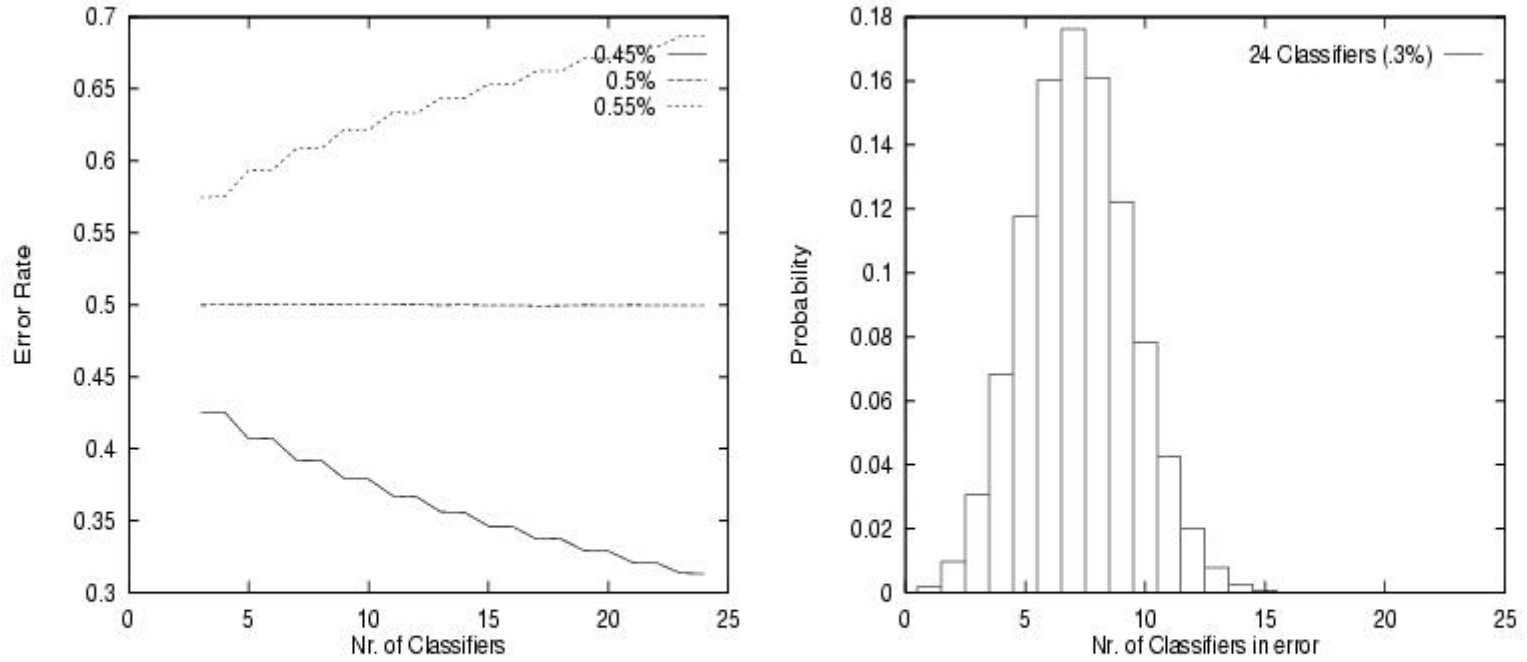


Figure 5.1: (a) Error rate versus nr. of classifiers in an ensemble. (b) Probability that exactly n of 24 classifiers will make an error.

Dietterich's reasons why multiple classifier may work better…

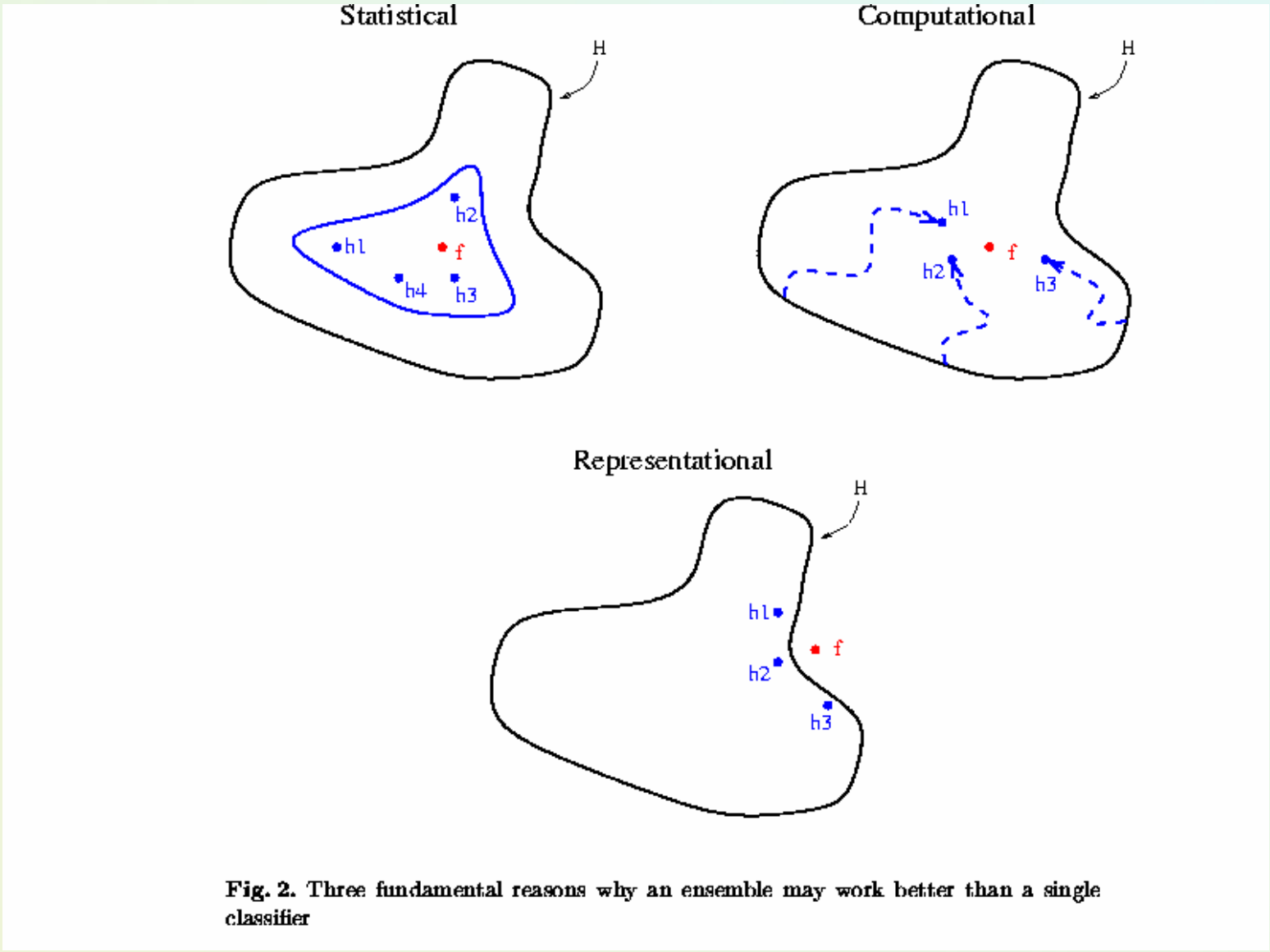


Fig. 2. Three fundamental reasons why an ensemble may work better than a single classifier

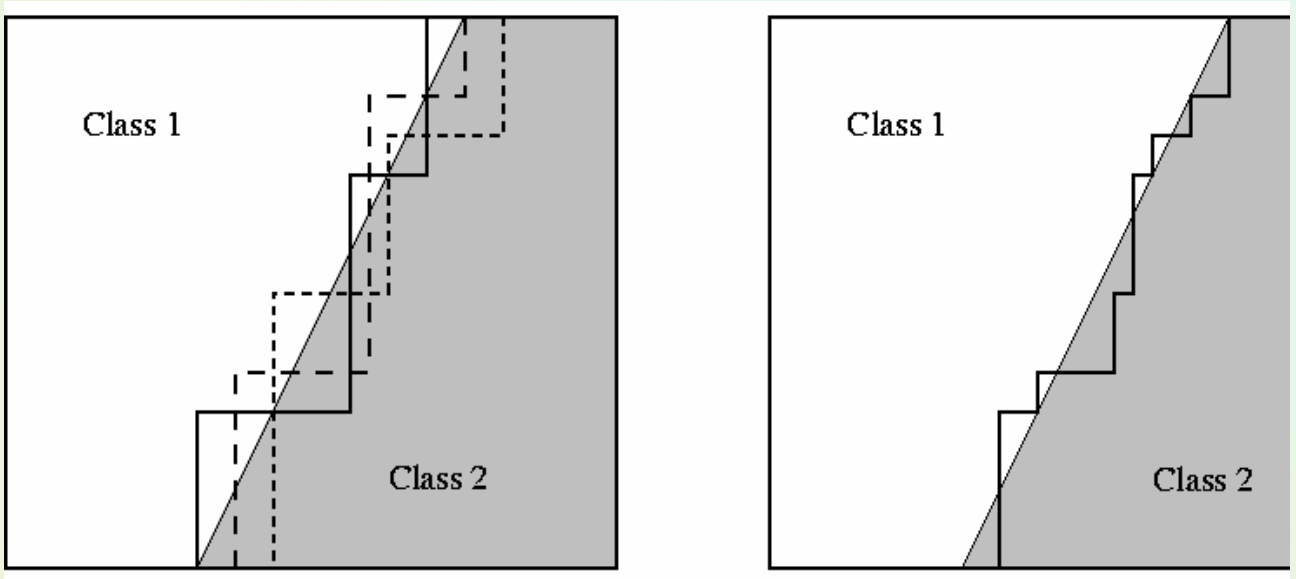
Why do ensembles work?

Dietterich(2002) showed that ensembles overcome three problems:

- ***The Statistical Problem*** arises when the hypothesis space is too large for the amount of available data. Hence, there are many hypotheses with the same accuracy on the data and the learning algorithm chooses only one of them! There is a risk that the accuracy of the chosen hypothesis is low on unseen data!
- ***The Computational Problem*** arises when the learning algorithm cannot guarantee finding the best hypothesis.
- ***The Representational Problem*** arises when the hypothesis space does not contain any good approximation of the target class(es).

Multiple classifier may work better than a single classifier.

- The diagonal decision boundary may be difficult for individual classifiers, but may be approximated by ensemble averaging.
- Decision boundaries constricted by decision trees → hyperplanes parallel to the coordinate axis - „staircases”.
- By averaging a large number of „staircases” the diagonal boundary can be approximated with some accuracy.



Combing classifier predictions

- Intuitions:
  - Utility of combining diverse, independent opinions in human decision-making
- Voting vs. non-voting methods
  - Counts of each classifier are used to classify a new object
  - The vote of each classifier may be weighted, e.g., by measure of its performance on the training data. (Bayesian learning interpretation).
- Non-voting $\rightarrow$ output classifiers (class-probabilities or fuzzy supports instead of single class decision)
  - Class probabilities of all models are aggregated by specific rule (product, sum, min, max, median,…)
  - More complicated $\rightarrow$ extra meta-learner

Group or specialized decision making

- Group (static) – all base classifiers are consulted to classify a new object.
- Specialized / dynamic integration – some base classifiers performs poorly in some regions of the instance space
  - So, select only these classifiers whose are „expertised” (more accurate) for the new object

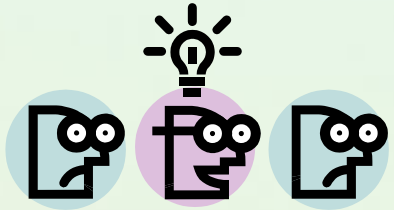
Dynamic voting of sub-classifiers

Change the way of aggregating predictions from sub-classifiers!

- Standard $\rightarrow$ equal weight voting.

Dynamic voting:

- For a new object to be classified:
  - Find its *h*-nearest neighbors in the original learning set.
  - Reclassify them by all sub-classifiers.
  - Use weighted voting, where a sub-classifier weight corresponds to its accuracy on the *h*-nearest neighbors.



Diversification of classifiers

- Different training sets (different samples or splitting,..)
- Different classifiers (trained for the same data)
- Different attributes sets

  (e.g., identification of speech or images)
- Different parameter choices

  (e.g., amount of tree pruning, BP parameters, number of neighbors in KNN,…)
- Different architectures (like topology of ANN)
- Different initializations

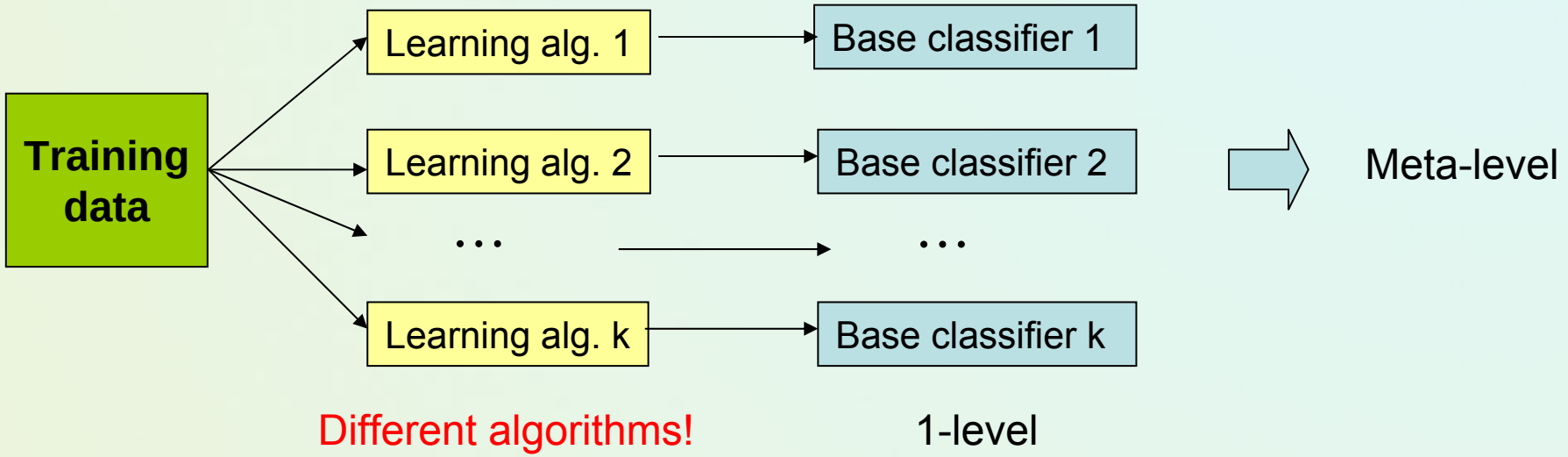
Different approaches to create multiple systems

- **Homogeneous classifiers** – use of the same algorithm over diversified data sets
  - Bagging (Breiman)
  - Boosting (Freund, Schapire)
  - Multiple partitioned data
  - Multi-class specialized systems, (e.g. ECOC pairwise classification)
- **Heterogeneous classifiers** – different learning algorithms over the same data
  - Voting or rule-fixed aggregation
  - Stacked generalization or meta-learning

Stacked generalization [Wolpert 1992]

- Use meta learner instead of averaging to combine predictions of base classifiers.
  - Predictions of base learners (*level-0 models*) are used as input for meta learner (*level-1 model)*
- Method for generating base classifiers usually apply different learning schemes.
- Hard to analyze theoretically.

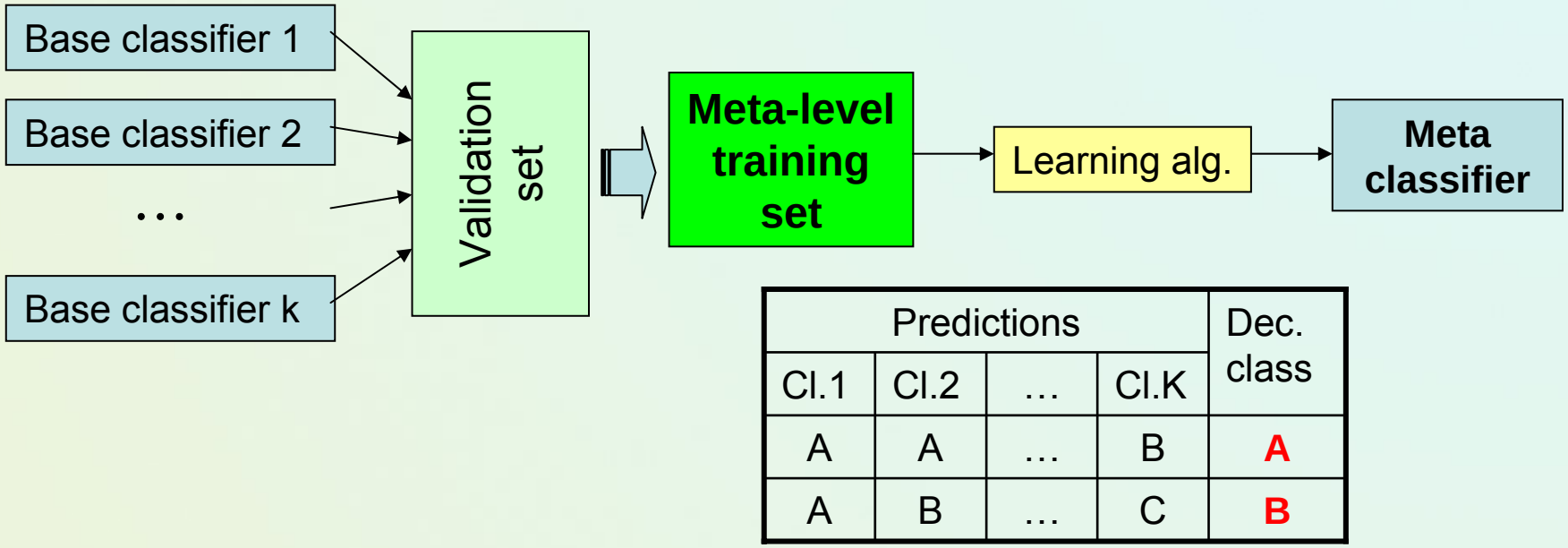
The Combiner - 1



Chan & Stolfo : *Meta-learning*.

- Two-layered architecture:
  - 1-level – base classifiers.
  - 2-level – meta-classifier.
- Base classifiers created by applying the different learning algorithms to the same data.

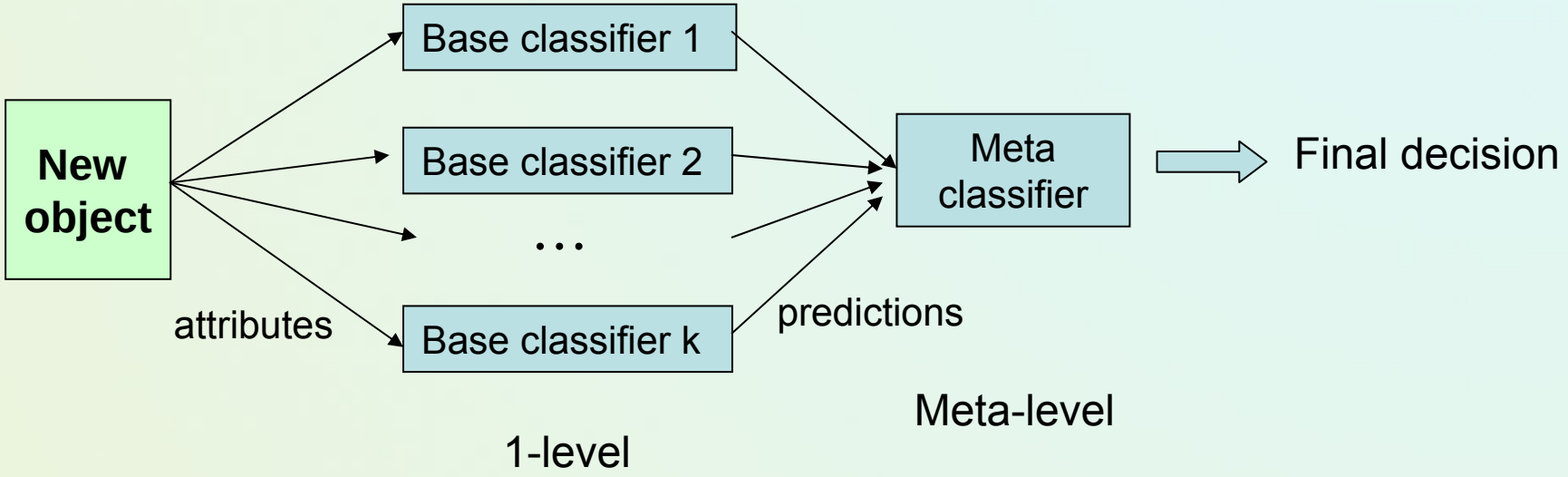
Learning the meta-classifier



Predictions				Dec. class
Cl.1	Cl.2	…	Cl.K	
A	A	…	B	**A**
A	B	…	C	**B**

- Predictions of base classifiers on an extra validation set (not directly training set – apply „internal" cross validation) with correct class decisions → a meta-level training set.
- An extra learning algorithm is used to construct a meta-classifiers.
- The idea → a meta-classifier attempts to learn relationships between predictions and the final decision;
  It may correct some mistakes of the base classifiers.

The Combiner - 2



Classification of a new instance by the combiner

- Chan & Stolfo [95/97] : experiments that their combiner ({CART,ID3,K-NN}→NBayes) is better than equal voting.

More on stacking

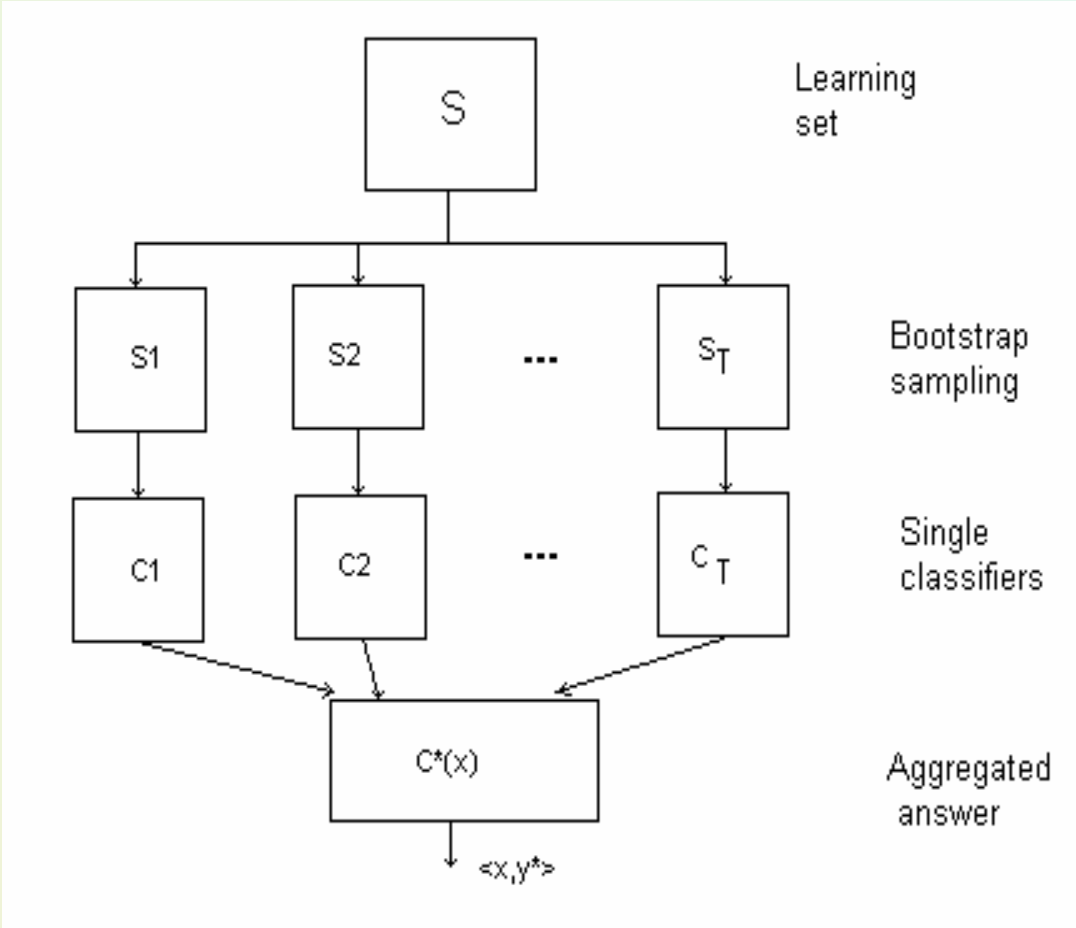
- Other 1-level solutions: use additional attribute descriptions, introduce an arbiter instead of simple meta-combiner.
- If base learners can output probabilities it's better to use those as input to meta learner
- Which algorithm to use to generate meta learner?
  - In principle, any learning scheme can be applied
  - David Wolpert:
    - Base learners do most of the work
    - Reduces risk of overfitting
- Relationship to more complex approaches: SCANN [Mertz] create a new attribute space for the metalearning.

Bagging [L.Breiman, 1996]

- Bagging = **B**ootstrap **agg**regation
  - Generates individual classifiers on bootstrap samples of the training set
- As a result of the sampling-with-replacement procedure, each classifier is trained on the average of 63.2% of the training examples.
  - For a dataset with N examples, each example has a probability of $1-(1-1/N)^N$ of being selected at least once in the N samples. For $N \rightarrow \infty$, this number converges to (1-1/e) or 0.632 [Bauer and Kohavi, 1999]
- Bagging traditionally uses component classifiers of the same type (e.g., decision trees), and combines prediction by a simple majority voting across.

More about „Bagging”

- Bootstrap aggregating – L.Breiman [1996]



input S – learning set, T – no. of bootstrap samples, LA – learning algorithm

output C* - multiple classifier

for i=1 **to** T **do**

begin

S_i:=bootstrap sample from S;

C_i:=$LA(S_i)$;

end;

$$C^*(x) = \text{argmax}_y \sum_{i=1}^{T} (C_i(x) = y)$$

Bagging Empirical Results

Misclassification error rates [Percent]

Data	Single	Bagging	Decrease
waveform	29.0	19.4	33%
heart	10.0	5.3	47%
breast cancer	6.0	4.2	30%
ionosphere	11.2	8.6	23%
diabetes	23.4	18.8	20%
glass	32.0	24.9	22%
soybean	14.5	10.6	27%

Breiman “Bagging Predictors” Berkeley Statistics Department TR#421, 1994

Bagging – how does it work?

- Related works – experiments Breiman [96], Quinlan [96], Bauer&Kohavi [99]; Conclusion – bagging improves accuracy for decision trees.
- The perturbation in the training set due to the bootstrap re+sampling causes different base classifiers to be built, particularly if the classifier is unstable
- Breiman says that this approach works well for unstable algorithms:
  - Whose major output classifier undergoes major changes in response to small changes in learning data.
- Bagging can be expected to improve accuracy if the induced classifiers are uncorrelated!

Bias-variance decomposition

- Theoretical tool for analyzing how much *specific* training set affects performance of a classifier
  - Total expected error: bias + variance
  - The *bias* of a classifier is the expected error of the classifier due to the fact that the classifier is not perfect
  - The *variance* of a classifier is the expected error due to the particular training set used

Why does bagging work and may hurt?

- Bagging reduces variance by voting/ averaging, thus reducing the overall expected error
  - Usually, the more classifiers the better but …
  - In the case of classification there are pathological situations where the overall error might increase
    - For smaller training samples and too stable classifiers …

Experiments with rules

- The single use of the MODLEM induced classifier is compared against bagging classifier (composed of rule sub-classifiers - also induced by MODLEM)
- Comparative studies on 18 datasets. Predictive accuracy evaluated by 10-fold cross-validation (stratified or random)
- An analysis of the change parameter *T* (number of sub-classifiers) on the performance of the bagging classifier

Comparing classifiers

Dataset	Single	Bagging - with different T			
	MODLEM	3	5	7	10
bank	93.81 ± 0.94	95.05 ± 0.91	94.95 ± 0.84	95.22 ± 1.02	93.95* ± 0.9
buses	97.20 ± 0.94	98.05* ± 0.97	99.54 ± 1.09	97.02* ± 1.15	97.45* ± 1.1
zoo	94.64 ± 0.67	93.82* ± 0.68	93.89* ± 0.71	93.47 ± 0.73	93.68 ± 0.70
hsv	54.52 ± 1.05	64.75 ± 1.21	65.94 ± 0.69	64.78 ± 0.57	64.53 ± 0.55
hepatitis	78.62 ± 0.93	82.00 ± 1.14	84.05 ± 1.1	81.05 ± 0.97	84.0 ± 0.49
iris	94.93 ± 0.5	95.13* ± 0.46	94.86* ± 0.54	95.06* ± 0.53	94.33* ± 0.5
auto	85.23 ± 1.1	82. 98 ± 0.86	83.0 ±0.99	82.74 ±0.9	81.39 ± 0.84
segmentation	85.71 ± 0.71	86.19* ± 0.82	87.62 ± 0.55	87.61 ± 0.46	87.14 ± 0.9
glass	72.41 ± 1.23	68.5 ± 1.15	74.81 ± 0.94	74.25 ± 0.89	76.09 ± 0.68
bricks	90.32* ± 0.82	90.3 * ± 0.54	89.84* ± 0.65	91.21* ± 0.48	90.77* ± 0.7
vote	92.67 ± 0.38	93.33* ± 0.5	94.34 ± 0.34	95.01 ± 0.44	96.01 ± 0.29
bupa	65.77 ± 0.6	64.98* ± 0.76	76.28 ± 0.44	70.74 ± 0.96	75.69 ± 0.7
election	88.96 ± 0.54	90.3 ± 0.36	91.2 ± 0.47	91.66 ± 0.34	90.75 ± 0.55
urolog1	62.4 ± 0.51	65.2 ± 0.25	63.1* ± 0.5	65.8 ± 0.35	65.2 ± 0.34
urolog2	63.80 ± 0.73	64.8 ± 0.83	65.0 ± 0.43	67.40 ± 0.46	67.0 ± 0.67
german	72.16 ± 0.27	73.07* ± 0.39	76.2 ± 0.34	75.62 ± 0.34	75.75 ± 0.35
crx	84.64 ± 0.35	84.74* ± 0.38	86.24 ± 0.39	87.1 ± 0.46	89.42 ± 0.44
pima	73.57 ± 0.67	75.78* ± 0.6	74.35* ± 0.64	74.88 ± 0.44	77.87 ± 0.39

Classification accuracy [%] – average over 10 f-c-v with standard deviations; Asterik – difference is not significant α =0.05

Some remarks

- Bagging outperformed the single classifiers on 14 of 18 datasets;
  - for others (easier e.g. *iris*, *bank*, *buses*) difference non-significant; the single classifier is better for *zoo* and *auto* data sets.
- The bagging is a „winner” for more difficult data and it improves for higher number of examples.
- We should expect good result as
  - The MODLEM is an unstable algorithm in the sense of Breiman’s postulate

    (the choice of the best elementary condition to the rule, the choice of thresholds for numerical attributes)
- The bagging additional computational costs – depends on *T*.

Analysis of the number of component classifiers

- For some data (e.g. *hsv*, *glass*, *pima*, *vote*) increasing T has lead to better accuracy
- For majority of data $T > 5$ but is seems to be difficult do indicate one the best value
- Breiman says: „more replicants are required with an increasing number of classes”

Table 3. Increase of accuracy with respect to a number of sub-classifiers T

Dataset	Number of sub-classifiers T					
	3	5	7	10	15	20
glass	-3.91	2.40	1.84	3.68	7.06	6.09
pima	2.21	0.78*	1.31	4.30	5.47	3.66
election	1.34	2.24	2.70	1.79	1.8	0.1*
crx	0.1*	1.6	2.46	4.78	2.61	2.18
hsv	10.23	11.42	10.26	10.01	11.35	14.64
german	0.91*	4.04	3.46	3.59	1.21	2.42
vote	0.66*	1.67	2.34	3.34	2.0	3.0

Boosting [Schapire 1990; Freund & Schapire 1996]

- In general takes a different weighting schema of resampling than bagging.
- Freund & Schapire: theory for “weak learners” in late 80’s
- **Weak Learner**: performance on ***any*** train set is slightly better than chance prediction
  - Schapire has shown that a weak learner can be converted into a strong learner by changing the distribution of training examples
- Iterative procedure:
  - The component classifiers are built sequentially, and examples that are misclassified by previous components are chosen more often than those that are correctly classified!
  - So, new classifiers are influenced by performance of previously built ones. New classifier is encouraged to become expert for instances classified incorrectly by earlier classifier.
- There are several variants of this algorithm – **AdaBoost** the most popular (see also arcing).

AdaBoost

- Weight all training examples equally (1/n)
- Train model (classifier) on train sample D_i
- Compute error e_i of model on train sample D_i
- A new training sample D_{i+1} is produced by decreasing the weight of those examples that were correctly classified (multiple by $e_i/(1-e_i)$)), and increasing the weight of the misclassified examples.
- Normalize weights of all instances.
- Train new model on re-weighted train set
- Re-compute errors on weighted train set
- The process is repeated until (# iterations or error stopping)
- Final model: weighted prediction of each classifier
  - Weight of class predicted by component classifier $\log(e_i/(1-e_i))$

Remarks on Boosting

- Boosting can be applied without weights using re-sampling with probability determined by weights;
  - Example weights might be harder to deal with some algorithms or packages.
  - Draw a bootstrap sample from the data with the probability of drawing each example is proportional to it's weight
- Boosting should decrease exponentially the training error in the number of iterations;
- Boosting works well if base classifiers are not too complex and their error doesn't become too large too quickly!

Boosting vs. Bagging with C4.5 [Quinlan 96]

	C4.5	Bagged C4.5 vs C4.5			Boosted C4.5 vs C4.5			Boosting vs Bagging	
	err (%)	err (%)	w-l	ratio	err (%)	w-l	ratio	w-l	ratio
anneal	7.67	6.25	10-0	.814	4.73	10-0	.617	10-0	.758
audiology	22.12	19.29	9-0	.872	15.71	10-0	.710	10-0	.814
auto	17.66	19.66	2-8	1.113	15.22	9-1	.862	9-1	.774
breast-w	5.28	4.23	9-0	.802	4.09	9-0	.775	7-2	.966
chess	8.55	8.33	6-2	.975	4.59	10-0	.537	10-0	.551
colic	14.92	15.19	0-6	1.018	18.83	0-10	1.262	0-10	1.240
credit-a	14.70	14.13	8-2	.962	15.64	1-9	1.064	0-10	1.107
credit-g	28.44	25.81	10-0	.908	29.14	2-8	1.025	0-10	1.129
diabetes	25.39	23.63	9-1	.931	28.18	0-10	1.110	0-10	1.192
glass	32.48	27.01	10-0	.832	23.55	10-0	.725	9-1	.872
heart-c	22.94	21.52	7-2	.938	21.39	8-0	.932	5-4	.994
heart-h	21.53	20.31	8-1	.943	21.05	5-4	.978	3-6	1.037
hepatitis	20.39	18.52	9-0	.908	17.68	10-0	.867	6-1	.955
hypo	.48	.45	7-2	.928	.36	9-1	.746	9-1	.804
iris	4.80	5.13	2-6	1.069	6.53	0-10	1.361	0-8	1.273
labor	19.12	14.39	10-0	.752	13.86	9-1	.725	5-3	.963
letter	11.99	7.51	10-0	.626	4.66	10-0	.389	10-0	.621
lymphography	21.69	20.41	8-2	.941	17.43	10-0	.804	10-0	.854
phoneme	19.44	18.73	10-0	.964	16.36	10-0	.842	10-0	.873
segment	3.21	2.74	9-1	.853	1.87	10-0	.583	10-0	.684
sick	1.34	1.22	7-1	.907	1.05	10-0	.781	9-1	.861
sonar	25.62	23.80	7-1	.929	19.62	10-0	.766	10-0	.824
soybean	7.73	7.58	6-3	.981	7.16	8-2	.926	8-1	.944
splice	5.91	5.58	9-1	.943	5.43	9-0	.919	6-4	.974
vehicle	27.09	25.54	10-0	.943	22.72	10-0	.839	10-0	.889
vote	5.06	4.37	9-0	.864	5.29	3-6	1.046	1-9	1.211
waveform	27.33	19.77	10-0	.723	18.53	10-0	.678	8-2	.938
average	*15.66*	*14.11*		*.905*	*13.36*		*.847*		*.930*

Table 1: Comparison of C4.5 and its bagged and boosted versions.

Boosting vs. Bagging

- Bagging doesn't work so well with stable models. Boosting might still help.
- Boosting might hurt performance on noisy datasets. Bagging doesn't have this problem.
- On average, boosting helps more than bagging, but it is also more common for boosting to hurt performance.
- In practice bagging almost always helps.
- Bagging is easier to parallelize.

Randomization Injection

- Inject some randomization into a standard learning algorithm (usually easy):
  - Neural network: random initial weights
  - Decision tree: when splitting, choose one of the top N attributes at random (uniformly)
- Dietterich (2000) showed that 200 randomized trees are statistically significantly better than C4.5 for over 33 datasets!

Feature-Selection Ensembles

- ***Key idea:*** Provide a different subset of the input features in each call of the learning algorithm.
- ***Example:*** Venus&Cherkauer (1996) trained an ensemble with 32 neural networks. The 32 networks were based on 8 different subsets of 119 available features and 4 different algorithms. The ensemble was significantly better than any of the neural networks!
- See also Random Subspace Methods by Ho.

Integrating attribute selection with bagging

- Diversification of classifiers by selecting subsets of attributes (some related works,…)
- What about integration of attribute selection (MFS) and bagging together?
- Study of P.Latinne *et al*. → encouraging results of simple random technique (BagFS, Bag vs. MFS)
- In my and M.Kaczmarek study → we have used different techniques of attribute subset selection (random choice, correlation subsets, contextual merit, Info-gain, χ^2 ,…) inside WEKA toolkit
- Dynamic selection of classifiers (nearest neighbor,…)
- Results → selection of attributes and classifiers + standard bagging – slightly improves the classification performance

Random forests [Breiman]

- At every level, choose a random subset of the attributes (not examples) and choose the best split among those attributes.
- Combined with selecting examples like basic bagging.
- Doesn’t overfit.

Data set	Adaboost	Selection	Forest-RI single input	One tree
Glass	22.0	20.6	21.2	36.9
Breast cancer	3.2	2.9	2.7	6.3
Diabetes	26.6	24.2	24.3	33.1
Sonar	15.6	15.9	18.0	31.7
Vowel	4.1	3.4	3.3	30.4
Ionosphere	6.4	7.1	7.5	12.7
Vehicle	23.2	25.8	26.4	33.1
German credit	23.5	24.4	26.2	33.3
Image	1.6	2.1	2.7	6.4
Ecoli	14.8	12.8	13.0	24.5
Votes	4.8	4.1	4.6	7.4
Liver	30.7	25.1	24.7	40.6
Letters	3.4	3.5	4.7	19.8
Sat-images	8.8	8.6	10.5	17.2
Zip-code	6.2	6.3	7.8	20.6
Waveform	17.8	17.2	17.3	34.0
Twonorm	4.9	3.9	3.9	24.7
Threenorm	18.8	17.5	17.5	38.4
Ringnorm	6.9	4.9	4.9	25.7

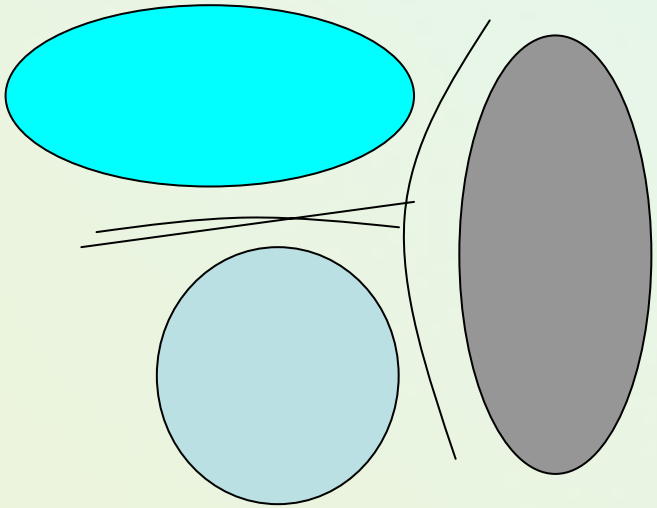
Breiman, Leo (2001). "Random Forests". Machine Learning 45 (1), 5-32

The n^2 classifier for multi-class problems

- Specialized approach for multi-class difficult problems.
  - Decompose a multi-class problem into a set of two-class sub-problems.
  - Combine them to obtain the final classification decision
- The idea based on pairwise coupling by Hastie T., Tibshirani R [NIPS 97] and J.Friedman 96.
- The n^2 version proposed by Jacek Jelonek and Jerzy Stefanowski [ECML 98].
- Other specialized approaches:
  - One-per-class,
  - Error-correcting output codes.

Solving multi-class problems

- The problem is to classify objects into a set of *n* decision classes (*n*>2)
- Some problems may be difficult to be learned (complex target concepts with non-linear decision boundaries).
- An example of three-class problem, where pairwise decision boundaries between each pairs of classes are simpler.

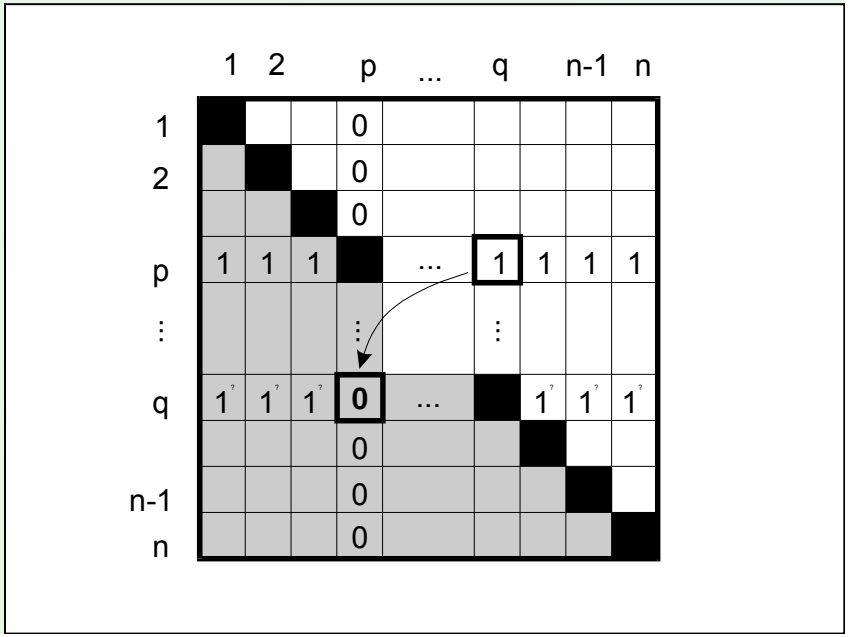


The n2-classifier

It is composed of $(n^2-n)/2$ *base binary classifiers* (all combinations of pairs of n classes).

- discrimination of each pair of the classes (i,j), where $i,j \in [1..\, n]$, $i \neq j$, by an independent binary classifier C_{ij}
- The specificity of training binary classifier C_{ij} - only examples from two classes i,j.
- classifier C_{ij} yields binary classification (1 or 0), classifiers C_{ij} and C_{ji} are equivalent

$$C_{ji}(\mathbf{x}) = 1 - C_{ij}(\mathbf{x})$$



Final classification decision of the n^2-classifier

- For an unseen example **x,** a final classification of the n^2-classifier is a proper aggregation of predictions of all base classifiers $C_{ij}(\mathbf{x})$
- Simplest aggregation - find a class that wins the most pairwise comparison
- The aggregation could be extended by estimating credibility of each base classifier (during learning phase) P_{ij}
- Final classification decision - a weighted majority rule:
  - choose such a decision class „***i***” that maximizes:

$$\sum_{j=1, i \neq j}^{n} P_{ij} \cdot C_{ij}(x)$$

Conditions of experiments

- We examine an influence of the learning algorithm on the classification performance of n^2-classifier:
  - Decision trees
  - Decision rules (MODLEM)
  - Artificial neural network (feed forward multi-layer network trained by Back-Propagation)
  - Instance based learning (k-nn, k=1, Euclidean distance)
- Computations on MLR-UCI benchmark data sets and our medical ones.
- The classification accuracy estimated by stratified 10-fold cross validation

Performance of n^2 classifier based on decision trees

Data set	Classification accuracy *DT* (%)	Classification accuracy n^2 (%)	Improvement n^2 vs. *DT* (%)
Automobile	85.5 ± 1.9	87.0 ± 1.9	**1.5***
Cooc	54.0 ± 2.0	59.0 ± 1.7	**5.0**
Ecoli	79.7 ± 0.8	81.0 ± 1.7	**1.3**
Glass	70.7 ± 2.1	74.0 ± 1.1	**3.3**
Hist	71.3 ± 2.3	73.0 ± 1.8	**1.7**
Meta-data	47.2 ± 1.4	49.8 ± 1.4	**2.6**
Primary Tumor	40.2 ± 1.5	45.1 ± 1.2	**4.9**
Soybean-large	91.9 ± 0.7	92.4 ± 0.5	**0.5***
Vowel	81.1 ± 1.1	83.7 ± 0.5	**2.6**
Yeast	49.1 ± 2.1	52.8 ± 1.8	**3.7**

Discussion of experiments with various algorithms

- Decision trees $\rightarrow$ significant better classification for 8 of all data sets; other differences non-significant
- Comparable results for decision rules
  Artificial neural networks $\rightarrow$ generally better classification for 9 of all data sets; some of highest improvements but difficulties in constructing networks
- However, k-nn does not result in improving classification performance of the n^2-classier with respect to single multi-class instance-based learner!
  - We proposed an approach to select attribute subsets discriminating each pair of classes $\rightarrow$ it improved a k-nn constructed classifier.

The n^2-classifier with decision rules induced by MODLEM

Table 2. Comparison of classification accuracies [%] and computation times [s] for the single MODLEM based classifier and the n^2-classifier also based on decision rules induced by MODLEM algorithm

Name of data set	Accuracy of single MODLEM (%)	Accuracy of n^2_{MODLEM} (%)	Time of comput. MODLEM	Time of comput. n^2_{MODLEM}
automobile	85.25 ± 1.3	87.96 ± 1.5	15.88 ± 0.4	5.22 ± 0.3
cooc	55.57 ± 2.0	59.30 ± 1.4	4148,7 ± 48.8	431.51 ± 1.6
ecoli	79.63 ± 0.8	81.34 ± 1.7	27.53 ± 0.5	11.25 ± 0.7
glass	72.07 ± 1.2	74.82 ± 1.4	45.29 ± 1.1	13.88 ± 0.4
hist	69.36 ± 1.1	73.10 ± 1.4	3563.79 ± 116.1	333.96 ± 0.8
meta-data	47.2 ± 1.3	49.83 ± 1.9	252.59 ± 78.9	276.71 ± 5.21
iris	94.2 ± 0.6	95.53* ± 1.2	0.71 ± 0.04	0.39 ± 0.04
soybean-large	91.09 ± 0.9	91.99* ± 0.8	26.38 ± 0.3	107.5 ± 5.7
vowel	81.81 ± 0.5	83.79 ± 1.2	3750.57 ± 30.4	250.63 ± 0.7
yeast	54.12 ± 0.7	55.74 ± 0.9	1544.3 ± 13.2	673.82 ± 9.4
zoo	94.64 ± 0.5	94.46* ± 0.8	0.30 ± 0.02	0.34 ± 0.12

Notice → improvements of classification accuracy
But also for computational costs

Few comments on the use of MODLEM

- Unlike other methods does not increase the computation time
  - In experiments time is even decreased (8 sets)
- Pairwise decision boundaries are easier to be learned
  - Smaller number of attributes (elementary conditions / rules) is sufficient to discriminate
- Properties of MODLEM sequential covering scheme
  - Given class against all other (n-1) classes vs. only pair
  - Smaller number of examples and attribute-value tests to be verified
- Analysis of the rules for Ecoli data set:
  - MODLEM $\rightarrow$ 46 rules (av. Length 3.7, strength 9.3)
  - n^2-classifier $\rightarrow$ 118 rules (av. Length 1.8, strength 26.5)

Some Practical Advices [Smirnov]

If the classifier is unstable (i.e, decision trees) then apply bagging!

If the classifier is stable and simple (e.g. Naïve Bayes) then apply boosting!

If the classifier is stable and very complex (e.g. Neural Network) then apply randomization injection!

If you have many classes and a binary classifier then try error-correcting codes! If it does not work then use a complex binary classifier!

Any questions, remarks?