An Operations Research Look at Voting

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Lets go back to 15th Century

Electing the Doge

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Enter Lewis Carrol

Charles Dodgson came up with a voting system (perhaps in order to "win" a vote on a belfry in Oxford, but also spurred by decisions on studentships) that involved finding the best way to flip adjacent candidates in preference orders to get a Condorcet winner (a candidate who beats every other candidate one-on-one).

Outline

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Much overlap with a CS view, particularly the part of CS that is willing to solve NP-complete problems (SAT, CP), but I will point out some directions that are more OR-ish.

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- 20 years? Really? But (I'll claim) this acts like a five year old idea
- And then I'll give you some more recent stuff (including results from last week)

INTRODUCTION

Problem: Given n alternatives, and v voters, each with an preference ordering on the alternatives, aggregate them into either
(a) a "winner" or winners, or
(b) a total ordering of the alternatives (where the "winner" is the first in the ordering).

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Huge number of voting rules. A voting rule should be "fair", decisive, and practical.

EXAMPLE

Suppose the preferences are:

3 voters	$a \succ b \succ e \succ c \succ d$
2 voters	$c \succ a \succ e \succ b \succ d$
4 voters	$d \succ b \succ e \succ c \succ a$
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Result from run-off elections (how?)?

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Independence of Irrelevant Alternatives: A voting system satisfies *independence of irrelevant alternatives* if the decision of c versus d depends only on the relative ranking of c and d in the voter preference profiles.

IMPOSSIBILITY THEOREMS

Theorem (Arrow): The only voting rule that satisfies Unanimity and Independence of Irrelevant Alternatives is Dictatorship

Many, many efforts build on this: is IIA relevant? should we allow any possible input? etc. etc.

ALGORITHMIC QUESTION

How quickly can we determine the result under a particular voting rule?

n candidates, v voters.

Plurality: $O(n)$
Borda and many others: $O(nv)$

Even low order polynomials would be a problem (U.S. election with an $\theta(v^3)$ algorithm?).
Can it get worse?

CONDORCET CRITERION

Definition: Given a voting instance, if a candidate c is preferred to each other candidate by a majority of voters, then c is the *Condorcet winner*.

If every instance had a Condorcet winner, then choosing it would satisfy Unanimity and IIA, but some instances do not
$(a \succ b \succ c, b \succ c \succ a, c \succ a \succ b)$.

(Re-)ENTER LEWIS CARROLL

Dodgson's Rule: The winner of an election is the candidate who requires the fewest preference switches (adjacent) to become the winner.

Theorem (Bartholdi, Tovey, and Trick (BTT), 1989): It is NP-Hard to determine the winner under Dodgson's Method.

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Impracticality Theorem (BTT, 1989). For any voting system that satisfies
(a) neutrality
(b) consistency
(c) Condorcet winner
it is NP-Hard to determine the winner.

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- Difficult to analyze aspects that require characterization of Kemeny or Dodgson winner for arbitrary sizes

VOTING THEORY ISSUE: MANIPULABILITY

Sometimes a voter can get a preferred result by misrepresenting his preferences. (Example: plurality election between a, b, and c. Without you a and b are tied, and c way behind. You prefer c but instead vote to break tie between a and b).

Definition. A voting system satisfies *non-manipulability* if no voter can ever get a preferred result by misrepresenting his preferences.

Many impossibility theorems (example: No voting system satisfies anonymity, neutrality, Condorcet winner, and non-manipulability). Many, many papers on this: what if everyone is changing? Weakening of conditions, etc.

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Theorem (BTT 1989). It is NP-complete for a voter to determine how to manipulate an election under second order Copeland score.

There are others, with Single Tranferable Vote the most natural (Bartholdi and Orlin).

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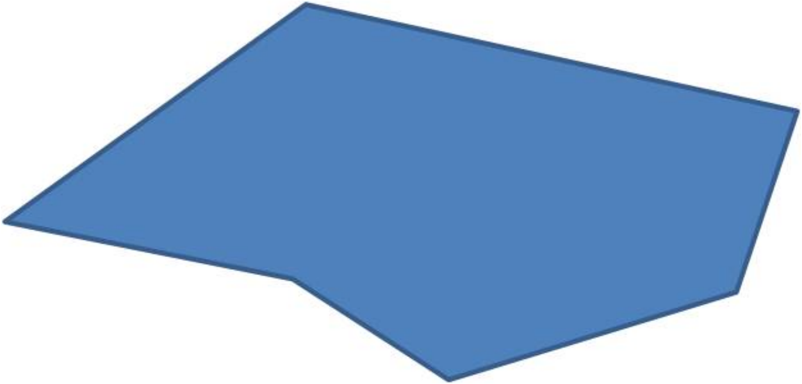
- Difficult to analyze manipulable instances since no characterization of such.
- Heuristics probably *are* useful.
- Likelihood of manipulation now depends on both opportunity and the recognition of the opportunity.
- Matches up with intuitive feel for difficulty of problem.

Opportunity/Recognition/Heuristics



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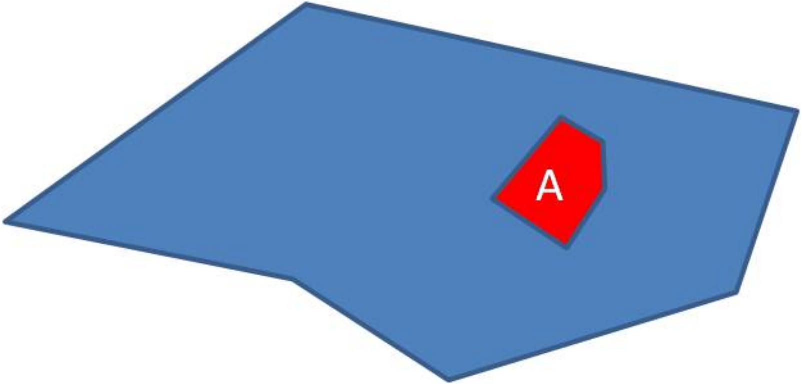

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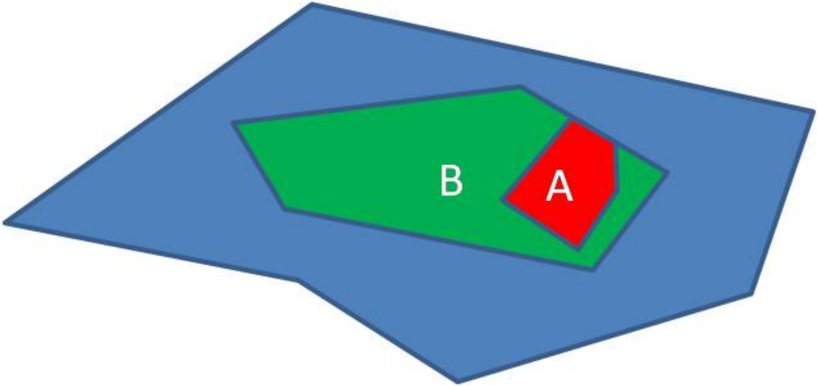

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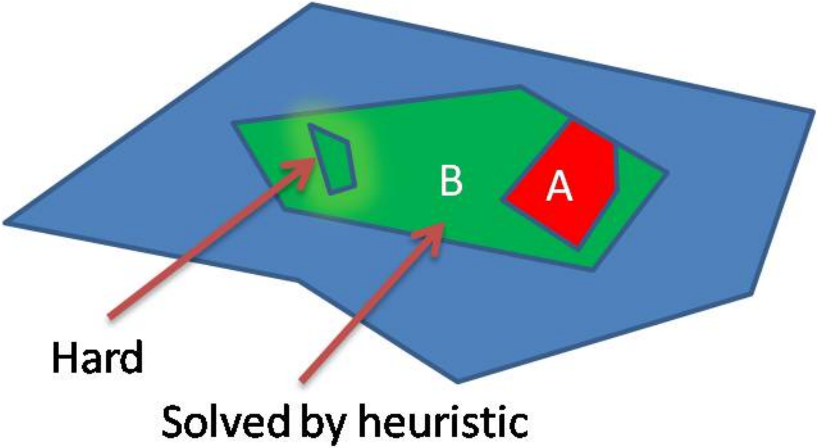

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Algorithmic issue: For which rules is this heuristic guaranteed to work (i.e. will always correctly determine if c can win)?

HEURISTIC ANALYSIS

Denote an ordering P, where iPj mean i is ordered before j.

Theorem (BTT 1989). Greedy-Manipulation will find an ordering P that will make candidate c a winner or conclude that it is impossible for any voting scheme that can be represented as a function $S(P) : C \rightarrow \mathbf{R}$ that is both
– "responsive": a candidate with the largest $S(P, i)$ is a winner.
– "monotone": for any two preference orders P and P' and for any candidate i, $\{j : iP'j\} \subseteq \{j : iPj\}$ implies that $S(P', i) \leq S(P, i)$.

Shows Plurality, Borda, Copeland, and many others are manipulable quickly.

MANIPULATION BY GROUPS

Suppose you lead a group of 100 people and wish to tell them how to vote in order to get your prefered candidate c to win.

Plurality: Easy! Vote for candidate c. Don't even need to know others preferences: if it works, fine; otherwise you can't make c the winner (need more knowledge to elect your highest possible candidate).

Borda: Much harder. Clearly put c in first slot. But who in second (who will get $n-1$ points)? May have to have some people put a_1 in second slot and others put a_2 in second slot.

Manipulating Borda count by Groups is NP-complete (and needs multiple profiles).

MANIPULATION BY CHAIRS

Chairs of committees may have a number of powers:

Changing the Candidates

- Adding Candidates

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Many "fairness conditions" address the question of whether a voting rule is vulnerable to these sort of manipulations.

COMPARISON of PLURALITY and CONDORCET

Can also ask the algorithmic question: how can a chair determine *how* to optimally use power (BTT 1992).

Control by ...	Plurality	Condorcet
adding candidates	resistant	immune
deleting candidates	resistant	vulnerable
partitioning candidates	resistant	vulnerable
adding voters	vulnerable	resistant
deleting voters	vulnerable	resistant
partitioning voters	vulnerable	resistant

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4. Many issues left to explore! And they have been over the last years, including
    - More detailed complexity classes
    - Average case behavior
    - Other hard/easy problems

INTERLUDE: Some Further History

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Citation History

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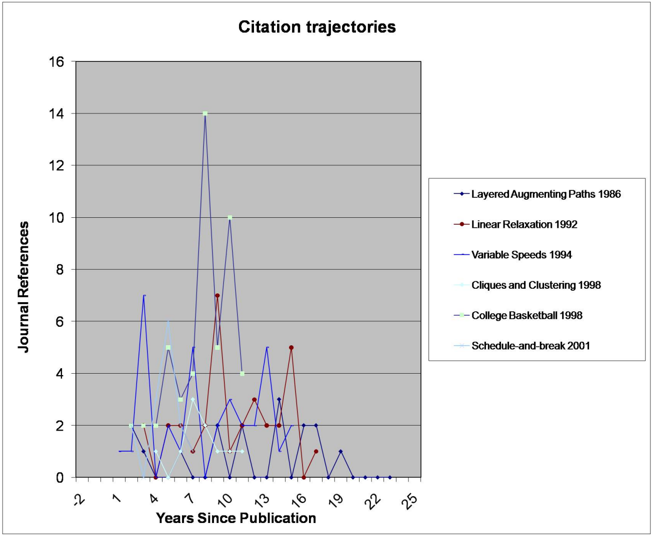

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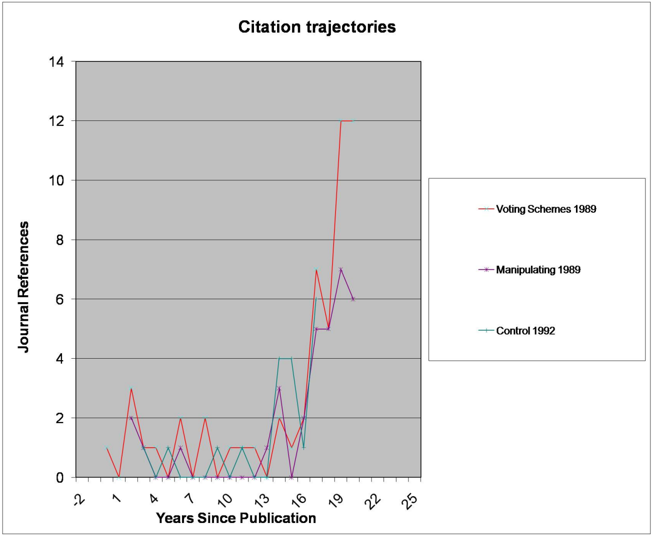

Current State

Currently the Winner NP-completeness and the Manipulation NP-completeness papers are number 5 and 8 *all-time* from *Social Choice and Welfare* in terms of google scholar citations (164 and 150). Only 3 papers in JET from 1989 have more cites.

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Thanks!

Don't give up on what you think is a good, but unrecognized, research direction. Its day *may* come.

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Another type of chair manipulation

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Medium open problem: 5 candidates. a is chair's favorite candidate: can the chair set the agenda so that a wins as long as a is in the top cycle? We'll see the 3 and 4 candidate cases.

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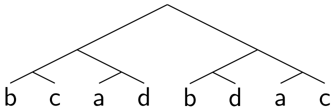

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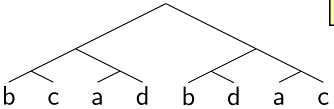

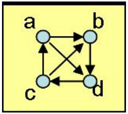

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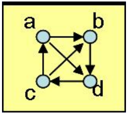

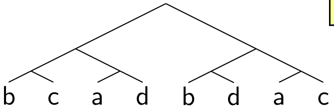

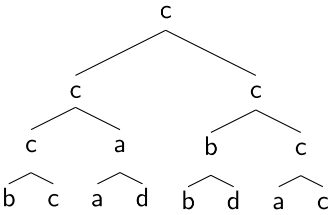

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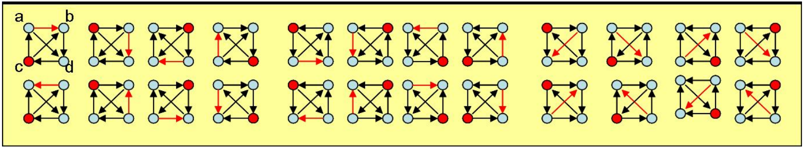

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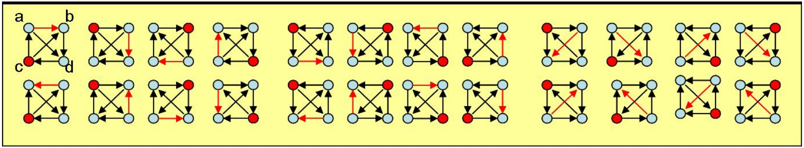

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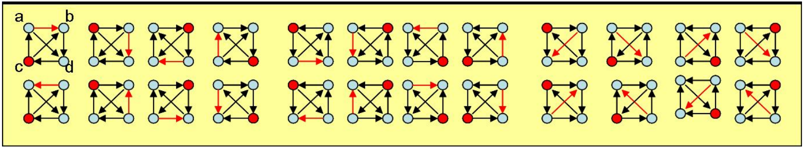

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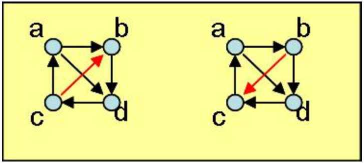
Question

What rules are implementable by voting trees?

In the economics literature, this is known as implementation by backwards induction and is a key open problem in mechanism design.

Implementable Rules

Assumption: all candidates appear in tree (rule is onto).
Clearly, rule must choose from top cycle of each tournament (including choosing Condorcet winner if it exists)
Sufficient?



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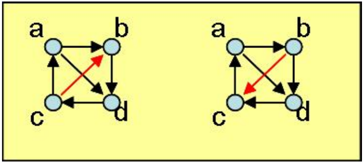

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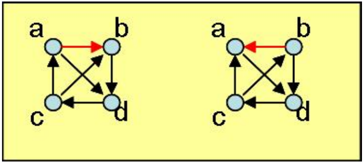


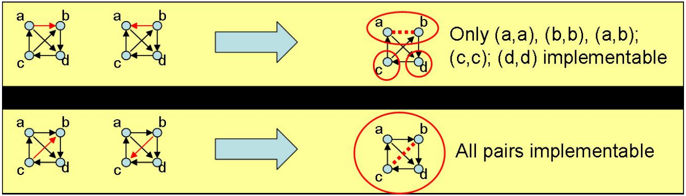
only $(a, a), (b, b), (c, c), (d, d)$ and (a, b) are implementable.

Pairwise Conjecture

Conjecture (Pairwise Conjecture)

A rule defined over all tournaments of n candidates is implementable if and only if it is implementable over all pairs of tournaments. (Srivasta and Trick, 1996)

Srivastava and Trick also give necessary and sufficient conditions for a rule to be implementable over a pair of tournaments.



If true, then this implies there is an agenda-control tree for all n.

Computational Procedure

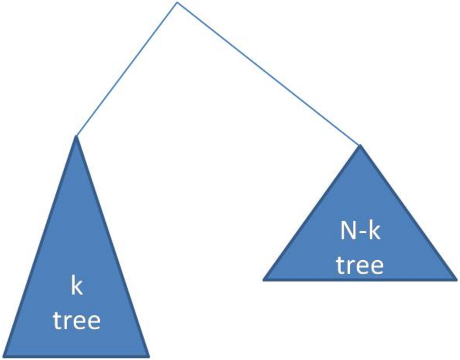
After thirteen years, little progress on conjecture (but no counterexamples either!)

Want a computational procedure to provide verification (or find counterexample)

Algorithm to generate all small rules over small number of candidates (number of rules increases quickly with number of candidates)

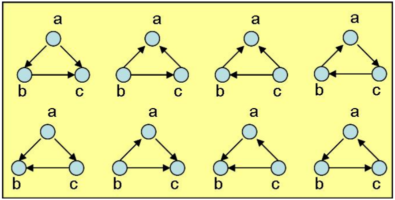
Computational Procedure: Dynamic Programming

Complete enumeration to generate minimum size trees for each rule.
Generate all trees of size up to N
Generate $N+1$ size trees by combining



There has to be a better way!

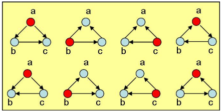
Rules on 3 Candidates

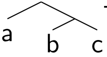


There are 8 tournaments on 3 candidates, so there are $3^8 = 6561$ rules over these tournaments. Of these, only 9 rules are Condorcet. The Pairwise conjecture requires each of these 9 to be implementable, and the computational procedure shows that to be the case.

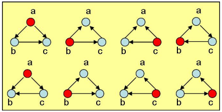
Tournaments on 3 Candidates

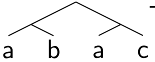
Always choose Condorcet candidate if it exists. Else:



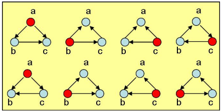


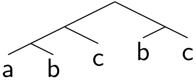
Type I: Choose *a*





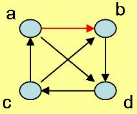
Type II: Choose candidate that beats *a*





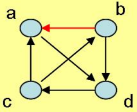
Type III: Choose candidate that loses to *a*

Structure of 4 Candidate Tournaments



Copeland: 2
a, c
Copeland: 1
b, d

Partner



Copeland: 2
b, c
Copeland: 1
a, d

Define
w_1: c
w_2: a(b)
l_1: b(a)
l_2: d

Implementable
w_1 : (c,c)
w_2 : (a,b)
l_2 : (d,d)

(w_2,l_1) : (a,a)
(l_1,w_2) : (b,b)

Number of Rules on 4 Candidates

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- Of these 4096 choose among the Copeland winners, and 1 chooses only Copeland losers (interesting to find these).

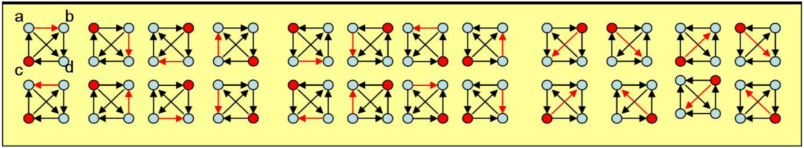
Results so far

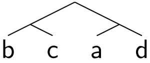
We have found about 66,835,958 rules so far, 3933 of the Copeland winner (out of 4096), and the Copeland Loser Rule (up to 31 leaves in the tree).

Size	Number	Copeland
4	15	3
5	102	0
6	424	0
7	1104	0
8	2377	19
9	5486	4
10	11232	18
11	21768	36
12	40420	36
13	70600	96
14	116670	60
15	187560	96
16	294510	240
17	439102	192

Size	Number	Copeland
18	633986	138
19	895648	292
20	1231551	368
21	1655920	148
22	2188704	240
23	2829882	318
24	3595685	276
25	4464020	296
26	5428012	224
27	6468312	220
28	7542497	366
29	8613668	88
30	9610118	76
31	10486540	84

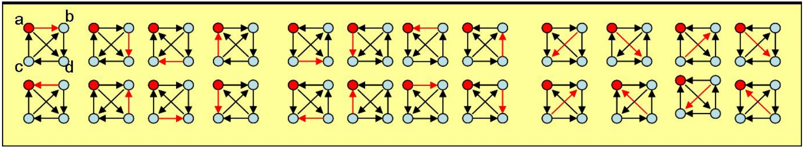
Smallest Rule, Choice from Copeland Winners

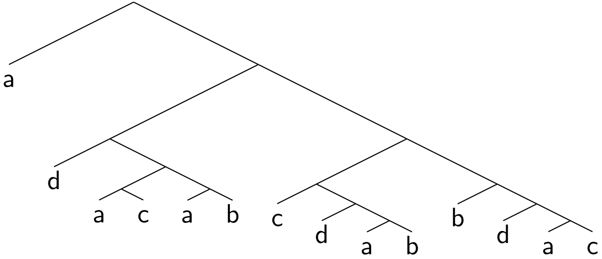




4 Node, Top Cycle, Lexicographic Tiebreak

Always choose a

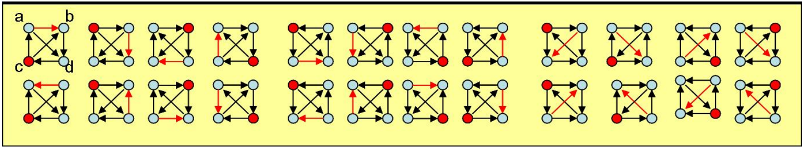


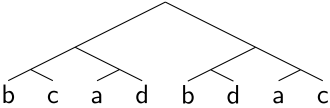


Compare with 3 candidate case! Agenda manipulation possible, but obvious due to complexity of the result.

4 Node, Copeland winner, 2nd Order Copeland tiebreak

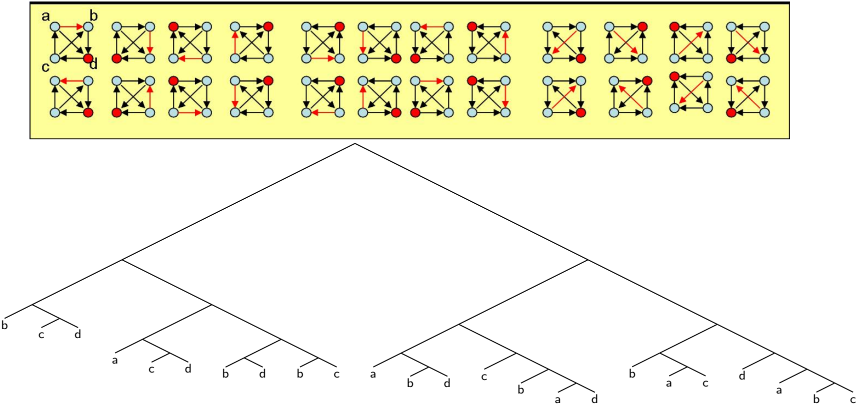
Always choose w_1 (neutral rule)





Copeland Loser in Top Cycle, 2nd Order Copeland tiebreak

Always choose l_2 (neutral rule)



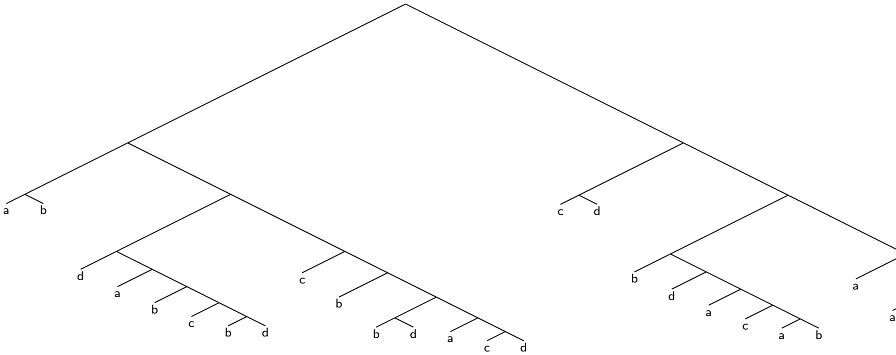
Copeland Winner in Top Cycle, Copeland loser tiebreak

Always choose w_2 (neutral rule)

Not yet found.

Sample 31 node tree

"Mainly" choose w_2 (correct 18 out of 24 times)



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- Would appreciate someone proving or disproving Pairwise Conjecture!

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- Need for more real instances: does any of this happen in practice?
- Breaking out of OR/CS and changing economics/finance. We have useful formalisms of "bounded rationality".

Questions or Comments?