Preference Handling: (a decision analysis perspective)

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Outline

Preferences

- Preferences are “rational” desires.
- Preferences are at the basis of any decision aiding activity.
- There are no decisions without preferences.
- Preferences, Values, Objectives, Desires, Utilities, Beliefs, ...

Decision Aiding

A client		An analyst
	A problem situation $\langle \mathcal{A}, \mathcal{O}, \mathcal{S} \rangle$	
	A problem formulation $\langle \mathbb{A}, \mathbb{V}, \Pi \rangle$	
	An evaluation model $\langle A, D, E, H, U, R \rangle$	
	A final recommendation	

What are the problems?

- How to learn preferences?
- How to model preferences?
- How to aggregate preferences?
- How to use preferences for recommending?

Basic information

- A: a set of alternatives (enumerative, combinatorial, product space ...)
- D: a set of dimensions (attributes) describing A.
- E: the “scales” used for the attributes in D.
- H: the set of criteria.
- U: uncertainties ....
- R: algorithms, procedures, protocols etc ...

Binary relations

- $\succeq$: binary relation on a set (A).
- $\succeq \subseteq A \times A$ or $A \times P \cup P \times A$.
- $\succeq$ is reflexive.

What is that?

If $x \succeq y$ stands for x is at least as good as y, then the asymmetric part of $\succeq$ ($x \succeq y \ \wedge \ \neg(y \succeq x)$) stands for strict preference. The symmetric part stands for indifference ($\sim$: $x \succeq y \ \wedge \ y \succeq x$) or incomparability ($? : \neg(x \succeq y) \ \wedge \ \neg(y \succeq x)$).

Numbers

$$x \succeq y \quad \Leftrightarrow \quad \Phi(x, y) \geq 0$$

where:

$\Phi : A \times A \mapsto \mathbb{R}$. Simple case $\Phi(x, y) = f(x) - f(y)$; $\;f : A \mapsto \mathbb{R}$

N.B.

Likelihoods can also be expressed under form of binary relations and their numerical representations ($\omega_1 \succeq \omega_2$: event 1 is likely to occur at least as much as event 2).

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Consider sentences of the type:

- I like red shoes.
- I do not like brown sugar.
- I prefer Obama to McCain.
- I do not want tea with milk.
- Cost is more important than safety.
- I prefer flying to Athens than having a suite at Istanbul.

What do we learn out of such sentences?

- Basic hypotheses about the structure of the evaluation model.
- Binary relations.
- Numerical values (exact or imprecise).
- Importance Parameters.
- Inconsistencies.

Preference Structures

Independently from the nature of the set A (enumerated, combinatorial etc.), consider $x, y \in A$ as whole elements. Then:

If $\succeq$ is a weak order then:

$\succ$ is a strict partial order, $\sim$ is an equivalence relation and ? is empty.

If $\succeq$ is an interval order then:

$\succ$ is a partial order of dimension two, $\sim$ is not transitive and ? is empty.

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What characterises such structures?

Characteristic Properties

Weak Orders are complete and transitive relations.
Interval Orders are complete and Ferrers relations.

Numerical Representations

w.o. $\Leftrightarrow$ $\exists f : A \mapsto \mathbb{R} : x \succeq y \leftrightarrow f(x) \geq f(y)$
i.o. $\Leftrightarrow$ $\exists f, g : A \mapsto \mathbb{R};\ f(x) > g(x) : x \succeq y \leftrightarrow f(x) \geq g(y)$

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What if A is multi-attribute described?

$$x = \langle x_1 \cdots x_n \rangle \quad y = \langle y_1 \cdots y_n \rangle$$

$$x \succeq y \quad \Leftrightarrow \quad \Phi([u_1(x_1) \cdots u_n(_n)], [u_1(y_1) \cdots u_n(y_n)] \geq 0$$

A special case is when Φ is increasing to its first n arguments and decreasing to the following n arguments: it then can be an additive function.

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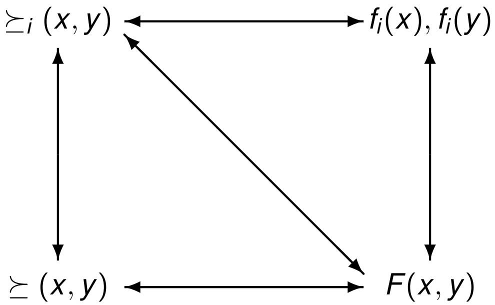
The Problem

Suppose we have n preference relations $\succeq_1 \cdots \succeq_n$ on the set A. We are looking for an overall preference relation $\succeq$ on A "representing" the different preferences.



Social choice approach

$x \succeq y$ **iff** it is the case for some majority rule among the criteria

A specific example:

$$x \succeq y \quad \Leftrightarrow \quad C(x, y) \ \wedge \ \neg D(x, y)$$

where:

- $C(x, y)$: a weighted majority rule;
- $D(x, y)$: veto condition

The goof and the bad news

Advantages

It applies always ... (almost)

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It does not turn always a useful result ...
It is not compact ...
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Value functions approach

Make out of the $\succeq_i$ some value functions $u_i : A \mapsto \mathbb{R}$ in such a way that:

- $u_i(x)$ makes sense (you can compare it to $u_i(y)$).
- $u_i(x) - u_i(y)$ makes sense (you can compare it to $u_i(z) - u_i(w)$).

Then:

$$x \succeq y \quad \Leftrightarrow \quad \sum_i u_i(x) \geq \sum_i u_i(y)$$

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It is elegant.
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It always give a clear result

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It does not apply always (independence, commensurability, compensation).
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When you get the information it might be too late ...

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What happens after aggregating preferences?

Suppose you use a majority rule in order to aggregate the $\succeq_i$ in a single $\succeq$.

Then $\succeq$ is just a reflexive binary relation and no other properties can be guaranteed to hold: <u>IT IS NOT AN ORDER</u>

You have to do something in order to construct an "ordering relation" $\succcurlyeq$ out of $\succeq$.

<u>Vast choice of procedures more or less axiomatised</u>

Critical Questions

1. Am I solving the right problem?
2. Am I asking the right questions (and getting the right answers)?
3. Do I know exactly what I am doing?
4. Is it easy to implement?

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