Algorithmic Decision Theory

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Outline

1. General
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   - Some History
   - Problem Statements
2. Basics
   - Preferences
   - Measurement
3. Methods
   - Optimisation, Constraint Satisfaction, MOMP
   - Social Choice Theory
   - Uncertainty
4. Reality and Future
   - Real Life
   - Research Agenda

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What is the problem?

You are told

The elevators are slow and we waste a lot of time ...

1. more powerful engines?
2. more elevators?
3. dedicated elevators?
4. rescheduling of functioning?

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What about putting mirrors at the sides of the elevators?

Deciding ...

- Decision Maker
- Decision Process
- Cognitive Effort
- Responsibility
- Decision Theory

... and Aiding to Decide

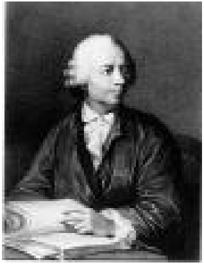
- A client and an analyst
- Decision Aiding Process
- Cognitive Artifacts
- Consensus
- Decision Aiding Methodology

Pre-History 1

From Aristotle

Preferences are rational desires

to Euler



Problems seen as graphs

Pre-History 2

From Borda

Social Choice

and Concorcet

Democratic Paradoxes

to Pareto

Economic Efficiency

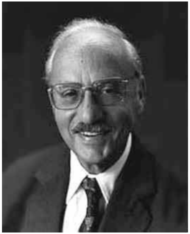
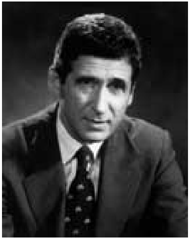
History 1

From radars and enigmas to production systems and networks

- Where to deploy the radars defending UK in the second world war?
- P.M.S. Blackett, Nobel Prize 1948
- Operational Research Office in the British Army

History 2

From radars and enigmas to production systems and networks

Georg Dantzig and Ralph Gomory “Founding Fathers” or Linear (1948) and Integer Programming (1960)

History 3

So many Nobel Prizes ...

Maurice Allais, Kenneth Arrow, Robert Aumann, Leonid Kantorovich, Daniel Khanemman, Tjalling Koopmans, Harry Markowitz, John Nash, Amartya Sen, Herbert Simon, George Stigler, ...

Decision Analysis and Artificial Intelligence 1

Common Background:

Problem Solving

Two different perspectives:

- model of rationality: Decision Analysis
- algorithmic efficiency: Computer Science and Artificial Intelligence

Decision Analysis and Artificial Intelligence 2

Since the 60s common research concerns:

- bounded rationality;
- heuristics;
- uncertainty modelling.

And then:

- preferential entailment;
- computational social choice;
- planning and scheduling;
- contraint programming;
- preference handling;
- learning and knowledge extraction

Decision Analysis and Artificial Intelligence 3

Still some differences:

- aiding human decision making: decision analysis;
- enhance decision autonomy of automatic devices: artificial intelligence.

Nevertheless, it is clear today that the two disciplines are working on very similar fields and there is a clear benefit in cross-fertilising them.

What is a decision problem?

Consider a set *A* established as any among the following:

- an enumeration of objects;
- a set of combinations of binary variables (possibly the whole space of combinations);
- a set of profiles within a multi-attribute space (possibly the whole space);
- a vector space in $\mathbb{R}^n$.

Technically:

A Decision Problem is an application on *A* partitioning it under some desired properties.

Examples

- Patients triage in emergency room;
- Identification of classes of similar DNA sequences;
- Star rankings of hotels;
- Waste collection vehicle routing;
- Vendor rating and bids assessment;
- Optimal mix of sausages;
- Chip-set lay out;
- Airplanes priority landing;
- Tennis tournament scheduling ...

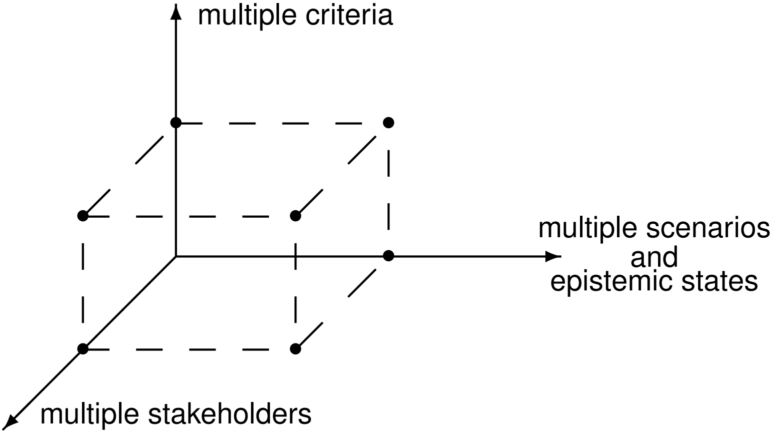
Partitioning? How?

Practically we partition *A* in *n* classes. These can be:

	Pre-defined wrt some external standard	Defined only through pairwise comparison
Ordered	Sorting	Ranking
Not Ordered	Classifying	Clustering

Two special cases:
- there are only two classes (thus complementary);
- the size (cardinality) of the classes is also predefined.

Partitioning? How?



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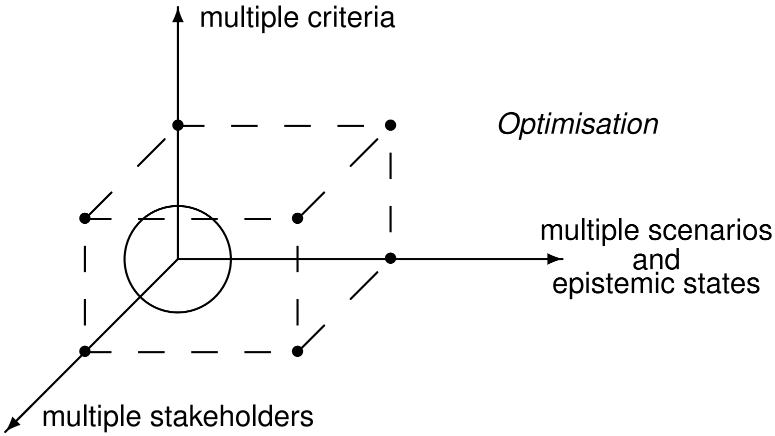

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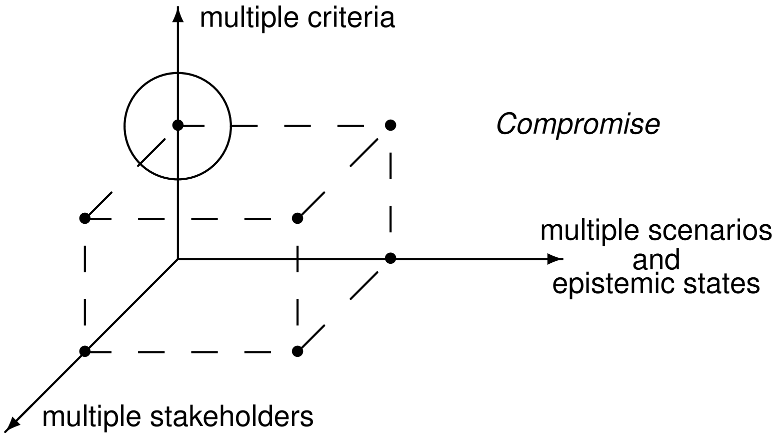

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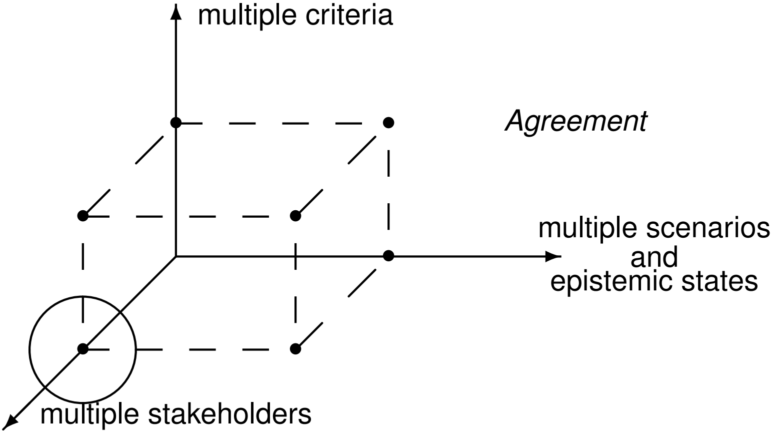

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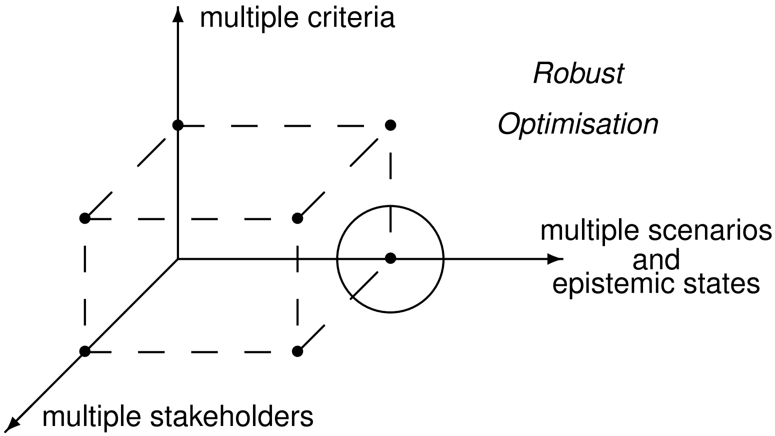

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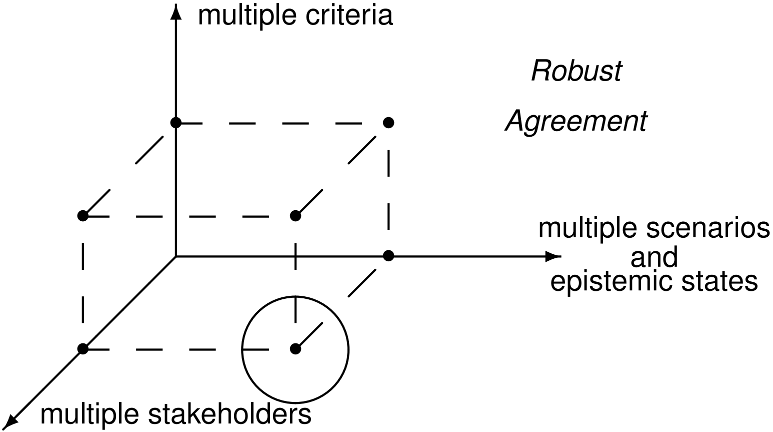
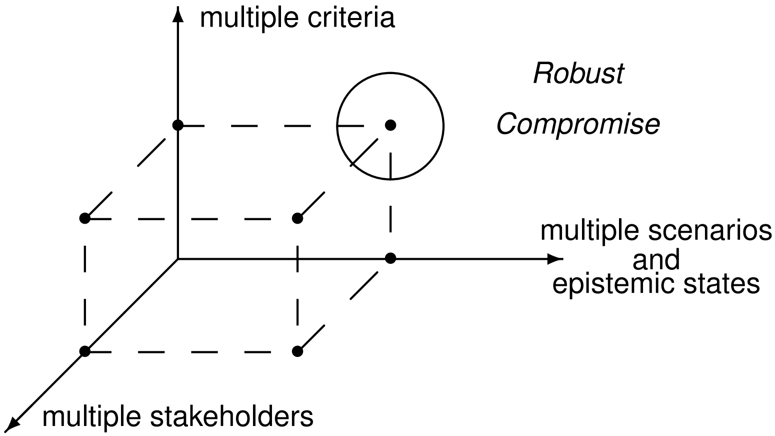


Robust

Agreement

Partitioning? How?



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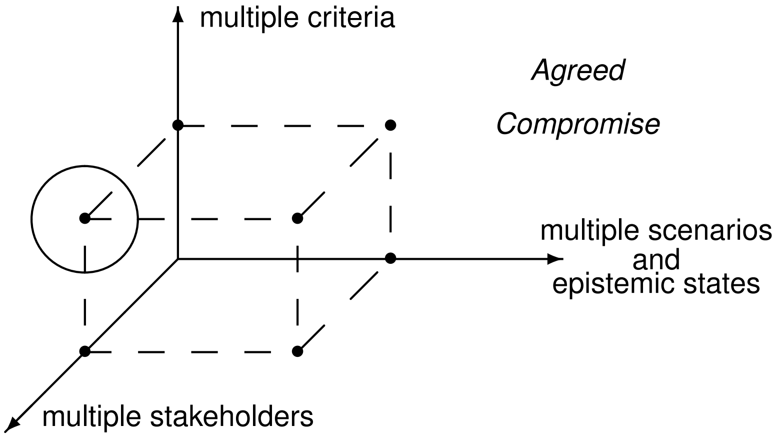

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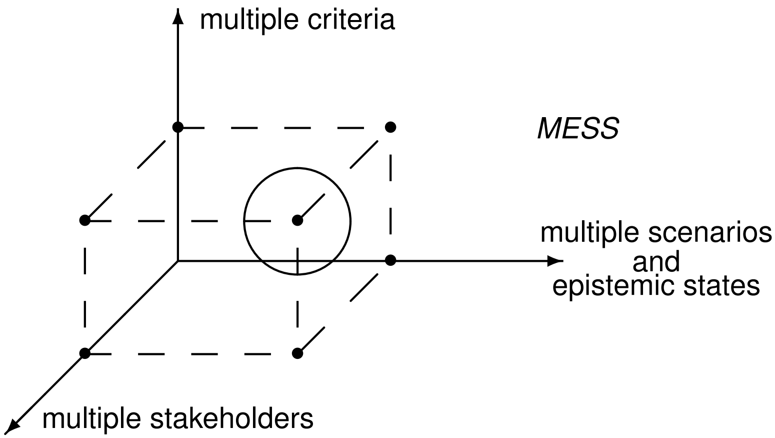

Examples

- Assigning patients to illness under multiple symptoms is a compromise classification to predefined not pre-ordered categories.
- Hiring 10 employees by a commission using elimination by aspects is a repeated agreed compromise sorting of the candidates in two ordered and predefined categories until the last's one size is 10.
- Airplanes priority landing is robust compromise ranking of aircrafts to ordered non predefined categories of size 1.
- Identifying similar DNA sequences is an optimal clustering to non predefined non pre-ordered categories.
- Establish a long term community water management plan is a MESS!!

What are the problems?

- How to learn preferences?
- How to model preferences?
- How to aggregate preferences?
- How to use preferences for recommending?

Binary relations

- $\succeq$: binary relation on a set (A).
- $\succeq \subseteq A \times A$ or $A \times P \cup P \times A$.
- $\succeq$ is reflexive.

What is that?

If $x \succeq y$ stands for x is at least as good as y, then the asymmetric part of $\succeq$ ($\succ$: $x \succeq y \ \wedge \ \neg(y \succeq x)$) stands for strict preference. The symmetric part stands for indifference ($\sim_1$: $x \succeq y \ \wedge \ y \succeq x$) or incomparability ($\sim_2$: $\neg(x \succeq y) \ \wedge \ \neg(y \succeq x)$).

More on binary relations

- We can further separate the asymmetric (symmetric) part in more relations representing hesitation or intensity of preference.

$$\succ = \succ_1 \cup \succ_2 \cdots \succ_n$$

- We can get rid of the symmetric part since any symmetric relation can be viewed as the union of two asymmetric relations and the identity.
- We can also have valued relations such that: $v(x \succ y) \in [0, 1]$ or other logical valuations ...

Binary relations properties

Binary relations have specific properties such as:

- Irreflexive: $\forall x \;\; \neg(x \succ x)$;
- Asymmetric: $\forall x, y \;\; x \succ y \;\rightarrow\; \neg(y \succ x)$;
- Transitive: $\forall x, y, z \;\; x \succ y \;\wedge\; y \succ z \;\rightarrow\; x \succ z$;
- Ferrers; $\forall x, y, z, w \;\; x \succ y \;\wedge\; z \succ w \;\rightarrow\; x \succ w \;\vee\; z \succ y$;

Numbers

One dimension

$$x \succeq y \quad \Leftrightarrow \quad \Phi(u(x), u(y)) \geq 0$$

where:

$\Phi : A \times A \mapsto \mathbb{R}$. Simple case $\Phi(x, y) = f(x) - f(y)$; $\quad f : A \mapsto \mathbb{R}$

Many dimensions

$$x = \langle x_1 \cdots x_n \rangle \quad y = \langle y_1 \cdots y_n \rangle$$

$$x \succeq y \quad \Leftrightarrow \quad \Phi([u_1(x_1) \cdots u_n({}_n)], [u_1(y_1) \cdots u_n(y_n)] \geq 0$$

More about Φ in Measurement Theory

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More about Φ in Measurement Theory

Preference Structures

A preference structure

is a collection of binary relations $\sim_1, \cdots \sim_m, \succ_1, \cdots \succ_n$ such that:

- they are pair-disjoint;
- $\sim_1 \cup \cdots \sim_m \cup \succ_1 \cup \cdots \succ_n = A \times A$;
- $\sim_i$ are symmetric and $\succ_j$ are asymmetric;
- possibly they are identified by their properties.

$\sim_1, \sim_2, \succ$ Preference Structures

Independently from the nature of the set A (enumerated, combinatorial etc.), consider $x, y \in A$ as whole elements. Then:

If $\succeq$ is a weak order then:

$\succ$ is a strict partial order, $\sim_1$ is an equivalence relation and $\sim_2$ is empty.

If $\succeq$ is an interval order then:

$\succ$ is a partial order of dimension two, $\sim_1$ is not transitive and $\sim_2$ is empty.

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If $\succeq$ is a *PQI* interval order then:

$\succ_1$ is transitive, $\succ_2$ is quasi transitive, $\sim_1$ is asymmetrically transitive and $\sim_2$ is empty.

If $\succeq$ is a pseudo order then:

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What characterises such structures?

Characteristic Properties

Weak Orders are complete and transitive relations.
Interval Orders are complete and Ferrers relations.

Numerical Representations

w.o. $\Leftrightarrow$ $\exists f : A \mapsto \mathbb{R} : x \succeq y \leftrightarrow f(x) \geq f(y)$
i.o. $\Leftrightarrow$ $\exists f, g : A \mapsto \mathbb{R} : f(x) > g(x); x \succeq y \leftrightarrow f(x) \geq g(y)$

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More about structures

Characteristic Properties

PQI Interval Orders are complete and generalised Ferrers relations.
Pseudo Orders are coherent bi-orders.

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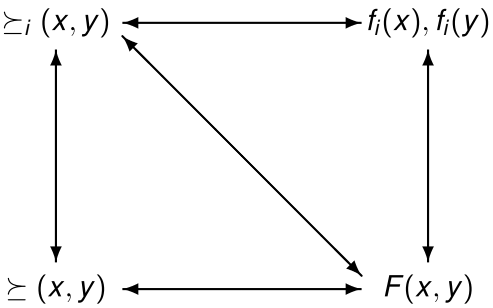
The Problem

- Meaningful numerical representations.
- Putting together numbers (measures).
- Putting together binary relations.
- Overall coherence ...
- Relevance for likelihoods ...

The Problem

Suppose we have n preference relations $\succeq_1 \cdots \succeq_n$ on the set A. We are looking for an overall preference relation $\succeq$ on A "representing" the different preferences.



What is measuring?

Constructing a function from a set of “objects” to a set of “measures”.

Objects come from the real world.

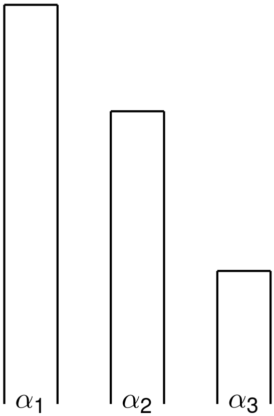
Measures come from empirical observations on some attributes of the objects.

The problem is: how to construct the function out from such observations?

Measurement

1. Real objects $(x, y, \cdots)$.
2. Empirical evidence comparing objects $(x \succeq y, \cdots)$.
3. First numerical representation $(\Phi(x, y) \geq 0)$.
4. Repeat observations in a standard sequence $(x \circ y \succeq z \circ w)$.
5. Enhanced numerical representation $(\Phi(x, y) = \Phi(x) - \Phi(y))$.

Example



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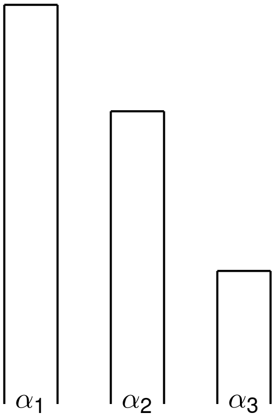

$$\alpha_1 \succ \alpha_2 \succ \alpha_3$$

α_1	α_2	α_3
10	8	6
97	32	12
3	2	1

Any of the above could be a numerical representation of this empirical evidence.

Ordinal Scale: any increasing transformation of the numerical representation is compatible with the EE.

Further Example

Consider putting together objects and observing:

$$\alpha_1 \circ \alpha_5 > \alpha_3 \circ \alpha_4 > \alpha_1 \circ \alpha_2 > \alpha_5 > \alpha_4 > \alpha_3 > \alpha_2 > \alpha_1$$

Consider now the following numerical representations:

	L_1	L_2	L_3
α_1	14	10	14
α_2	15	91	16
α_3	20	92	17
α_4	21	93	18
α_5	28	99	29

L_1, L_2 and L_3 capture the simple order among α_{1-5}, but L_2 fails to capture the order among the combinations of objects.

Further Example

NB

For L_1 we get that $\alpha_2 \circ \alpha_3 \sim \alpha_1 \circ \alpha_4$
while for L_3 we get that $\alpha_2 \circ \alpha_3 > \alpha_1 \circ \alpha_4$.
We need to fix a “standard sequence”.

Length

If we fix a “standard” length, a unit of measure, then all objects will be expressed as multiples of that unit.

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Scales

Ratio Scales

All proportional transformations (of the type αx) will deliver the same information. We only fix the unit of measure.

Interval Scales

All affine transformations (of the type $\alpha x + \beta$) will deliver the same information. Besides the unit of measure we fix an origin.

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More complicated

Consider a Multi-attribute space:

$$X = X_1 \times \cdot X_n$$

to each attribute we associate an ordered set of values:

$$X_j = \langle x_j^1 \cdots x_j^m \rangle$$

An object x will thus be a vector:

$$x = \langle x_1^l \cdots x_n^k \rangle$$

Generally speaking ...

$$x \succeq y$$

$$\Longleftrightarrow$$

$$\langle x_1^l \cdots x_n^k \rangle \succeq \langle y_1^i \cdots y_n^j \rangle$$

$$\Longleftrightarrow$$

$$\Phi(f(x_1^l \cdots x_n^k), f(y_1^i \cdots y_n^j)) \geq 0$$

What that means?

	Commuting Time	Clients Exposure	Services	Size	Costs
a	20	70	C	500	1500

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For what value of δ_1 a and a_1 are indifferent?

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a	20	70	C	500	1500
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For what value of δ_2 a_1 and a_2 are indifferent?

What that means?

	Commuting Time	Clients Exposure	Services	Size	Costs
a	20	70	C	500	1500
a_1	25	80	C	500	1500
a_2	25	80	C	700	1700

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The trade-offs introduced with δ_1 and δ_2 allow to get $a \sim a_1 \sim a_2$

What do we get?

Standard Sequences

Length: objects having the same length allow to define a unit of length;

Value: objects being indifferent can be considered as having the same value and thus allow to define a “unit of value”.

Remark 1: indifference is obtained through trade-offs.
Remark 2: separability among attributes is the minimum requirement.

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The easy case

IF

1. restricted solvability holds;
2. at least three attributes are essential;
3. $\succeq$ is a weak order satisfying the Archimedean condition $\forall x, y \in \mathbb{R}, \exists n \in \mathbb{N} : ny > x$.

THEN

$$x \succeq y \Leftrightarrow \sum_j u_j(x) \geq \sum_j u_j(y)$$

General Usage

The above ideas apply also in

- Economics (comparison of bundle of goods);
- Decision under uncertainty (comparing consequences under multiple states of the nature);
- Inter-temporal decision (comparing consequences on several time instances);
- Social Fairness (comparing welfare distributions among individuals).

Is optimisation rational?

General Setting

$$\min F(x)$$
$$x \in S \subseteq K^n$$

where:
- x is a vector of variables
- S is the feasible space
- K^n is a vector space, $(\mathbb{Z}^n, \mathbb{R}^n, \{0,1\}^n)$.
- $F : S \mapsto R^m$

Well known specific cases: m=1

- $F(x)$ is linear, S is a n-dimensional polytope: linear programming
  $\min cx$, $Ax \leq b$, $x \geq 0$.
- S is a n-dimensional polytope, but $F : \mathbb{R}^{n+m} \mapsto \mathbb{R}$: constraint satisfaction
  $\min y$, $Ax + y \leq b$, $x, y \geq 0$.
- $F(x)$ is linear, $S \subseteq \{0, 1\}^n$: combinatorial optimisation.
- $F(x)$ is convex and S is a convex subset of $\mathbb{R}^n$: convex programming

More challenging cases

- Instead of $\min_{x \in S} F(x)$ we get $\sup_{x \in S} x$. Practically we only have a preference relation on S (and thus we cannot define any "quantitative" function of x).

NB

The problem becomes tricky when the preference relation cannot be represented explicitly (for instance when $S \subseteq \{0, 1\}^n$)

- $m > 1$. We get

$$F(x) = \langle f_1(x) \cdots f_n(x) \rangle$$

Practically a problem mathematically undefinable ...

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- Instead of $\min_{x \in S} F(x)$ we get $\sup_{x \in S} x$. Practically we only have a preference relation on S (and thus we cannot define any "quantitative" function of x).

NB

The problem becomes tricky when the preference relation cannot be represented explicitly (for instance when $S \subseteq \{0, 1\}^n$)

- $m > 1$. We get

$$F(x) = \langle f_1(x) \cdots f_n(x) \rangle$$

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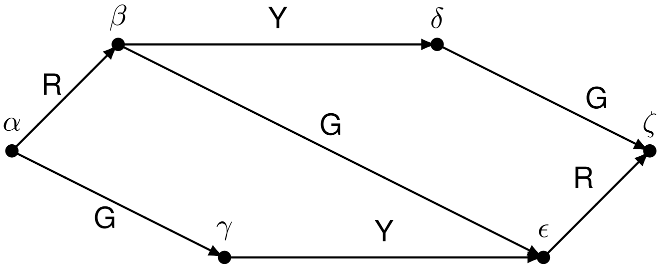
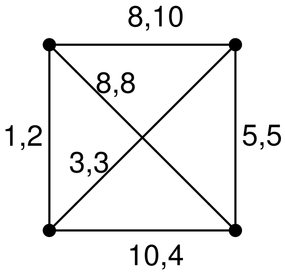
Example



- R: dangerous
- Y: fairly dangerous
- G: not dangerous

Which is the safest path in the network?

Example 2



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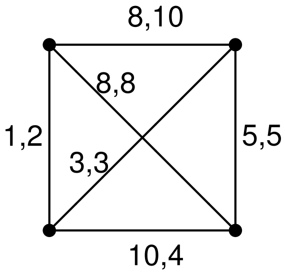

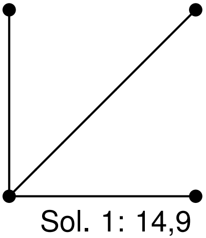

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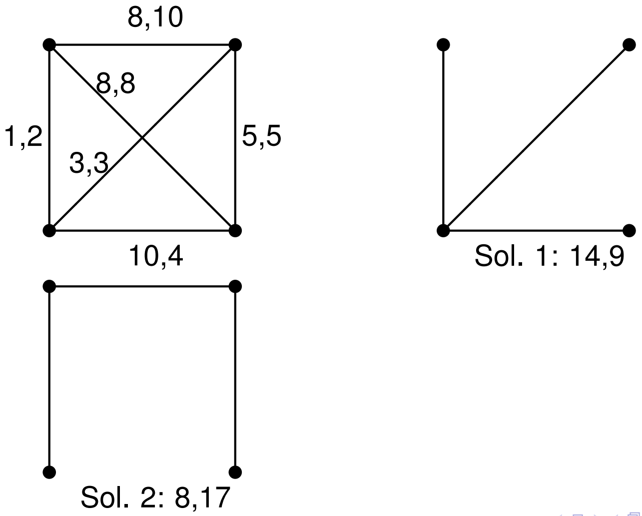

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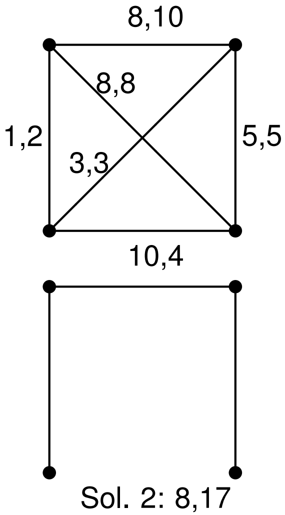

First Idea

Find all “non dominated solutions” and then explore it appropriately (straightforward or interactively) until a compromise is established. BUT:

- The set of all such solutions can be extremely large, an explicit enumeration becoming often intractable.
- Depending on the shape and size of the size of the “non dominated solutions”, exploring the set can be intractable.

Further Ideas

- Instead trying to construct the whole set of "non dominated solutions", concentrate the search of the compromise in an "interesting" subset. Problem: how to define and describe the "interesting" subset?
- Aggregate the different objective functions (the criteria) to a single one and then apply mathematical programming:
  - scalarising functions;
  - distances.

Scalarising Functions

We transform

$$\min_{x \in S} [f_1(x) \cdots f_n(x)]$$

to the problem

$$\min_{x \in S} \lambda^T F(x)$$

λ being a vector of trade-offs. Problem: how we get them?

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Add Constraints

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Tchebychev Distances

We transform

$$\min_{x \in S}[f_1(x) \cdots f_n(x)]$$

to the problem

$$\min_{x \in S}[\max_{j=1 \cdots m} w_j(f_j(x) - y_j)]$$

w_j being a vector of trade-offs. Problem: how we get them? y_j being a special point (for instance the ideal point) in the objective space

Combinatorial Optimisation

What happens if we have to choose among collections of objects, while we only know the values of the objects?

1. Knapsack Problems
2. Network Problems
3. Assignment Problems

Combinatorial Optimisation

What happens if we have to choose among collections of objects, while we only know the values of the objects?

1. Knapsack Problems
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What if there are interactions (positive or negative synergies) among the chosen objects?

The Choquet Integral

Given a set N, a function $v : 2^N \mapsto [0, 1]$ such that:
- $v(\emptyset) = 0$, $V(N) = 1$
- $\forall A, B \in 2^N : A \subseteq B \;\; v(A) \leq v(B)$
is a capacity

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We use the Choquet Integral

$$C_v(f) = \sum_{i=1}^{n} [f(\sigma(i)) - f(\sigma(i-1))] v(A_i)$$

which is a measure of a capacity where:
- f represent the value function for x;
- $\sigma(i)$ represents a permutation on A_i such that:
$f(\sigma(0)) = 0$ and $f(\sigma(1)) \leq \cdots f(\sigma(n))$

Several Models Together

The Choquet Integral contains as special cases several models:

- The weighted sum.
- The k-additive model
- The expected utility model.
- The Ordered Weighted Average model
- The Rank Depending Utility model

Lessons Learned

- Optimising is not necessary “rational”.
- Optimising multiple objectives simultaneously is ill defined and “difficult”.
- We can improve using preference based models.
- We need to (and we can) take into account the possible interactions among objects or among objectives.
- We need “good” approximation algorithms.

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Borda vs. Condorcet

Four candidates and seven examiners with the following preferences.

	a	b	c	d	e	f	g
A	1	2	4	1	2	4	1
B	2	3	1	2	3	1	2
C	3	1	3	3	1	2	3
D	4	4	2	4	4	3	4

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B(x)
15
14
16
25

The Borda count gives B>A>C>D

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If D is not there then A>B>C, instead of B>A>C

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The Condorcet principle gives A>B>C>A !!!!

Arrow's Theorem

Given N rational voters over a set of more than 3 candidates can we found a social choice procedure resulting in a social complete order of the candidates such that it respects the following axioms?

- Universality: the method should be able to deal with any configuration of ordered lists;
- Unanimity: the method should respect a unanimous preference of the voters;
- Independence: the comparison of two candidates should be based only on their respective standings in the ordered lists of the voters.

YES!

There is only one solution: the dictator!!

If we add no-dictatorship among the axioms then there is no solution.

Gibbard-Satterthwaite's Theorem

When the number of candidates is larger than two, there exists no aggregation method satisfying simultaneously the properties of universal domain, non-manipulability and non-dictatorship.

Why MCDA is not Social Choice?

Social Choice	MCDA
Total Orders	Any type of order
Equal importance of voters	Variable importance of criteria
As many voters as necessary	Few coherent criteria
No prior information	Existing prior information

General idea: coalitions

Given a set A and a set of $\succeq_i$ binary relations on A (the criteria) we define:

$$x \succeq y \Leftrightarrow C^+(x,y) \trianglerighteq C^+(y,x) \text{ and } C^-(x,y) \trianglelefteq C^-(y,x)$$

where:
- $C^+(x,y)$: “importance” of the coalition of criteria supporting x wrt to y.
- $C^-(x,y)$: “importance” of the coalition of criteria against x wrt to y.

Specific case 1

Additive Positive Importance

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$$C^{+}(x,y) = \sum_{j \in J^{\pm}} w_j^{+}$$

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w_j^{+}: “positive importance” of criterion i
$J^{\pm} = \{h_j : x \succeq_j y\}$

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Where "positive importance" comes from?

Specific case 2

Max Negative Importance

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$$C^{-}(x,y) = \max_{j \in J^{-}} w_j^{-}$$

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$J^{-} = \{h_j : v_j(x,y)\}$

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Max Negative Importance

$$C^-(x,y) = \max_{j \in J^-} w_j^-$$

where:
w_j^-: "negative importance" of criterion i
$J^- = \{h_j : v_j(x,y)\}$

Then we can fix a veto threshold γ and have

$$x \succeq^- y \Leftrightarrow C^-(x,y) \geq \gamma$$

Where "negative importance" comes from?

Example

The United Nations Security Council

Positive Importance

15 members each having the same positive importance $w_j^+ = \frac{1}{15}$, $\delta = \frac{9}{15}$.

Negative Importance

10 members with 0 negative importance and 5 (the permanent members) with $w_j^- = 1$, $\gamma = 1$.

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Outranking Principle

$$x \succeq y \Leftrightarrow x \succeq^{+} y \text{ and } \neg(x \succeq^{-} y)$$

Thus:

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NB

The relation $\succeq$ is not an ordering relation. Specific algorithms are used in order to move from $\succeq$ to an ordering relation $\succcurlyeq$

What is importance?

Where w_j^+, w_j^- and δ come from?

Further preferential information is necessary, usually under form of multi-attribute comparisons. That will provide information about the decisive coalitions.

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Example

Given a set of criteria and a set of decisive coalitions ($J^{\pm}$) we can solve:

$$\max \delta$$
$$\text{subject to}$$
$$\sum_{j \in J^{\pm}} w_j \geq \delta$$
$$\sum_j w_j = 1$$

Lessons Learned

- We can use social choice inspired procedures for more general decision making processes.
- Care should be taken to model the majority (possibly the minority) principle to be used. The key issue here is the concept of “decisive coalition”.
- We need to “learn” about decisive coalitions, since it is unlike that this information is available. Problem of learning procedures.
- The above information is not always intuitive. However, the intuitive idea of importance contains several cognitive biases.
- A social choice inspired procedure will not deliver automatically an ordering. We need further algorithms (graph theory).

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What is Probability?

A measure of uncertainty, of likelihood ...
of subjective belief ...

Consider a set N and a function $p : 2^N \mapsto [0, 1]$ such that:
- $p(\emptyset) = 0$;
- $A \subseteq A \subseteq N$, then $p(A) \leq p(B)$;
- $A \subseteq A \subseteq N$, $A \cap B = \emptyset$, then $p(A \cup b) = p(A) + p(B)$;

Then the function p is a "probability".

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Then the function p is a “probability”.

A probability is an additive measure of capacity

Decision under risk

	θ_1	θ_2	states of the nature	θ_n
a_1	x_{11}	x_{12}	$\cdots$	x_{1n}
a_2	x_{21}	x_{22}	$\cdots$	x_{2n}
actions	$\cdots$	$\cdots$	outcomes	$\cdots$
a_m	x_{m1}	x_{m2}	$\cdots$	x_{mn}
	p_1	p_2	probabilities	p_n

$$\langle p_1, x_{i1}; p_2, x_{i2}; \cdots p_n, x_{in} \rangle$$

is a lottery associated to action a_i.

Expected Utility

Von Neuman and Morgenstern Axioms

- A1 There is a weak order $\succeq$ on the set of outcomes X.
- A2 If $x \succ y$ implies that $\langle x, P; y, 1-P\rangle \succ \langle x, Q; y, 1-Q\rangle$, then $P > Q$.
- A3 $\langle x, P; \langle y, Q; z, 1-Q\rangle, 1-P\rangle \sim \langle x, P; y, Q(1-P); z, (1-Q)(1-P)\rangle$
- A4 If $x \succ y \succ z$ then $\exists P$ such that $\langle y, 1\rangle \sim \langle x, P; z, 1-P\rangle$

If the above axioms are true then

$$\exists v : X \mapsto \mathbb{R} : \ a_l \succeq a_k \ \Leftrightarrow \sum_{j=1}^{n} p_j x_{lj} \geq \sum_{j=1}^{n} p_j x_{kj}$$

Problems

Expected Utility Theory is falsifiable under several points of view

- Gains and losses induce a different behaviour of the decision maker when facing a decision under risk.
- Independence is easily falsifiable.
- Rank depending utilities.
- What happens if probabilities are “unknown”?
- Where probabilities come from?
- What is subjective probability?

Probability does not exist!!!

Ramsey and De Finetti

If the option of α *for certain is indifferent with that of* β *if p is true and* γ *if p is false, we can define the subject's degree of belief in p as the ratio of the difference between* α *and* γ *to that between* β *and* γ (Ramsey, 1930, see also De Finetti, 1936).

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Savage will give a normative characterisation of von Neuman's expected utility, but the axioms remain empirically falsifiable

Idea: Qualitative Decision Theory

Consider a capacity (a measure of uncertainty) for which $v(A \cup b) = max(v(A), v(B))$. We call that a *possibility distribution* π.

Under conditions relaxing Savages's axioms we get (Dubois, Prade, 1995)

$$v_*(a_i) \; \min_{\theta_j} \max(u(\pi(\theta_j)), v(x_{ij})))$$

The above formula extends the min-max decision rule. It also "replaces" Bayesian conditioning with a form of non-monotonic inference.

However

- The possibility equivalent of the min-max rule requires that possibilities and utilities are commensurable (which can be arguable although reasonable).
- Working generally will just ordinal preferences and likelihoods results either in overconfident rules or in indecisive ones.

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- Working generally will just ordinal preferences and likelihoods results either in overconfident rules or in indecisive ones.

The reason is that as soon as we lose the "density" of the structure imposed by Savage we fell in the case of "social choice" aggregations and thus Arrow's theorem holds.

Another option

- Cumulative Prospect Theory (Kahneman and Tversky).
- Rank Dependant Utility (Quiggin).
- Choquet Expected Utility (Schmeidler).

Once again we are within a framework we already saw in measurement theory:

$$a_k \succeq a_l \Leftrightarrow \Phi(u_j(x_{kj}), u_j(x_{lj})) \geq 0$$

Replacing “attributes” with “states of the nature” we come back to conjoint measurement theory.

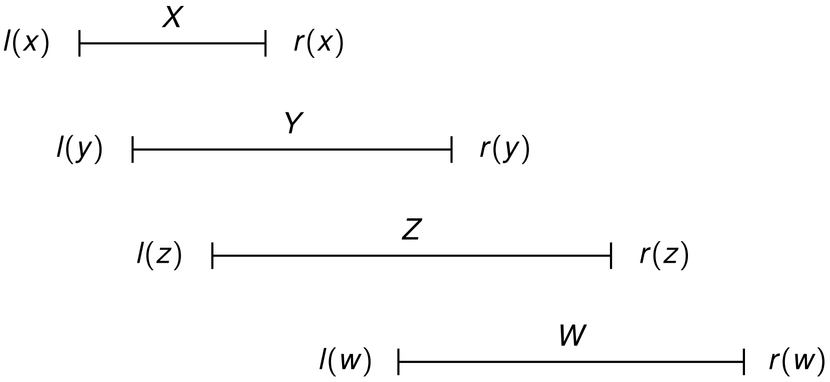
Different option: Intervals

Consider a set $A = \{x, y, z \cdots\}$ and an attribute h.

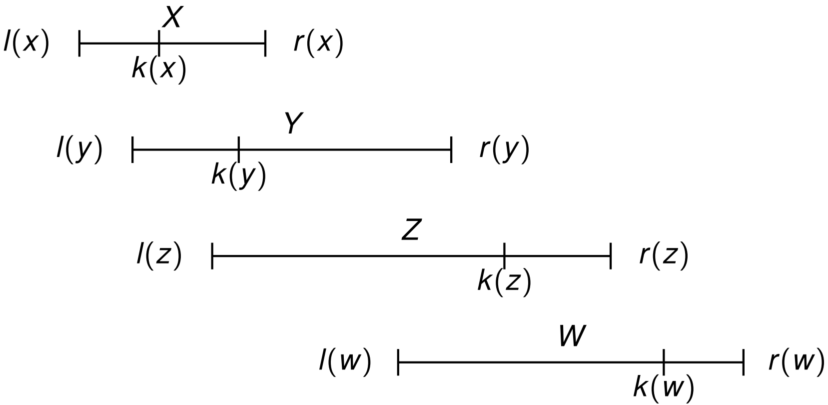
Assume that $h(x) \subseteq \mathbb{R}$ and denote $\min(h(x)) = l(x)$ and $\max(h(x)) = r(x)$.

To each element of A we associate an interval $[l(x), r(x)]$ which contains the “real” value of x, but who is presently unknown. Possibly we may consider intermediate points of the interval: $k(x)$.

An example: 2 points



An example: 3 points



$\langle P, I \rangle$ Models

$$
\begin{aligned}
P_1(x,y) &\Leftrightarrow l(x) > r(y) \\
P_2(x,y) &\Leftrightarrow l(x) > k(y) \\
P_3(x,y) &\Leftrightarrow l(x) > l(y \wedge r(x) > r(y)) \\
I(x,y) &\Leftrightarrow \text{the rest}
\end{aligned}
$$

$\langle P, Q, I \rangle$ Models

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Representation Theorems

Interval Orders:	$P_1 P_1 I \subseteq P_1$
$\langle P, Q, I \rangle$ Interval Orders:	$I = I_l \cup I_l^{-1} \cup I_o$
	$(P_1 \cup Q_1 \cup I_l) P_1 \subseteq P_1$
	$P_1 (P_1 \cup Q_1 \cup I_l^{-1}) \subseteq P_1$
	$(P_1 \cup Q_1 \cup I_l) Q_1 \subseteq P_1 \cup Q_1 \cup I_l$
	$Q_1 (P_1 \cup Q_1 \cup I_l^{-1}) \subseteq P_1 \cup Q_1 \cup I_l^{-1}$
Double Threshold Orders	$Q_2 I Q_2 \subseteq P_2 \cup Q_2$
	$Q_2 I P_2 \subseteq P_2$
	$P_2 I P_2 \subseteq P_2$
	$P_2 Q_2^{-1} P_2 \subseteq P_2$

Issues Raising

- Create a general framework for intervals comparison.
- Introduce the general idea using positive and negative reasons when comparing intervals.
- Generalise the concept of interval considering the “length” and the “mass” associated to an interval.
- How to aggregate such ordering relations?

Lessons Learned

- Uncertainty can be represented in several different ways.
- A purely ordinal representation of preferences and likelihoods is possible, but not that operational.
- There are strong similarities (in the good and the bad sense) between multiple attributes and multiple states of the nature.
- Conjoint measurement theory can be used also in this case as a general theoretical framework.
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- Uncertainty can be represented in several different ways.
- A purely ordinal representation of preferences and likelihoods is possible, but not that operational.
- There are strong similarities (in the good and the bad sense) between multiple attributes and multiple states of the nature.
- Conjoint measurement theory can be used also in this case as a general theoretical framework.
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What happens in reality?

Real Case Studies in MCDA, forthcoming with Springer

- Siting a university kindergarten in Madrid
- A multi-criteria decision support system for hazardous material transport in Milan
- An MCDA approach for evaluating H2 storage systems for future vehicles
- A multi-criteria application concerning sewers rehabilitation
- Multicriteria Evaluation-Based Framework for Composite Web Service Selection
- A multicriteria model for evaluating confort in TGV
- Coupling GIS and Multi-criteria Modeling to support post-accident nuclear risk evaluation
- Choosing a cooling system in a power plant: an ex post analysis
- Decision support for the choice of road pavement and surfacing
- An MCDA approach for personal financial planning
- Criteria evaluations by means of fuzzy logic, Case study: The cost of a nuclear-fuel repository
- Road Maintenance Decisions in Madagascar
- A Multicriteria Approach to Bank Rating
- Participative and multicriteria localisation of a wind farm in Corsica

Hazardous Material Transportation

Courtesy of A. Colorni and A. Lué

There are two problems:

1. Define a route for a shipment at a given time slot;
2. Manage the shipments (lot sizing and scheduling).

In this presentation we are going to talk about the first problem.

Representation

Topology

Given an area Y of the city interested by the shipment of a hazardous material m, we represent this area as a directed graph $G_Y = \langle N, A \rangle$ where N represent the road intersections and A the road segments between the intersections.

Information

For each arc of the graph G_Y we can retrieve the following information: population, infrastructures (power distribution, telecom network, railways, pipelines etc.), natural elements (water resources, green areas, cultural heritage) and critical elements (potential targets of a terrorist attack).

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Problem Formulation

Given a shipment of a hazardous material m at a time slot f, define a route within G_Y minimising the risk

Four risks are considered:

$R^{POP}(h, f, m)$: risk for the population.

$R^{INF}(f, m)$: risk for the infrastructures.

$R^{NAT}(f, m)$: risk for the nature.

$R^{CRI}(h, f, m)$: risk for critical installations.

where h represents the population in area Y.

Risk for the Population

$$\sigma_{x,y}^{POP} = p_x \times POP_{y,f} \times e^{-\phi[L(x,y)]^{\eta}}$$

where:
- p_x: probability that an accident occurs at point x
- $L(x, y)$: distance between x and y
- $POP_{y,f}$: population at point y at time slot f
- ϕ, η: parameters depending on the type of shipment m

$$R^{POP}(h, f, m) = \sum_x \sum_y \sigma_{x,y}^{POP}$$

Resolution

A multi-objective shortest path problem. In this precise case the method adopted consists in scalarising the different risks.

$$
\begin{aligned}
C(h,f,m) = \quad & \alpha\bar{T}(h,f) + \beta\bar{R}^{POP}(h,f,m) + \\
& \gamma\bar{R}^{INF}(f,m) + \delta\bar{R}^{NAT}(f,m) + \epsilon\bar{R}^{CRI}(h,f,m)
\end{aligned}
$$

where
$\bar{R}^J$ is the normalised risk for J
$\bar{T}$ is the normalised cost
$\alpha, \beta, \gamma, \delta, \epsilon$ are the trade-offs among risks and costs.

The Niguarda Hospital

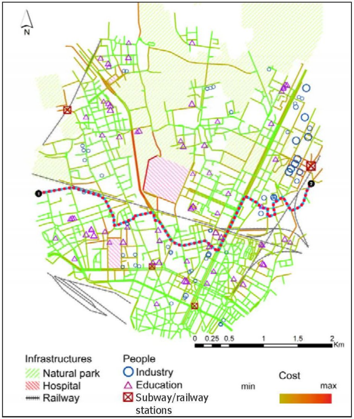


Figure 7. The result of the toolbox developed in ArcGIS is a map where a cost is calculated for each link as a function of travel time and risk. The figure shows an example of path at minimum cost (dotted line), which avoids population and infrastructures.

Road Maintenance in Madagascar

AGETIPA is “Maitre d’Ouvrage” for the public works on behalf of the Minister of Infrastructure in Madagascar. In that capacity they have to establish a medium term plan for the maintenance of the rural road network of the country. For this purpose they manage a grant (from the BEI) to be used for the covering (possibly partially) the cost of the maintenance programme.

This has also been seen an an opportunity to enhance AGETIPA’s capacity in OR and project management.

Road Maintenance in Madagascar: Who?

The Actors

- The State
- The Management Agency (the client)
- The local Mayors
- Other local actors
- The Funding Agencies

Road Maintenance in Madagascar: Why?

The Concerns

- Improve Road Maintenance
  - Network Connections
  - Accessibility
  - Local Economy
  - Robustness against climate
- Improve Local Involvement
- Justify wrt to Funding Agencies

Road Maintenance in Madagascar: What?

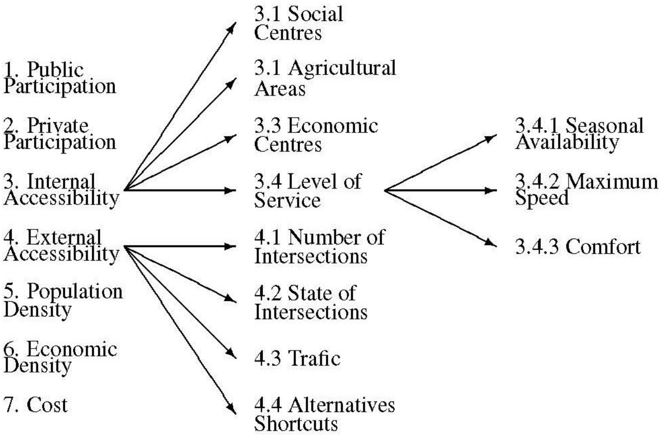
The Problem Formulation

Γ: Given a set of possible road maintenance projects choose the ones to fund within the current budget so that strategic planning priorities are met and local involvement is pursued.

The Evaluation Model

$\mathcal{M}$: Assess the projects submitted to the Agency in order to classify them in “accepted”, “negotiable” and “rejected”. Use the criteria and the “negotiable” class in order to pursue the local involvement strategy.

Attributes and Criteria Structure



Example

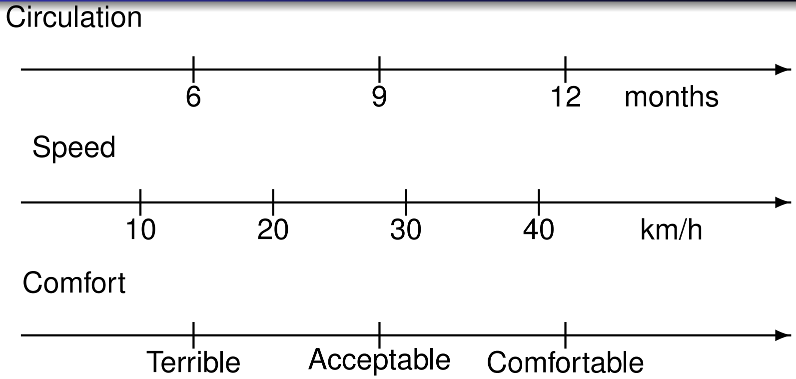
You have a number of rural road maintenance projects and you want to assess the "service level" of each road concerned by the projects.

Such an assessment takes into account:

- how many months the road is accessible;
- what is the maximum speed you can use safely;
- how confortable is the road at that speed.

The "service level" can be 0, 1, 2, 3, 0 being the worst and 3 being the best.

How do you do that?



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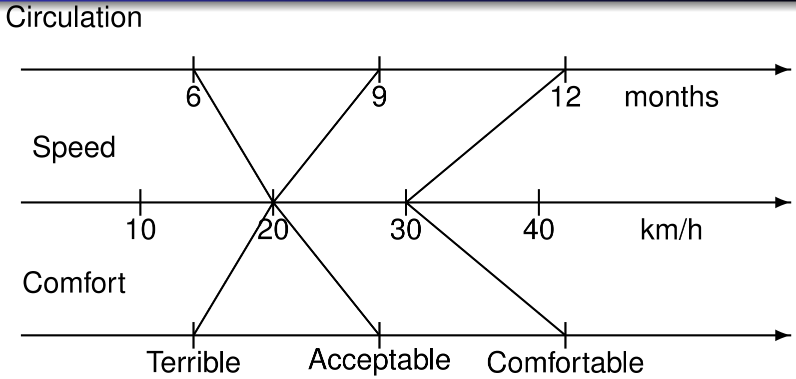
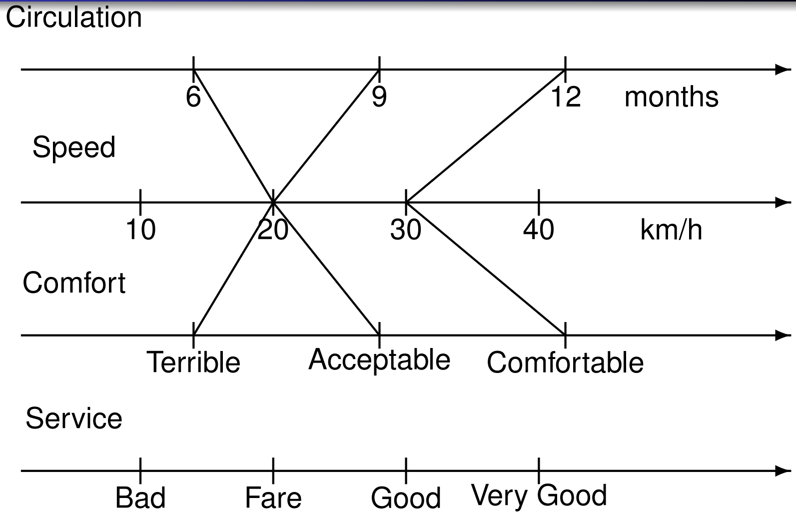


How do you do that?



Results

The method has been tested in a pilot study in an area near Antananarivo. 4 real projects already submitted for funding were considered as alternatives. Information has been retrieved from AGETIPA's databases on all relevant dimensions of the model.

The projects have been compared to the profiles of the categories of “acceptable”, “negotiable”, “to reject” and then classified to one among these classes.

Details

Feedback

2 years of experience

Two years later in a feedback meeting the model has been adapted to a number of remarks from the field experience without changing the approach. The method is routinely used in order to fund rural road maintenance projects.

Further applications

AGETIPA is further investing today in increasing its capacity in decision support.

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Further applications

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Where do we go know?

Context

- Large information sets.
- Conflicting opinions, criteria and scenario.
- Strong interdependencies.
- Strong uncertainties and ambiguous information.
- Rigour and usefulness.
- Models of Rationality

Where do we go know?

What is the problem?

- Formal methods for aiding formulating a problem.
- Explanations and Justifications.
- Argumentation

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What is the problem?

- Formal methods for aiding formulating a problem.
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How do we learn what we model?

- Learning Algorithms.
- Constructive Learning.
- Update and Revision.

Where do we go know?

Extended Preference Models

- Positive and Negative Reasons.
- From Preference Statements to Models.
- Conjoint Measurement Theory.

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- From Preference Statements to Models.
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Extended Optimisation

- Search Algorithms using “preferences”.
- Compromise Programming.
- Robustness

Where do we go know?

Computational Social Choice

- Aggregating preferences, votes, judgements, beliefs ...
- Fairness, Efficiency and Reliability.
- Information Fusion.

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New Models of Uncertainty

- Extreme Risks.
- Beyond Probability.
- Soft Computing.

Where do we go know?

Better Decision Aiding Processes

Where do we go know?

Better Decision Aiding Processes

Better Decision Processes

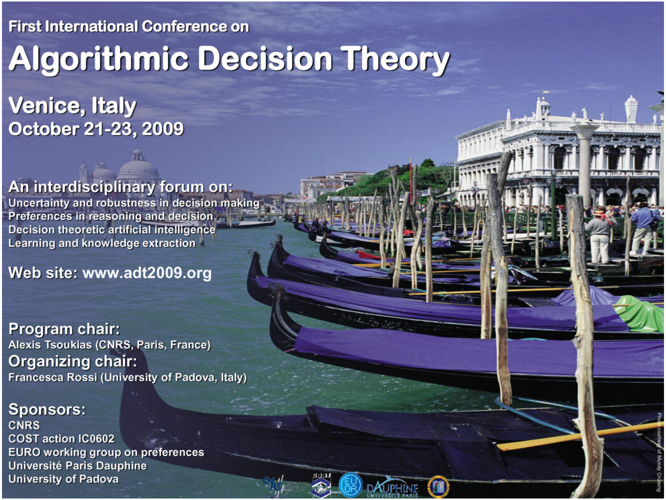
Where do we go know?

Better Decision Aiding Processes

Better Decision Processes

Better Decisions

DO NOT MISS


First International Conference on
Algorithmic Decision Theory
Venice, Italy
October 21-23, 2009
An interdisciplinary forum on:
Uncertainty and robustness in decision making
Preferences in reasoning and decision
Decision theoretic artificial intelligence
Learning and knowledge extraction
Web site: www.adt2009.org
Program chair:
Alexis Tsoukias (CNRS, Paris, France)
Organizing chair:
Francesca Rossi (University of Padova, Italy)
Sponsors:
CNRS
COST action IC0602
EURO working group on preferences
Université Paris Dauphine
University of Padova
Photo courtesy of Mundy Cruising

Resources

- http://www.algodec.org
- http://www.inescc.pt/∼ewgmcda
- http://decision-analysis.society.informs.org/
- http://www.mcdmsociety.org/
- http://www.euro-online.org
- http://www.informs.org

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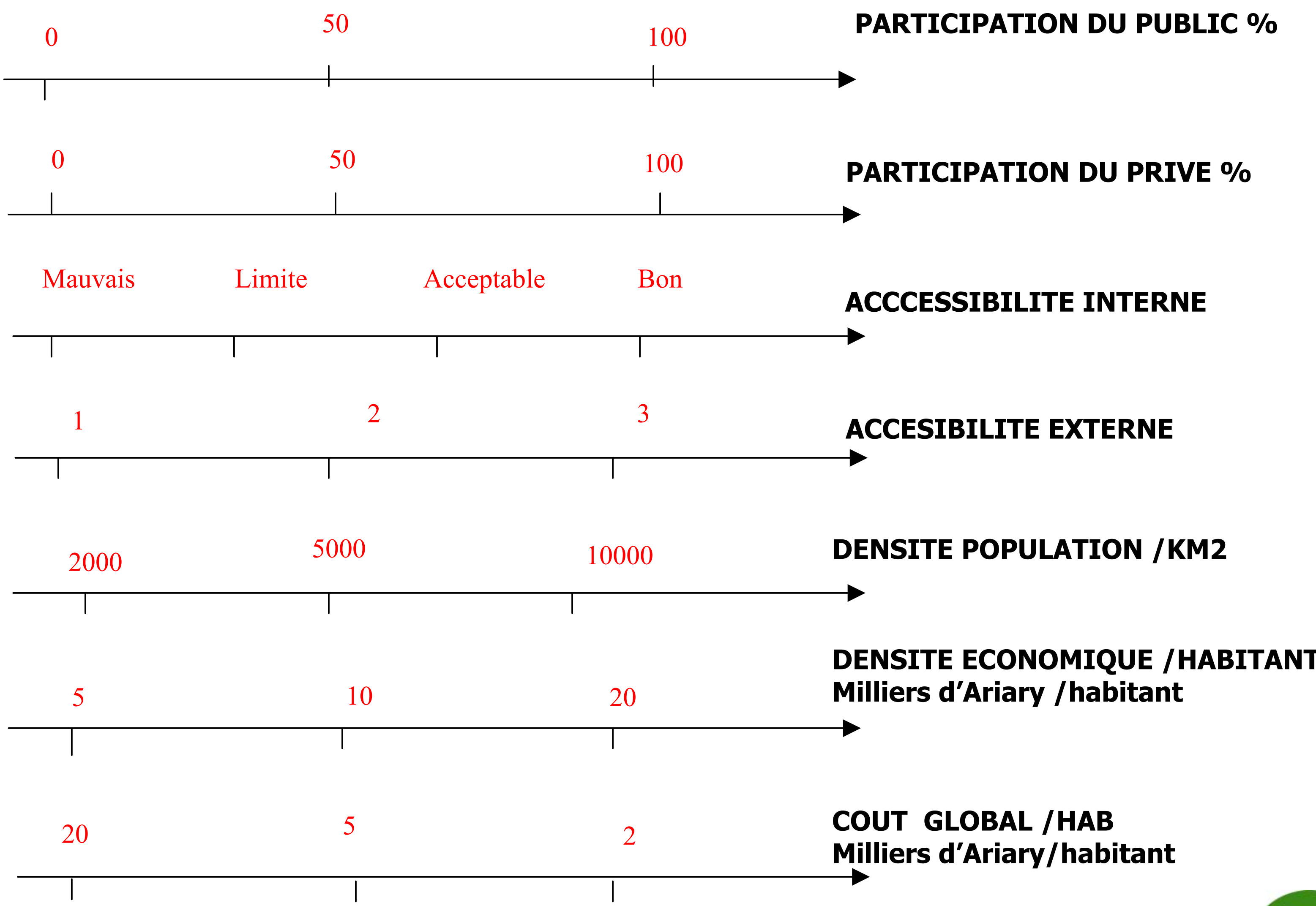
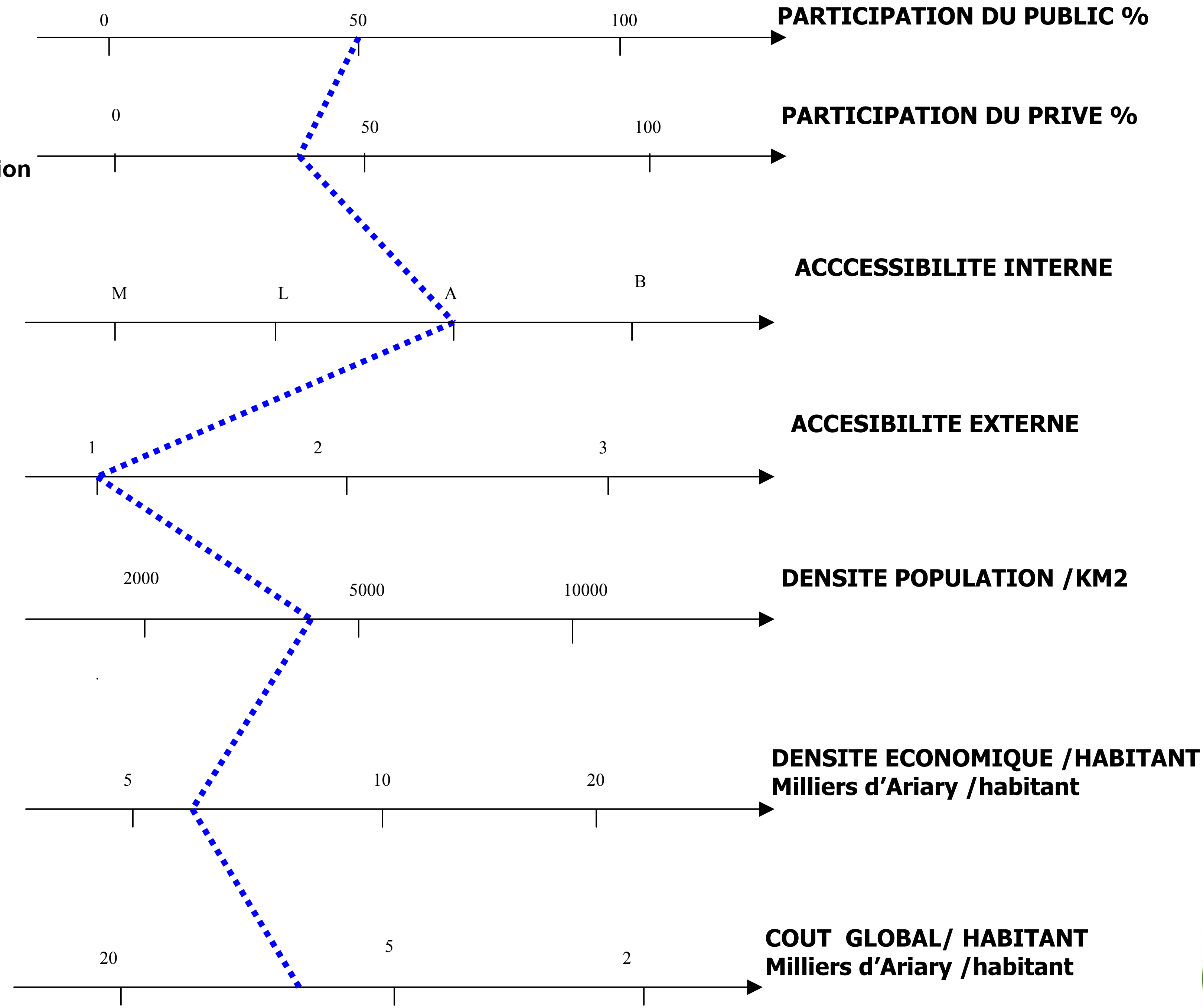
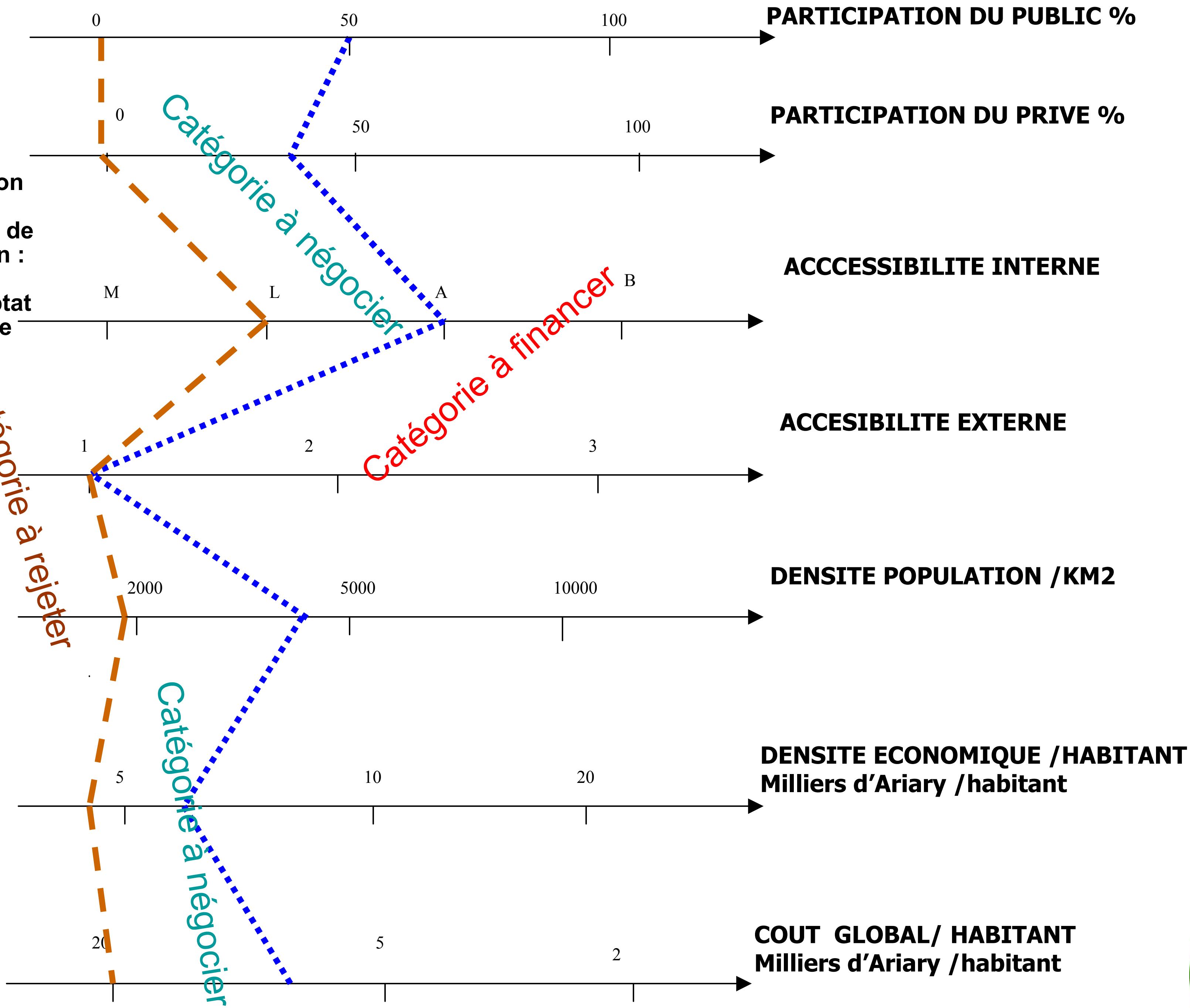
6. Schématisation des échelles de valeur



7. Définition des profils

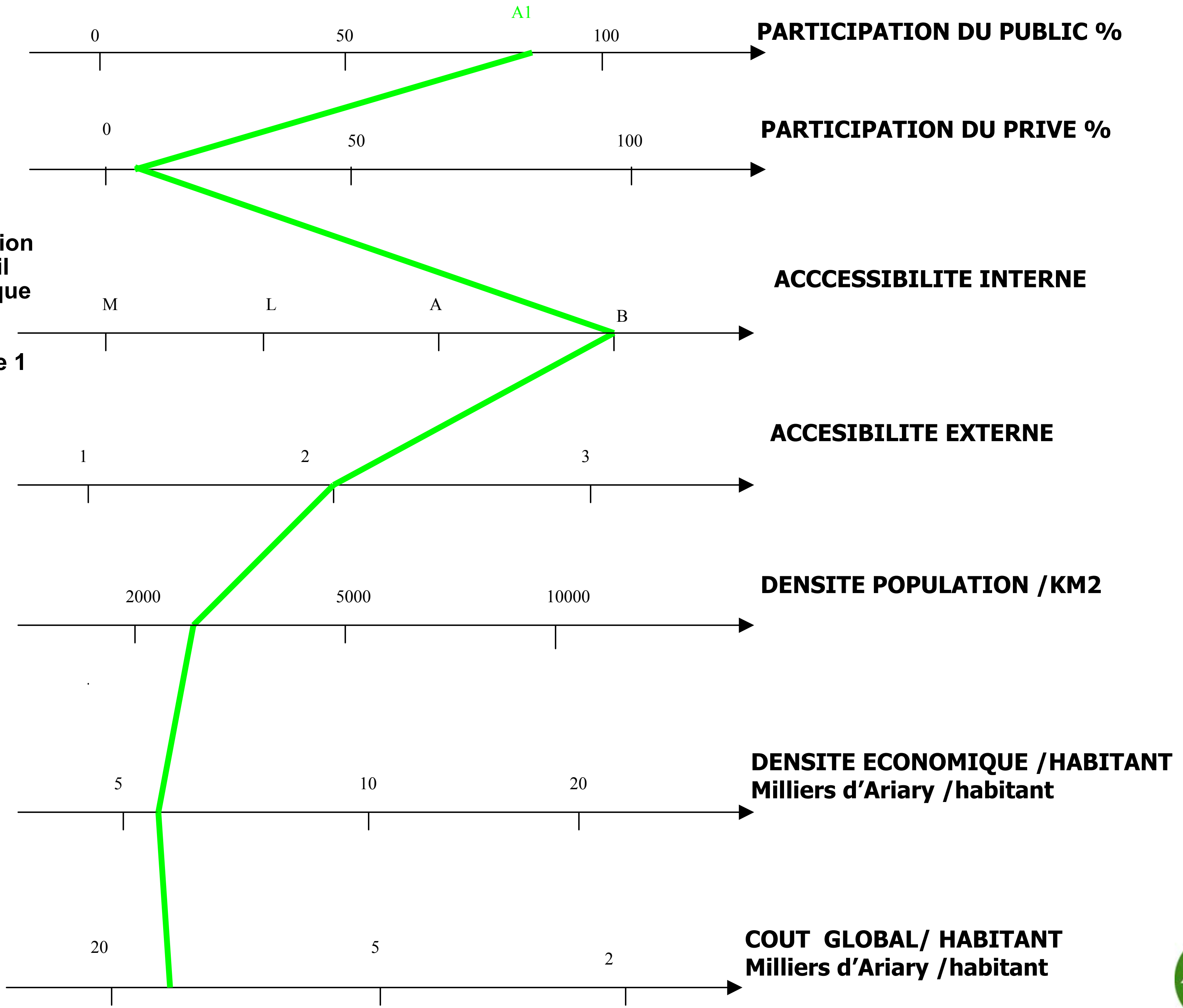


8. Définition des classes de décision : seuil d'acceptation et de rejet



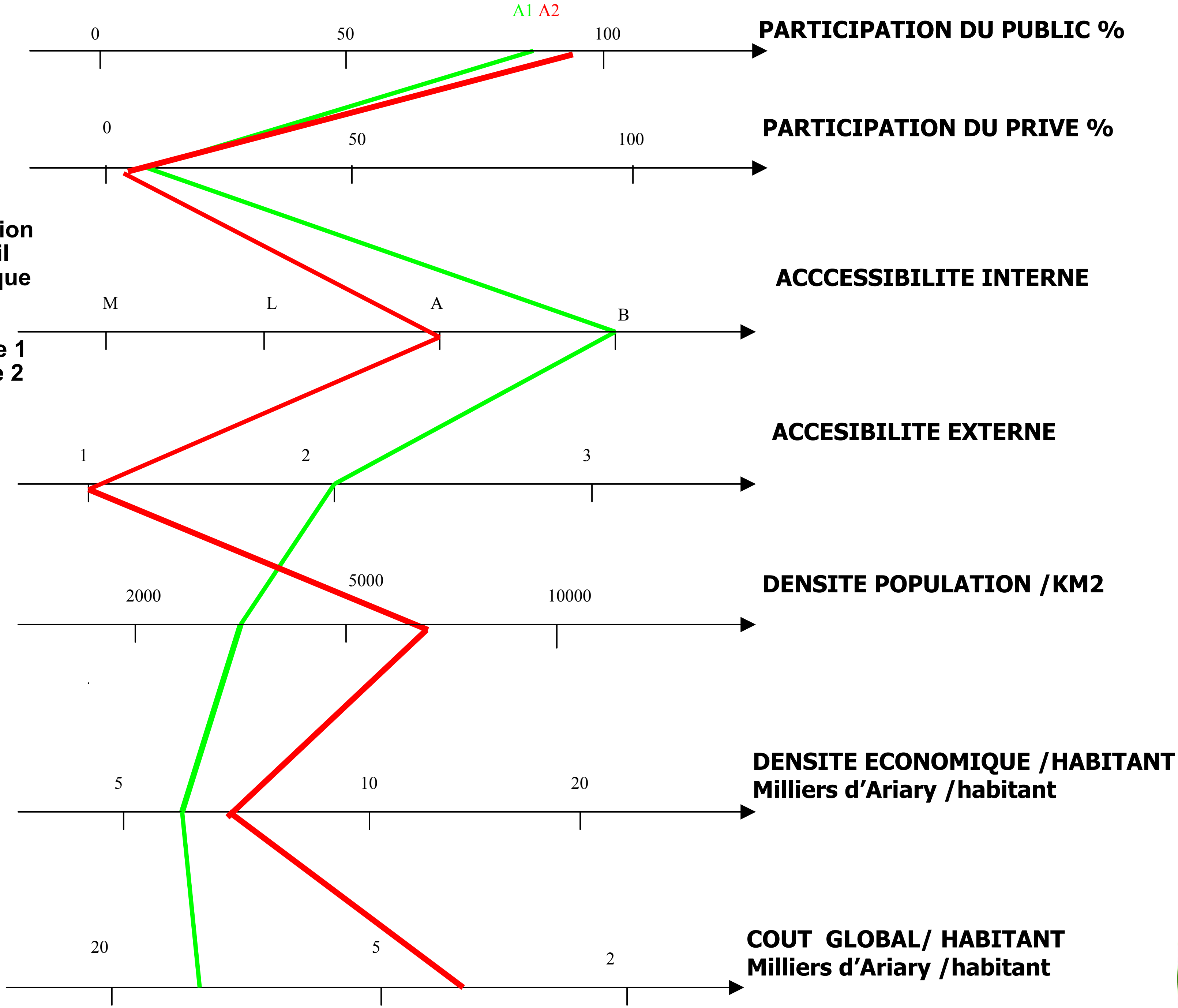
9. Évaluation du profil de chaque axe:

Cas Axe 1



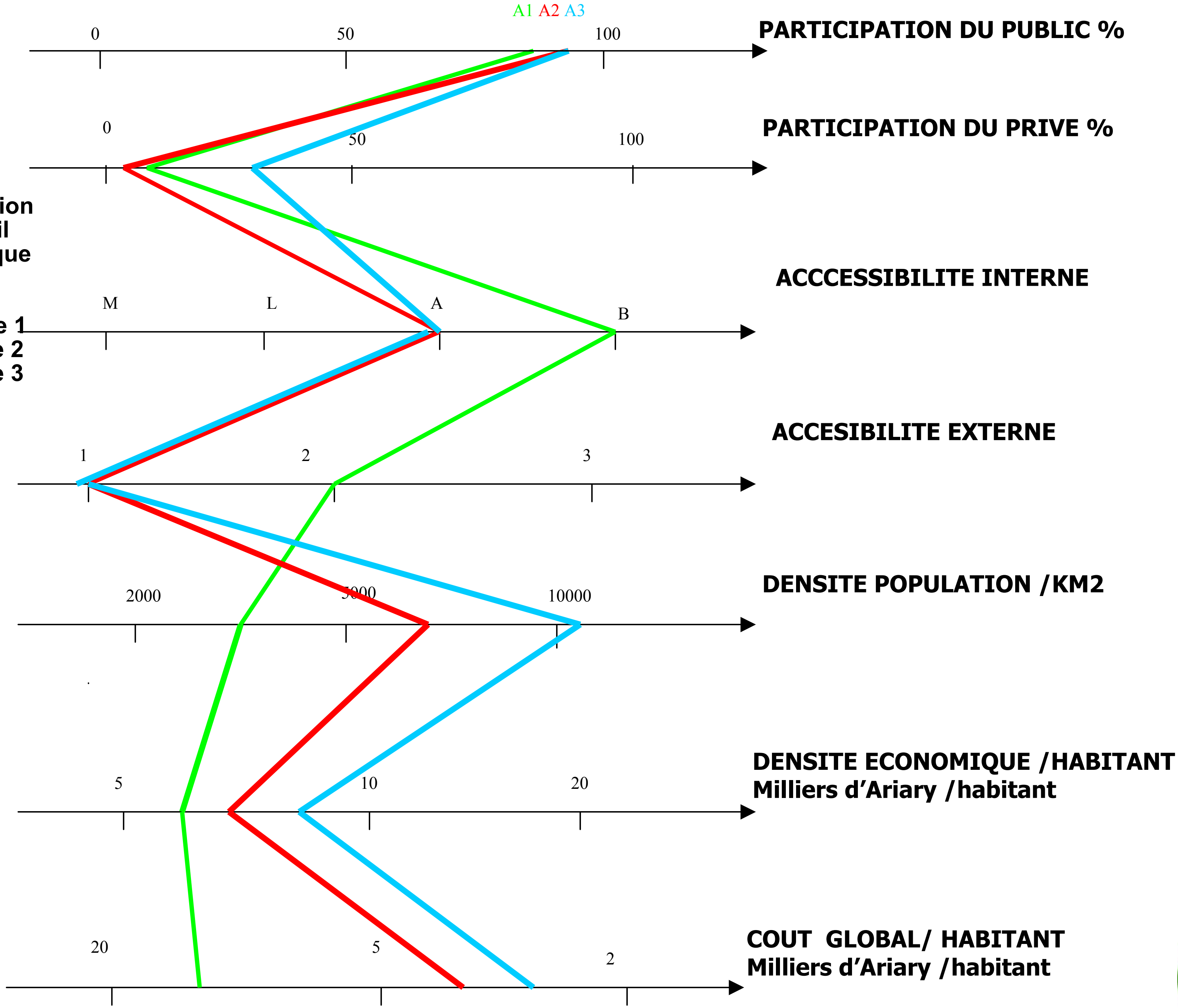
10. Évaluation du profil de chaque axe:

Cas Axe 1
Axe 2



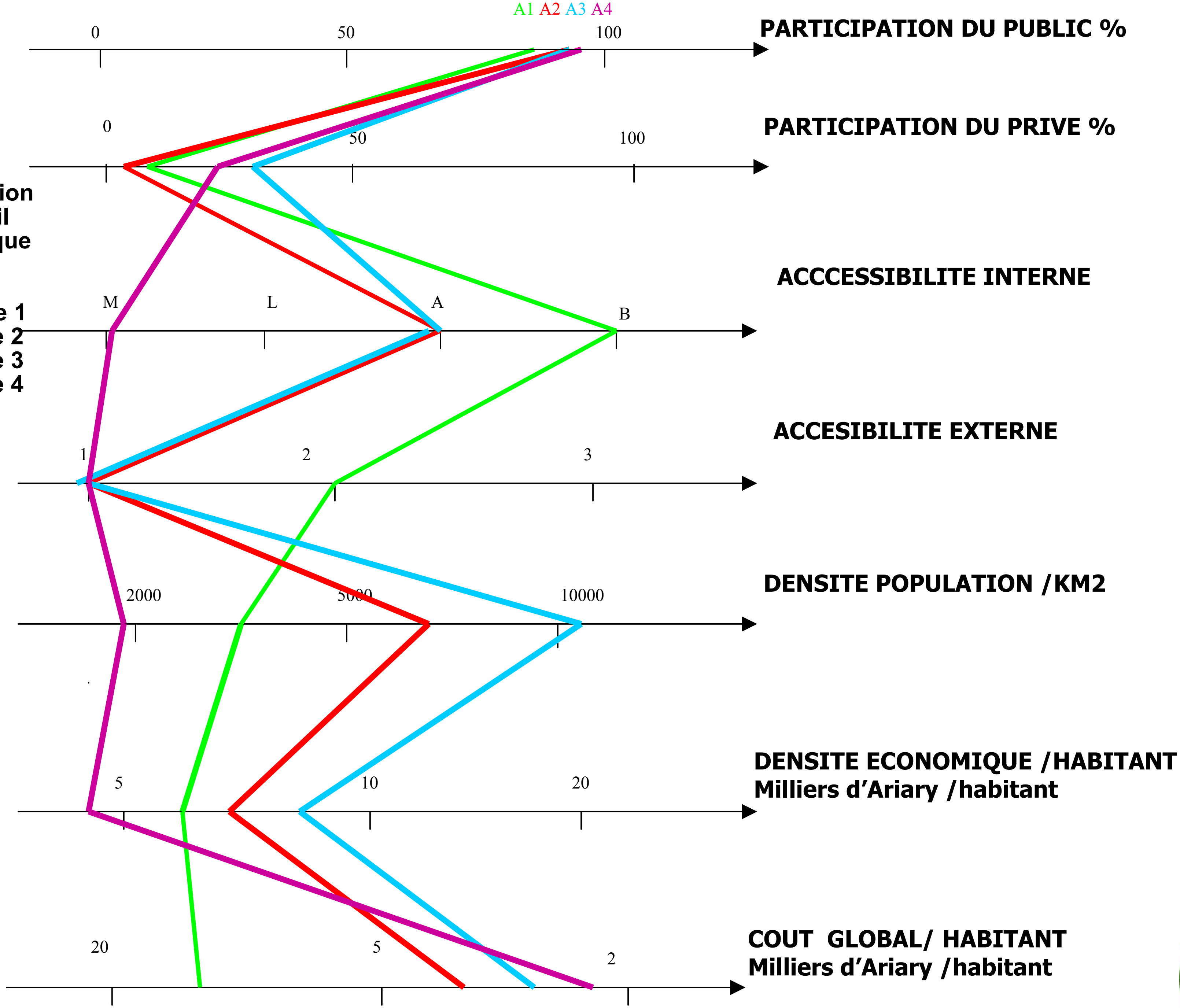
11. Évaluation du profil de chaque axe:

Cas Axe 1
Axe 2
Axe 3

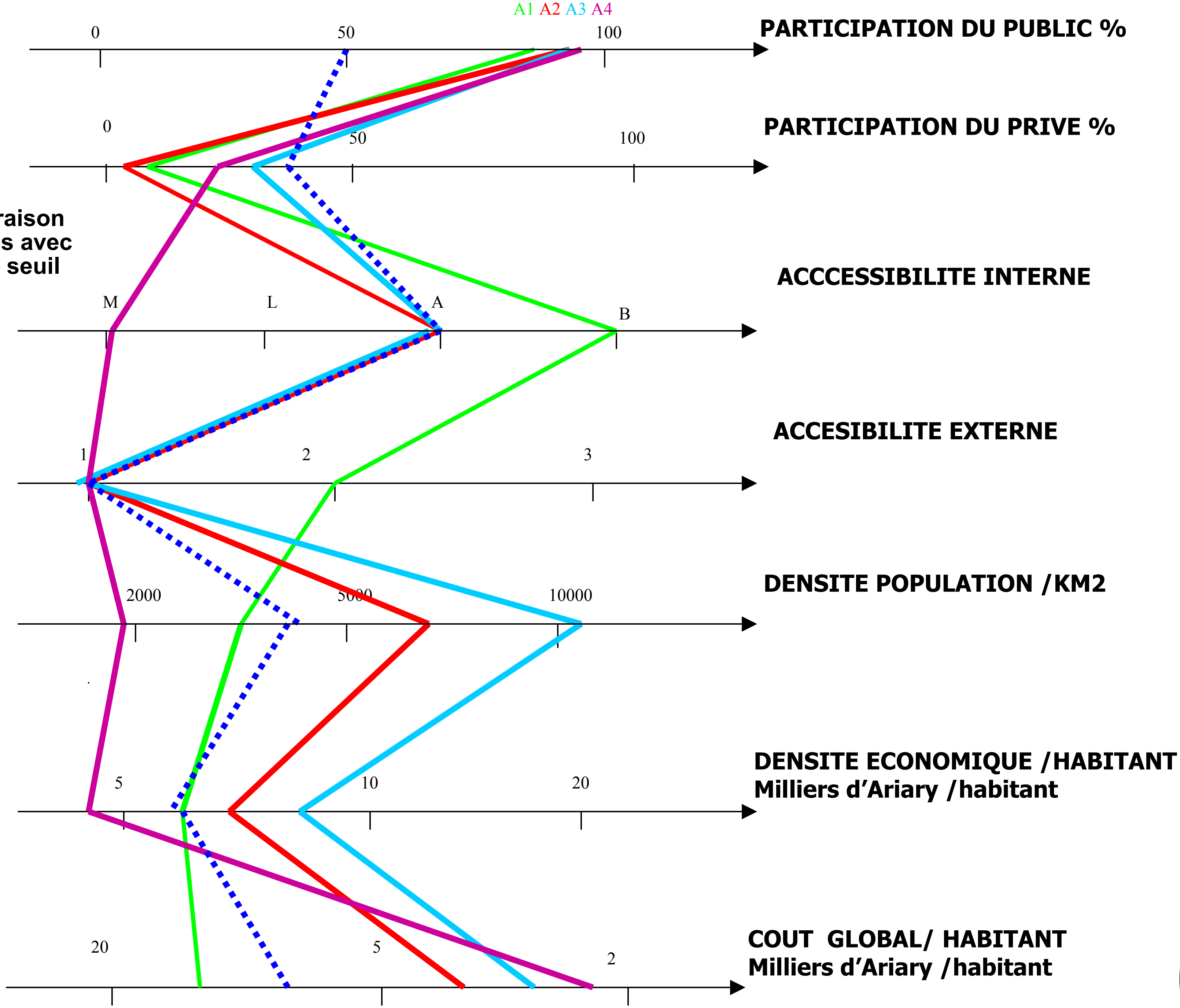


12. Évaluation du profil de chaque axe:

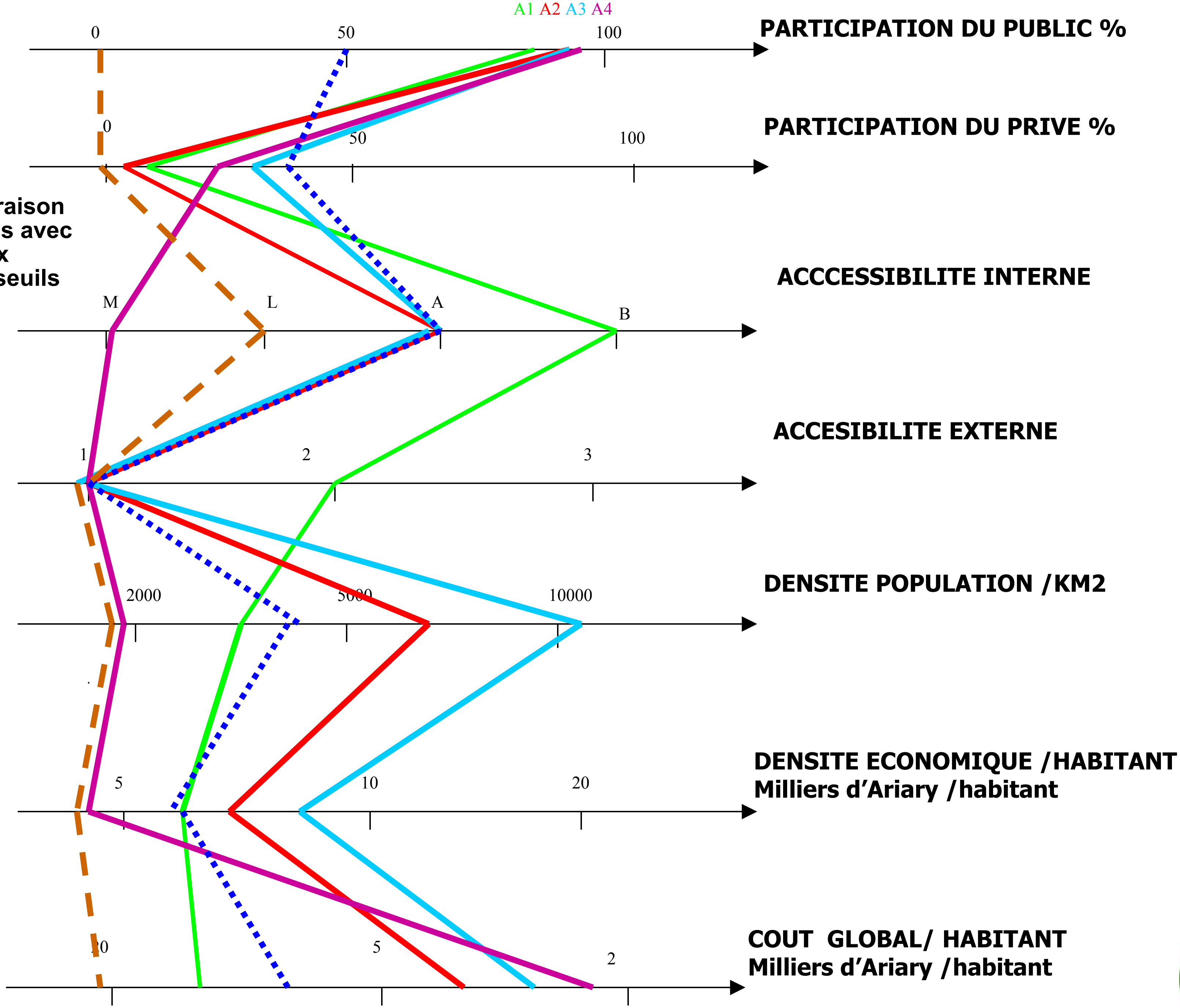
Cas Axe 1
Axe 2
Axe 3
Axe 4



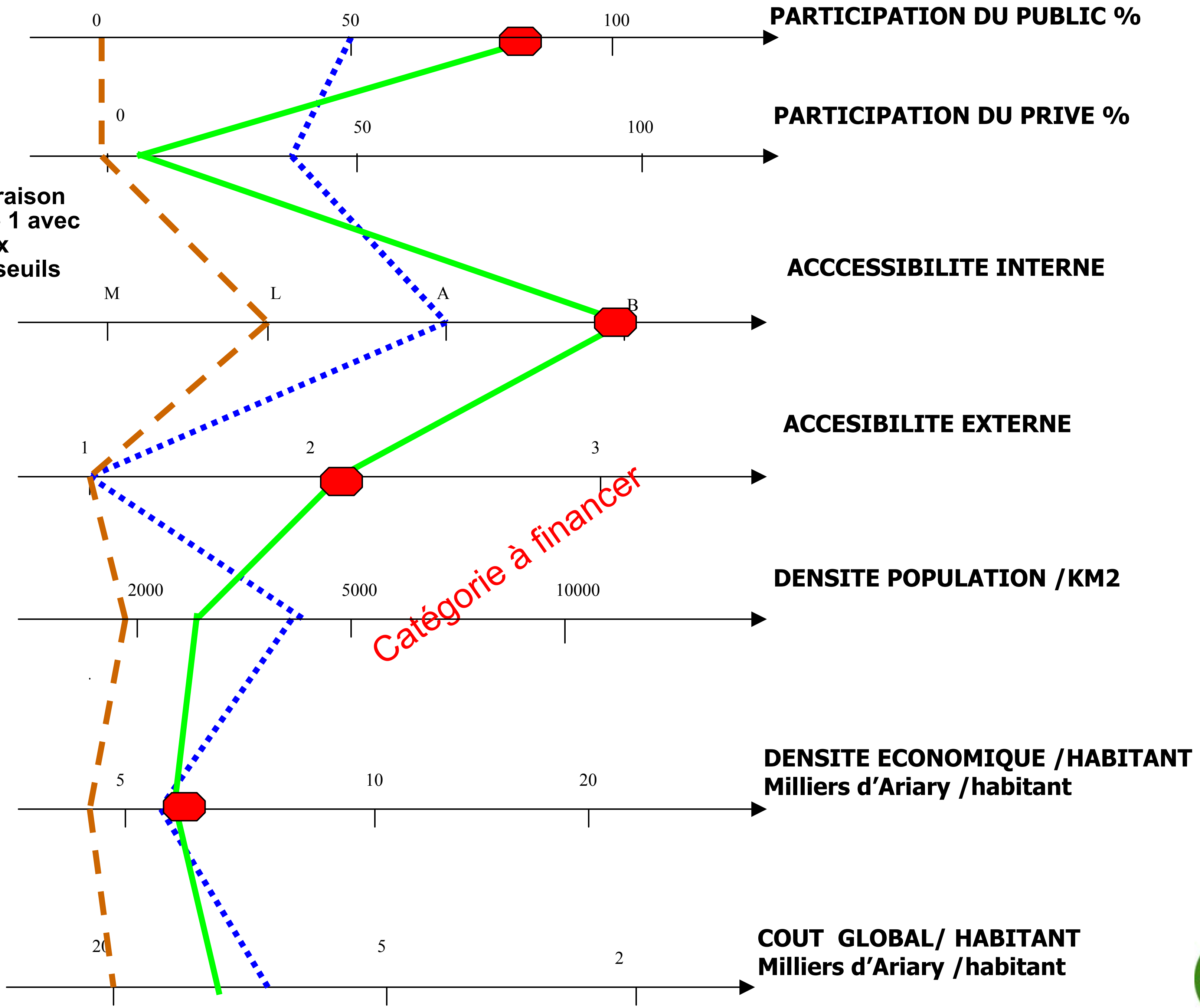
13. Comparaison des axes avec le profil seuil



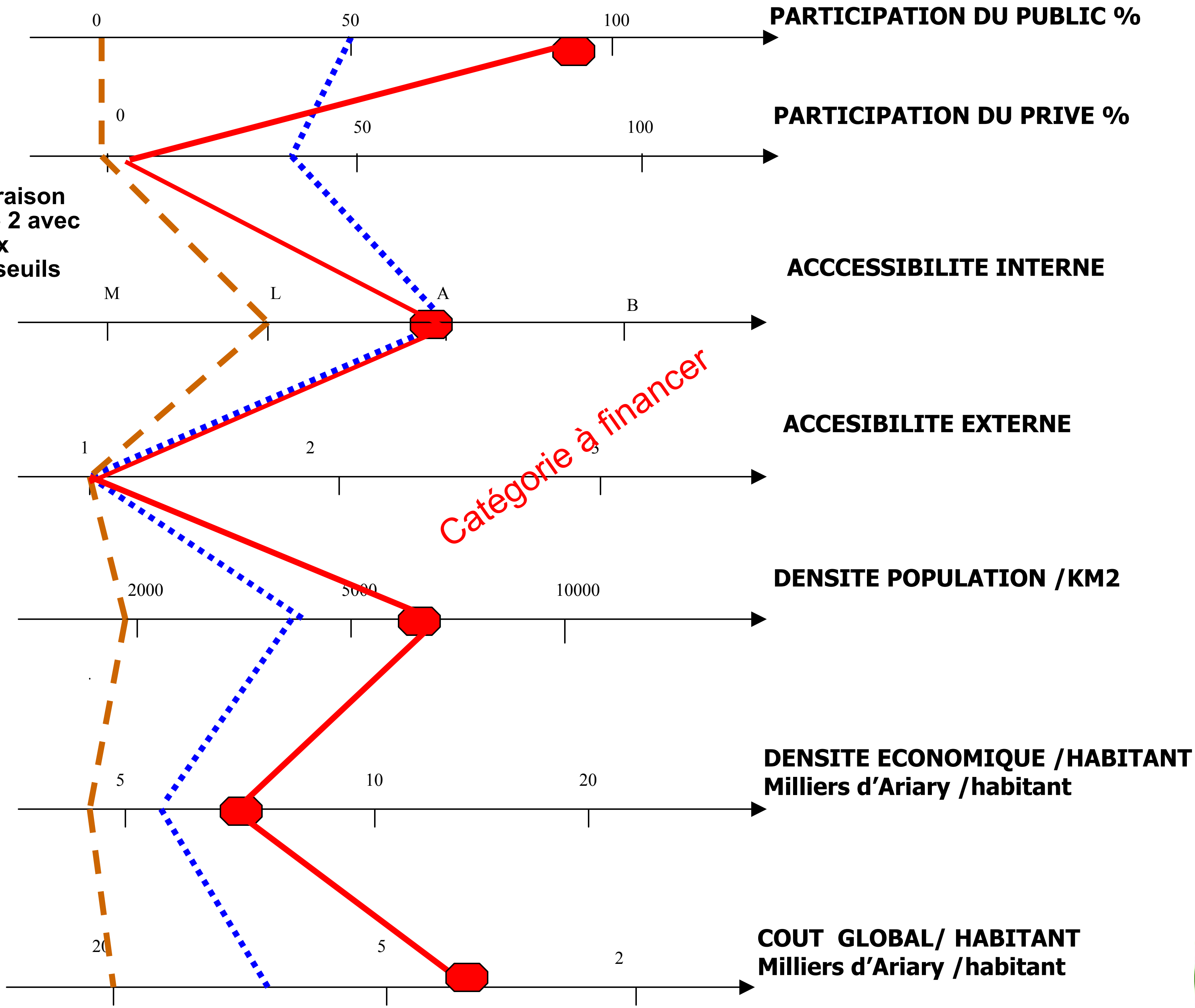
14. Comparaison des axes avec les deux profils seuils

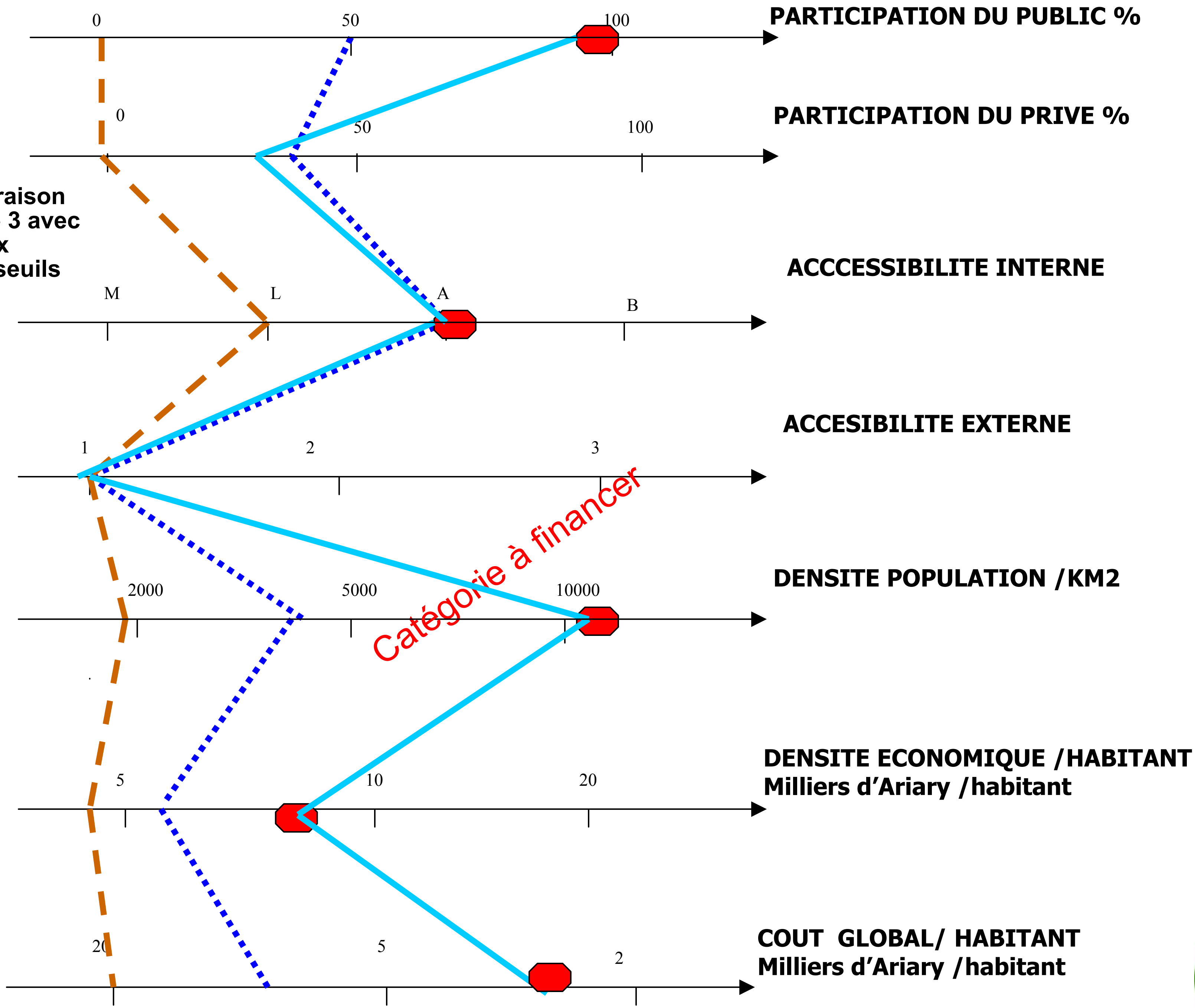


15. Comparaison de l'axe 1 avec les deux profils seuils



15. Comparaison de l'axe 2 avec les deux profils seuils





15. Comparaison de l'axe 3 avec les deux profils seuils

15. Comparaison de l'axe 4 avec les deux profils seuils