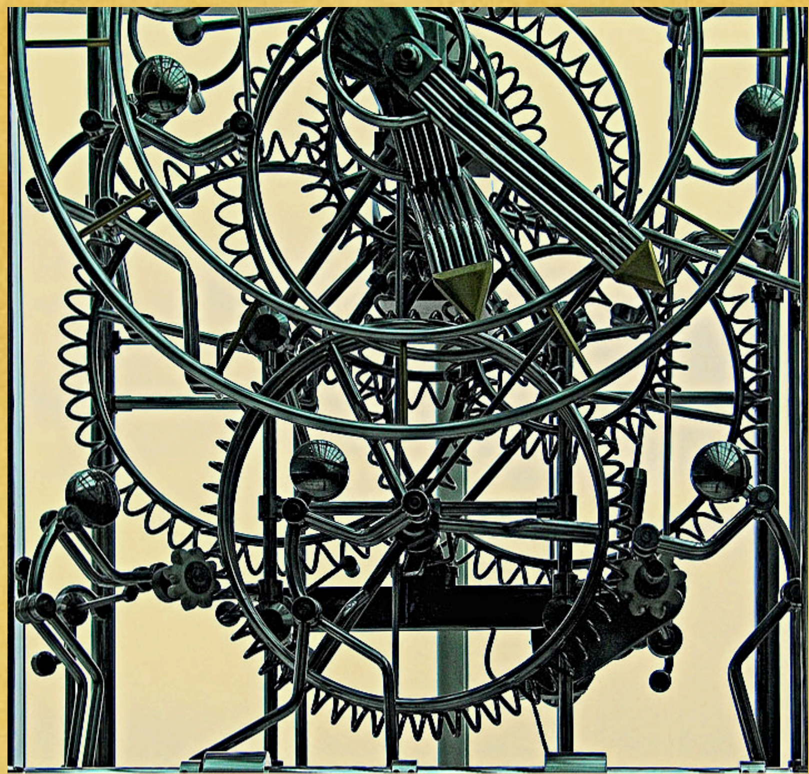
Empirical Methods in Algorithmic Decision Theory

Toby Walsh
NICTA and UNSW
Sydney, Australia

ADT, Venice, Oct 2009

Why? An example

- Perhaps complexity can be an escape from Gibbard-Sattertwhaite's theorem?
  - Some voting rules (2nd order Copeland, STV, ..) are NP-hard to manipulate
    [Bartholdi, Tovey & Trick 89, Bartholdi & Orlin 91



Why? An example

- Is complexity an escape from Gibbard-Sattertwhaite's theorem?
  - But recent results raise doubts
  - NP-hardness is only worst case
  - Manipulation might be easy in practice

Why? An example

- Is complexity an escape from Gibbard-Sattertwhaite's theorem?
  - For instance, with STV, either a coalition is too small O(√n) to change result or so large Ω(√n) they easily can [Xia & Conitzer 08]
    - Only question is when coalition is Θ(√n)?
    - We can run some experiments!

Outline

- Where are the *hard* problems?
  - Phase transition behaviour
  - Sharp/smooth transitions
- A *close* look at the data
  - Early mistakes
  - Discrepancy search & restarts
- What makes problems *hard*?
  - Backbones, backdoors, …
  - Structure

Where are the really *hard* problems?

- Influential IJCAI-91 paper by Cheeseman, Kanefsky & Taylor
  - 857 citations on Google Scholar

"… for many NP problems one or more "order parameters" can be defined, and hard instances occur around particular critical values of these order parameters … the critical value separates overconstrained from underconstrained …"

Where are the really *hard* problems?

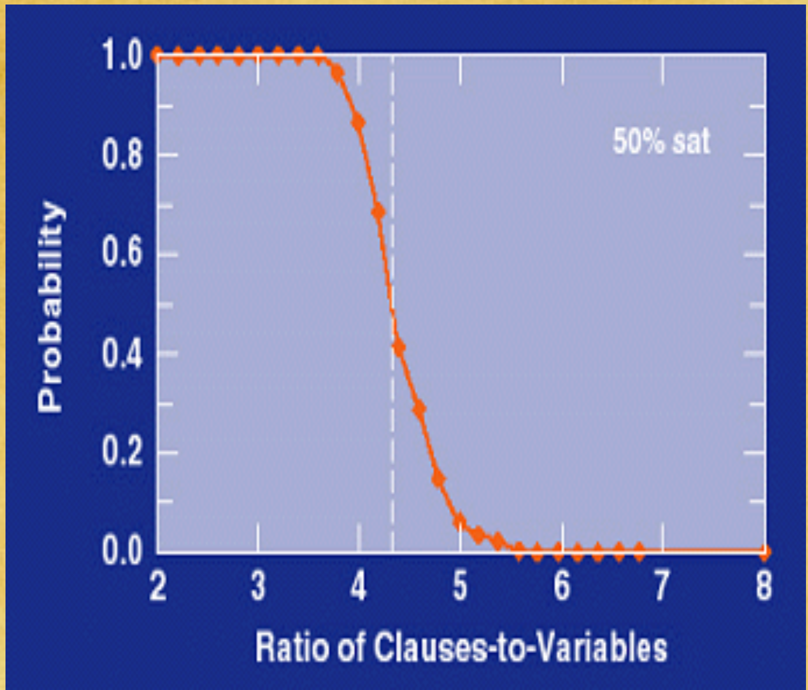
- Influential IJCAI-91 paper by Cheeseman, Kanefsky & Taylor
  - 857 citations on Google Scholar

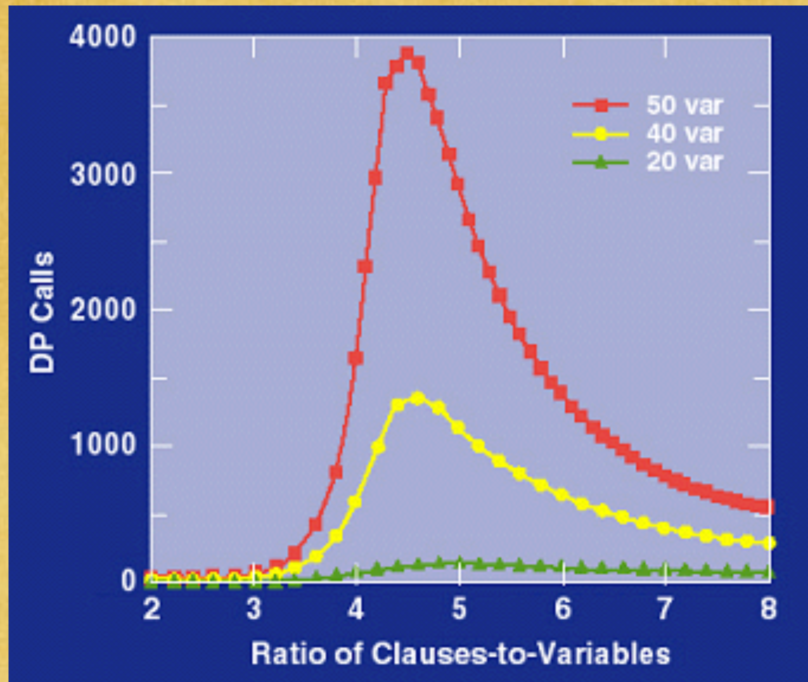
“We expect that in future computer scientists will produce "phase diagrams" for particular problem domains to aid in hard problem identification”

Where are the really *hard* problems?

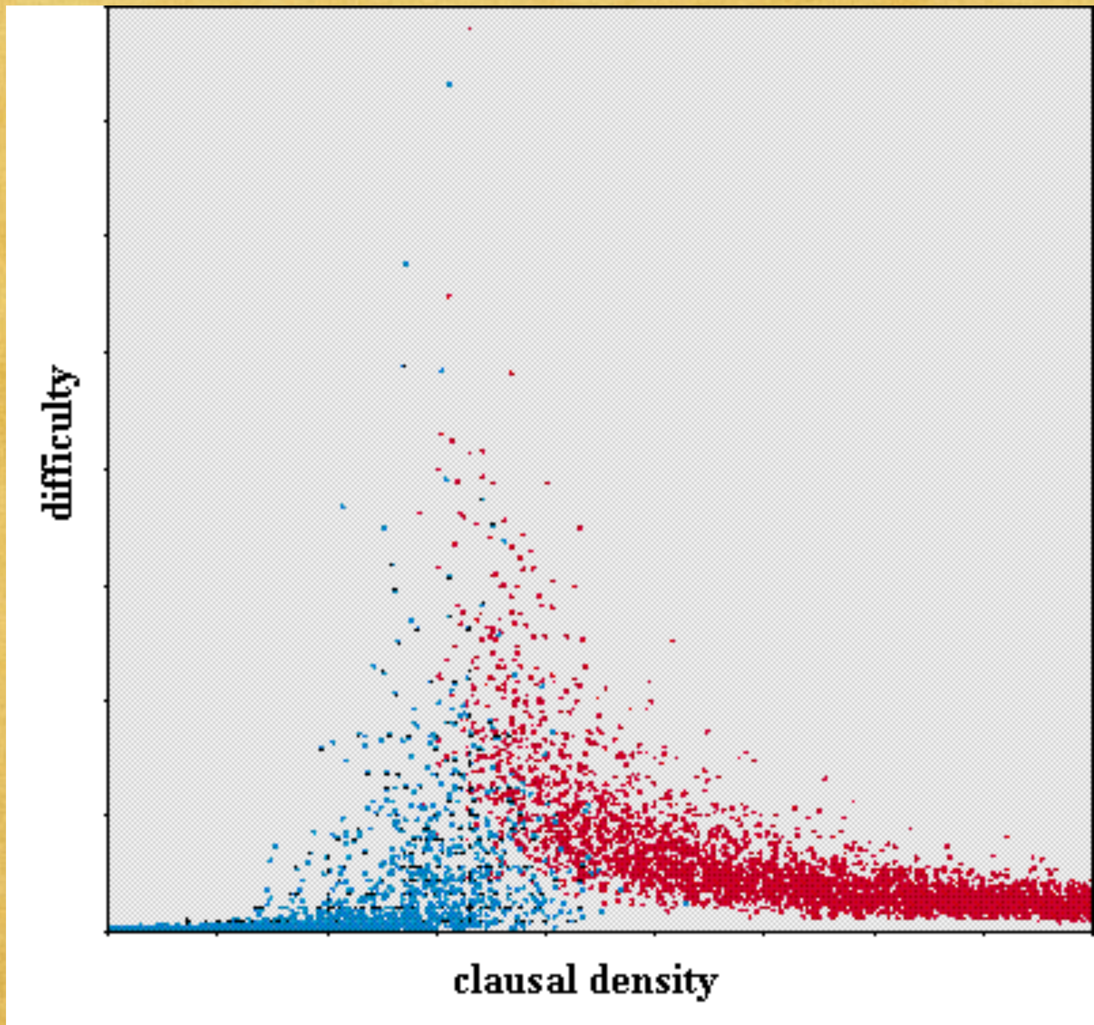
- 1980s
  - Difficulty finding hard satisfiability problems using constant probability model
- 1992 (one year after Cheeseman et al)
  - *Hard & Easy Distributions of SAT Problems*, Mitchell, Selman & Levesque
  - 804 citations on Google Scholar

3-SAT phase transition





3-SAT phase transition



Phase transitions

- Polynomial problems
  - 2-SAT, arc consistency, …
- NP-complete problems
  - SAT, COL, k-Clique, HC, TSP, number partitioning, ..
- Higher complexity classes
  - QBF, planning, …

So where are the hard manipulation problems?

[Walsh, IJCAI-09]

Manipulating the veto rule

- Simple rule to analyse
  - Each voter gets one veto
  - Candidate with least vetoes wins
- But on border of complexity
  - NP-hard to manipulate constructively with 3 or more candidates
  - Polynomial to manipulate destructively

Manipulating veto rule

- Manipulation not possible with 2 candidates
  - If the coalition want A to win then veto B

Manipulating veto rule

- Manipulation possible with 3 candidates
  - Voting strategically can improve the result

Manipulating veto rule

- Suppose
  - A has 4 vetoes
  - B has 2 vetoes
  - C has 3 vetoes
- Coalition of 5 voters
  - Prefer A to B to C

Manipulating veto rule

- Suppose
  - A has 4 vetoes
  - B has 2 vetoes
  - C has 3 vetoes
- Coalition of 5 voters
  - Prefer A to B to C
  - If they all veto C, then B wins

Manipulating veto rule

- Suppose
  - A has 4 vetoes
  - B has 2 vetoes
  - C has 3 vetoes
- Coalition of 5 voters
  - Prefer A to B to C
  - Strategic vote is for 3 to veto B and 2 to veto C

Manipulating veto rule

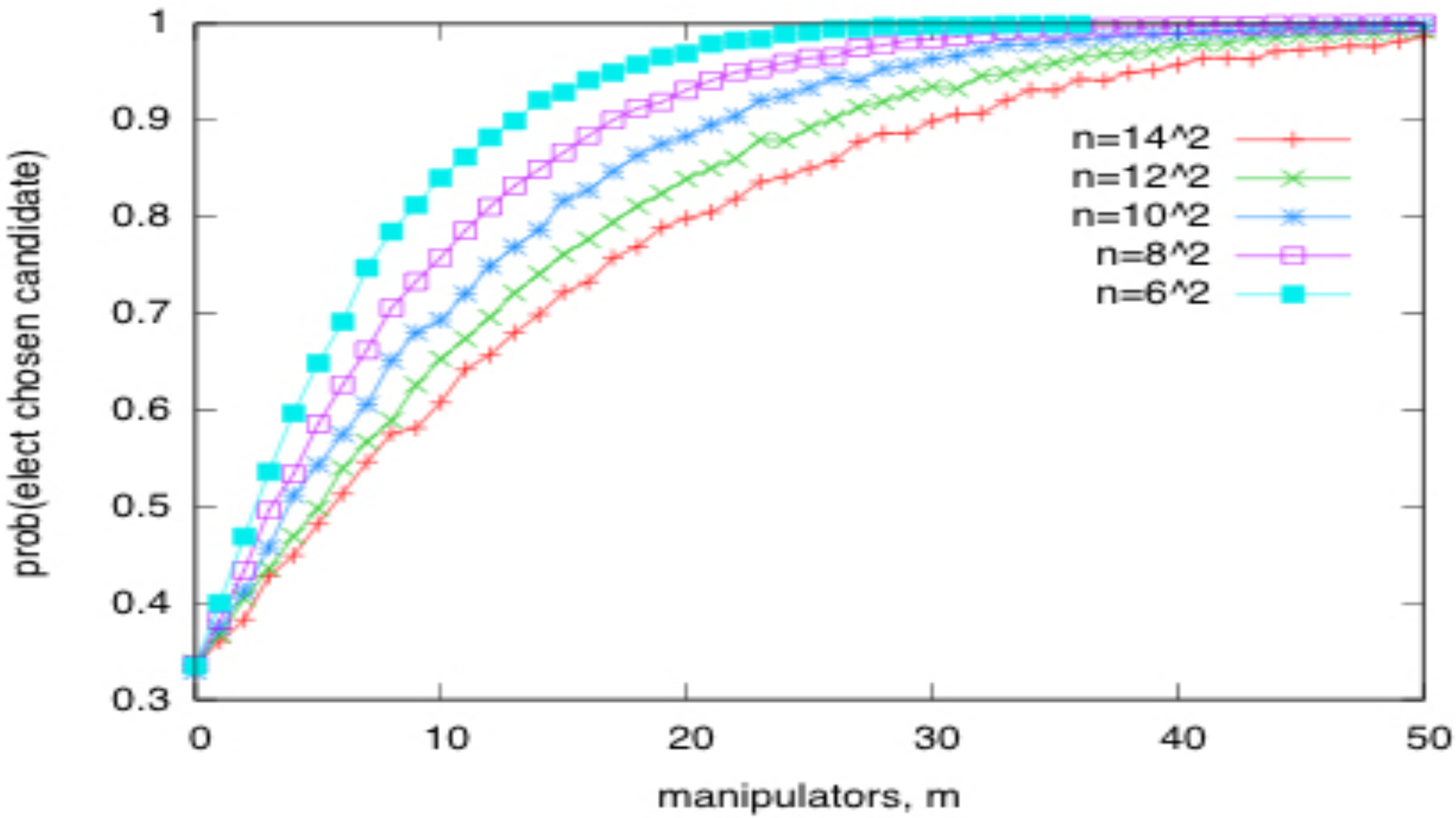
- With 3 or more candidates
- Unweighted votes
  - Manipulation is polynomial to compute
- Weighted votes
  - Destructive manipulation is polynomial
  - Constructive manipulation is NP-hard (=number partitioning)

Uniform votes

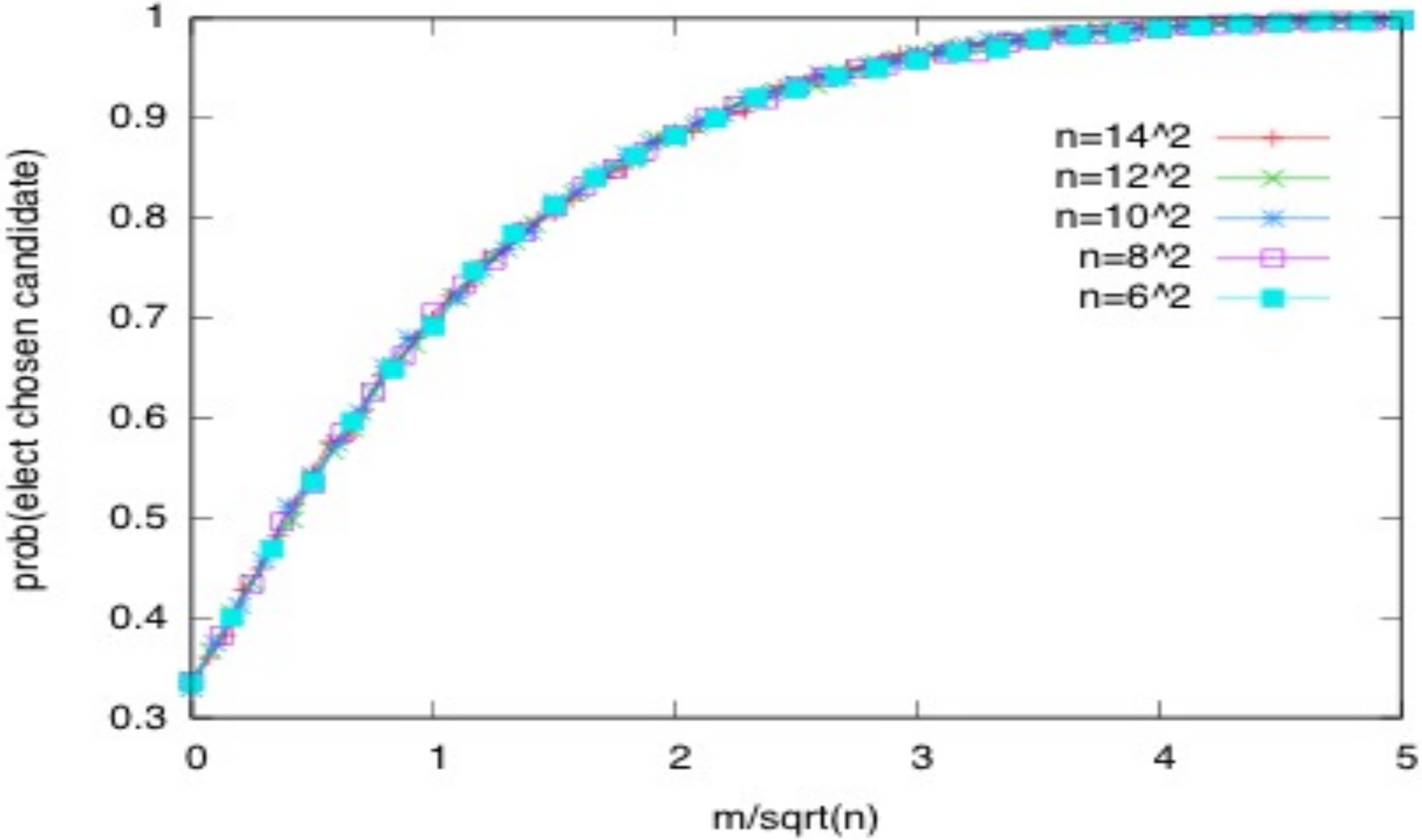
- *n* agents
- 3 candidates
- coalition of size *m*
- weights from [0,*k*]

aka “Impartial Culture”

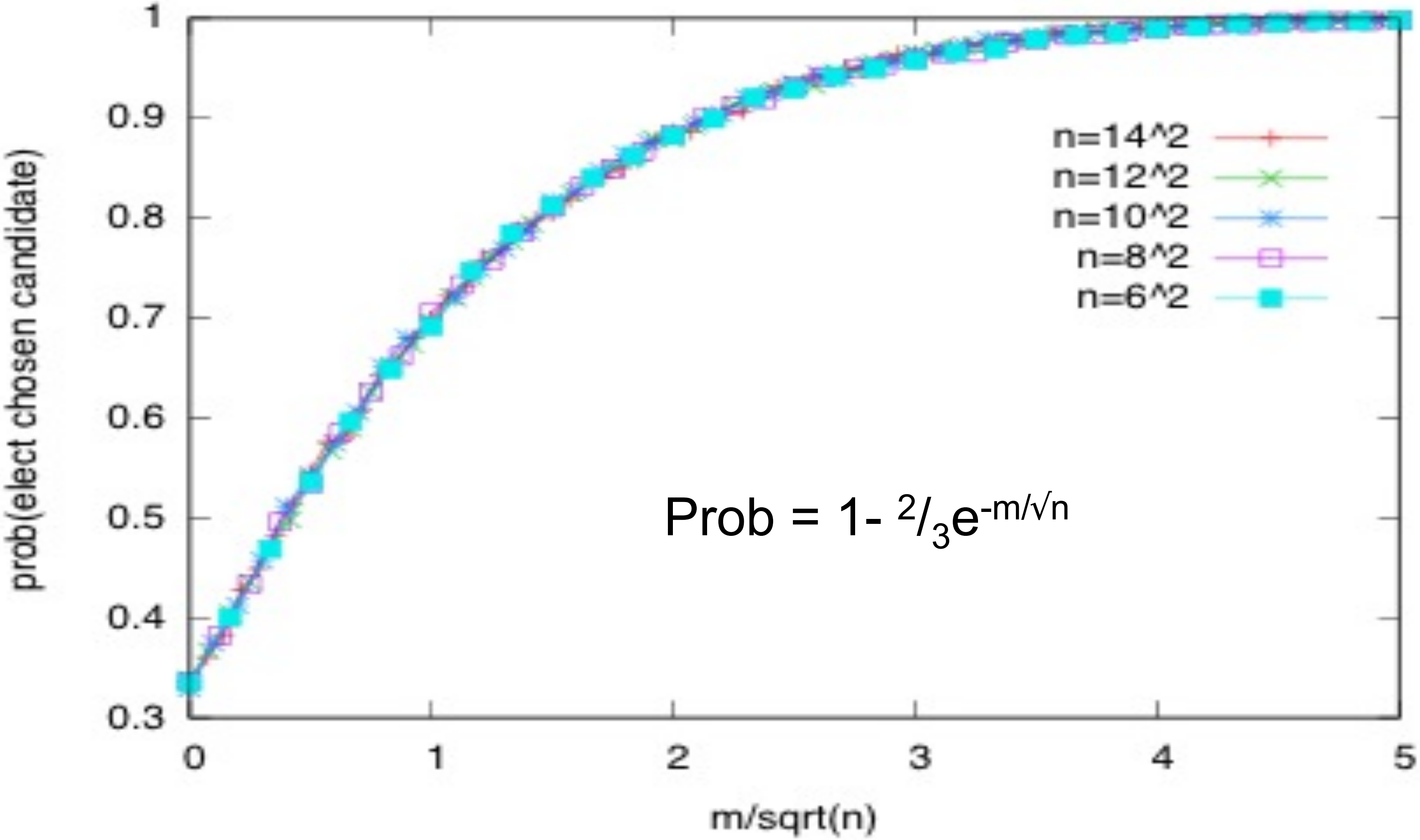
Phase transition



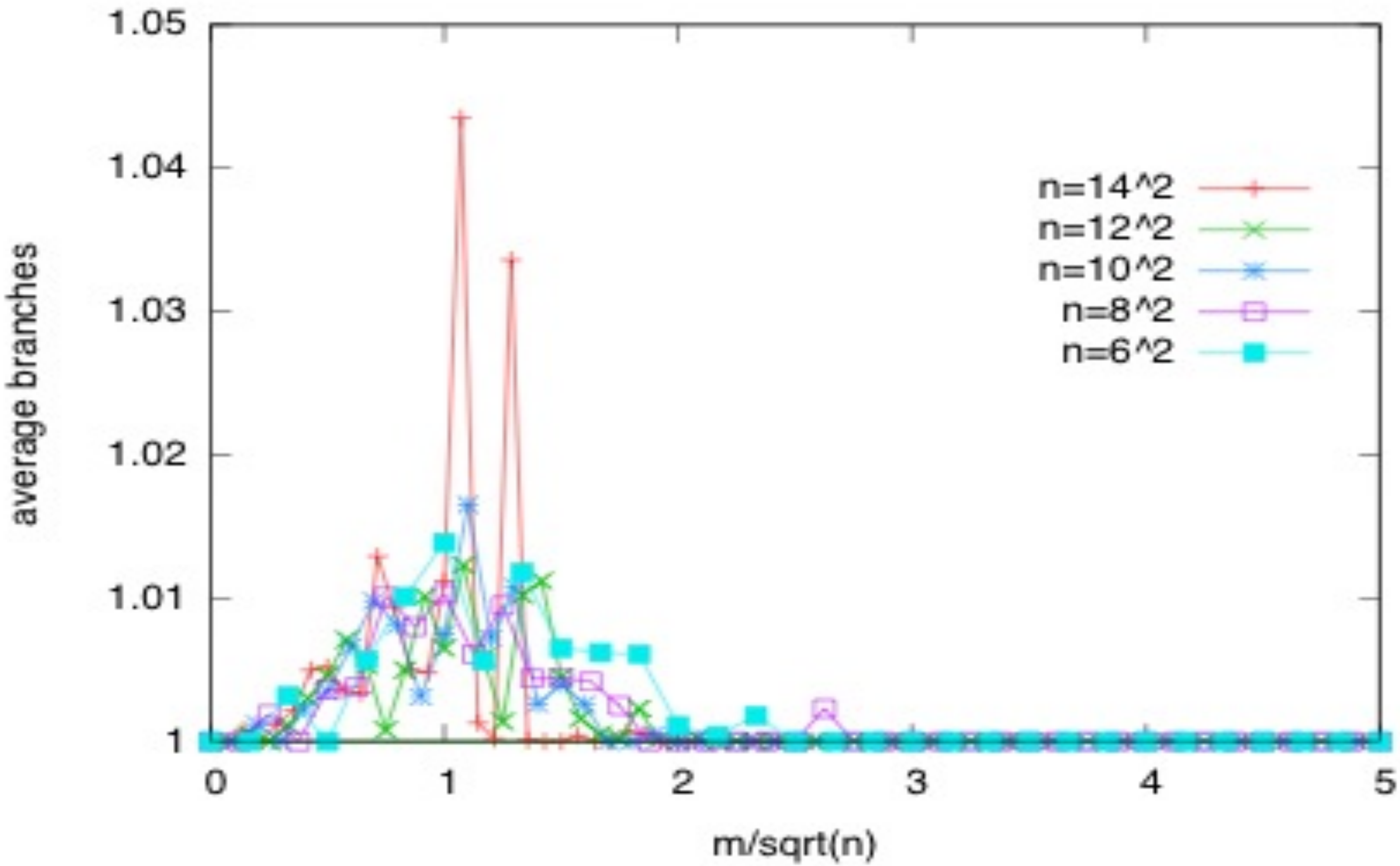
Phase transition



Phase transition



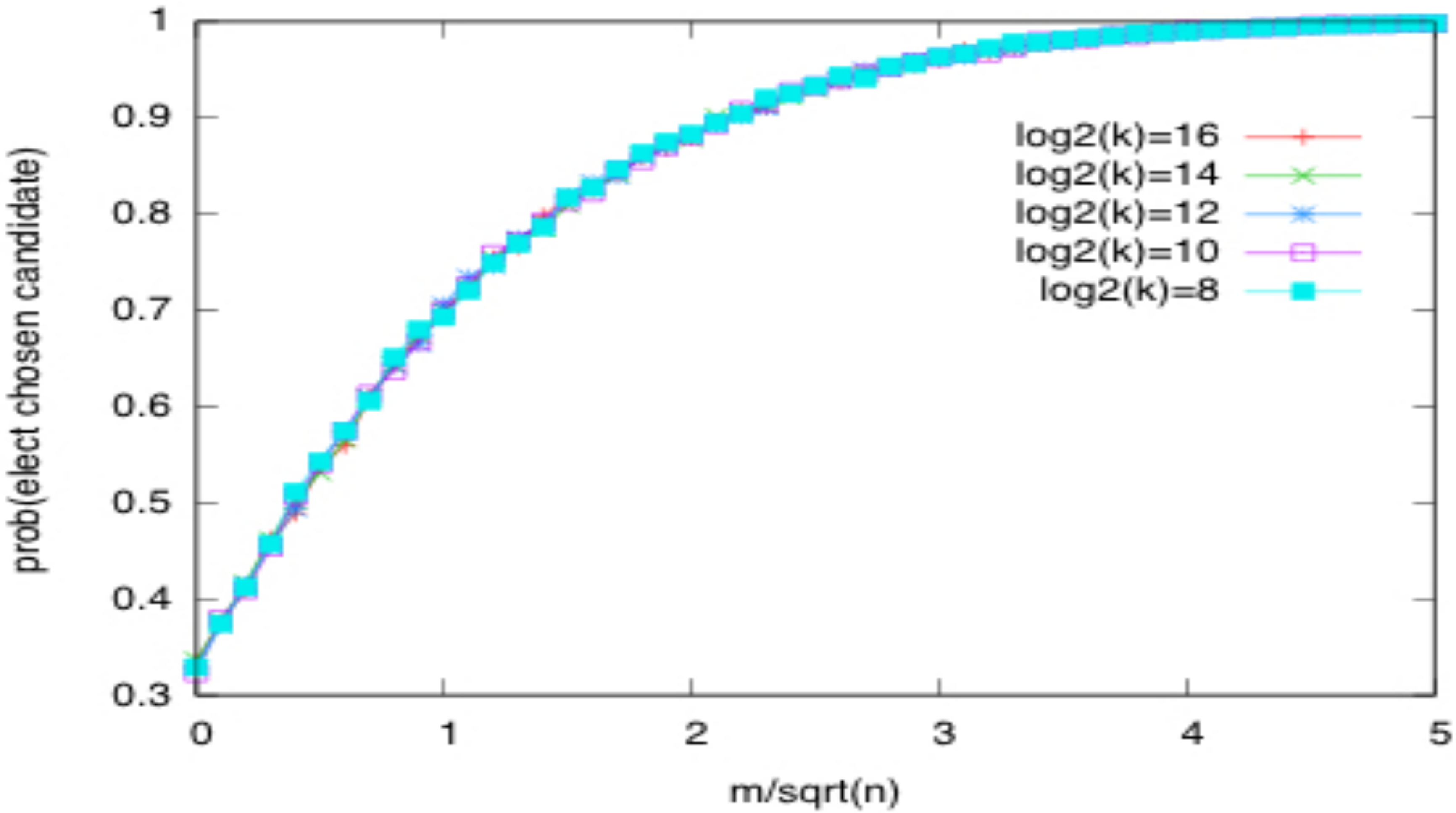
Phase transition



Phase transition

- Same result with other distributions of votes
  - Different size weights
  - Normally distributed weights
  - ..

Phase transition

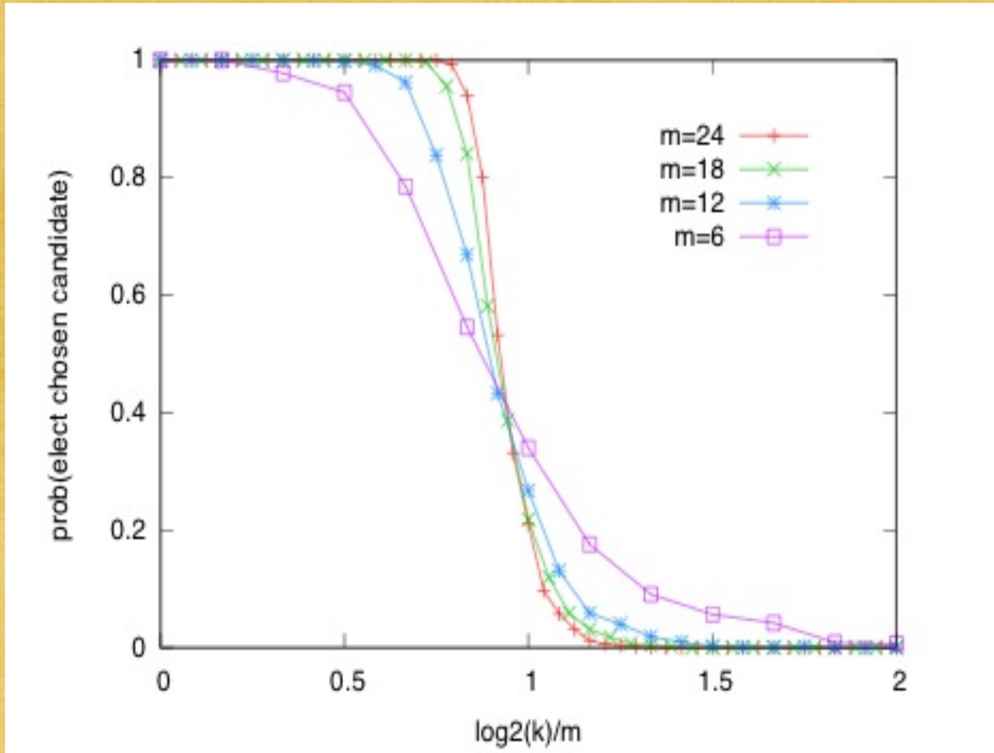


Why is manipulation easy?

- Coalition needs to be large enough to be able to change result
  - Coalition size m = O(√n)
- But if the coalition is large
  - Variance in number of vetoes is large, O(m)
- Easy to find a partition of votes or to prove none exists
  - Greedy heuristic solves problem
  - Or simple bound proves it is impossible

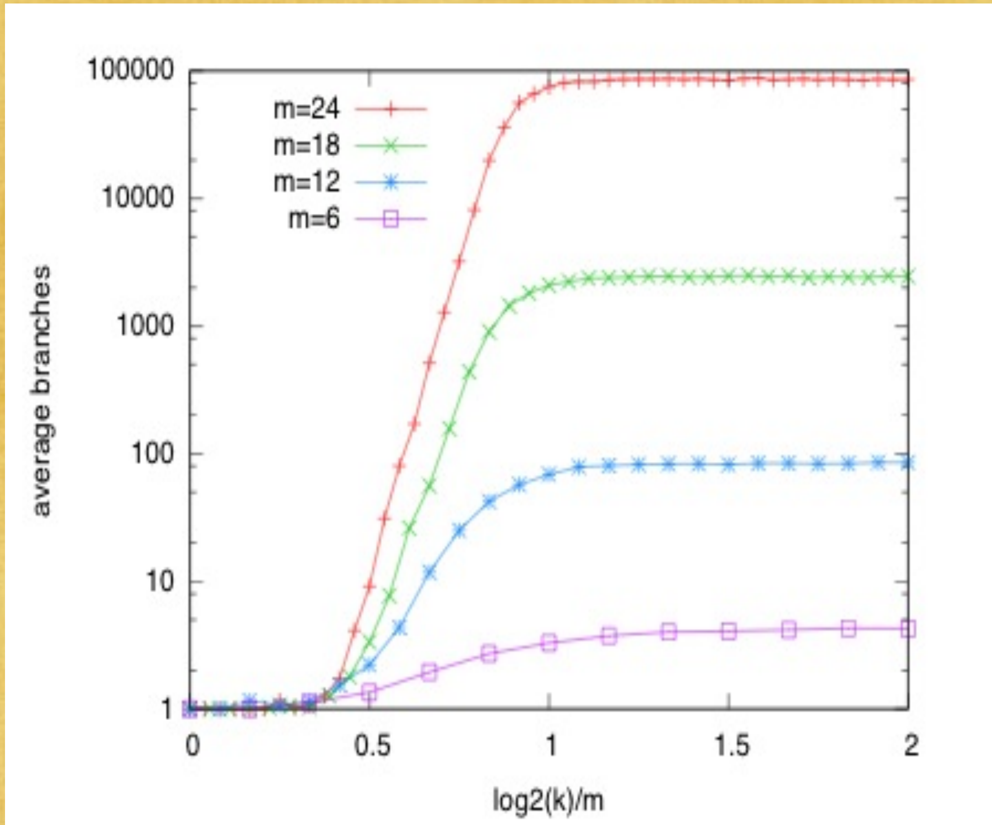
So where are the hard manipulation problems?

- Hung election
  - *n* voters have vetoed one candidate
  - coalition of size *m* has twice weight of these *n* voters



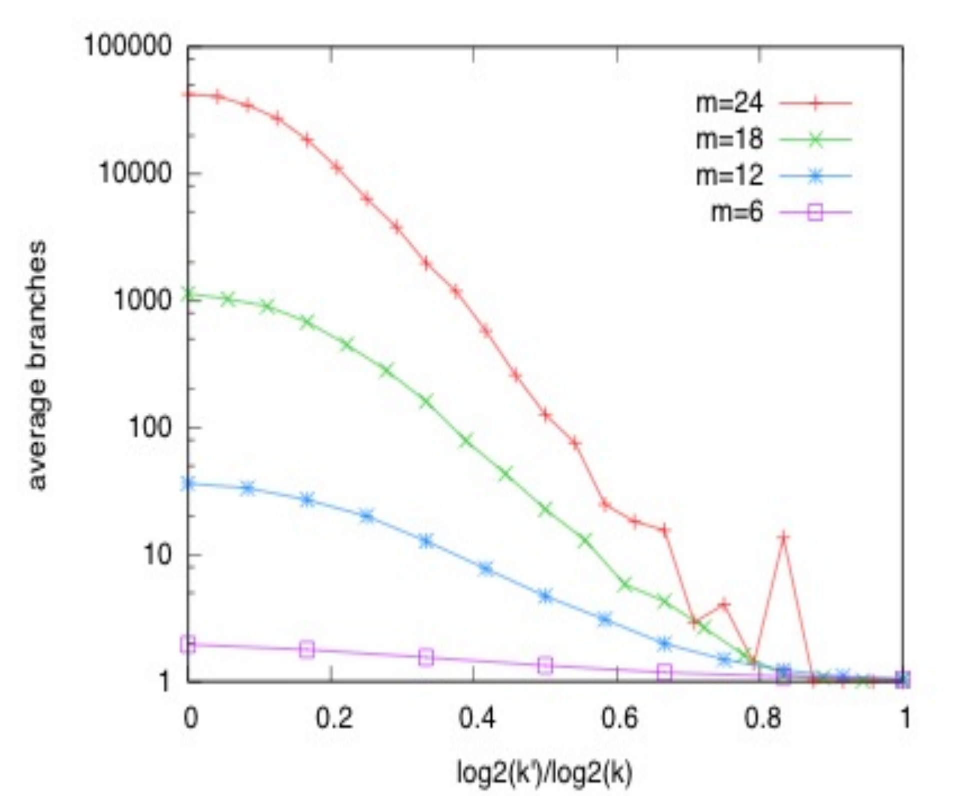
So where are the hard manipulation problems?

- Hung election
  - *n* voters have vetoed one candidate
  - coalition of size *m* has twice weight of these *n* voters



So where are the hard manipulation problems?

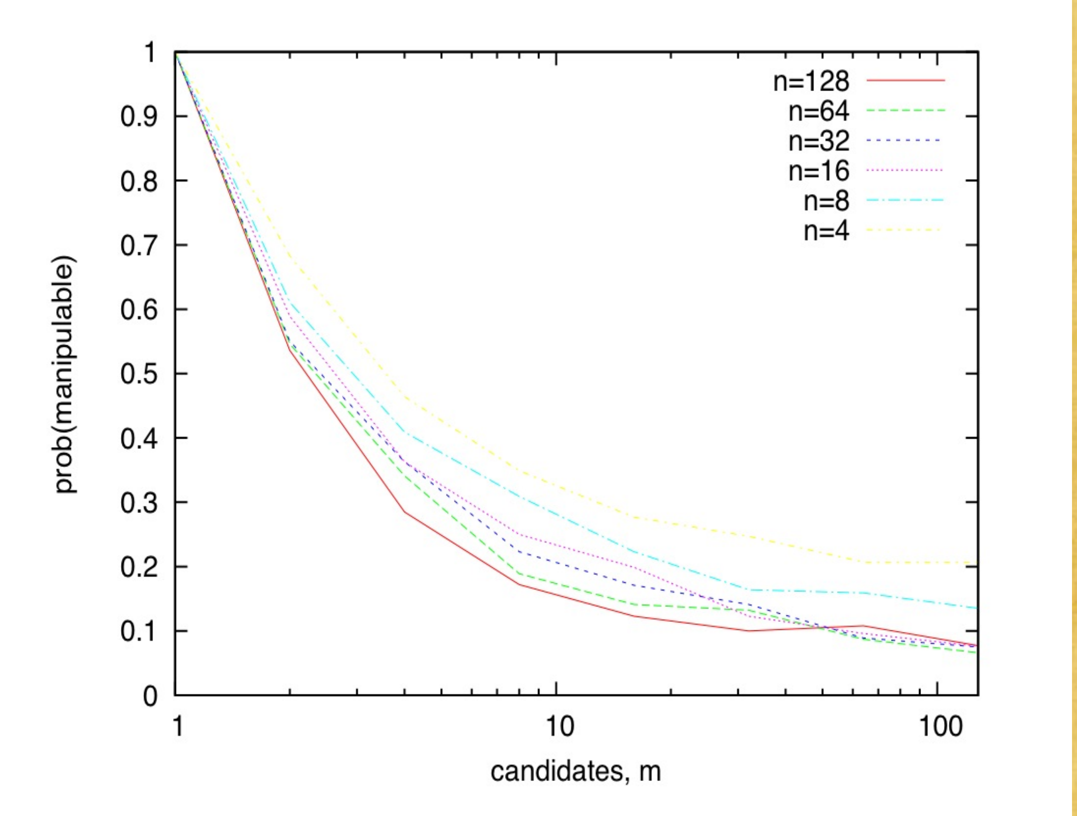
- Hung election
  - *n* voters have vetoed one candidate
  - coalition of size *m* has twice weight of these *n* voters
  - But one random voter with enough weight makes it **easy**



What about unweighted votes?

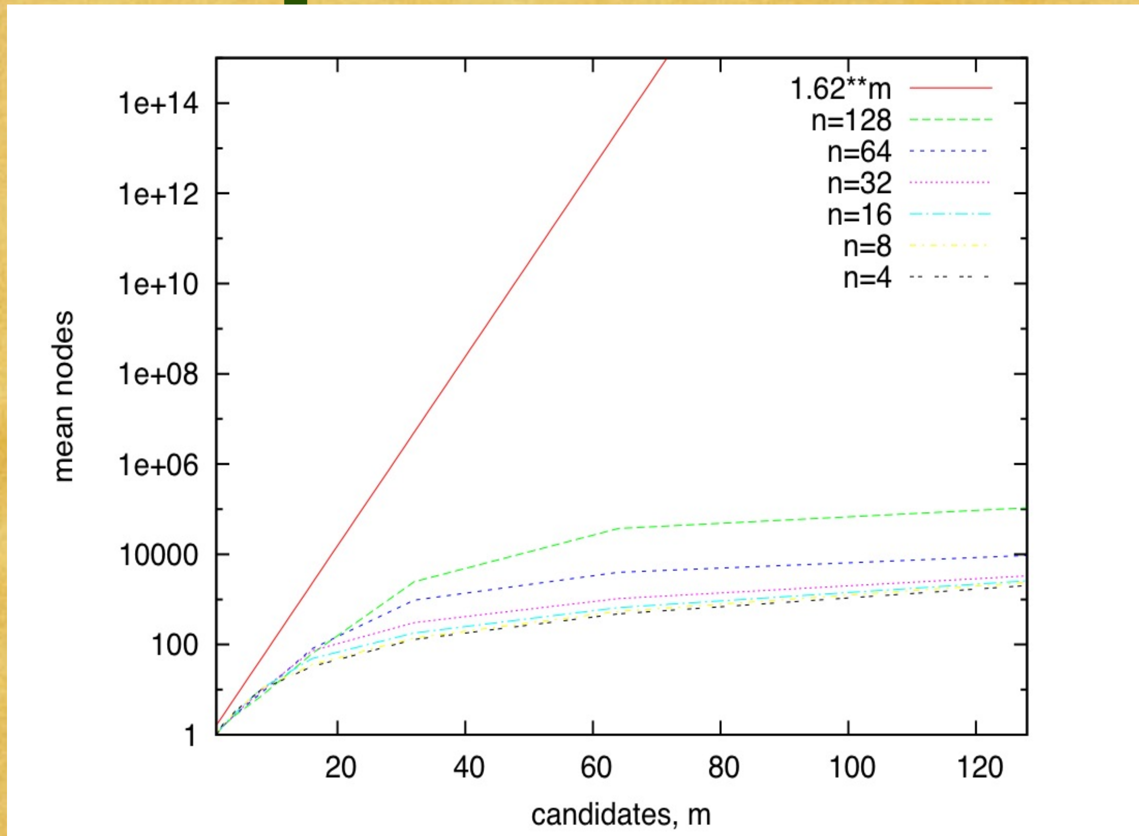
- STV is one of the few voting rules where manipulation by a single agent is NP-hard without weights
  - Unbounded number of candidates
- Conitzer gives an $O(n1.62^m)$ procedure to compute this
  - *How does this perform in practice?*

STV phase transition



- Smooth not sharp?
  - Other smooth transitions: 2-COL, 1in2-SAT, …

STV phase transition



- Fits 1.008^m with coefficient of determination R^2=0.95

Outline

- Where are the *hard* problems?
  - Phase transition behaviour
  - Sharp/smooth transitions
- A *close* look at the data
  - Early mistakes
  - Discrepancy search & restarts
- What makes problems *hard*?
  - Backbones, backdoors, …
  - Structure

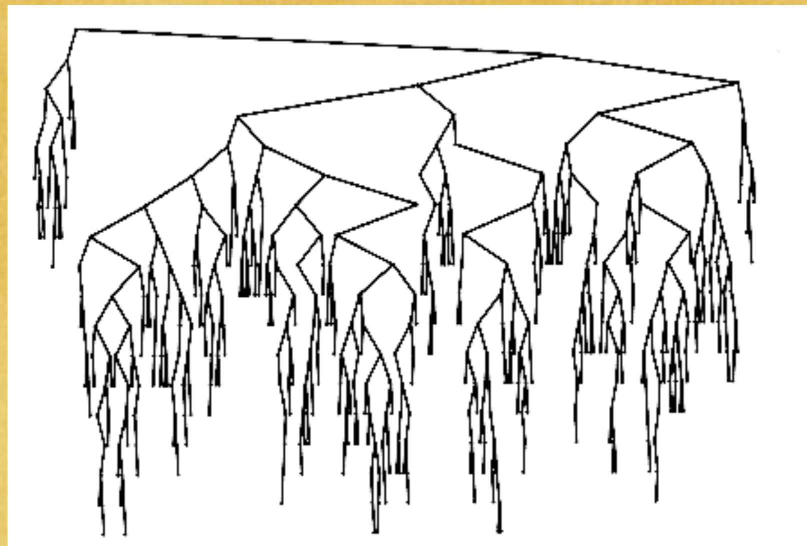
A close look at the data …

- Often worth having a look at the data
- Sometimes interesting items jump out
  - Outliers are always interesting!
  - Where complexity can hide

```
(1054 SAT 10 0.01)
(1055 SAT 11 0.01)
(1056 SAT 10 0.01)
(1067 SAT 21 0.01)
(1067 SAT 17059238 52653)
(1068 SAT 10 0.01)
(1069 SAT 10 0.01)
…
```

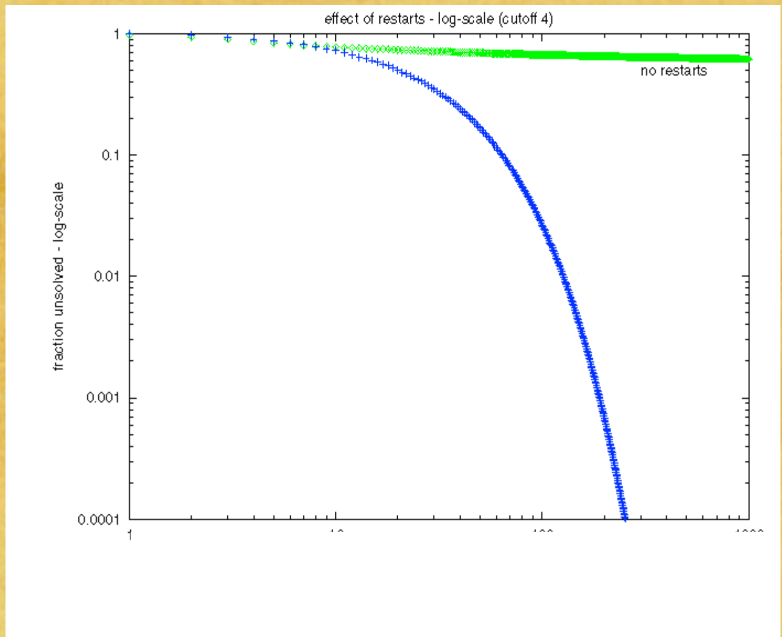
Easy problems are sometimes hard

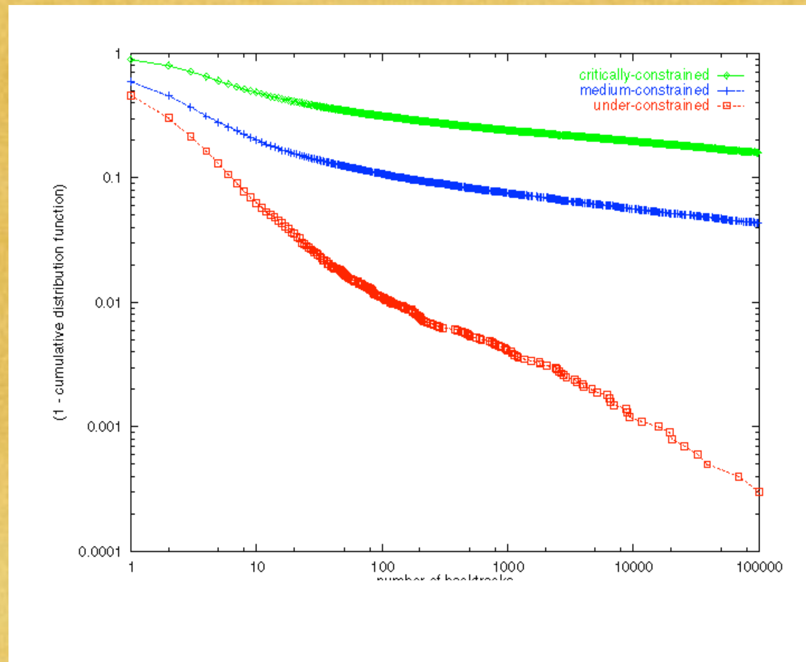
- Under-constrained
  - Little information for branching
  - Little pruning
- Early branching mistakes can be very costly



[Gent & Walsh, AIJ 1994]

Easy problems are sometimes hard





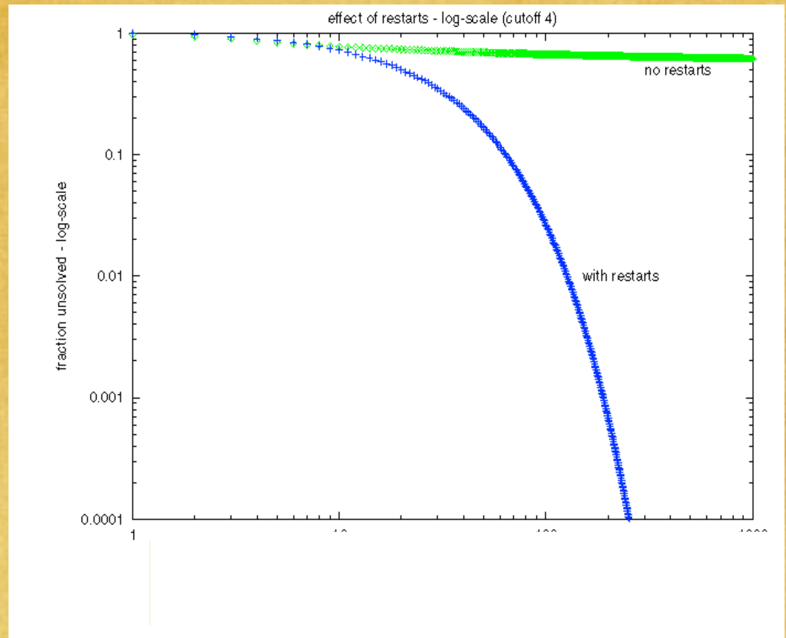
Normal exponential distribution

Heavy-tailed distribution

[Gomes et al, JAR 2000]

Two attacks on this problem!

- Randomization & Restarts
  - Give up and start again from the root
  - Provably eliminates heavy-tails



[Gomes, Selman, Kautz AAAI-98]

Two attacks on this problem!

Deepest Discrepancy=0

- Discrepancy search
  - Discrepancy = branch against heuristic
- Several flavours
  - LDS=search according to #discrepancies
  - DDS=search according to deepest discrepancy
  - …

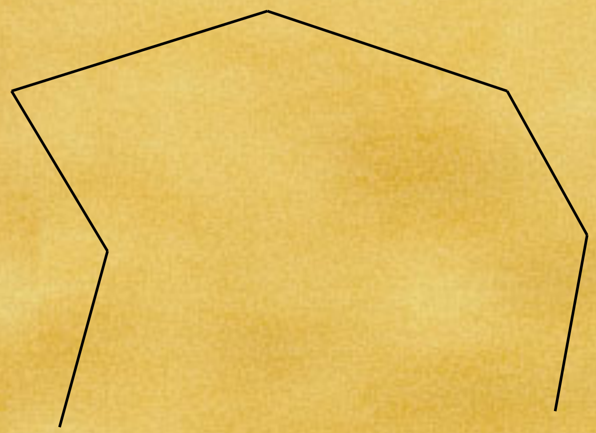
[Ginsberg IJCAI 95, Walsh IJCAI 97]

Two attacks on this problem!

Deepest Discrepancy=1

- Discrepancy search
  - Discrepancy = branch against heuristic
- Several flavours
  - LDS=search according to #discrepancies
  - DDS=search according to deepest discrepancy
  - …

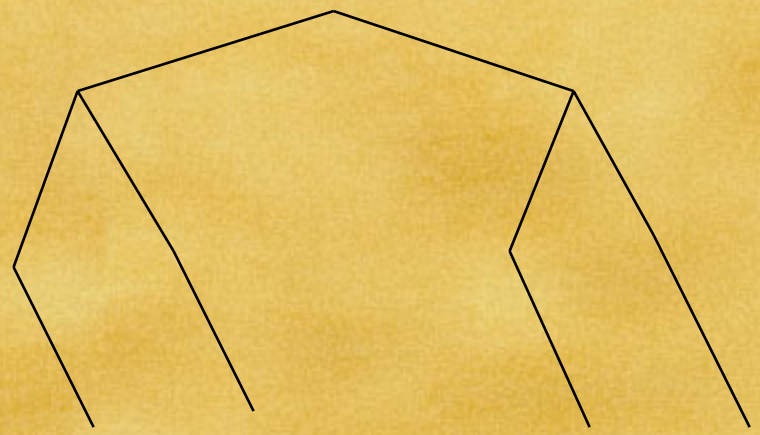
Two attacks on this problem!



Deepest Discrepancy=2

- Discrepancy search
  - Discrepancy = branch against heuristic
- Several flavours
  - LDS=search according to #discrepancies
  - DDS=search according to deepest discrepancy
  - …

Two attacks on this problem!



Deepest Discrepancy=3

- Discrepancy search
  - Discrepancy = branch against heuristic
- Several flavours
  - LDS=search according to #discrepancies
  - DDS=search according to deepest discrepancy
  - …

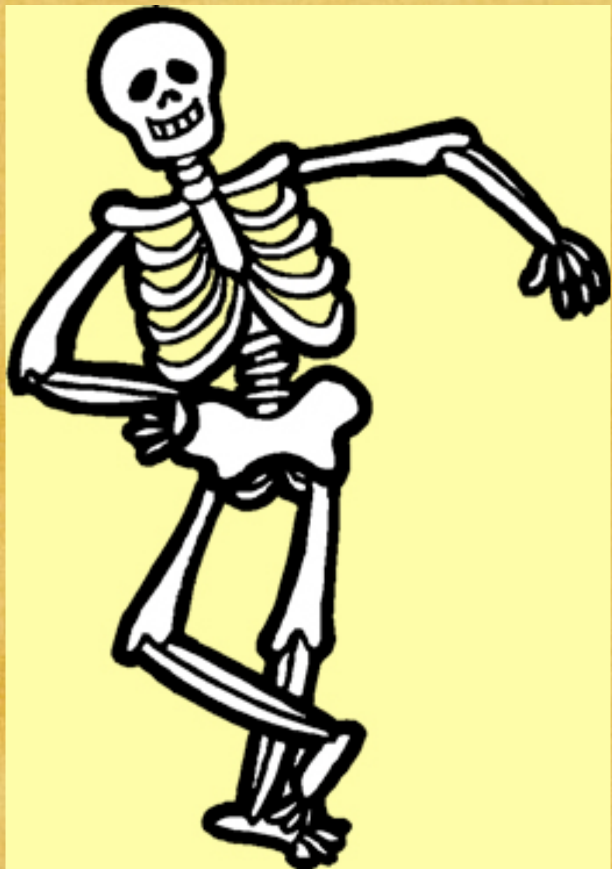
Outline

- Where are the *hard* problems?
  - Phase transition behaviour
  - Sharp/smooth transitions
- A *close* look at the data
  - Early mistakes
  - Discrepancy search & restarts
- What makes problems *hard*?
  - Backbones, backdoors, …
  - Structure

What makes problems hard?

- Large backbones
- Absence of backdoors
- Presence of certain structures
  - Balance
  - Small worldiness
  - High degree nodes
  - …

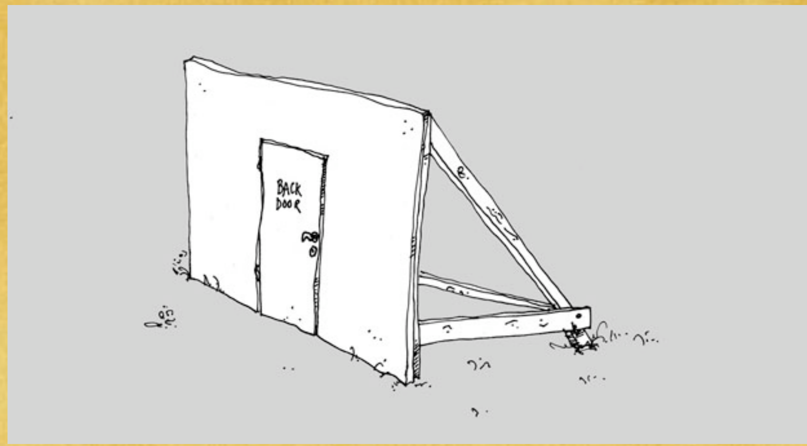
Backbone



- Fixed decisions in all solutions
  - Large backbone = high chance heuristic will get one wrong
- Problem hardness correlated with backbone size

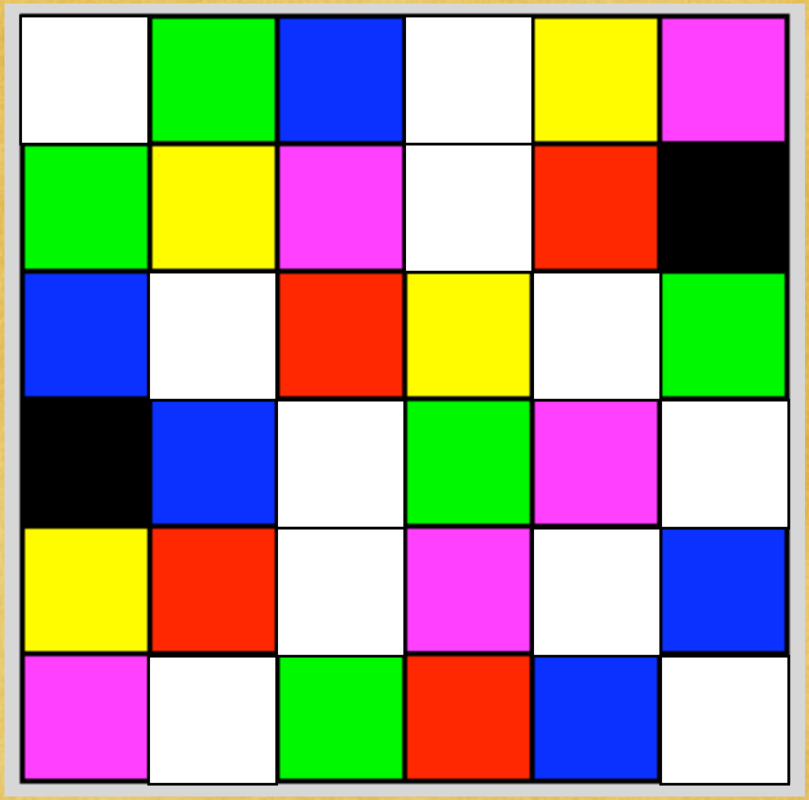
Backdoor

- Decisions that give polynomial subproblem
  - Weak = particular decisions
  - Strong = all possible decisions for a set of vars
- Problem hardness exponential in size of backdoor
  - c.f. fixed parameter tractability



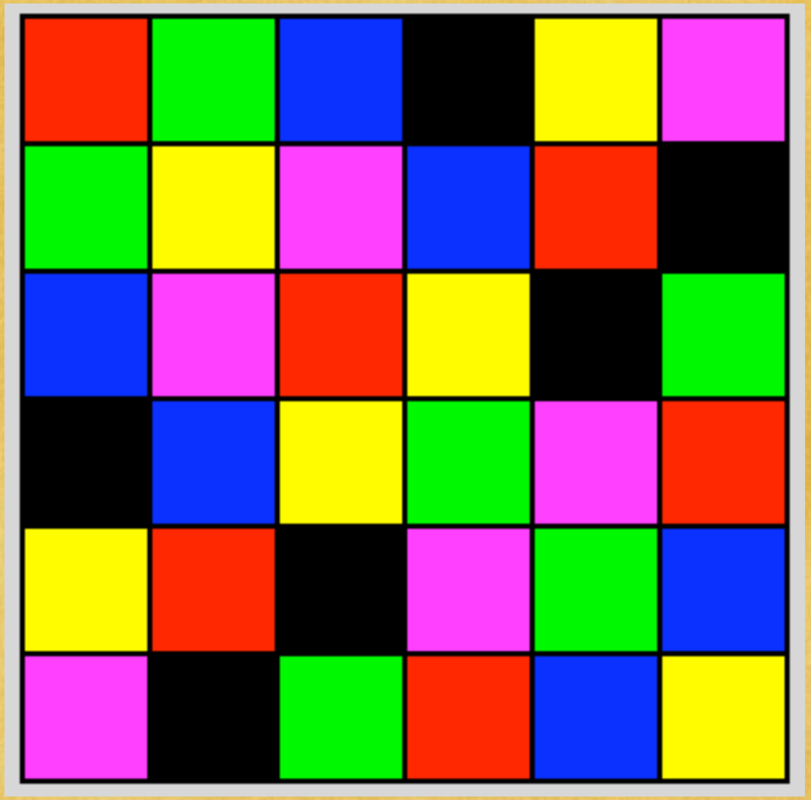
Balance

- SAT
  - All literals occur with same frequency
- COL
  - All nodes have same degree
- Quasigroup completion problems
  - Same number of holes on each row/column



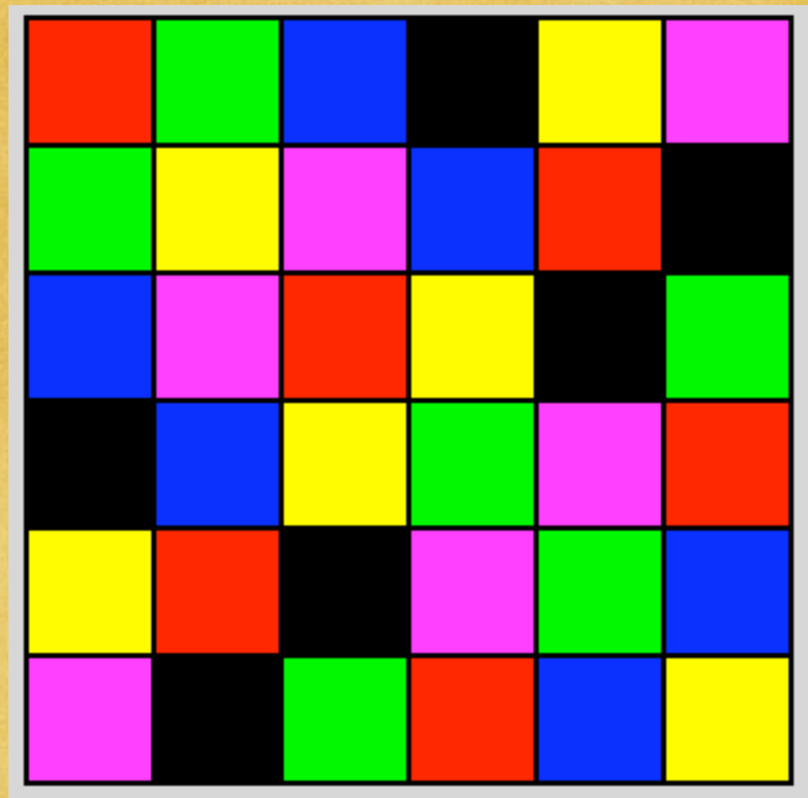
Balance

- SAT
  - All literals occur with same frequency
- COL
  - All nodes have same degree
- Quasigroup completion problems
  - Same number of holes on each row/column



Balance

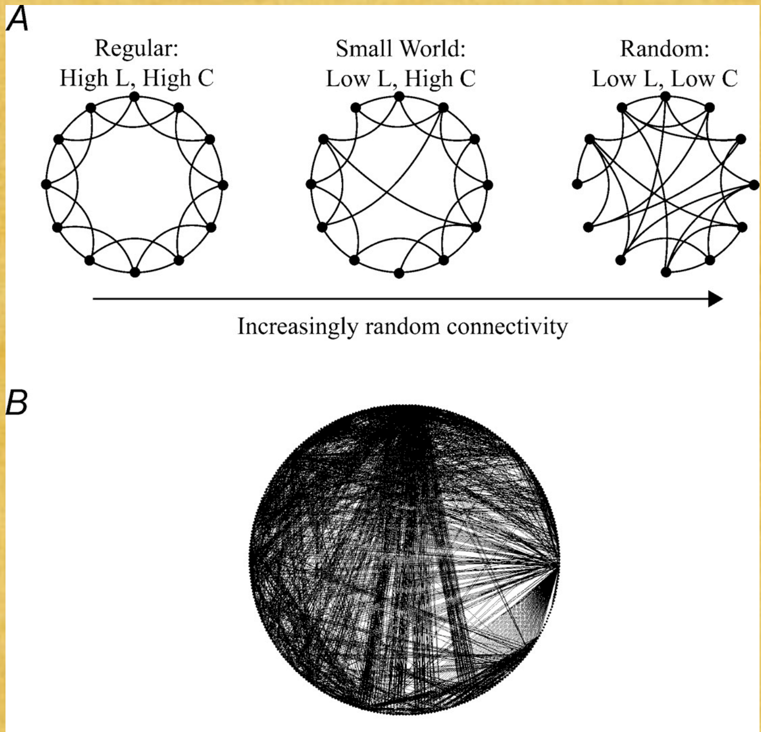
- Balance can make problems harder
  - No information for branching heuristics
  - Often necessary to make problems with "hidden" solutions hard
    - Hiding solutions often makes problems very easy



Small worlds

- Real problems are not random
  - Structure can make problems hard/easy
- For instance, graphs often have small world structure
  - Short path lengths
  - Highly clustered nodes

Small worlds



	L_{actual}	L_{random}	C_{actual}	C_{random}
C elegans	2.65	2.25	0.28	0.05

- Random graphs
  - Short paths
  - Not clustered
- Ring lattice
  - Long paths
  - Clustered
- Small world graph
  - Morph between ring lattice & random graph

Small worlds

- Impact on search
  - For example, consider colouring small world graphs
  - Heavy tailed distribution in search cost
  - Randomization and restarts appears empirically to eliminate heavy tail!

[Walsh IJCAI-99]

Morphing

- Often have only a few real data sets
  - E.g. few voting records available
  - One such data set is voting record of 10 scientific teams for 32 Mariner trajectories
- How do we run large experiments from such data?
  - Sample this voting record
    - More/fewer voters, more/fewer candidates
  - Morph it with some randomness
    - Apply random permutations to these votes

Structure in social choice

- Single peakedness
  - Makes manipulation harder
    - Escapes GS theorem
    - Fewer manipulations
- Balance
  - Hung elections

…

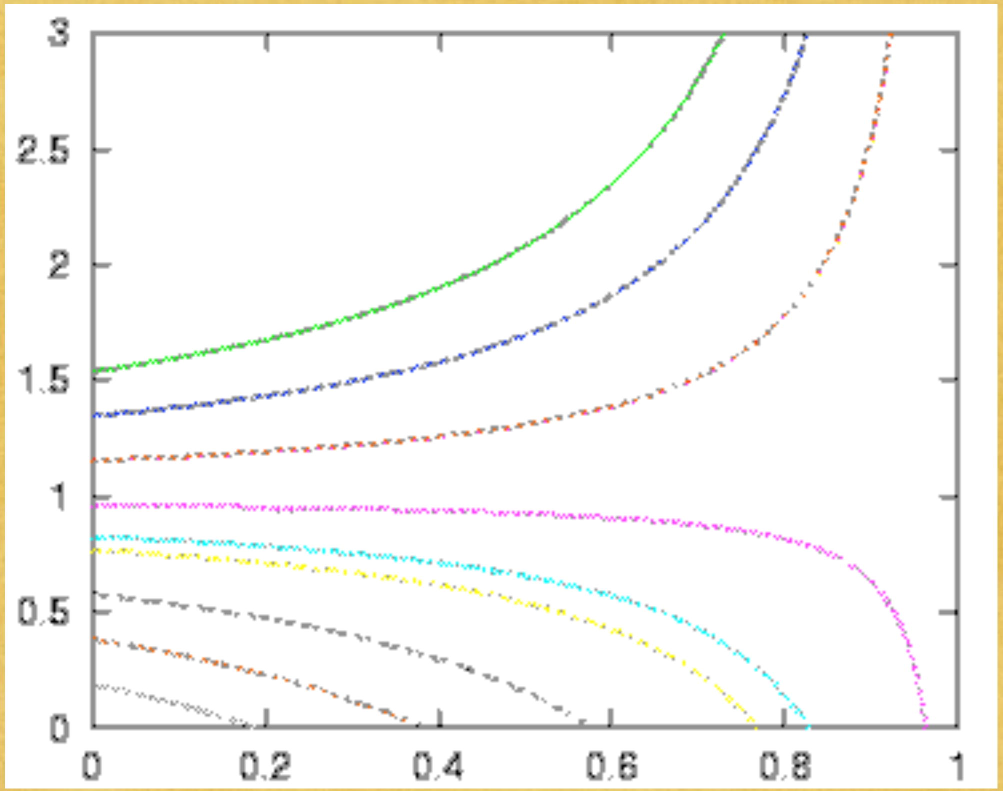
Outline

- Where are the *hard* problems?
  - Phase transition behaviour
  - Sharp/smooth transitions
- A *close* look at the data
  - Early mistakes
  - Discrepancy search & restarts
- What makes problems *hard*?
  - Backbones, backdoors, …
  - Structure

Conclusions

- Empirical studies can complement theoretical results
  - Hidden constants
  - Finite size effects
  - Theoretical conjectures
- Empirical studies can be fun!
  - Knife edge graph

Constrainedness knife-edge



[Walsh AAAI 98]