# Extended Preference Structures in MultiCriteria Decision Aid

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**Abstract**: Uncertainty and ambiguity are common situations in decision aid and therefore in preference modelling and aggregation. A basic distinction between uncertainty reasons is advanced in the paper, that is between lack of information and contradictory information. A semantical investigation of this approach is conducted and the PC preference structure is adopted in order to represent the different preference situations that may occur. This type of preference structure, besides offering an enhanced granularity in preference modelling, is strongly axiomatized, enabling a precise calculus. Some of its potentialities are discussed using a preference aggregation problem.

## Introduction

It is a well known problem that when preferences have to be aggregated one necessarily faces a dilemma. Either you accept to lose information, reducing the complexity of the aggregation problem, in order to gain in efficiency and clarity of the results or you keep as much as possible of the original information and then you lose in efficiency and effectiveness (see Vincke 1982a and 1982b).

Suppose that you do not always accept a reductionist approach because of the critical nature of some problem situations. You need to distinguish different situations of preferences that may arise, mainly in order to model the hesitation of the decision maker. In these cases the conventional preference structure appears to be unsuitable even from a probabilistic point of view (for a discussion see Roy, 1977; Bell et al., 1988). The decision maker's hesitation may have different origins (see Roy, 1988; Bouyssou, 1989). In this paper we propose a basic distinction between lack of information and excess of information. Even when the result is the same (confusion and/or hesitation) it seems important to be able to explain to the decision maker why such situations occur.

The basic idea introduced in this paper is that each time two alternatives $x$ and $y$ have to be compared, then it is quite natural to try to find the "positive reasons" supporting one of the two alternatives and the "negative reasons" that go against it. Under this reasoning (comparable with the concept of concordance and discordance introduced by Roy, 1985) it is possible to have either situations where such reasons are absent (ignorance situations) or situations where all are present (contradictory situations). Using such concepts it is possible to identify new "extended preference structures" in the sense of a higher "granularity" of the usual relations.

The paper is therefore organized as follows. In section 1, a small example is given in order to present the kind of problems that appear in preference aggregation. In section 2, a new approach is presented and the PC preference structure is introduced. In section 3 the example of section 1 is exploited by the new approach in order to present its advantages and disadvantages.

# 1 An example

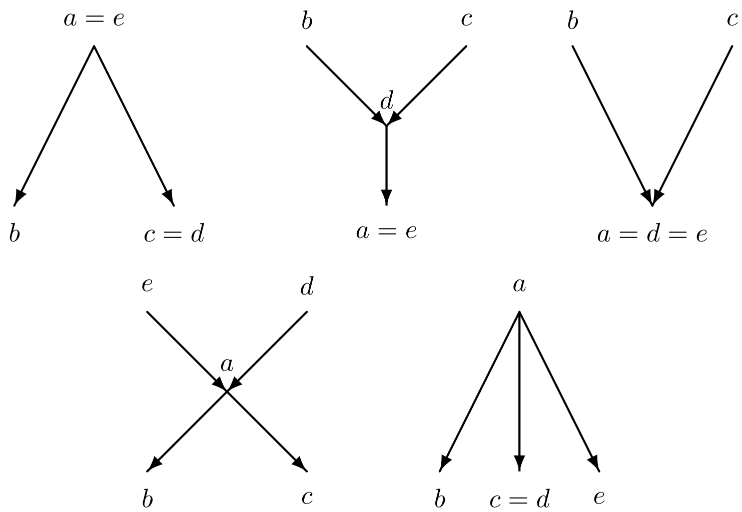



Figure 1: five quasi orders which must be aggregated
(transitive arcs are omitted)

Suppose there are five examinators and a set $A$ of five candidates ($A = \{a, b, c, d, e\}$). Each examinator used a multicriteria model to evaluate the candidates. The results are presented in figure 1 where each arc should be read as "strict preference", the sign of equality as "indifference" and the absence of arcs as "incomparability". These are well known quasi orders that can be easily obtained when preferences are aggregated on the basis of several criteria.

We are interested in obtaining a final ranking. The aggregation rule is the following: "for all $x, y \in A$, $x$ is globally at least as good as $y$ **iff** there exists a majority in this sense and there is no strong opposition against it". More formally:

$$\forall x, y \;\; s(x,y) \;\Leftrightarrow\; \mid s_j(x,y) \mid \geq \alpha \text{ and } \mid p_j^{-1}(x,y) \mid \leq \beta$$

where
$s(x,y)$ means that *"x is globally at least as good as y"*,
$\mid s_j(x,y) \mid$ is the number of preorders in which $x$ is at least as good as $y$ (preferred or indifferent),
$\mid p_j^{-1}(x,y) \mid$ is the number of preorders in which $y$ is strictly preferred to $x$.

It is well known that:
- "$x$ is globally better than $y$" **iff** "$x$ is globally at least as good as $y$" ($s(x,y)$ and "$y$ is not globally at least as good as $x$" ($\neg s^{-1}(x,y)$);
- "$x$ and $y$ are globally equivalent" **iff** "$x$ is globally at least as good as $y$" ($s(x,y)$ and "$y$ is globally at least as good as $x$" ($s^{-1}(x,y)$);
- "$x$ and $y$ are globally incomparable" **iff** "$x$ is not globally at least as good as $y$" ($\neg s(x,y)$ and "$y$ is not globally at least as good as $x$" ($\neg s^{-1}(x,y)$);

Let us consider for example $\alpha = 3$ (a strict majority is required) and $\beta = 1$ (no more than one opposition is required). There are different ways to obtain a result. The first one is to reduce these partial orders to one of their underlying complete orders and then aggregate the obtained rankings. This can be done using a simple exploiting procedure (reduction of circuits and then dominance; see Vincke 1992). The result is shown in Table 1. We obtain $c, b \leq a, d, e$. This is a reasonable result from a technical point of view, but it can be strongly argued by any decision maker as the couple $b, c$ which appears as the best is the worst for three of the five examinators. However this is the usual solution in such cases. In any case, it presents the inconvenience of eliminating useful information as the incomparabilities and to impose technical transformations that may be obscure for the decision maker.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| a | - | 0 | 0 | 0 | 1 |
| b | 0 | - | 1 | 0 | 0 |
| c | 0 | 1 | - | 1 | 0 |
| d | 0 | 0 | 0 | - | 1 |
| e | 1 | 0 | 0 | 1 | - |

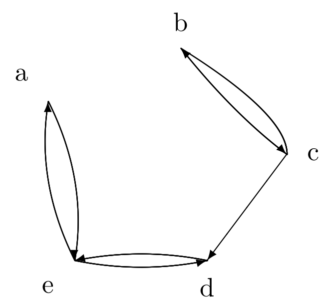
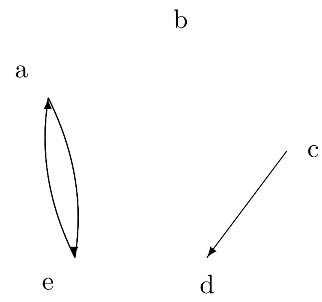



Table 1: the graph obtained after aggregating the five complete orders

Another way is to try to directly aggregate the five partial orders (using the rule previously defined). The result is shown in Table 2. The information contained in this result is rather poor. No reasonable conclusion can be obtained here.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| a | - | 0 | 0 | 0 | 1 |
| b | 0 | - | 0 | 0 | 0 |
| c | 0 | 0 | - | 1 | 0 |
| d | 0 | 0 | 0 | - | 0 |
| e | 1 | 0 | 0 | 0 | - |

Table 2: the graph obtained aggregating the five partial orders




What can we do now? Where is the problem? In fact there exist several problems. It is obvious that the preference structure we are using forces different situations to be represented by the same preference relation, creating confusion. For instance in Table 1, $b$ and $c$ are considered indifferent while they are incomparable. Moreover, the incomparability holding between $a$ and $c$ does not have the same meaning as the one between $c$ and $e$. The first case presents some reasons (not sufficient for the decision rule) to conclude $s(a, c)$ while it is certain that it will never be $s(c, a)$. On the other hand there exists a symmetrically conflicting situation between $c$ and $e$. These are differences that appear clearly in Table 2. Here, all the incomparabilities are detected, but they are all considered in the same way. For the decision maker(s) it may be interesting to know that $b$ and $c$ are incomparable because of lack of information while $e$ and $d$ are in a strong conflicting position.

Concequently exists an expresssiveness problem of the conventional preference structure $< P, I, R >$ towards particular decision problems, such as the ones that naturally occur in multicriteria decision aid and social choice. We claim that we need preference structures presenting a higher "granularity". It might be thought that we could just add the particular preference relations we need in order to take in account our specific situations. There are two objections to such a direction.

1. The new relations cannot be completely arbitrary, because in the extreme case one may end up defining a preference relation for each ordered couple in $A \times A$. It is necessary to follow some general rules or axioms which define the set of possible preference relations.

2. The new set of preference relations should have an axiomatization, that

is should fulfill some basic properties considered relevant in order to have sound and reliable models.

We therefore claim that a preference structure used for decision aid purposes should fulfill the following axioms (this may not be always valid; it is not a normative axiom set):

**A1** any preference structure should be a f.r.s.p. (fundamental relational system of preferences), that is should define a partition on $A \times A$ for any given $A$; in other words the preference relations included in the preference structure should be exhaustive for all possible situations and not redundant;

**A2** the preference structure should obey the axiom of "independence from irrelevant alternatives"; in a more general version the evaluation, if a specific ordered couple belongs (and in which way) to a specific relation, should depend on information concerning only this ordered couple;

**A3** the preference structure should be "well founded" in the sense that any binary relation in it should be univoquely defined by its properties.

Under these axioms, the problem of extending the conventional preference structure becomes more complex. We consider that the main difficulty in this direction is the language adopted as a basic formalism for preference modeling. Not surprisingly the language generally used is formal logic. The conventional preference structure $< P, I, R >$ is based on classic two valued logic which also supports the usual mathematical notation adopted in this case. In Tsoukiàs and Vincke, 1995 we proved that, using a language with a finite number of truth values, a limited number of possible preference relations can be defined, at least under the three axioms previously defined. Using classic logic, only three preference relations can be defined, exactly the $< P, I, R >$ preference structure. In fact it is easy to prove that using classic logic the $< P, I, R >$ preference structure is a maximal, well founded f.r.s.p.

So, if we want to extend a preference structure and retain the three axioms we have to change the language. The best known alternative is to use fuzzy sets and logics or the so called "valued binary relations". In this case, it is possible to imagine the existence of an infinity of preference relations each one defined for a particular value of the membership function. This is the direction chosen by the majority of researchers working in non conventional preference modeling (see Tsoukiàs and Vincke, 1992), but is not the one followed here. The reason for this choice is the following: in extending the set of preference relations we want to have a clear distinction between situations where the information is missing, not satisfactory, very poor and situations in which the information is too rich, contradictory, conflictual, ambiguous. We consider these situations as completely different so they should be treated in different ways. This is not the case when using the valued extensions of the conventional preference structure $< P, I, R >$. The use of the concept of credibility (or membership of each ordered couple in a binary relation) does not make any difference in the above mentioned situations. Low credibility can occur either because of strong evidence in the negation or because of ignorance. The fuzzy approach formally treats these cases in the same way.

Under these considerations we definitely chose to adopt a four valued paraconsistent logic, named DDT, as the basic formalism under which preference modeling could be represented (see Doherty et al., 1993; Tsoukiàs and Vincke, 1995). This logic enables an explicit representation for situations under lack of information and situations under conflicting information. The first order extension of this logic is immediate (necessary for the preference modeling case). The use of this formalism will be explained in the next section.

# 2 A new approach

Suppose you must compare two actions $x$ and $y$. Naturally you may compare the consequences of adopting one of the two different actions. Now if you have very clear and sufficient information you can conclude the following situations:

1 $x$ is definitely better than $y$ as the consequences of $x$ are judged definitely better than those of $y$;

2 $x$ and $y$ are equivalent because their consequences are equivalent from any point of view;

3 $x$ and $y$ are in a conflicting position as you have clear different and conflicting consequences between them.

However these conclusions are possible because we made the assumption of complete and sure information. They do not cover all possible situations a decision maker can be faced with. Consider now the situations in which you have symmetrically lack of information and excess of information in the sense that you have contradictory inputs. You may introduce two new situations:

4 $x$ and $y$ could be either equivalent or conflicting, but you hesitate because you have no relevant information in any of the two directions;

5 $x$ and $y$ could be either equivalent or conflicting, but you hesitate because you have contradictory informations in both directions.

Another possibility may occur when it is easy to positively compare the consequences of $x$ with these of $y$, but the inverse is more difficult again either because you do not know or because you know too much. We have two more situations:

6 $x$ could be better than $y$, but you hesitate to conclude if it is preferred or equivalent, because you do not have all the relevant information;

7 $x$ could be better than $y$, but you hesitate between preference and equivalence due to some conflicting information.

Another possibility occurs when it is easy to conclude something in a negative way, but the inverse is more difficult. So you may have two more situations:

8 $y$ cannot be better than $x$, but you hesitate whether they are conflicting or if $x$ is better than $y$, because you do not have sufficient information;

9 $y$ cannot be better than $x$, but you hesitate wheter they are conflicting or if $x$ is better than $y$, because you have contradictory information.

Finally you find yourself in the most ambiguous situation, that is having simultaneously both some lack of information and some contradictory information so that:

10 $x$ could be better than $y$, but you hesitate before concluding anything, because you have only bad information.

By these definitions all the situations of hesitation are represented. The question is now wheter it is possible to formalize these situations in a way which constitutes a sound preference modeling theory satisfying the three axioms we introduced above.

In order to build a new extended preference structure we will start with a preference relation that we call "outranking", which we will denote by $s(x, y)$ and read as *"$x$ is at least as good as $y$"*. In order to satisfy axioms **A2, A3** the only logical evaluations to perform will concern $s(x, y)$ and $s^{-1}(x, y)$ where

$$s^{-1}(x, y) \equiv s(y, x)$$

We generally accept that there may exist a set of "positive reasons" which support the sentence $s(x, y)$ (or $s^{-1}(x, y)$) and a set of negative reasons against this sentence or that supports the sentence $\neg s(x, y)$ ($\neg s^{-1}(x, y)$ respectively). This kind of reasoning can be generally accepted in many human problem situations. In any case, it seems reasonable when decision aid must be provided. The balance between reasons supporting a certain assertion and reasons against it is a natural "common sense" situation. Moreover, when a prescription has to be presented and explained to a decision maker a clear presentation of the positive and negative reasons behind it eases the discussion, the acceptance and the reflection about the problem situation and the action to be undertaken. There are therefore four possible states:

- there exist sufficient positive reasons to establish $s(x, y)$ and there are not enough negative reasons to establish $\neg s(x, y)$; $s(x, y)$ is "true";

- there exist sufficient positive reasons to establish $s(x,y)$ and sufficient negative reasons to establish $\neg s(x,y)$; $s(x,y)$ is "contradictory";
- there do not exist sufficient positive reasons to establish $s(x,y)$ and there are not enough negative reasons to establish $\neg s(x,y)$; $s(x,y)$ is "unknown";
- there do not exist sufficient positive reasons to establish $s(x,y)$ and there exist enough negative reasons to establish $\neg s(x,y)$; $s(x,y)$ is "false";

More formally, we accept that $s(x,y)$ and $\neg s(x,y)$ are not complementary or that the extensions of the two predicates may overlap and that they do not cover the whole set of possible situations. We can express this idea by introducing (using the DDT language) the sentence $\triangle s(x,y)$ to be read as "there is presence of truth in saying $s(x,y)$". The definitions of the sentences $\triangle\neg s(x,y)$, $\neg\triangle s(x,y)$, $\neg\triangle\neg s(x,y)$ are straightforward, with the corresponding meanings. Using these sentences we can define also the following (the formal machinery of this language is described in Doherty at al., 1993 and Tsoukiàs and Vincke, 1995):
- $\mathbf{T}s(x,y) \equiv \triangle s(x,y) \wedge \neg\triangle\neg s(x,y)$: the "true" extension of the predicate $s(x,y)$;
- $\mathbf{K}s(x,y) \equiv \triangle s(x,y) \wedge \triangle\neg s(x,y)$: the "contradictory" extension of the predicate $s(x,y)$;
- $\mathbf{U}s(x,y) \equiv \neg\triangle s(x,y) \wedge \neg\triangle\neg s(x,y)$: the "unknown" extension of the predicate $s(x,y)$;
- $\mathbf{F}s(x,y) \equiv \neg\triangle s(x,y) \wedge \triangle\neg s(x,y)$: the "false" extension of the predicate $s(x,y)$.

Let us now give names to the ten situations we previously introduced. We will have: case 1: relation $P$; case 2: relation $I$; case 3: relation $R$; case 4: relation $U$; case 5: relation $J$; case 6: relation $K$; case 7: relation $H$; case 8: relation $V$; case 9: relation $Q$; case 10: relation $L$;

It is easy to see that semantically the true extension of the relation $S$ will be the union of the true extensions of the relations $P$, $I$, $K$ and $H$; the ones where in fact we are at least sure that "$x$ is at least as good as $y$". Remember that in $P$ "$x$ is definitely better than $y$", that in $I$ "$x$ and $y$ are equivalent" and that in $K$ and $H$ "$x$ could be better than $y$", but you hesitate for the inverse relation. By the same reasoning we have that the false extension of the relation $S$ will be the union of the true extensions of the relations $P^{-1}$, $Q^{-1}$, $V^{-1}$ and $R$. The unknown extension of $S$ will be the union of the true extensions of the relations $V$, $U$, $K^{-1}$ and $L^{-1}$. The contradictory extension of $S$ will be the union of the true extensions of the relations $Q$, $L$, $J$ and $H^{-1}$. We can give a formal representation of these concepts with the following DDT formulas:
- $\forall x,y \in A \;\; \mathbf{T}s(x,y) \equiv \mathbf{T}p(x,y) \vee \mathbf{T}i(x,y) \vee \mathbf{T}k(x,y) \vee \mathbf{T}h(x,y)$.
- $\forall x,y \in A \;\; \mathbf{K}s(x,y) \equiv \mathbf{T}q(x,y) \vee \mathbf{T}l(x,y) \vee \mathbf{T}j(x,y) \vee \mathbf{T}h^{-1}(x,y)$.
- $\forall x,y \in A \;\; \mathbf{U}s(x,y) \equiv \mathbf{T}v(x,y) \vee \mathbf{T}u(x,y) \vee \mathbf{T}k^{-1}(x,y) \vee \mathbf{T}l^{-1}(x,y)$.
- $\forall x,y \in A \;\; \mathbf{F}s(x,y) \equiv \mathbf{T}r(x,y) \vee \mathbf{T}p^{-1}(x,y) \vee \mathbf{T}q^{-1}(x,y) \vee \mathbf{T}v^{-1}(x,y)$.

We summarize the formulas given above in Table 3.

| $A \times A$ | $\mathbf{T}s^{-1}(x,y)$ | $\mathbf{F}s^{-1}(x,y)$ | $\mathbf{U}s^{-1}(x,y)$ | $\mathbf{K}s^{-1}(x,y)$ |
|---|---|---|---|---|
| $\mathbf{T}s(x,y)$ | $\mathbf{T}i(x,y)$ | $\mathbf{T}p(x,y)$ | $\mathbf{T}k(x,y)$ | $\mathbf{T}h(x,y)$ |
| $\mathbf{F}s(x,y)$ | $\mathbf{T}p^{-1}(x,y)$ | $\mathbf{T}r(x,y)$ | $\mathbf{T}v^{-1}(x,y)$ | $\mathbf{T}q^{-1}(x,y)$ |
| $\mathbf{U}s(x,y)$ | $\mathbf{T}k^{-1}(x,y)$ | $\mathbf{T}v(x,y)$ | $\mathbf{T}u(x,y)$ | $\mathbf{T}l^{-1}(x,y)$ |
| $\mathbf{K}s(x,y)$ | $\mathbf{T}h^{-1}(x,y)$ | $\mathbf{T}q(x,y)$ | $\mathbf{T}l(x,y)$ | $\mathbf{T}j(x,y)$ |

Table 3: the PC preference structure

We call these ten different basic preference relations the PC preference structure $< P, Q, K, L, I, J, H, R, U, V >$. In Tsoukiàs and Vincke (1995) we proved that the PC preference structure is a maximal well founded f.r.s.p.. What are the semantics of the ten preference relations defined if we consider the relation $S$ as the basic information to ask to the decision maker or to compute? We give a brief discussion hereunder.

1. $p(x, y)$ (strict preference):

   we can definitely establish that "$x$ is at least as good as $y$" as there exist sufficient positive reasons supporting it and there are not enough negative reasons against it, while we can surely establish that "$y$ is not as least as good as $x$" for the opposite reasons; therefore "$x$ is strictly better than $y$".

2. $q(x, y)$ (weak preference):

   we have doubts concerning "$x$ is at least as good as $y$" as there exist both sufficient positive reasons supporting it and sufficient negative reasons against it, while we can surely establish that "$y$ is not as least as good as $x$" because we do not have sufficient positive reasons to assume the opposite and there exist sufficient negative reasons against it; therefore "$x$ could be better than $y$, but we have some doubts due to the presence of strong negative reasons about $s(x, y)$".

3. $k(x, y)$ (semi preference):

   we can surely establish that "$x$ is at least as good as $y$" as there exist sufficient positive reasons supporting it and there are not sufficient negative reasons against it, while we cannot establish that "$y$ is not as least as good as $x$" or the opposite because we have neither sufficient positive nor negative reasons; therefore "$x$ could be better than $y$, but we have some doubts because we do not know what happens with $s^{-1}(x, y)$".

4. $l(x, y)$ (semi weak preference):

   we have doubts concerning "$x$ is at least as good as $y$" as there exist both sufficient positive reasons supporting it and sufficient negative reasons against it, while we cannot establish that "$y$ is not as least as good as $x$" or the opposite because we have neither sufficient positive nor negative reasons; therefore "$x$ could be better than $y$, but we have some doubts both because we have evidence indicating that may hold $\neg s(x, y)$ and we do not know anything about $s^{-1}(x, y)$".

5. $i(x, y)$ (strict indifference):

   we can surely establish that "$x$ is at least as good as $y$" as there exist sufficient positive reasons supporting it and there are not sufficient negative reasons against it, while we can also surely establish that "$y$ is as least as good as $x$" for the same reasons; therefore "$x$ and $y$ are strictly indifferent as they could be considered equivalent".

6. $j(x, y)$ (semi indifference):

   we have doubts concerning "$x$ is at least as good as $y$" as there exist both sufficient positive reasons supporting it and sufficient negative reasons against it, while we have also doubts to establish that "$y$ is at least as good as $x$" for the same reasons; therefore "$x$ and $y$ could be indifferent", but we doubt due to the presence of strong evidence against it in both directions.

7. $h(x, y)$ (weak indifference):

   we can surely establish that "$x$ is at least as good as $y$" as there exist sufficient positive reasons supporting it and there are not sufficient negative reasons against it, while we have doubts about "$y$ is as least as good as $x$" because of the presence of both positive and negative reasons; therefore "$x$ could be indifferent to $y$ (notice in this case that "indifference" is is not symmetric), but we have some doubts to about $s^{-1}(x, y)$".

8. $r(x, y)$ (incomparability):

   we can surely establish that "$x$ is not at least as good as $y$" as there are not sufficient positive reasons supporting the opposite and there are sufficient negative reasons against it, while we can also surely establish that "$y$ is not as least as good as $x$" for the same reasons; therefore "$x$ and $y$ are in conflicting position due to strong contrasting information".

9. $u(x, y)$ (semi incomparability):

   we cannot establish that "$x$ is at least as good as $y$" as there exist neither sufficient positive reasons supporting it nor sufficient negative reasons against it, while we cannot establish that "$y$ is as least as good as $x$" for the same reasons; therefore "we cannot establish what holds between $x$ and $y$ because of the absence of relevant information".

10. $v(x, y)$ (weak incomparability):

we can surely establish that "$x$ is not at least as good as $y$" as there are not sufficient positive reasons supporting the opposite and there are sufficient negative reasons against it, while we cannot establish that "$y$ is as least as good as $x$" because of the absence of either positive or negative reasons; therefore "$x$ could be in opposition to $y$, but we have some doubts due to the absence of all the necessary information".

To summarize, comparing $x$ to $y$ and $y$ to $x$ in order to establish a preference between them we have the ten following possible situations:
- $x$ is strictly better than $y$ (strict preference $p(x, y)$);
- $y$ is not better than $x$, but we are not sure that $x$ is better than $y$ because of some evidence against it (weak preference $q(x, y)$);
- $x$ could be better than $y$, but we are not sure due to lack of all the necessary information (semi preference $k(x, y)$);
- $x$ is possibly better than $y$, but we have both contradictory information and lack of all the necessary information (semi weak preference $l(x, y)$);
- $x$ and $y$ are strictly equivalent (strict indifference $i(x, y)$);
- $x$ and $y$ could be indifferent, but there exist contradictions in both directions (semi indifference $j(x, y)$);
- $x$ could be indifferent to $y$, but there is some contradictory information if $y$ is also indifferent to $x$ (weak indifference $h(x, y)$);
- $x$ and $y$ are in strong opposition (incomparability $r(x, y)$);

we cannot establish what holds between $x$ and $y$ (semi incomparability $u(x, y)$);
- $x$ could be in opposition to $y$, but we are not sure due to lack of all the necessary information (weak incomparability $v(x, y)$).

These ten situations cover all the possibilities that may occur when comparing $x$ and $y$.

# 3 The example again

Let us now see what happens if we apply the previously developed theory to the preference aggregation problem presented in section 1. We shall make for the moment a basic assumption: the preferences expressed by each examinator are modelled by the conventional preference structure $< P, I, R >$. We can now translate the decision rule by the following DDT formula:

$$\begin{cases} \triangle s(x, y) \ \Leftrightarrow \ \mid s_j(x, y) \mid \geq \alpha, \\ \triangle \neg s(x, y) \ \Leftrightarrow \ \mid p_j^{-1}(x, y) \mid \geq \beta. \end{cases}$$

Table 4 gives the results for all ordered couples in $A \times A$ on the basis of the definitions of the **PC** preference structure.

Let us dicuss the result. We recall here that the result of the direct aggregation of the five partial orders in section 1 was:
$P = \{(c, d)\}$, $I = \{(a, e)\}$, $R = A \times A \setminus (P \cup I)$

Using the **PC** preference structure we have instead:
$P = K = L = \emptyset$, $Q = \{(a,b),(a,c)\}$, $R = \{(c,e),(b,e)\}$, $U = \{(c,b),(d,e)\}$, $V = \{(b,d)\}$, $I = \{(a,e)\}$, $J = \{(a,d)\}$, $H = \{(c,d)\}$

| | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $a$ | $I$ | $Q$ | $Q$ | $J$ | $I$ |
| $b$ | $Q^{-1}$ | $I$ | $U$ | $V^{-1}$ | $R$ |
| $c$ | $Q^{-1}$ | $U$ | $I$ | $H$ | $R$ |
| $d$ | $J$ | $V$ | $H^{-1}$ | $I$ | $U$ |
| $e$ | $I$ | $R$ | $R$ | $U$ | $I$ |

Table 4: The **PC** preference structure applied to set $A$

We note that only two couples ($(b,e)$ and $(c,e)$) are effectively incomparable because of symmetric conflicting reasons (two examinators for part). The other couples included in the incomparability relation have been assigned to other relations, more precisely:
- for both couples $(a,b)$ and $(a,c)$ $a$ could be better, but the existence of "strong" opposition (as valued by the decision rule: two examinators against) leads to a hesitation between preference and incomparability;
- for both couples $(b,c)$ and $(d,e)$ there is a strong lack of information (in the first case no examinator compares them and in the second case only three examinators compare them, but in a conflicting way) and therefore they could be either indifferent or incomparable;
- for couple $(b,d)$ we have some evidence that "$d$ cannot be better than $b$", but we have no other relevant information, so we hesitate between incomparability and preference (but this situation is obviously different from the case of couples $(a,b)$ and $(a,c)$);
- for couple $(a,d)$ we have reasons both to conclude equivalence and strong opposition (two examinators for each and one considering them equivalent) so we hesitate between equivalence and incomparability (again this is a different situation from the one concerning couples $(b,c)$ and $(d,e)$);

Finally it should be noted that couple $(c,d)$ which was in the preference relation, has now been assigned to an hesitation situation. Observing the five quasi orders it is possible to see that in fact two examinators consider them equivalent, two prefer $c$ and one prefers $d$. It is more reasonable to hesitate between equivalence and preference rather to express a definite preference and this is our conclusion.

At this point one must remember that until now we considered for each examinator a conventional model based on the $< P, I, R >$ preference structure. It is easy to observe that if each examinator reported his preferences in a more "rich" model (reporting his positive and negative reasons) the final evaluations may be different (see for example Mainka et al., 1993).

So what are the final prescriptions? The most important ones are to ask the committee to clarify the uncertain situations (if possible) providing the missing information or eliminating contradictions. A more thorough investigation should be pursued, but the direction in which such investigation should take place is now clear. And what if, by any mean a final ranking must be given? No time or no will for further investigation. What could be a final prescription?

A basic idea could be the following:
- build the graphs of the "presence of truth in outranking" relation and of the "presence of truth in not outranking" relation (these are classic binary relations);
- compute the two rankings, one for each graph, using any appropriate procedure and then combine them.

It is possible to give some considerations about this final problem. Assuming the idea of working on the two graphs previously introduced, there are two directions of research in order to obtain a more reliable answer to an eventual demand of a final ranking (or prescription).

1. Define "a priori" the properties that the final ranking should fulfill and then try to identify which exploiting procedure fits the requirements. In this sense the results obtained by Vincke (1992) can be used.

2. Bouyssou (1994) recently proposed some results about the existence of structural properties in outranking relations. This result applies in conventional definitions of the outranking relation based on the $< P, I, R >$ preference structure. It may be interesting to verify if the outranking relation, as defined by the PC preference structure, has any structural properties (conditioning the exploiting procedure) and under which conditions.

# 4 Conclusions

The problem of extending the conventional preference structure has been addressed in this paper in order to take into account situations of uncertainty, imprecision, ambiguity etc.. A basic distinction between hesitation due to ignorance and hesitation due to contradiction is introduced and it is applied to the case where a preference model must be established. Under this perspective, all the possible situations are reviewed and formalized, using the DDT language, in a new preference structure named PC. Such a preference structure fulfils some basic axioms, that is a maximal well founded f.r.s.p.. An example of aggregation of partial orders is introduced in the paper in order to present the limits of the conventional approach and the eventual advantages of the new one. Some future research directions conclude the paper.

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