Bi-oriented Graphs and Four Valued Logic for preference modelling

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Abstract

In this paper we show the relations between 4-valued logics (and more precisely the DDT logic) and the use of bi-oriented graphs. Further on we focus on the use of bi-oriented graphs for non conventional preference modelling. More specifically, we show how bi-oriented graphs can be used in order to represent extended preference structures of the type definable using the DDT logic which has been created with the purpose of modelling hesitation in preference statements. We then study how transitive closure can be extended within such extended preference structures.

Keywords: DDT logic, bi-oriented graphs, signed graphs, transitive closure, preference modelling

1. Introduction

Preference modelling is an important issue for many domains: decision analysis, social choice theory, game theory, fair division, etc. (see [17]). A classical way to represent preferences is to use a binary relation defined on the set of alternatives, such as "alternative a is at least as good as alternative b" (which we denote by aSb). The usual semantics associated to binary relations is that the sentence aSb can either be true or false and nothing more. However, such a semantic does not really fit our intuition, because it can be the case that we cannot clearly state if aSb is the case or not, either because of ignorance (we have very little/poor/unreliable information) or because of contradictions (we actually have to much information ...).

Such further epistemic states (ignorance and contradiction) cannot be captured by classic logic which only admits true or not true (false) interpretations for any sentence (propositional or first order).

In order to be able to explicitly distinguish the four epistemic states (true false, unknown, contradiction) we may use Belnap's logic ([2], [3]) and more precisely an extension of Belnap's logic. The DDT logic ([10, 11, 25, 30]) is a first order four valued logic (a logic accepting 4 values i.e. true, false, true and false, neither true nor false, epistemic states) including a weak negation. In this logic, negation does not coincide with complementation and the reasons for which an expression can be regarded as true are not complementary to the reasons for which it can be regarded as false. Therefore it could be seen as a logic about uncertainty and hesitation. The principal idea introduced by Belnap was to define a logic where the truth values are partially ordered on a bilattice. The DDT logic has been explicitly conceived as a language for preference modelling, a language aiming at capturing hesitation and qualitative uncertainty (see [27]) when decision makers express their preferences.

Graphs have been extensively used as a language for preference modelling (see [24])[1]. A preference relation S on a set of alternatives A may be represented by a directed graph $G = (A, E)$, A being the set of nodes/alternatives and E being the set of edges related to S. The holding convention is that the presence of an arc from a to b stands for the relation aSb being true, while the absence of the arc stands for aSb being not true, hence false (no other epistemic state being considered). Under such a perspective, graph theory, as we know, fits conventional preference modelling where only these two epistemic states are considered, but does not fit for more sophisticated models allowing an explicit representation of ignorance and/or contradiction as with DDT. For this reason, in order to pursue the use of the DDT semantics in preference modelling it turned out natural to consider bi-oriented graphs as an appropriate formalism.

Bi-oriented graphs were introduced by Tutte [31]. A bi-oriented graph is a graph, where each edge is regarded as a set of two half-edges, each half-edge of the graph being equipped with a sign + or -. This concept was already studied for a long time in the theory of homology and algebraic topology [16]. In the 50s the combinatorial aspects of bi-oriented graphs were studied by Harary [14] who defined in 1953 the notion of signed graph (see also [33]).

In our article, we start by showing the relation between DDT semantics and bi-oriented graphs. After this, we analyse the consequences for preference modelling. A special attention is given to the notion of transitive closure in this new representation. Transitive closure is commonly used in preference modelling in order to transform a generic preference relation to an ordering relation. Being in a four valued case, there exist different notions of transitivity that can be defined. In this article we propose eight types of transitivity and we analyse the use of different combinations, some of them providing unique representations for transitive closures.

The paper is organised as follows. In Section 2 we briefly introduce four valued logics as well as the specific language DDT which is a first order language of this type. We also show how this language is used for preference modelling purposes (actually it has been developed for this reason: see [25], [27]). In Section 3, we introduce the principal elements of bi-oriented graphs. In Section 4, we show the existing connections between DDT logic and bi-oriented graphs by just extending the latter to directed graphes. We show, however, that the definition of transitive closure, as conceived in the classical theory of signed graphs is not appropriate for preference modelling purposes. In Section 5, we introduce new types of transitivities, and we generalise and characterise them in Section 6, showing where and how these apply. Section 6 pay special attention to the closure of combination of transitivities. In Section 7, we show the application of the previously introduced concepts to the transitive closure of a whole

[1]including some cases of non conventional preference structures, using valued graphs (see [15]; for a general survey see [21])

graph. We conclude with some remarks and further research directions.

2. The DDT logic (four-valued logic)

2.1. Generalities

Belnap's original proposition ([2]) aimed at capturing situations where hesitation in establishing the truth of a sentence can be associated either to ignorance (poor information) or to contradiction (excess of information). In order to distinguish these two types of uncertainty, he suggested the use of four values forming a bi-lattice. The DDT logic ([25]) is a four-valued first order language extending Belnap's logic in two ways:
- introducing a weak negation which allows to establish a Boolean algebra (an idea inspired to the work of Dubarle; see [12]);
- introducing first order semantics, thus allowing to work with variables.

The language is based on a net distinction between the "negation" which represents the part of the universe of discourse verifying a negated predicate and the "complement" which represents the part of the universe which does not verify a predicate since the two concepts do not necessarily coincide. The four values t (true), f (false), u (unknown) and k (contradiction), capture four epistemic states derived from the presence of information supporting or not a certain sentence. If α is a sentence then:
- α is true (t): there is evidence supporting α and there is no evidence against it;
- α is false (f): there is no evidence supporting α and there is evidence against it;
- α is unknown (u): there is neither evidence supporting α nor against it;
- α is contradictory (k): there is both evidence supporting α and against it.

The differences between the strong negation ($\neg$), the complement ($\sim$) and the weak negation ($\nsim$) are presented in Table 1. The reader will note the Boolean algebra properties this structure allows. It is easy to check that $\sim \alpha \equiv \neg \nsim \neg \nsim \alpha$. Binary connectives are established using the usual Boolean algebra principle (conjunction being the *glb* (see [3], [25]) and disjunction being the *lub*[2] (see [3], [25]) on the bi-lattice of the truth values; for details see [25]).

α	$\neg\alpha$	$\sim \alpha$	$\nsim \alpha$	$\sim\nsim \alpha$	$\neg \nsim \alpha$	$\neg \sim\nsim \alpha$	$\neg \sim \alpha$
t	f	f	k	u	k	u	t
k	k	u	t	f	f	t	u
u	u	k	f	t	t	f	k
f	t	t	u	k	u	k	f

Table 1: The truth tables of $\sim$, $\nsim$ and $\neg$ and their combinations

We give now the definition of some strong monadic operators enabling to obtain "non contradictory" (only true or false) statements for a sentence α.

[2] *glb*: greatest lower bound, *lub*: least upper bound

Definition 2.1.
$\mathbf{T}\alpha \equiv \alpha \ \wedge \ \sim\neg\alpha$*: α is true*
$\mathbf{K}\alpha \equiv \not\sim \alpha \ \wedge \ \not\sim \neg\alpha$*: α is contradictory*
$\mathbf{U}\alpha \equiv \neg \not\sim \alpha \ \wedge \ \neg \not\sim \neg\alpha$*: α is unknown*
$\mathbf{F}\alpha \equiv \neg\alpha \ \wedge \ \sim \alpha$*: α is false*
$\Delta\alpha \equiv \mathbf{T}\alpha \ \vee \ \mathbf{K}\alpha$*: there is presence of truth in claiming α*
$\Delta\neg\alpha \equiv \mathbf{F}\alpha \ \vee \ \mathbf{K}\alpha$*: there is presence of truth in claiming $\neg\alpha$*
$\neg\Delta\alpha \equiv \mathbf{F}\alpha \ \vee \ \mathbf{U}\alpha$*: there is no presence of truth in claiming α*
$\neg\Delta\neg\alpha \equiv \mathbf{T}\alpha \ \vee \ \mathbf{U}\alpha$*: there is no presence of truth in claiming $\neg\alpha$*

Obviously we get:
$\mathbf{T}\alpha \equiv \Delta\alpha \wedge \neg\Delta\neg\alpha$
$\mathbf{F}\alpha \equiv \neg\Delta\alpha \wedge \Delta\neg\alpha$
$\mathbf{U}\alpha \equiv \neg\Delta\alpha \wedge \neg\Delta\neg\alpha$
$\mathbf{K}\alpha \equiv \Delta\alpha \wedge \Delta\neg\alpha$

2.2. DDT and preference modelling

As already mentioned, the DDT logic has been conceived as a language aiming at capturing hesitation when preference statements need to be considered in decision making settings (see [1], [9], [13], [19], [22], [26], [27], [28], [29]).

The basic idea is simple. Consider the typical binary relation used in preference modelling: $S(x, y)$, to be read as "x is at least as good as y". Given a set A on which S applies, we can define a universe of discourse $A \times A$ for the predicate S. If now we allow the interpretations of S in $A \times A$ to be four valued, instead of binary valued as in conventional preference modelling, we obtain a more rich preference modelling language where:
- hesitation about a preference statement can be explicitly considered (for instance $\Delta S(x, y)$ will stand for "there is presence of truth in claiming that x is at least as good as y" or that there are sufficient positive reasons to claim it, see [27]);
- it is possible to construct richer preference structures beyond the well known $\langle P, I, J\rangle$ (preference, indifference, incomparability) ones, allowing for explicit preference relations about conflicting preferences, ignorance about preference etc. (see [28]);
- it is possible to give new and/or more elegant proofs for representation theorems allowing for numerical representations for interval preference structures (see [26], [29]);
- it is possible to conceive new procedures aiming at exploiting such rich preference structures in order to produce a recommendation (see [13], [19], [22]).

With respect to this framework, the reader will note that many of the representation theorems as well as many of the decision support procedures explicitly need to consider extended notions of transitivity, either well known ones such as "semi-transitivity" and "Ferrers" ([21]), or new ones ([23], [18], [20], [26]). Under such a perspective a problem still open in the relevant literature concerns the extension and/or generalisation of the concept of transitive closure, a key issue in many decision support procedures. However, this calls for extending graph theory in order to be able to take into account the new preference structures the DDT logic allows. For this purpose it turned out natural to consider bi-oriented graphs as an appropriate extension of graph theory functional to the DDT language.

3. Bi-oriented graphs

Bi-oriented graphs were introduced by Tutte [31]. Then they were studied by several researchers ([7], [16], [34]), interested in the study of the flows in the bi-oriented graphs, but the notations and the results used in this paper are those used by Bessouf (see [4]and [5]), in which the notions of paths, connectivity and transitive closure were studied.

Consider an undirected graph $G = (V, E)$, V being the set of vertices and E being the set of edges. We denote the edge e between nodes x and y by xy. The set of the half-edges of G is a set $\Phi(G)$ defined as follows:

$$\Phi(G) = \{(xy, x) \in E \times V\}$$

Thus, each edge e between any two vertices x and y is represented by its two half-edges (xy, x) and (xy, y).

Definition 3.1 (Signature). *[4], [5]. A bi-orientation of G is a signature of its half-edges*

$$\tau : \Phi(G) \mapsto \{-1, +1\}$$

A bi-oriented graph is a graph endowed with a bi-orientation τ, denoted as $G_\tau = (V, E; \tau)$ or simply (if there is no ambiguity) by $G_\tau = (V, E)$.

The **Four** possible bi-orientations of the edge xy are shown in Figure 1.

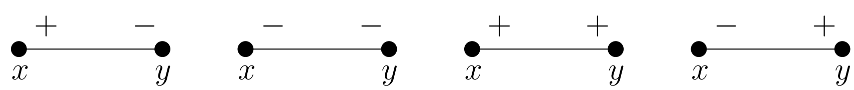


Figure 1: Possible bi-orientations

Example 3.2. *Let $G_\tau = (V, E)$ be a bi-oriented graph with $V = \{1, 2, 3, 4\}$, $E = \{e_1, e_2, e_3, e_4\}$ and $\tau(e_1, 1) = \tau(e_1, 2) = +1, \tau(e_2, 2) = -\tau(e_2, 3) = -1$, $\tau(e_3, 4) = -\tau(e_3, 3) = +1, \tau(e_4, 1) = \tau(e_4, 4) = -1$*

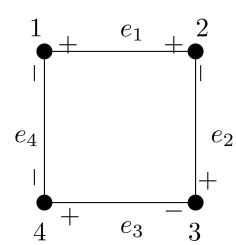


Figure 2: Example of bi-oriented graph

We present in the following some notions that we will use in the rest of the article.

Definition 3.3 (Reorientation, Rotation). *Let $G_\tau = (V, E)$ be a bi-oriented graph with $x, y \in V$ and $xy \in E$.*

- ***Reoriention of*** *xy denoted by $R(.)$: take the opposite sign of each half-edge:*

$$R(\tau(xy, x)) = -\tau(xy, x), R(\tau(xy, y) = -\tau(xy, y)$$

- ***Rotation of*** *xy denoted by $O(.)$: permute the signatures of half-edges.*

$$O(\tau(xy, x)) = \tau(xy, y), O(\tau(xy, y) = \tau(xy, x)$$

Definition 3.4 (Positive-negative edge). *[4], [5]. Let $G_\tau = (V, E)$ be a bi-oriented graph and W (resp. $\overline{W}$) be a function defined on V (resp. E) as follows:*

$W : V \to \mathbb{Z}$ $\qquad$ $\overline{W} : E \to \{-2, 0, 2\}$

$W(x) = \sum_{e \in E, x \in e} \tau(e, x)$ $\qquad$ $\overline{W}(e) = \sum_{x \in V, x \in e} \tau(e, x)$

An edge e of E is called a positive (resp. negative) edge , if $\overline{W}(e) = 0$ (resp. $\overline{W}(e) = \pm 2$). G_τ is called all positive (resp. all negative) bi-oriented graph, if $\forall e \in G_\tau$: $\overline{W}(e) = 0$ (resp. $\overline{W}(e) = \pm 2$). An elementary cycle C in G_τ is called a negative cycle, if the number of its edges such that $\overline{W}(e) = \pm 2$ is odd.

Briefly, $W(x)$ represents the sum of the signatures of half-edges leaving x and $W(e)$ represents the sum of signatures of half-edges of e.

Transitivity is an important concept in preference modeling, since it facilitates the construction of orders and rankings. When a graph representing preferences does not satisfy transitivity it is often the case that we apply transitive closure in order to introduce some "rationality principle". Before presenting the definition of the transitivity in bi-oriented graphs, we need to introduce the notion of "b-path".

Definition 3.5 (b-path). *[4],[5] Let $G_\tau = (V, E)$ be a bi-oriented graph, and x, y, $x_1, x_2, \ldots, x_k$ be nodes in G_τ. Let $P : (x, x_1, x_2, \ldots, x_k, y)$ be a chain connecting x and y in G_τ ($\forall i, (x_i x_{i+1}) \in E$ and $(xx_1), (x_k y) \in E$, with possible cycles) such that $\tau(xx_1, x) = \alpha$ and $\tau(x_k y, y) = \beta$, then P is denoted by $P_{(\alpha,\beta)}(x, y)$ and is called* ***b-path*** *from x^α to y^β if the following conditions hold:*

i. $\tau(x_{i-1}x_i, x_i) + \tau(x_i x_{i+1}, x_i) = 0$, $\forall i \in \{2, k-1\}$

ii. $\tau(xx_1, x_1) + \tau(x_1 x_2, x_1) = 0$ *and* $\tau(x_{k-1}x_k, x_k) + \tau(x_k y, x_k) = 0$

iii. $P_{(\alpha,\beta)}(x, y)$ *is minimal wrt to k for the property (i)-(ii).*

Examples of b-paths with some cycles can be seen in Figure 3. Note that (a, a_1, b) is not a b-path since $\tau(aa_1, a_1) + \tau(a_1 b, a_1) \neq 0$ and there are two b-paths between x and y with different signatures: $P_{(+,+)}(x, y)$ with (x, x_1, x_2, x_3, y) and $P_{(-,+)}(x, y)$ with (x, x_3, y). Remark that if $P_{(\alpha,\beta)}(x, y)$ is a b-path from x^α to y^β, then $P_{(\beta,\alpha)}(y, x)$ is also a b-path from y^β to x^α.

We can now introduce a first notion of transitivity in bi-oriented graphs.

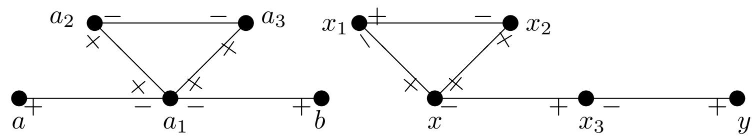
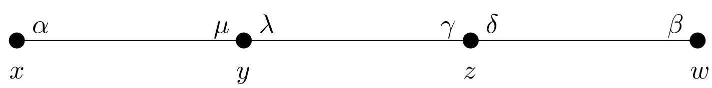


Figure 3: b-paths : $P_{(+,+)}(a,b)$ with $(a, a_1, a_2, a_3, a_1, b)$, $P_{(+,+)}(x,y)$ with (x, x_1, x_2, x, x_3, y) and $P_{(-,+)}(x,y)$ with (x, x_3, y)

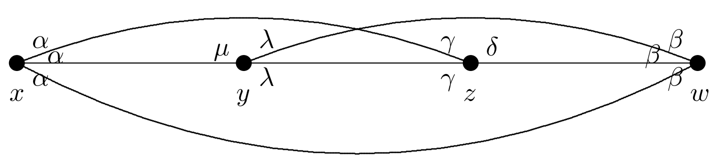
Definition 3.6. *(***Transitivity in Bi-oriented Graphs***) [6]. Let $G_\tau = (V,E)$ be a bi-oriented graph with $|V| \geq 3$. G_τ is transitive if for any vertices x and y (not necessarily distinct) such that there is an (α,β) b-path from x to y in G_τ, there is an edge ($x^\alpha y^\beta$) in G_τ.*

The two graphs represented in Figure 3 are not transitive because of many violations. For instance, in the graph on the left, we see that there is a b-path $P_{(+,+)(a,b)}$ but there is no arc (a^+, b^+). Note that there are many other violations $((a^+, a_2^+), (a^+, a_3^-), (a_1^+, a_3^-), ...)$. A way to handle non transitive graphs is to find a transitive closure of this graph by adding "missing arcs". We present in the following the definition of a transitive closure on a bi-oriented graph given by Bessouf and her colleagues ([6]).

Definition 3.7. *(***Transitive Closure in Bi-oriented Graphs***) [6]. Let $G_\tau = (V,E)$ be a bi-oriented graph with $|V| \geq 3$. The transitive closure of G_τ is the graph denoted Ft(G_τ) = (V, E', τ) such that for all b-paths $P_{(\alpha,\beta)}(x,y)$ of G_τ there is an edge $(xy) \in E'$ such that $\tau(xy,x) = \alpha$, $\tau(xy,y) = \beta$.*



initial bi-oriented graph G_τ with $\mu + \lambda = \gamma + \delta = 0$



transitive closure $Ft(G_\tau)$

Figure 4: An example of transitive closure of bi-oriented graph

Remark 3.8. *The transitive closure adds a unique arc between two nodes x and y if they are only connected by a unique b-path. The uniqueness is due to the fact that the signatures of the added arc between x and y does not depend on the order of intermediate added arcs in the b-path. Let us explain it with an example: Consider the graph G_τ presented in Figure 4. G_τ is not transitive. $Ft(G_\tau)$ represents it's transitive closure where three new arcs are added.*

Note that if a chain (x, y, z, w) is a $P_{\alpha,\beta}(x, w)$ b-path then

- x, y, z is a $P_{\alpha,\gamma}(x, z)$ b-path with $\gamma = \tau(yz, z)$,
- y, z, w is $P_{\lambda,\beta}(y, w)$ b-path with $\lambda = \tau(yz, y)$.

Hence, in Figure 4 the edges (x^α, z^γ) and (y^λ, w^β) must be added in the transitive closure. Remark that adding (x^α, z^γ) creates a new b-path $P_{\alpha,\beta}(x, w)$ with nodes (x, z, w). Similarly adding (y^λ, w^β) creates a new b-path $P_{\alpha,\beta}(x, w)$ with nodes (x, y, w). This shows us that the transitive closure is **associative** in the sense that when there is a b-path with the chain (x, y, z, w), we can first do the transitive closure on the chain (x, y, z) and do the transitive closure of this with the edge (zw) or we can first do the transitive closure on the chain (y, z, w) and do the transitive closure of this with the edge (xy). This notion of associativity can be easily generalized to the case of b-path with k nodes. In other terms, the transitive closure of any sequence of arcs, if exists, is unique when the nodes are connected by not more then one b-path (it does not depend on the order of added edges for a b-path containing smaller b-paths).

Remark 3.9. *Let us analyse now what happens if there are more then one b-path between two nodes or if there is already an arc between them: One can not no more guarantee the uniqueness of added signatures. The right hand graph of Figure 3 shows that the transitive closure provides two different signatures between x and y since there are $P_{(+,+)}(x, y)$ and $P_{(-,+)}(x, y)$.*

Note that preference modelling is based on binary relations which need a representation with directed graphs (x being preferred to y is different from y being preferred to x). Hence, we need to extend the notion of bi-oriented graph to directed ones. This is essentially straightforward: given any two nodes x and y of G_τ we keep the notation xy for any arc, but we distinguish the edges xy and yx as two different arcs: we get two edges xy and yx and four signatures $\tau(xy, x), \tau(xy, y), \tau(yx, y), \tau(yx, x)$ which are independent (for examples see Figure 6 and 7 in section 4). We can easily translate the definition of transitive-closure in the directed case, except that the existence of a b-path $P_{\alpha,\beta}(x, y)$ does not imply the existence of a b-path $P_{\beta,\alpha}(y, x)$ in the directed graph. The operations of re-orientation and rotation are identical as in Definition 3.3. We introduce two new operations as in the following.

Definition 3.10 (Left-switching, Right-switching). *Let $G_\tau = (V, E)$ be a bi-oriented graph with $x, y \in V$ and $(xy) \in E$.*

- ***Left-switching of*** *xy denoted by $H_l(.)$: take the opposite sign for left*

$$H_l(\tau(xy,x)) = -\tau(xy,x), (H_l(\tau(xy,y)) = \tau(xy,y)$$

• ***Right-switching of*** *xy denoted by $H_r(.)$: take the opposite sign for right*

$$H_r(\tau(xy,x)) = \tau(xy,x), (H_r(\tau(xy,y)) = -\tau(xy,y)$$

4. The relations between bi-oriented graphs and the four valued logic (DDT logic)

Let A be a discrete countable set and let S be the binary relation "at least as good as" applied upon A. The four strong monadic operators on S, $\mathbf{T}S(x,y)$, $\mathbf{F}S(x,y)$, $\mathbf{U}S(x,y)$ and $\mathbf{K}S(x,y)$, are defined as follows:
TS(x,y): there exist sufficient positive reasons to establish $S(x,y)$ and there are not enough negative reasons to establish $\neg S(x,y)$; $S(x,y)$ is true
FS(x,y): there do not exist sufficient positive reasons to establish $S(x,y)$ and there exist enough negative reasons to establish $\neg S(x,y)$; $S(x,y)$ is false.
US(x,y): there do not exist sufficient positive reasons to establish $S(x,y)$ and there are not enough negative reasons to establish $\neg S(x,y)$; $S(x,y)$ is unknown.
KS(x,y): there exist sufficient positive reasons to establish $S(x,y)$ and sufficient negative reasons to establish $\neg S(x,y)$; $S(x,y)$ is contradictory.

More formally, we accept that S and $\neg S$ are not complementary and they do not cover the whole set of possible situations. We can express this idea by introducing the sentence $\Delta S(x,y)$:
• $\Delta S(x,y)$: there is presence of truth in claiming that x is at least as good as y (presence of positive reasons)
• $\Delta\neg S(x,y)$: there is presence of truth in claiming that x is not at least as good as y (presence of negative reasons)
• $\neg\Delta S(x,y)$: there is no presence of truth in claiming that x is at least as good as y (absence of positive reasons)
• $\neg\Delta\neg S(x,y)$: there is no presence of truth in claiming that x is not at least as good as y (absence of negative reasons)

Consequently we have:
$\mathbf{T}S(x,y) \Leftrightarrow \Delta S(x,y) \wedge \neg\Delta\neg S(x,y)$
$\mathbf{F}S(x,y) \Leftrightarrow \neg\Delta S(x,y) \wedge \Delta\neg S(x,y)$
$\mathbf{U}S(x,y) \Leftrightarrow \neg\Delta S(x,y) \wedge \neg\Delta\neg S(x,y)$
$\mathbf{K}S(x,y) \Leftrightarrow \Delta S(x,y) \wedge \Delta\neg S(x,y)$

The result of such definitions is that we can extend the notion of preference structure (see [27], [28]) and obtain precise definitions for structures such as strict preference ($\mathbf{T}S(x,y) \wedge \mathbf{F}S(y,x)$) and weak preference ($\mathbf{K}S(x,y) \wedge \mathbf{F}S(y,x)$). We add to Tsoukiàs and Vincke's results a graphic representation given as follows: Let S be the binary relation given above defined on A and G be a bi-oriented digraph with nodes in A. $\forall x, y \in A$ we put:

• $\Delta S(x,y) \rightarrow \tau(xy,x) = +1$
• $\neg\Delta S(x,y) \rightarrow \tau(xy,x) = -1$
• $\Delta\neg S(x,y) \rightarrow \tau(xy,y) = +1$
• $\neg\Delta\neg S(x,y) \rightarrow \tau(xy,y) = -1$

Henceforth, a bi-oriented digraph endowed with the relation S will be noted $G_\tau = (A, S)$. The graphic representations of $\mathbf{T}S(x,y), \mathbf{F}S(x,y), \mathbf{K}S(x,y)$ and $\mathbf{U}S(x,y)$ are given in Figure 5, the orientation of the edges means that the relation $S(x,y)$ is from x to y.

Figure 5: Bi-oriented graphs of $\mathbf{T}S$, $\mathbf{F}S$, $\mathbf{K}S$, $\mathbf{U}S$ from x to y

Generally speaking, in preference modelling there is no relation between xSy and ySx. Considering that each of these sentences is four valued, there are 16 different possible combinations between xSy and ySx. Some of them are easy to interpret, for instance, $(\mathbf{T}S(x,y), \mathbf{T}S(y,x))$ shows an indifference between x and y while others are more complicated such as $(\mathbf{K}S(x,y), \mathbf{U}S(y,x))$. Other examples are given in Figure 6 (the strict preference relation $(\mathbf{T}S(x,y) \wedge \mathbf{F}S(y,x))$, according to [27]) and in Figure 7 (the weak preference relation $(\mathbf{K}S(x,y) \wedge \mathbf{F}S(y,x)$, according to [22],[27]).

Remark 4.1. *For the rest of our article, we suppose that our graphs are fully "signed", in the sense that for any two nodes x and y, the signatures $\tau(xy,x)$, $\tau(xy,y)$, $\tau(yx,y)$, $\tau(yx,x)$ are known. For the sake of visibility, if it is not necessary to emphasize, we will omit to draw arcs which are unknown ($\tau(xy,x) = -1$, $\tau(xy,y) = -1$). This is consistent with the convention adopted in conventional digraphs: non true arcs (false arcs) are omitted, but we know (or conventionally consider them) they are false.*

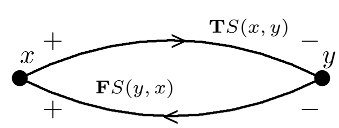


Figure 6: x is strictly preferred to y

Let ϕ be a predicate admitting a representation through a bi-oriented digraph (in other terms, a binary relation admitting four truth values). We indicate by:

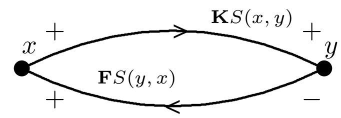


Figure 7: x is weakly preferred to y

- $H_r(\phi)$ $(resp.H_l(\phi))$ the right switching (resp. left switching) of ϕ.
- $R(\phi)$ the reorientation of ϕ.
- $T(\phi)$ the rotation of ϕ.

Proposition 4.2. *Let $\psi \in \{\mathbf{T}, \mathbf{F}, \mathbf{K}, \mathbf{U}\}$ and S be the usual relation, then :*

1. $H_r(\psi(S)) = \psi(\nsim S)$
2. $H_l(\psi(S)) = \psi(\sim\nsim S)$
3. $R(\psi(S)) = \psi(\sim S)$
4. $T(\psi(S)) = \psi(\neg S)$

Proof. From Definition 2.1 it is easy to show that for any given formula ϕ:
- $\mathbf{T}\neg\phi \equiv \mathbf{F}\phi$, $\mathbf{T} \nsim \phi \equiv \mathbf{K}\phi$, $\mathbf{T} \sim \phi \equiv \mathbf{F}\phi$, $\mathbf{T} \sim\nsim \phi \equiv \mathbf{U}\phi$
- $\mathbf{K}\neg\phi \equiv \mathbf{K}\phi$, $\mathbf{K} \nsim \phi \equiv \mathbf{T}\phi$, $\mathbf{K} \sim \phi \equiv \mathbf{U}\phi$, $\mathbf{K} \sim\nsim \phi \equiv \mathbf{F}\phi$
- $\mathbf{U}\neg\phi \equiv \mathbf{U}\phi$, $\mathbf{U} \nsim \phi \equiv \mathbf{F}\phi$, $\mathbf{U} \sim \phi \equiv \mathbf{K}\phi$, $\mathbf{U} \sim\nsim \phi \equiv \mathbf{T}\phi$
- $\mathbf{F}\neg\phi \equiv \mathbf{T}\phi$, $\mathbf{F} \nsim \phi \equiv \mathbf{U}\phi$, $\mathbf{F} \sim \phi \equiv \mathbf{T}\phi$, $\mathbf{F} \sim\nsim \phi \equiv \mathbf{K}\phi$

Applying the definition of H_r, H_l, R and T of an edge from Definition 3.3 and from Figure 5 we complete the proof. ■

The result can be shown in the following table.

ψ	$H_r(\psi(S))$	$H_l(\psi(S))$	$R(\psi(S))$	$T(\psi(S))$
$\mathbf{T}S$	$\mathbf{K}S$	$\mathbf{U}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{F}S$	$\mathbf{U}S$	$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{U}S$	$\mathbf{K}S$
$\mathbf{U}S$	$\mathbf{F}S$	$\mathbf{T}S$	$\mathbf{K}S$	$\mathbf{U}S$

Once established a first correspondence between the extended preference structures introduced in [27] and [28], it is tempting to check whether the definition of transitivity and of transitive closure as introduced in Definition 3.7, can be used for preference modelling and decision making purposes.

We showed in the last section that the transitive closure is associative and the transitive closure of any sequence of arcs forming a unique b-path, if exists, is unique (see Remark 3.8). However, as we will show in the next proposition, the computing of the signatures of the arcs added due to the transitive closure is not at all satisfactory with respect to the semantics of preference modelling.

Proposition 4.3. *Let (G_τ) be a bi-oriented digraph endowed with the binary relation S, such that $G_\tau = (A, S)$. Adopting the operation of transitivity in $Ft(G_\tau)$ as introduced in Definition 3.6 $\forall x, y$ and $z \in A$ we get the following*[3]*:*

$\mathbf{T}S(x,y) \wedge \mathbf{T}S(y,z) \to \mathbf{T}S(x,z)$
$\mathbf{T}S(x,y) \wedge \mathbf{K}S(y,z) \to \mathbf{K}S(x,z)$
$\mathbf{F}S(x,y) \wedge \mathbf{F}S(y,z) \to \mathbf{F}S(x,z)$
$\mathbf{F}S(x,y) \wedge \mathbf{U}S(y,z) \to \mathbf{U}S(x,z)$
$\mathbf{U}S(x,y) \wedge \mathbf{T}S(y,z) \to \mathbf{U}S(x,z)$
$\mathbf{U}S(x,y) \wedge \mathbf{K}S(y,z) \to \mathbf{F}S(x,z)$
$\mathbf{K}S(x,y) \wedge \mathbf{F}S(y,z) \to \mathbf{K}S(x,z)$
$\mathbf{K}S(x,y) \wedge \mathbf{U}S(y,z) \to \mathbf{T}S(x,z)$

Proof. Obvious. ■

Remark that only half of the 16 situations between $S(x,y), S(y,z)$ are covered by the previous proposition since within the other 8 situations, the chain xyz is not a b-path. This has some non intuitive conclusions for preference modelling:

- One can not conclude $\mathbf{U}S(x,z)$ even if $\mathbf{U}S(x,y)$ and $\mathbf{U}S(y,z)$ hold. The same remark is also valid for the case $\mathbf{K}S(x,y)$ and $\mathbf{K}S(y,z)$.
- Implications shown in Proposition 4.3 are not symmetric with respect to the left-hand components of the implication. For instance, while $\mathbf{T}S(x,y) \wedge \mathbf{K}S(y,z) \to \mathbf{K}S(x,z)$ holds, there is NO implication in form of "$\mathbf{K}S(x,y) \wedge \mathbf{T}S(y,z) \to \mathbf{K}S(x,z)$" since the chain xyz is not a b-path.

We think that such a symmetry of the left-hand components is necessary and coherent with the semantic of the preference modelling.

As a direct consequence of Remark 3.9, one can not guarantee the uniqueness of added/changed signatures at the end of the transitive closure. For instance, if we have $\mathbf{F}S(x,y), \mathbf{U}S(y,z), \mathbf{T}S(x,t)$ and $\mathbf{K}S(t,z)$. The transitive closure implies $\mathbf{U}S(x,z)$ because of the b-path xyz and $\mathbf{K}S(x,z)$ because of the b-path xtz.

Moreover, the fact that a transitive closure between x, y and z needs $\tau(xy,y) + \tau(yz,y) = 0$ has no meaning for preference modeling (why do we need $\Delta\neg S(x,y) + \Delta S(x,y) = 0$ in order to define the transitive in preference modelling?).

For all these reasons, in the next section, we will propose some new interpretations and definitions of transitivity distinguishing positive and negative reasons. Our propositions will be motivated from a preference modelling point of view and we will use bi-oriented digraph modelling.

5. New Transitivities

As commented at the end of the last section, we introduce a number of transitivity definitions based on the DDT language, all of them satisfying the symmetry on the left-hand components of the transitivity. $\forall x, y, z$ such that $x \neq y, x \neq z$ and $y \neq z$:

[3]Note that the remaining 8 cases do not provide a b-path, hence they are not concerned by transitive closure.

1. $\Delta S(x,y) \wedge \Delta S(y,z) \rightarrow \Delta S(x,z)$ (transitivity of positive reasons ΔS)
2. $\neg\Delta S(x,y) \wedge \neg\Delta S(y,z) \rightarrow \neg\Delta S(x,z)$ (negative transitivity of positive reasons ΔS)

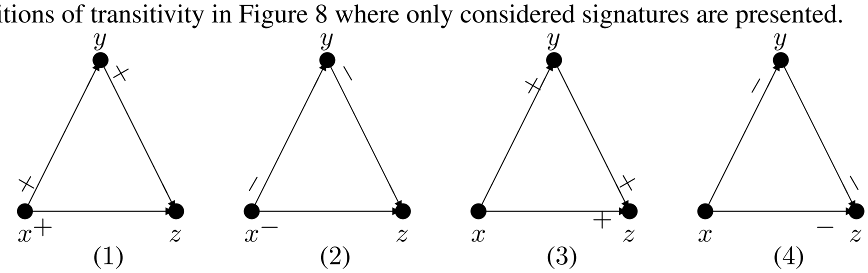
The above definitions represent how transitivity applies to the positive part of the preference modelling reasoning. Substituting to ΔS, $\Delta\neg S$ we get the equivalent notion of transitivity for the negative part of the preference modelling reasoning.

3. $\Delta\neg S(x,y) \wedge \Delta\neg S(y,z) \rightarrow \Delta\neg S(x,z)$ (transitivity of negative reasons $\Delta\neg S$)
4. $\neg\Delta\neg S(x,y) \wedge \neg\Delta\neg S(y,z) \rightarrow \neg\Delta\neg S(x,z)$ (negative transitivity of negative reasons $\Delta\neg S$)

These four definitions of transitivity combine the same type of information (presence or absence of positive or negative reasons), hence they respect the symmetry condition that we are looking for by definition. We denote these as "direct transitivities". We further introduce four new definitions of transitivity, hereby shown as cases 5, 6, 7 and 8 where we combine positive and negative reasons aiming at creating (positive or negative) reasons. Such a definition being not symmetric on the left-hand components of the transitivity, we will impose the symmetry. $\forall x, y, z$ such that $x \neq y, x \neq z$ and $y \neq z$:

5. $\Delta S(x,y) \wedge \neg\Delta\neg S(y,z) \rightarrow \Delta S(x,z)$ (creating positive reasons)
   $\neg\Delta\neg S(x,y) \wedge \Delta S(y,z) \rightarrow \Delta S(x,z)$
6. $\Delta\neg S(x,y) \wedge \neg\Delta S(y,z) \rightarrow \neg\Delta S(x,z)$ (eliminating positive reasons)
   $\neg\Delta S(x,y) \wedge \Delta\neg S(y,z) \rightarrow \neg\Delta S(x,z)$
7. $\Delta\neg S(x,y) \wedge \neg\Delta S(y,z) \rightarrow \Delta\neg S(x,z)$ (creating negative reasons)
   $\neg\Delta S(x,y) \wedge \Delta\neg S(y,z) \rightarrow \Delta\neg S(x,z)$
8. $\Delta S(x,y) \wedge \neg\Delta\neg S(y,z) \rightarrow \neg\Delta\neg S(x,z)$ (eliminating negative reasons)
   $\neg\Delta\neg S(x,y) \wedge \Delta S(y,z) \rightarrow \neg\Delta\neg S(x,z)$

We will denote such type of transitivity as indirect and present graphically the eight definitions of transitivity in Figure 8 where only considered signatures are presented.



Let's see now what happens if we assume any of the above 8 transitivity holding. There are 255 of such combinations ($2^8 - 1$). The interested reader can check them in Annex A. Table 2 presents the result when each of the above transitive closures holds alone. Each type of transitivity is represented by its number; for instance the first table numbered 1 represents the transitivity of positive reasons. Table 3 presents the results when the four direct and the four indirect definitions of transitivity hold simultaneously.

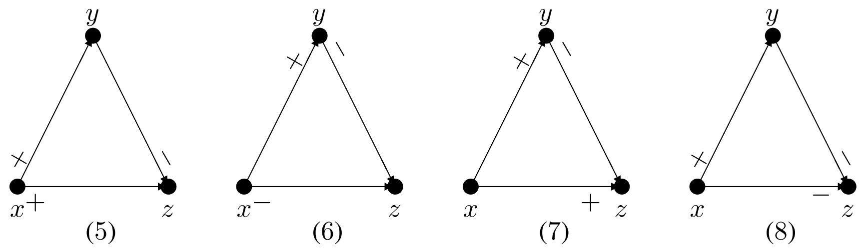


Figure 8: Graphical representation of the 8 transitive closures

Finally, the case where all 8 definitions of transitivity hold simultaneously is presented in Table 4. The tables stand for sentences of the type $\psi S(x,y) \ \wedge \ \psi S(y,z)$ where $\psi \in \{\mathbf{T},\mathbf{K},\mathbf{U},\mathbf{F}\}$ (rows will stand for $S(x,y)$ and columns for $S(y,z)$).

Discussion. First of all the reader should note that there are no other possible "rational" definitions of transitivity we can define within this framework. The four direct definitions of transitivity represent the natural extension of the notion of transitivity within our framework (symmetric combination of the presence or the absence of positive or negative reasons). The four indirect ones combine asymmetrically the presence (or absence) of positive (or negative reasons), but with a symmetric result (on the left-hand components). Analysing the different combinations of such definitions of transitivity we can observe that:

- Direct transitivity of $\mathbf{T},\mathbf{F},\mathbf{K},\mathbf{U}$ [4] is obtainable by using the definitions labelled 1,2,3 and 4 simultaneously. The only simultaneous uses of transitivity satisfying this property contain definitions 1,2,3 and 4.
- Definitions labelled 5,6,7 and 8 when applied simultaneously they will yield only true or false statements (see Table 3).
- All the tables are symmetric with respect to the diagonal.
- Definitions 5 and 6 have a special attitude when we have to combine unknown cases with contradictory ones. They provide contradictory conclusions. For instance, with ($\mathbf{K}$ and $\mathbf{U}$), 5 implies ΔS while 6 implies $\neg\Delta S$. Hence, we conclude that if we impose 5 and 6 together, we will not have any conclusion for $\mathbf{K}$ and $\mathbf{U}$ (similarly for $\mathbf{U}$ and $\mathbf{K}$). Because of a similar reasoning, there are no conclusion for ($\mathbf{K}$ and $\mathbf{U}$) or ($\mathbf{U}$ and $\mathbf{K}$) when 7 and 8 are imposed together.
- More detailed analysis of the 255 combinations allows to reveal which are the minimal conditions in order to obtain a precise result (i.e. transitivity of ΔS or of $\mathbf{K}S$ etc.).

[4]meaning that $\mathbf{T}S(x,y)$ and $\mathbf{T}S(y,z)$ implies $\mathbf{T}S(x,z)$, ...

1	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	ΔS	-	ΔS	-
$\mathbf{U}S$	-	-	-	-

2	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	-	-	-	-
$\mathbf{U}S$	-	$\neg\Delta S$	-	$\neg\Delta S$

3	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{U}S$	-	-	-	-

4	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	-	-	-	-
$\mathbf{U}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$

5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	ΔS	-	-	ΔS
$\mathbf{U}S$	ΔS	-	ΔS	-

6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	-

7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	-	$\Delta\neg S$	-	$\Delta\neg S$
$\mathbf{U}S$	-	$\Delta\neg S$	$\Delta\neg S$	-

8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-

Table 2: The eight basic transitive closures

1234	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$

5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-

Table 3: Applying transitive closures 1,2,3 and 4 or 5,6,7 and 8 simultaneously

12345678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\mathbf{U}S$

Table 4: Combining all eight transitive closures

6. Transitive Closure in Bi-oriented Di-graphs

Let's try to establish a more general and formal framework about transitivity. As a concept transitivity implies the capacity to transfer pieces of information along the arcs of a graph. If an information holds (or not) in a sequence of arcs we make the hypothesis that this information also holds (or not) for the arc connecting the extremes of the path. Presence of information can be transferred (transitivity) as well the absence of information (negative transitivity). This notion of "information transfert" is introduced in the case of classical graphs by the definition of a transitive closure. Let us remind how transitive closure works in the case of directed graphs $G(A, S)$: for all x, y, z, if $S(x, y)$ and $S(y, z)$ hold, then we will add $S(x, z)$ if the arc does not exist[5]. In the same way one can define "negative transitive closure" meaning that for all x, y, z, if neither $S(x, y)$ nor $S(y, z)$ hold, then we will remove $S(x, z)$ if the arc does already exist.

The aim of this section is to try to define transitive closures when bi-oriented digraphs are used. We saw in the previous section that different type of definitions of transitivity can be defined with bi-oriented digraphs. We also saw that the definition of transitive closure (Definition 3.7) is not satisfying for preference modelling. In this section we will first define the type of chains where an information transfer is meaningful, then we will analyse the consequences of a generalisation of transitive closure where the definitions of transitivity introduced in Section 5 are used.

Let us start by "acceptable" chains for an information transfer. As it will be clear in Definition 6.1, we will accept all the chains except the ones containing consecutive $\mathbf{T}$ and $\mathbf{F}$ (or the inverse, $\mathbf{F}$ and $\mathbf{T}$) or $\mathbf{K}$ and $\mathbf{U}$ (or the inverse, $\mathbf{U}$ and $\mathbf{K}$).

Definition 6.1. *A t-path is any sequence of directed bi-oriented edges which does not contain any sequence x, y, z such that:*

- $\tau(xy, x) = 1$; - $\tau(xy, y) = -1$; - $\tau(yz, y) = -1$; - $\tau(yz, z) = 1$;

or

- $\tau(xy, x) = -1$; - $\tau(xy, y) = 1$; - $\tau(yz, y) = 1$; - $\tau(yz, z) = -1$;

or

- $\tau(xy, x) = -1$; - $\tau(xy, y) = -1$; - $\tau(yz, y) = 1$; - $\tau(yz, z) = 1$;

or

- $\tau(xy, x) = 1$; - $\tau(xy, y) = 1$; - $\tau(yz, y) = -1$; - $\tau(yz, z) = -1$.

Remark: The edges of the sequence x, y, z in the Definition 6.1 are positive edges in the first case and are negative edges in the second case (see Definition 3.4). The reader will note that it does not make any sense to establish a transitive closure among edges

[5]If we interpret this by classical logic, saying that the existence of an arc $S(x, y)$ means that the affirmation $S(x, y)$ is true and the absence $S(x, y)$ is false, then we can say that the transitive closure replace the value false of $S(x, z)$ by the value true if $S(x, y)$ and $S(y, z)$ are true.

which do not form a t-path. Why to combine a true value with a false one or a contradictory information with an unknown one? As we showed in the previous section, there exist 255 possible combinations of the eight definitions of transitivity. Hence, we will not formally define the closures of different combinations of transitivities one by one, but present a general idea of transitive closure when a combination of definitions of transitivity is used. Remark that all the definitions are in form of an implication: $if....then.....$ Hence, we say that a transitive closure of a combination of definitions of transitivity consists on adding the signature defined in the right hand of the implication if the left hand is satisfied ; and we will do this only on t-paths. If the new signatures are in contradiction with the existing ones, the existing ones are replaced by the new ones. Before analysing closures in bi-oriented digraphs, let us remind that in the case of classical digraphs, a simultaneous use of transitive closure and negative transitive closure[6] generally leads to inconsistencies. Let us show it by a simple example. Consider the case $A = \{x, y, z, t\}$ with only $S(x, z), S(z, y)$. The positive transitive closure will add $S(x, y)$, while the negative transitive closure will keep $\neg S(x, y)$ because of $\neg S(x, t)$ and $\neg S(t, y)$.

Let us analyse know, what happens if we want to use all the 8 definitions of transitivity simultaneously. Let us denote "generalized transitivity" the combination of the eight definitions of transitivity presented in the previous section. Definition 6.2 presents "generalized transitivity" in form of implications.

Definition 6.2. *Let $G_\tau = (A, S)$ be a bi-oriented digraph, S satisfies generalized transitivity if and only if $\forall x, y, z \in A$:*
- ***IF*** $\tau(xy, x) = \tau(yz, y)$ ***AND*** $\tau(xy, y) = \tau(yz, z)$ ***THEN***
- $\tau(xz, x) = \tau(xy, x) = \tau(yz, y)$ ***AND*** $\tau(xz, z) = \tau(xy, y) = \tau(yz, z)$
- ***OTHERWISE***
- $\tau(xz, x) = \max(\tau(xy, x), \tau(yz, z)) * \tau(xy, x) * \min(\tau(xy, y), \tau(yz, y)) * \tau(xy, y)$
- $\tau(xz, z) = \min(\tau(xy, x), \tau(yz, z)) * \tau(xy, x) * \max(\tau(xy, y), \tau(yz, y)) * \tau(xy, y)$

Proposition 6.3. *Let $G(A, S)$ be a bi-oriented digraph, S satisfies generalized transitivity if and only if S satisfies any combination of the eight definitions of transitivity presented in Section 5.*

Proof. The proof is left to the reader. ■

Let us analyse what happens if we want to use a generalised transitivity closure. We start by a simple case which is a graph in form of a chain.

Proposition 6.4. *Let $G(A, S)$ be a bi-oriented digraph in form of a chain. The paths which will be created if the generalized transitive closure is applied within a t-path, are not always t-paths.*

Proof. It is sufficient to consider a sequence of three edges from x to y to z to w such that $\tau(xy, x) = 1, \tau(xy, y) = -1, \tau(yz, y) = -1, \tau(yz, z) = -1, \tau(zw, z) =$

[6] $\forall x, y, z\ \neg S(x, y) \wedge \neg S(y, z) \implies \neg S(x, z)$

$-1, \tau(zw, w) = 1$. The sequence $xyzw$ is a t-path, but establishing the transitive closure between x and z provides $\mathbf{T}S(x, z)$ then the chain xzw is no more a t-path (in the same way adding $\mathbf{F}S(y, w)$ will result in a sequence xyw which is not a t-path). ■

In other terms it might be the case that despite a sequence of arcs is a t-path it may happen that we can not apply the closure on all the arcs through successive transitive closures (for instance, in the example of the proof we can not add signatures on the arc between x and w even if $xyzw$ is a t-path.) Another interesting proposition concern chains which are not t-paths.

Proposition 6.5. *Let $G(A, S)$ be a bi-oriented digraph in form of a chain. A sequence of arcs which is not a t-path can admit "shortest" paths which are t-paths, constructed if generalized transitive closures is applied.*

Proof. It is sufficient to consider a sequence of three edges from x to y to z to w such that $\tau(xy, x) = 1, \tau(xy, y) = 1, \tau(yz, y) = -1, \tau(yz, z) = -1, \tau(zw, z) = -1, \tau(zw, w) = 1$. The sequence $xyzw$ is not a t-path, but the sequence yzw it is. The transitive closure will yield the signature $\tau(yw, y) = -1, \tau(yw, w) = 1$ and this allows to establish a t-path xyw. The transitive closure will yield the signature $\tau(xw, x) = -1, \tau(xw, w) = 1$. ■

We can introduce categories of graphs for which the generalized transitive closure does not induce the previously mentioned situations.

Definition 6.6. *We define as strong t-path (and we denote it a t_s-path) any t-path which remains such under any sequence of generalized transitive closures.*

The reader can easily verify that given a bi-oriented digraph $G_\tau = (A, S)$ with all negative signatures, every t-path is a t_s-path. Same result applies in case of all positive signatures.

As we already mentioned, when new signatures are added because of a closure, it is important to know if there could be created multiple inconsistent signatures. Proposition 6.7 will show that when the graph is in form of a chain then this situation is impossible because the new computed signatures are unique. Before presenting this-Proposition 6.7, let us analyse the associativity of a closure on a chain $(xyzt)$ where new arcs will be added in two steps:

- first, an arc between x and z (because of $S(x, y)$ and $S(y, z)$) and another arc between y and t (because of $S(y, z)$ and $S(z, t)$),

- then, an arc between x and t (because of $S(x, y)$ and $S(y, t)$, let us denote it by $S'(x, t)$) and also an arc between x and t (because of $S(x, z)$ and $S(z, t)$, let us denote it by $S''(x, t)$.

An exhaustive analysis of 64 possibles cases (4 values for each of the three arcs of $(xyzt)$), shows that $S'(x, t) = S''(x, t)$ for all chains if generalized transitivy is used, meaning that the order to complete the chain is not important for a closure in a chain. Table 6 recapitulates these 64 cases where $V(S(x, y), S(y, z), S(z, t))$ represents the values of the three arcs of the chain $(xyzt)$ and $S(x, t)$ a new added arc

$S(x,t)$	$V(S(x,y),S(y,z),S(z,t))$
K	**(K, K, K)**
U	**(U, U, U)**
F	**(F, F, F)** **(F, F, K), (F, K, F), (K, F, F)** **(F, F, U), (F, U, F), (U, F, F)** **(F, K, K), (K, K, F), (K, F, K)** **(F, U, U), (U, U, F), (U, F, U)** **(U, F, K), (U, K, F), (K, F, U), (K, U, F), (F, K, U), (F, U, K)**
T	**(T, T, T)** **(T, T, K), (T, K, T), (K, T, T)** **(T, T, U), (T, U, T), (U, T, T)** **(T, K, K), (K, K, T), (K, F, T)** **(T, U, U), (U, U, T), (U, T, U)** **(U, T, K), (U, K, T), (K, T, U), (K, U, T), (T, K, U), (T, U, K)**
no added arc j	**(T, T, F),(T, F, T), (F, T, T)** **(T, F, F), (F, T, F),, F, F, T)** **(T, F, K),(T, K, F), (K, F, T),(K, T, F), (F, K, T),(F, T, K)** **(T, F, U),(T, U, F), (U, F, T),(U, T, F), (F, U, T),(F, T, U)** **(U, K, K),(K, U, K), (K, K, U),** **(K, U, U),(U, K, U), (U, U, K)**

Table 5: Given a chain $(xyzt)$, the value of $S(x,t)$ obtained by a closure with generalized transitivity.

because of a closure with generalized transitivity. Interestingly the order of values in $V(S(x,y),S(y,z),S(z,t))$ is not important for the the determination of the value of the added arc $S(x,t)$.

Proposition 6.7. *Let $G(A,S)$ be a bi-oriented graph in form of a chain. The closure of generalized transitivity applied on t-paths of $G(A,S)$ yields a unique signature for any pair of arcs of $G(A,S)$.*

Proof. In Table 6 we show the signature of the transitive closure when all 8 possible closures apply simultaneously on a chain with three nodes: the signature is unique (or it does not exist because of the presence of a non t-path). The reader will note that this table corresponds exactly to Table 4. In order to complete the proof we need to check what happens for all possible combinations of the 8 possible transitive closures and use the fact that the procedure of closure construction is associative (the order of adding new arcs is not important). But this is exactly what the 255 tables in Annex show. For each of them we can construct the corresponding signature table as in Table 6. And this completes the proof. ■

We saw that when the generalized transitive closure is applied to a digraph in form of a chain then the new signatures are unique. However, we will see that this is not always the case if the graph is not a simple chain.

$\tau(xy,x)$	$\tau(xy,y)$	$\tau(yz,y)$	$\tau(yz,z)$	$\tau(xz,x)$	$\tau(xz,z)$
+	+	+	+	+	+
+	+	+	-	+	-
+	+	-	+	-	+
+	+	-	-	not t-path	
+	-	+	+	+	-
+	-	+	-	+	-
+	-	-	+	not t-path	
+	-	-	-	+	-
-	+	+	+	-	+
-	+	+	-	not t-path	
-	+	-	+	-	+
-	+	-	-	-	+
-	-	+	+	not t-path	
-	-	+	-	+	-
-	-	-	+	-	+
-	-	-	-	-	-

Table 6: Table of signatures

Let $G(A,S)$ be a bi-oriented digraph with $A = \{x,y,z,t\}$ and $\mathbf{T}S(x,y), \mathbf{T}S(y,z)$, $\mathbf{K}S(x,t), \mathbf{K}S(t,z)$. If we use a transitive closure combining all 8 definitions of transitivity we have to add $\mathbf{T}S(x,z)$ because of $\mathbf{T}S(x,y)$, $\mathbf{T}S(y,z)$ but also $\mathbf{K}S(x,z)$ because of $\mathbf{K}S(x,t)$, $\mathbf{K}S(t,z)$.

This remind us the remark related to the simultaneous use of transitive closure and negative transitive closure in the case of classical digraphs. The reason for which we get an inconsistency is related to the fact that some definitions of transitivity add positive signatures while other negative ones. For instance, if we want to use transitivity 1 and 2 together (see Table 2) and if there exist two different t-paths between x and y, one with $\mathbf{T}S(x,t), \mathbf{K}S(t,y)$ and another with $\mathbf{F}S(x,w), \mathbf{U}S(w,y)$, a transitive closure combining the definitions of transitivity 1+2 will result in inconsistency since we have to add $\Delta S(x,y)$ and $\neg\Delta S(x,y)$. Miming what happens with classical digraphs, in order to define transitive closures for an ordinary bi-oriented digraph we need to introduce a distinction between "positive" and "negative" transitive closures.

In Section 5 we showed that transitive closures labelled 1,3,5 and 7 introduce reasons (positive or negative): we will call such closures "positive". Instead, transitive closures labelled 2,4,6 and 8 eliminate reasons (positive or negative): we will call such closures "negative". We extend here the notion of transitivity and negative transitivity in regular bi-oriented digraphs. With this distinction in mind we can now state the following result.

Proposition 6.8. *Let $G(A,S)$ be a bi oriented digraph. The simultaneous application of any combination of "positive transitive closures" (definitions 1, 3, 5 and 7, see Table 7) or of any combination of "negative transitive closures" (definitions 2,4,6,8, see Table 8) yields a unique signature for $G(A,S)$.*

Proof. Consider any sequence of 3 edges xy, yz, zw such that they form a t-path (admitting transitive closures). Then consider Table 6 and take into account only the positive (resp. negative) signs. Positive (resp. negative) transitive closures will simply add positive (resp. negative) signatures to the arcs xz, yw and xw because of proposition 6.7 and Table 5

■

Proposition 6.9. *There does not exist a combination of definitions of transitivity with more than 4 definitions guarantying the uniqueness of signatures after the closure. The largest combinations are the following: (1,3,5,7), (2,4,6,8), (1,4,5,8) and (2,3,6,7) (Tables 7, 8, 9).*

Proof. The necessary condition in order to have a unique signature is to use definitions of transitivity which do not add and remove the same type of reasons (positive or negative ones). For instance, transitivity 1 can not be used with transitivity 2 or 6, since transitivity 1 adds positive reasons while transitivity 2 and 6 remove positive reasons; however 1 can be used together with 4 since 1 adds positive reasons and 4 removes negative reasons. Using similar reasoning, one can conclude in an exhaustive way that

- 1 is not compatible with 2 and 6
- 2 is not compatible with 1 and 5
- 3 is not compatible with 4 and 8
- 4 is not compatible with 3 and 7
- 5 is not compatible with 2 and 6
- 6 is not compatible with 1 and 5
- 7 is not compatible with 4 and 8
- 8 is not compatible with 3 and 7

Respecting these incompatiblities, combinations (1,3,5,7), (2,4,6,8), (1,4,5,8) and (2,3,6,7) (see Tables 7, 8 and 9) and their sub-sets are the only combinations for which the uniqueness of signatures is guaranteed by transitive closures. ■

1357	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	-

Table 7: Applying transitive closures 1,3,5 and 7 simultaneously

Discussion

Let's discuss briefly these results as far as two potential consequences are concerned.

2468	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\mathbf{U}S$

Table 8: Applying transitive closures 2,4,6 and 8 simultaneously

1458	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$

2367	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$

Table 9: Applying transitive closures 1,4,5 and 8 or 2,3,6 and 7 simultaneously

1. On the one hand we have a result consisting in being sure that if a signature is added after a closure in a chain then it is unique. On the other hand we still have the possibility that a signature can be computed even in case part of the sequence is not a t-path (see Proposition 6.5). This may have a practical consequence. We can design a *"strict"* algorithm computing signatures for arcs who connect nodes for which there is no path containing a non t-path. This reduces the number of computable signatures to a minimum. We can also design a *"large"* algorithm which will compute all signatures even in the case part of the path contains a "non t-path". The former can be considered a computing of "necessary" signatures through transitive closure, while the latter should be viewed as the computing of "possible" signatures through transitive closure. The reader should note that the difference between necessary and possible signatures is independent from the definition of transitive closure: it only depends on restricting or not the application of the algorithm implementing the definition.
2. Another practical consequence of using the arcs signatures is the generalisation of the concept of *"preference flow"*, a concept introduced for decision making purposes in [8]. This concept has been introduced in preference modelling in order to exploit a preference graph for prescriptive purposes (computing the maximal subset of the graph, a kernel, a minimal covering subset etc.) and essentially computes the difference between the incoming and outcoming arcs of any node in the graph (see also [32]).

   Considering the usual signature notation for bi-oriented digraphs we denote as a flow between two vertices x and y the function
   $\varphi(xy) : A \times A \mapsto \{-4, -2, 0, 2, 4\}$ such that:
   $\varphi(xy) = \tau(xy, x) - \tau(xy, y) - \tau(yx, y) + \tau(yx, x)$.
   Clearly $\varphi(xy) = -\varphi(yx)$.

   Given a pair of vertices x and y within a bi-oriented digraph we can now distinguish three possible situations:

- strong asymmetry such that $\varphi(xy) = 4$ ($\varphi(yx) = -4$);
- weak asymmetry such that $\varphi(xy) = 2$ ($\varphi(yx) = -2$);
- symmetry such that $\varphi(xy) = 0$ ($\varphi(yx) = 0$);

This is coherent with the intuition (see [28]) that preferences with no hesitation should "count more" with respect to preferences where the decision maker may have (for several different reasons) some hesitation. In case we consider a decision problem where a ranking is expected to be constructed out of a graph of preferences (which may include hesitation) this idea turns to become very useful since it allows to count differently strong asymmetric relations and weak asymmetric relations, allowing a more fine ranking of the set of alternatives (see also the method suggested in [13]).

7. Direct and indirect transitive closures

In this paper we started introducing a definition of transitive closure (see definitions 3.6 and 3.7 already existing in the relevant literature about bi-oriented graphs. We have shown that such definitions do not fit our requirements for using such graphs for preference modelling purposes. Because of this reason we introduced the definitions of transitivity in Section 5. Remains to be understood if there are any connections, at least for some simple cases between the two sets of definitions. In the following G_τ is a bi-oriented digraph representing the relation S upon A, $G_\tau = (A, S)$.

Proposition 7.1. *Let G_τ be an all positive bi-oriented digraph (i.e, $\forall e \in A : \overline{W}(e) = 0$). The transitive closure of G_τ, is the graph denoted $Ft(G_\tau) = (A, Ft(S))$ such that exists $e = \{x^\alpha, y^\beta\} \in Ft(S)$ if there exist a b-path $P_{(\alpha,\beta)}(x, y)$ in G_τ .*

Proof. The edges of $P_{(\alpha,\beta)}(x, y)$ can only be of type $\mathbf{T}$ or only of type $\mathbf{F}$, and according to the directed transitive closures of $\mathbf{F}$ and $\mathbf{T}$ which are given in tables 3 and 4, we get the result. ■

Corollary 7.2. *$Ft(G_\tau)$ is an all positive bi-oriented digraph, such that $\forall e \in Ft(S) : \overline{W}(e) = 0$.*

Proof. The transitive closure of $P_{(\alpha,\beta)}(x, y)$ is the positive edge $e = \{x^\alpha, y^\beta\}$ which is of type $\mathbf{T}$ or $\mathbf{F}$, and $\overline{W}(\mathbf{F}) = \overline{W}(\mathbf{T}) = 0$. ■

In other terms in case of a graph where arcs are only $\mathbf{T}$ (or only $\mathbf{F}$) we can apply a direct positive (or negative) transitive closure. The result will be adding exclusively $\mathbf{T}$ arcs (or $\mathbf{F}$ arcs) and the result is the same if we apply definition 3.7.

Proposition 7.3. *Let G_τ be an all negative bi-oriented digraph such that $\forall e \in S : \overline{W}(e) = -2$. The transitive closure of G_τ, is the graph denoted $Ft(G_\tau) = (A, Ft(S))$ such that exists $e = \{x^-, y^-\} \in Ft(S)$ if there exist a t_s-path, $Q(x^-, y^-) : x^- e_1 x_1 e_2 x_2 \dots x_k e_k y^-$ in G_τ.*

Proof. According to the directed transitive closure of $\mathbf{U}$ given in tables 3 and 4, from where the result. ■

Corollary 7.4. *$Ft(G_\tau)$ is an all negative bi-oriented digraph, such that $\forall e \in Ft(S)$: $\overline{W}(e) = -2$.*

Proof. The closure of $Q(x^-, y^-)$ is a negative edge $e = \{x^-, y^-\}$ which is of type $\mathbf{U}$ and $\overline{W}(\mathbf{U}) = -2$. ■

Proposition 7.5. *Let G_τ be an all negative bi-oriented digraph such that $\forall e \in S : \overline{W}(e) = +2$. The transitive closure of G_τ, is the graph denoted $Ft(G_\tau) = (A, Ft(S))$ such that exists $e = \{x^+, y^+\} \in Ft(S)$ if there exist a t_s-path, $Q(x^+, y^+) : x^+ e_1 x_1 e_2 x_2 \ldots x_k e_k y^+$ in G_τ.*

Proof. According to the directed transitive closure of $\mathbf{K}$ given in tables 3 and 4, from where the result. ■

Corollary 7.6. *$Ft(G_\tau)$ is a negative graph, such that $\forall e \in Ft(S) : \overline{W}(e) = +2$.*

Proof. The closure of $Q(x^+, y^+)$ is a negative edge $e = \{x^+, y^+\}$ which is of type $\mathbf{K}$ and $\overline{W}(\mathbf{K}) = +2$. ■

In other terms the results we obtained in case we have chains of $\mathbf{T}$ arcs (and we apply positive transitive closures) or of $\mathbf{F}$ arcs (and we apply negative transitive closures) hold also in case we we have chains of $\mathbf{K}$ arcs or chains of $\mathbf{U}$ arcs. These results are obtained using simply "direct" transitive closures. The introduction of "indirect" transitive closures allows to extend the above results. As already mentioned in Section 6 it is generally impossible to close transitively any type of chain of arcs (when signed arbitrarily: $\mathbf{T}$, $\mathbf{F}$, $\mathbf{K}$, $\mathbf{U}$). There are two special cases though: chains with no $\mathbf{T}$ arcs (and we need to apply indirect negative transitive closure) and chains with no $\mathbf{F}$ arcs (and we need to apply indirect positive transitive closures).

Proposition 7.7. *Let G_τ be a bi-oriented digraph. The negative transitive closure of G_τ,is the graph denoted $Ft(G_\tau) = (A, Ft(S))$ such that exists $e = \{x^-, y^+\} \in Ft(S)$ if there exist a t-path, $Q(x^\alpha, y^\beta) : x^\alpha e_1 x_1 e_2 x_2 \ldots x_k e_k y^\beta$ in $G_\tau$and does not admit edges of type $\mathbf{T}$.*

Proof. According to the indirect transitive closures of $\mathbf{F}$, $\mathbf{U}$ and $\mathbf{K}$ given in tables 3 and 4, from where the result. ■

Practically: in case we have chains of arcs with no $\mathbf{T}$ arcs we obtain as transitive closure arcs which will be all signed as $\mathbf{F}$ (using the indirect negative transitive closure definitions).

Proposition 7.8. *Let G_τ be a bi-oriented digraph. The positive transitive closure of G_τ, is the graph denoted $Ft(G_\tau) = (A, Ft(S))$ such that exists $e = \{x^+, y^-\} \in Ft(S)$ if there is a t-path, $Q(x^\alpha, y^\beta) : x^\alpha e_1 x_1 e_2 x_2 \ldots x_k e_k y^\beta \in G_\tau$and does not admits edges of the type $\mathbf{F}$.*

Proof. According to the indirect transitive closures of $\mathbf{T}$, $\mathbf{U}$ and $\mathbf{K}$ given in tables 3 and 4, from where the result. ■

Practically: in case we have chains of arcs with no $\mathbf{F}$ arcs we obtain as transitive closure arcs which will be all signed as $\mathbf{T}$ (using the indirect positive transitive closure definitions).

The last two results confirm the idea that using the positive or the negative transitive closures we can obtain a transitively closed bi-oriented digraph in case we have chains which do not contain at the same time $\mathbf{T}$ and $\mathbf{F}$ arcs.

Conclusions

In this paper we propose a first study about the use of signed graphs (more precisely bi-oriented digraphs) in order to complement the use of logical languages explicitly designed for preference modelling under hesitation. More precisely we show how bi-oriented digraphs can be used as graphical representation for preference structures based upon the DDT language (a first order four valued logic). In order to complete such new tool we need to introduce new forms of transitive closures (more precisely 8 different forms of transitivity). The result is the establishment of graphical representation tools which enable to use graph theory when preferences are expressed under hesitation and with multiple epistemic states. Two research directions can be followed from these findings. The first concerns the development of ranking and rating procedures exploiting directly the preference structure including hesitation. The second concerns the generalisation of the notion of transitivity as a process for tranfering/creating/revising knowledge included in preference statements.

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Annexe A: the complete list of combinations

1	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	ΔS	-	ΔS	-
$\mathbf{U}S$	-	-	-	-

2	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	-	-	-	-
$\mathbf{U}S$	-	$\neg\Delta S$	-	$\neg\Delta S$

3	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{U}S$	-	-	-	-

4	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	-	-	-	-
$\mathbf{U}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$

5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	ΔS	-	-	ΔS
$\mathbf{U}S$	ΔS	-	ΔS	-

6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	-

7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	-	$\Delta\neg S$	-	$\Delta\neg S$
$\mathbf{U}S$	-	$\Delta\neg S$	$\Delta\neg S$	-

8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-

1234567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{F}S\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$

123456 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\mathbf{U}S$

12345 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\mathbf{U}S$

1234 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\mathbf{U}S$

123 5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\neg\Delta S$

12 45678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\mathbf{U}S$

1 345678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\neg\Delta\neg S$

2345678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\mathbf{U}S$

12	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	ΔS	-	ΔS	-
$\mathbf{U}S$	-	$\neg\Delta S$	-	$\neg\Delta S$

1 3	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	-
$\mathbf{K}S$	ΔS	$\neg\Delta S$	$\mathbf{K}S$-	-
$\mathbf{U}S$	-	-	-	-

1 4	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	ΔS	-	ΔS	-
$\mathbf{U}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$

1 5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	ΔS	-	ΔS	ΔS
$\mathbf{U}S$	ΔS	-	ΔS	-

1 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	-

1 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	ΔS	$\Delta\neg S$
$\mathbf{U}S$	-	$\Delta\neg S$	$\Delta\neg S$	-

1 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-

78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	-
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\Delta\neg S$	-

23	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{U}S$	-	$\neg\Delta S$	-	$\neg\Delta S$

2 4	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	-	-	-	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$

2 5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	ΔS	-	-	ΔS
$\mathbf{U}S$	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$

2 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$

2 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\Delta\neg S$	-	$\Delta\neg S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\Delta\neg S$

2 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	-	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	-

6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	-

34	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	-	$\Delta\neg S$	ΔS	-
$\mathbf{U}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$

3 5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{U}S$	ΔS	-	-	-

3 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	-

3 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{U}S$	-	$\Delta\neg S$	$\Delta\neg S$	-

3 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-

56	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	-	-
$\mathbf{U}S$	ΔS	$\neg\Delta S$	-	-

5 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	-	$\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	-

5 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\mathbf{T}S$	-	-	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	-

45	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	ΔS	-	-	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$

4 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$

4 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	-	$\Delta\neg S$	-	$\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$

4 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$

1 45	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	ΔS	-	ΔS	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$

1 4 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$

1 4 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	ΔS	$\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$

1 4 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$

123	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	-
$\mathbf{U}S$	-	$\neg\Delta S$	-	$\neg\Delta S$

12 4	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	ΔS	-	ΔS	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$

12 5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	ΔS	-	ΔS	ΔS
$\mathbf{U}S$	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$

12 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$

12 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	ΔS	$\Delta\neg S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\Delta\neg S$

12 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

1 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	ΔS	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	-

1 68	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	-

1 34	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	$\neg\Delta\neg S$	-	ΔS	$\neg\Delta\neg S$

1 3 5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	ΔS	-	ΔS	-

1 3 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	-

1 3 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\Delta\neg S$
$\mathbf{U}S$	-	$\Delta\neg S$	$\Delta\neg S$	-

1 3 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-

1 56	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	ΔS	$\neg\Delta S$	$\neg\Delta S$	-

1 57	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	ΔS	$\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	-

1 58	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	-

234	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\Delta\neg S$	ΔS	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$

23 5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\Delta\neg S$	ΔS
$\mathbf{U}S$	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$

23 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$

23 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{U}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta S$

23 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

2 56	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	-	-
$\mathbf{U}S$	ΔS	$\neg\Delta S$	-	$\neg\Delta S$

2 5 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	-	$\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$

2 5 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	-	-	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\mathbf{T}S$	$\neg\Delta S$

2 45	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	Δ	-	-	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\mathbf{U}S$

2 4 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\mathbf{U}S$

2 4 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\Delta\neg S$	-	$\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$

2 4 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\mathbf{U}S$

1 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	-

2 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\neg\Delta S$

2 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	-	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$

2 68	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\neg\Delta S$

345	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	Δ	$\Delta\neg S$	$\Delta\neg S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$

34 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$

34 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$

34 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$

3 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	-

3 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	-

4 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	-	$\mathbf{F}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta\neg S$

4 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\neg\Delta\neg S$

3 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	-

3 56	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	-
$\mathbf{U}S$	ΔS	$\neg\Delta S$	-	-

3 5 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\Delta\neg S$	$\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	-

3 5 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	-

4 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	$\neg\Delta\neg S$

456	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	-	-
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\neg\Delta\neg S$

45 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	-	$\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$

45 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\mathbf{T}S$	-	-	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$

1234	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$

123 5	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$

123 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$
$\mathbf{U}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$

123 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\Delta\neg S$
$\mathbf{U}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta S$

123 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

12 56	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	ΔS	-
$\mathbf{U}S$	ΔS	$\neg\Delta S$	-	$\neg\Delta S$

12 5 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	ΔS	$\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$

12 5 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\mathbf{T}S$	$\neg\Delta S$

12 45	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	Δ	-	ΔS	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\mathbf{U}S$

12 4 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\neg\Delta S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\mathbf{U}S$

12 4 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\Delta\neg S$	ΔS	$\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$

12 4 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\mathbf{U}S$

678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	-

12 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\neg\Delta S$

12 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	ΔS	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$

12 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\neg\Delta S$

1 345	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	Δ	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$

1 34 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$

1 34 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$

1 34 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$

1 3 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	-

1 3 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	-

1 4 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	ΔS	$\mathbf{F}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta\neg S$

1 4 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\neg\Delta\neg S$

1 3 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	-

1 3 56	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	ΔS	$\neg\Delta S$	-	-

1 3 5 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	-

1 3 5 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	-

1 4 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	$\neg\Delta\neg S$

1 456	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	ΔS	-
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\neg\Delta\neg S$

1 45 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	ΔS	$\mathbf{K}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\neg\Delta\neg S$

1 45 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	-	-	-
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$

2345	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	Δ	$\Delta\neg S$	$\Delta\neg S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{U}S$

234 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\mathbf{U}S$

234 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$

234 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\mathbf{U}S$

23 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	-	-	-	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$

23 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\neg\Delta S$

2 4 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	-	$\mathbf{F}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{U}S$

2 4 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\neg\Delta S$	-	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\mathbf{U}S$

23 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	-

23 56	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	-
$\mathbf{U}S$	ΔS	$\neg\Delta S$	-	$\neg\Delta S$

23 5 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\Delta\neg S$	$\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$

23 5 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\mathbf{T}S$	$\neg\Delta S$

2 4 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\mathbf{U}S$

2 456	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\neg\Delta S$	-	-
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\mathbf{U}S$

2 45 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	-	$\mathbf{K}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$

2 45 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	-	-	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\mathbf{T}S$	$\mathbf{U}S$

567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	-	$\mathbf{F}S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\mathbf{F}S$	-

56 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	-

5 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	-	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	-

5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-

1 567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	ΔS	-
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	-	-

1 56 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	-

1 5 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	-

1 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	-

2 567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	-	-
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	-	$\neg\Delta S$

2 56 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

2 5 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	-	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$

2 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\neg\Delta S$

3 567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	-
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	-	-

3 56 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	-

3 5 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	-

3 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	-

12345	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	Δ	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\mathbf{U}S$

1234 6	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\mathbf{U}S$

1234 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$

1234 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\mathbf{U}S$

123 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	-
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{F}S$
$\mathbf{U}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$

123 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\neg\Delta S$

12 4 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	ΔS	$\mathbf{F}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{U}S$

12 4 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\mathbf{U}S$

123 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\neg\Delta S$

123 56	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	ΔS	$\neg\Delta S$	-	$\neg\Delta S$

123 5 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\Delta\neg S\mathbf{K}S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$

123 5 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\mathbf{T}S$	$\neg\Delta S$

12 4 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\mathbf{U}S$

12 456	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\mathbf{U}S$

12 45 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	ΔS	$\mathbf{K}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$

12 45 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	-	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\mathbf{T}S$	$\mathbf{U}S$

4567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta\neg S$

456 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta\neg S$

45 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	-	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	$\neg\Delta\neg S$

4 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\neg\Delta\neg S$

12 567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	ΔS	$\Delta\neg S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$

12 56 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

12 5 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$

12 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\neg\Delta S$

1 3 567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\Delta\neg S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	-

1 3 56 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	-

1 3 5 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	-

1 3 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{F}S$	-

1 4567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta\neg S$

1 456 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta\neg S$

1 45 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	$\neg\Delta\neg S$

1 4 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\neg\Delta\neg S$

34 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	$\neg\Delta\neg S$

3456	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\neg\Delta\neg S$

345 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\Delta\neg S$	$\mathbf{K}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\neg\Delta\neg S$

345 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$

34 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta\neg S$

34 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\neg\Delta\neg S$

1 34 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\Delta\neg S$	-	$\neg\Delta\neg S$

1 3456	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\neg\Delta\neg S$

1 345 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\mathbf{K}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\neg\Delta\neg S$

1 345 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$

1 34 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{F}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta\neg S$

1 34 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\neg\Delta\neg S$

1 34567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta\neg S$

1 3456 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta\neg S$

1 345 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	$\neg\Delta\neg S$

1 34 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\neg\Delta\neg S$

23 567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$

23 56 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

23 5 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$

23 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\neg\Delta S$

2 4567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$

2 456 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\mathbf{U}S$

2 45 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	-	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\mathbf{U}S$

2 4 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\mathbf{U}S$

34567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta\neg S$

3456 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$

345 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	$\neg\Delta\neg S$

34 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\neg\Delta\neg S$

1 5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-

2 5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\neg\Delta S$

3 5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\neg\Delta S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-

45678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\neg\Delta\neg S$

234 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\mathbf{U}S$

23456	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\mathbf{U}S$

2345 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\Delta\neg S$	$\mathbf{K}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$

2345 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\mathbf{T}S$	$\mathbf{U}S$

234 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{U}S$

234 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\mathbf{U}S$

1234 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	-	$\mathbf{U}S$

123456	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	-	$\mathbf{U}S$

12345 7	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\Delta\neg S$	$\mathbf{K}S$	$\mathbf{K}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$

12345 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	$\mathbf{T}S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\mathbf{T}S$	$\mathbf{U}S$

1234 67	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	ΔS	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{F}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{U}S$

1234 6 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\neg\Delta S$	$\mathbf{U}S$	$\mathbf{U}S$

123 567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	ΔS	-	ΔS	ΔS
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	ΔS	$\mathbf{F}S$	$\mathbf{K}S$	$\Delta\neg S$
$\mathbf{U}S$	ΔS	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta S$

123 56 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

123 5 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\mathbf{K}S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$

123 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\neg\Delta S$

12 4567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$

12 456 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\neg\Delta S$	ΔS	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\mathbf{U}S$

12 45 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	ΔS	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\mathbf{U}S$

12 4 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\mathbf{U}S$

234567	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{U}S$

23456 8	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\neg\Delta S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{U}S$	$\mathbf{T}S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\mathbf{U}S$

2345 78	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\Delta\neg S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\Delta\neg S$	$\Delta\neg S$	ΔS
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	$\mathbf{U}S$

234 678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	ΔS	$\neg\Delta S$
$\mathbf{U}S$	$\neg\Delta\neg S$	$\mathbf{F}S$	$\neg\Delta S$	$\mathbf{U}S$

12 5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\neg\Delta S$

1 3 5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-

345678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\neg\Delta S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\neg\Delta\neg S$

1 45678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	ΔS	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\neg\Delta\neg S$

23 5678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\Delta\neg S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\neg\Delta S$

2 45678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\mathbf{U}S$

12345678	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	$\mathbf{U}S$
$\mathbf{T}S$	$\mathbf{T}S$	-	$\mathbf{T}S$	$\mathbf{T}S$
$\mathbf{F}S$	-	$\mathbf{F}S$	$\mathbf{F}S$	$\mathbf{F}S$
$\mathbf{K}S$	$\mathbf{T}S$	$\mathbf{F}S$	$\mathbf{K}S$	-
$\mathbf{U}S$	$\mathbf{T}S$	$\mathbf{F}S$	-	$\mathbf{U}S$