Extending Variable Importance in Preference Networks

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Abstract

In many application domains we need to find solutions that satisfy, apart from a set of hard constraints, a set of user defined preferences. *Ceteris Paribus* (CP)-networks have been proposed as an intuitively appealing framework for expressing preference statements. CP-nets have been further extended to incorporate information on the relative importance of the variables, resulting in a formalism called TCP-nets. Despite their high expressive power, TCP-nets do not capture certain types of preference statements that seem to arise naturally in practice.

In this paper we extend TCP-networks with variable importance statements that specify that a variable is more important than its ancestors in the network. These importance statements may induce preference relations on the set of outcomes that contain conflicting pairs. To handle such cases we propose a new semantics that aggregates preference and variable important information in such a way that preferences on more important variables override preferences on less important variables.

1 Introduction

In many application domains we need to find solutions that satisfy, apart from a set of hard constraints, a set of user defined preferences. Such application domains include decision support systems, product configuration software, constraint optimization, planning and scheduling and many others.

Ceteris Paribus networks or *CP-nets* [Boutilier *et al.*, 1999; Domshlak and Brafman, 2002; Brafman and Dimopoulos, 2004] have been proposed as a powerful yet simple graphical tool for representing preference statements that most people find intuitive and easy to express. In a CP-net the user describes how her preferences for the values of one variable depend on the values of other variables. For example, a user may want to state that her preference between two car manufacturer depends on the car type. She may prefer the first manufacturer if the car is a 4x4 while she may prefer the second if the car is a saloon. This information can be represented in a CP-net by two nodes, one for the car type and one for the manufacturer and an edge that goes from the first node to the second.

Another kind of preferential statements that are useful in practice are *relative importance* statements. These are statements of the form: "It is more important that the value of X is high than that the value of Y is high" [Brafman and Domshlak, 2002]. This led to the development of TCP-nets [Brafman and Domshlak, 2002], an extension of CP-nets that can capture relative importance statements. In our car domain, we may want to express that safety is more important than speed, meaning that a better value for safety is more important than a better value for speed. TCP-nets allow us to represent *conditional relative importance* statements that are more useful than simple relative importance statements. For instance, in a TCP-net we can state that in family cars safety is more important than speed, while in sports cars speed is more important than safety.

The semantics of TCP-nets is based on a *ceteris paribus* [Doyle and Wellman, 1994; Hanson, 1996] comparison of solutions. This means that if the user states that she prefers the value x_1 over the value x_2 for some variable X, this is taken to mean that between two solutions (or outcomes) that assign *identical values* to all other variables, she prefers the one that assigns the value x_1 to X over the one that assigns x_2 to the same variable.

Despite their high expressive power and clear semantics, TCP-nets also have limitations. Indeed, the variable importance semantics introduced in [Brafman and Domshlak, 2002] permits information on the relative importance between two variables only if these variables are conditionally independent. There are however situation where there is importance information about variables that are conditionally dependent. Consider the following simple example.

Maria has the following dinning preferences. For food she prefers meat to fish, while her preference for wine depends on the choice of food. If the food is meat she prefers red wine over white wine, whereas when fish is served she prefers white wine to red wine. Moreover, wine is more important to Maria than food. This problem situation can not be captured in TCP-nets as this networks contains information that states that the child is more important than the parent.

In this paper we extend TCP-networks with variable importance statements as the one of the previous example, that specify that a variable is more important than its ancestors in the network. As this extended importance statements may introduce cycles in the outcome graph of the TCP-network, we employ the semantics introduced in [Brafman and Dimopoulos, 2004] for cyclic CP-nets. In order to be able to capture arbitrary cyclic TCP-nets, we define the semantics of variable importance statements that assert that a variable is more important than its immediate ancestors in the network.

A TCP-network is a collection of preference and variable importance statements on the set of variables of the network. Each of these statements can be regarded as a *criterion* that induces a binary relation on the set of outcomes. Under this perspective a semantics for the TCP-networks defines the exact meaning of these statements or criteria but also a method for *aggregating* these criteria. As we show in the following, in the case of acyclic TCP-networks a simple disjunctive aggregation of the criteria suffices.

However, in the extended TCP-networks, preference statements may induce preference relations that contain contradictory pairs of the form "outcome a is preferred over outcome b" and "outcome b is preferred over outcome a". In this context, we need a stronger form of criteria aggregation that takes into account variable importance. More specifically, we take a variable importance statement of the form "X is more important than Y" to mean not only mean that a good value for X is more important than a good value for Y, but also that the criteria defined on the values of variable X are more important than the criteria defined on the values of Y. Consequently, preference information defined on the values of X takes precedence over preference information defined on the values of Y.

The paper is organized as follows. Section 2 reviews CP and TCP-networks and states some of their basic properties. Section 3 presents the extended TCP-networks and defines the new semantics, and section 4 concludes.

2 CP and TCP-networks

In this section we review the basic CP-net and TCP-net semantics in the spirit of [Wilson, 2004].

Assume a set of variables $\mathbf{V} = \{X_1, \ldots, X_n\}$ with domains $dom(X_1), \ldots, dom(X_n)$ respectively. The set of possible outcomes of $\mathbf{V}$ is the set $dom(X_1) \times \ldots \times dom(X_n)$. We assume that variable domains are pairwise disjoint, ie. for every $X_i, X_j \in \mathbf{V}$ it holds that $dom(X_i) \cap dom(X_j) = \emptyset$. Given the value assignments $\mathbf{x}$ and $\mathbf{y}$ to the set of variables X and Y respectively with $Y \subseteq X \subseteq \mathbf{V}$, we write $\mathbf{x} \models \mathbf{y}$ to denote that the projection of $\mathbf{x}$ to the variables of Y equals $\mathbf{y}$.

A preference statement is an expression of the form $\mathbf{p} : q_i \succ q_j$ where $\mathbf{p}$ is an assignment to a set of variables $\mathbf{P} \subseteq \mathbf{V}$ and q_i, q_j values of a variable Q such that $Q \cap \mathbf{P} = \emptyset$. We then write $q_i \succ_{\mathbf{p}} q_j$. A CP-network is set of preference statements. We define the set of parent $Pa(X)$ of a variable X in a CP-net N as $Pa(X) = \{y | y \in \mathbf{V}$ and N contains a statement of the form $\mathbf{p} : x_i \succ x_j$ where $x_i, x_j \in dom(X)$ and $\mathbf{p}$ contains some value for $y\}$. We assume that in each preference statements of the form $\mathbf{p} : x_i \succ x_j$ on the values of variable X, the assignment $\mathbf{p}$ is a complete assignment to the set of variables $Pa(X)$.

A CP-net N induces a graph G_N that contain a node for every variable of N and an edge from the node associated with variable Y to the node of variable Y if $X \in Pa(Y)$. The notation $tr(R)$ denotes the transitive closure of a binary relation R.

Definition 1 *[Wilson, 2004]. Let $s = \mathbf{p} : q_i \succ q_j$ be a preference statement. The* relation induced *by s on a set of outcomes O wrt s is a binary relation $R_s = \{(o_i, o_j) | o_i, o_j \in O$ and $o_i = \mathbf{w}q_i$ and $o_j = \mathbf{w}q_j$ and $\mathbf{w} \models \mathbf{p}\}$. The* relation induced *by a CP-net $N = \{s_1, s_2, \ldots s_n\}$ is the relation $R_N = tr(R_{s_1} \cup R_{s_2} \cup \ldots \cup R_{s_n})$.*

Therefore, the criteria aggregation method used in the CP-networks is the *disjunctive* aggregation.

If N is an acyclic CP-net the relation R_N is a *strict partial order*, ie. it is irreflexive, asymmetric and transitive. The next theorem proves the first two properties of R_N as transitivity follows from its definition.

Theorem 1 *Let N be an acyclic CP-net. Then, the relation R_N is irreflexive and asymmetric.*

If N is an acyclic CP-network we say that the outcome o_i is *strictly preferred* to outcome o_j wrt N, denoted by $o_i \succ_N o_j$, if $(o_i, o_j) \in R_N$. We drop N form $\succ_N$ when the CP-network to which we refer is clear from the context. The following example illustrates the ranking relation imposed on the set outcomes by the CP-networks.

Example 1 *Let N be the CP-network defined on the variables X, Y, Z as follows:*

$s_1 =: x_1 \succ x_2$ $\quad$ $s_2 = x_1 : y_1 \succ y_2$
$s_3 = x_2 : y_2 \succ y_1$ $\quad$ $s_4 = y_1 : z_1 \succ z_2$
$s_5 = y_2 : z_2 \succ z_1$

The relation induced by each of the above statements are the following:

$R_{s_1} = \{(x_1y_1z_1, x_2y_1z_1), (x_1y_1z_2, x_2y_1z_2), (x_1y_2z_1, x_2y_2z_1), (x_1y_2z_2, x_2y_2z_2)\}$
$R_{s_2} = \{(x_1y_1z_1, x_1y_2z_1), (x_1y_1z_2, x_1y_2z_2)\}$
$R_{s_3} = \{(x_2y_2z_1, x_2y_1z_1), (x_2y_2z_2, x_2y_1z_2)\}$
$R_{s_4} = \{(x_1y_1z_1, x_1y_1z_2), (x_2y_1z_1, x_2y_1z_2)\}$
$R_{s_5} = \{(x_1y_2z_2, x_1y_2z_1), (x_2y_2z_2, x_2y_2z_1)\}$

The ranking induced by the relation $R_N = \cup_{i=1}^{5} R_{s_i}$ specifies that $x_1y_1z_1 \succ x_1y_1z_2 \succ x_1y_2z_2 \succ \{x_1y_2z_1, x_2y_2z_2\} \succ x_2y_2z_1 \succ x_2y_1z_1 \succ x_2y_1z_1$. Note that the outcomes $x_1y_2z_1$ and $x_2y_2z_2$ are incomparable.

TCP-networks [Brafman and Domshlak, 2002] extend CP-networks with *relative variable importance* statements. A relative variable importance (or relative importance) statement is of the form $\mathbf{p} : X \triangleright Y$ where $X, Y \subseteq \mathbf{V}$, and the sets $Pa(X)$, $\{X\}$, and $\{Y\}$ are pairwise disjoint. Intuitively, the meaning of such a sentence is when $\mathbf{p}$ is true we prefer a good value for X over a good value to variable Y. A variable importance statement induces a binary relation on the set of possible outcomes.

Definition 2 *Let $v = \mathbf{p} : X \triangleright Y$ be a variable importance statement of a TCP-net N. The* relation induced by *v on a set of outcomes O is a binary relation $R_v = \{(o_i, o_j) | o_i, o_j \in O$, $o_i = \mathbf{wz}x_iy_a$, $o_j = \mathbf{wz}x_jy_b$, $x_i \succ_{\mathbf{z}} x_j$, and $\mathbf{wz} \models \mathbf{p}\}$, where $x_i, x_j \in dom(X)$ and $y_a, y_b \in dom(Y)$. The* relation induced by *a TCP-net N that contains the preference statements $s_1, s_2, \ldots s_n$ and the variable importance statements $v_1, v_2, \ldots v_m$ is $R_N = tr(R_{s_1} \cup R_{s_2} \cup \ldots \cup R_{s_n} \cup R_{v_1} \cup R_{v_2} \cup \ldots \cup R_{v_m})$.*

We can extend the notion of the graph G_N associated with a CP-nets to the graph associated with a TCP-net N, also denoted by G_N, by adding to the graph an edge from the node that corresponds to X to the node that corresponds to Y for every variable importance statement of the form $\mathbf{p} : X \triangleright Y$. We can now extend theorem 1 and show that for a TCP-network N with an acyclic graph G_N the relation R_N is a strict partial order.

Theorem 2 *Let N be an acyclic TCP-net. Then, the relation R_N is irreflexive and asymmetric.*

As in the case of acyclic CP-networks we say that the outcome o_i is *strictly preferred* to outcome o_j wrt to an acyclic TCP-net N, denoted by $o_i \succ_N o_j$, if $(o_i, o_j) \in R_N$. The next example illustrates the semantics of TCP-networks.

Example 2 *Consider the CP-network of example 1 extended with the variable importance $v = X \triangleright Z$. The associated binary relation is $R_v = \{(x_1y_1z_1, x_2y_1z_2), (x_1y_1z_2, x_2y_1z_1), (x_1y_2z_1, x_2y_2z_2), (x_1y_2z_2, x_2y_2z_1)\}$. Note that the relation $tr(R_{s_1} \cup R_{s_2} \cup R_{s_3} \cup R_{s_4} \cup R_{s_5} \cup R_v)$ is antisymmetric and that the new relation includes the pair $(x_1y_2z_1, x_2y_2z_2)$, ie. outcomes that were previously incomparable now become comparable.*

3 Cyclic TCP-Networks

The work of [Brafman and Dimopoulos, 2004] extends the semantics of CP-networks from acyclic to cyclic networks. The semantics of a cyclic CP-network N is defined again by the relation R_N, which is now not a strict partial order, as it needs not be irreflexive or asymmetric. Instead, in order to be able to capture the semantics of cyclic CP-nets the relation R_N is required to be a *pre-order*, that is, a reflexive and transitive binary relation. We can easily turn the relation induced by a TCP-network N into a pre-order by defining it as $R_N = tr(R_{s_1} \cup R_{s_2} \cup \ldots \cup R_{s_n} \cup R_{v_1} \cup R_{v_2} \cup \ldots \cup R_{v_m}) \cup \{(o, o) | o$ is a value assignment to all variables of $N\}$. In the following we omit pairs of the form (o, o) from the relations.

Following [Brafman and Dimopoulos, 2004] we define the semantics of cyclic CP-networks as follows. Given a TCP-network N and two outcomes o, o' we say that o is *weakly preferred* to o', denoted by $o \succeq o'$ if $(o, o') \in R_N$. We say that o is *strongly preferred* to o', denoted by $o \succ o'$ if $(o, o') \in R_N$ and $(o', o) \notin R_N$. Finally, we say that outcomes o and o' are *equally preferred*, denoted by $\sim$, if $o \succeq o'$ and $o' \succeq o$. The next example illustrates the new semantics.

Example 3 *Consider the CP-net N' on the variables*
$STARTER = \{soupe, salat\}$
$MAIN = \{fish, meat\}$
$WINE = \{white, red\}$
with the following preferences:
$: salat \succ soup$
$salat : meat \succ fish$ $\quad$ $soup : fish \succ meat$
$meat : red \succ white$ $\quad$ $fish : white \succ red$

The ranking on the outcomes of N' induced by the relation $R_{N'}$ is given below, where each variable value is represented by its initial (sa stands for salad and so for soup):
$\{sa, m, r\} \succ \{sa, m, w\} \succ \{sa, f, w\} \succ \{\{sa, f, r\}, \{so, f, w\}\} \succ \{so, f, r\} \succ \{so, m, r\} \succ \{so, m, w\}$.
We extend N' into a TCP-net N by adding the variable importance statement $v = WINE \triangleright STARTER$. This statement induces the binary relation $R_v = \{(\{so, m, r\}, \{sa, m, w\}), (\{sa, m, r\}, \{so, m, w\}), (\{so, f, w\}, \{sa, f, r\}), (\{sa, f, w\}, \{so, f, r\})\}$, whereas the relation induced by N is $R_N = R'_N \cup R_v$. The outcome $\{sa, m, r\}$ is strictly preferred over all other outcomes, whereas the outcomes $\{sa, m, w\}$, $\{sa, f, w\}$, $\{sa, f, r\}$, $\{so, f, w\}$, $\{so, f, r\}$, and $\{so, m, r\}$ are equally preferred.

The simple extension of the semantics of TCP-networks described above captures a wide class of cyclic TCP-networks, but not all. Consider for instance the simple example presented in the introduction.

Example 4 *Maria prefers meat to fish. When the food is meat she prefers red wine over white wine, whereas when fish is served she prefers white wine to red wine. Moreover, wine is more important to Maria than food. The TCP-network N that represents Maria's preferences will contain the variables $FOOD$ and $WINE$ with $dom(FOOD) = \{meat, fish\}$ and $dom(WINE) = \{white, red\}$. Network N contains the preference statements*
$s_1 =: meat \succ fish$
$s_2 = meat : red \succ white$ $\quad$ $s_3 = fish : white \succ red$
The variable importance statement is $v = WINE \triangleright FOOD$, which is not an acceptable statement in the language of TCP-nets, as it violates the restriction that $Pa(WINE)$ and $\{FOOD\}$ must be disjoint, whereas $Pa(WINE) \cap \{FOOD\} = \{FOOD\}$.

In order to be able to handle TCP-networks with cycles of length two, as those of the previous example, we need to extend the the semantics of variable importance statements. This is accomplished by the following definition.

Definition 3 *Let $v = \mathbf{p} : X \triangleright Y$ be a variable importance statement of a TCP-net N. The* relation induced by *v on a set of outcomes O is a binary relation $R_v = \{(o_i, o_j) | o_i, o_j \in O$, $o_i = \mathbf{wz}x_iy_a$, $o_j = \mathbf{wz}x_jy_b$, $x_i \succ_{\mathbf{z}} x_j$, $\mathbf{Z} \cap \{Y\} = \emptyset$, and $\mathbf{wz} \models \mathbf{p}\} \cup \{(o_i, o_j) | o_i, o_j \in O$, $o_i = \mathbf{wz}xy_a, o_j = \mathbf{wz}xy_b$, $\exists x' \in dom(X)$ such that $x \succ_{\mathbf{z}y_a} x'$ and $x' \succ_{\mathbf{z}y_b} x$, with $\mathbf{wz} \models \mathbf{p}\}$.*

The variable importance statement $v = WINE \triangleright STARTER$ of network N of example 4 induces the relation $R_v = \{(\{meat, red\}, \{fish, red\}), (\{fish, white\}, \{meat, white\})\}$. The induced relation is $R_N = \{(\{meat, red\}, \{fish, red\}), (\{meat, red\}, \{meat, white\})$ $(\{meat, red\}, \{fish, white\}), (\{fish, white\}, \{fish, red\}), (\{meat, white\}, \{fish, white\}), (\{meat, white\}, \{fish, red\}), (\{fish, white\}, \{meat, white\})\}$. The relation R_N renders the outcomes $\{meat, white\}$ and $\{fish, white\}$ equally preferred. The reason is that there are two criteria, one that postulates that meat is preferred over fish and ranks the outcome $\{meat, white\}$ higher than the outcome

$\{fish, white\}$, and a second one that postulates that a good value for the wine is more important than a good choice for food, and therefore ranks $\{fish, white\}$ higher than $\{meat, white\}$. Intuitively, however, we would expect the second preference to override the first. The reason for this is that preference over combinations of values override preferences over single variable values. This can be seen as an instance of the general principle, widely used in AI, that specific information overrides more general information.

More generally, cyclic TCP-networks may induce preference relations on the set of outcomes that are cyclic, ie. contain pairs of the form (o, o') and (o', o). To be able to cope with such cases, whenever this is possible, we need to develop a method for aggregating the various criteria of a TCP-network that is more powerful than the simple disjunctive aggregation. To accomplish this we can exploit the variable importance information present in a TCP-network. Intuitively, an importance statement of the form "X is more important than Y" is understood as asserting that the criteria defined on variable X are more important than those defined on variable Y. Therefore, preference information defined on X overrides contradictory information coming from Y. Before we proceed with the formal definition of this intuition we establish some necessary notions.

The set D_s of a preference statement s on a variable X contains the variables that are more important than X. The importance relation is assumed to be transitive, ie. if X is more important than Y and Y more important than Z, then X is more important than Z. The definition of D_v for a variable importance statement v is analogous.

Definition 4 *Let s be a preference statement of a TCP-net N on a variable X. We define the set D_s as $D_s = \{Y|Y$ is a variable of N such that there is a sequence of variable importance statements in N of the form $\mathbf{p_1} : Y \triangleright X_1$, $\mathbf{p_2} : X_1 \triangleright X_2, \ldots, \mathbf{p_n} : X_{n-1} \triangleright X\}$.*

Similarly, if v is a variable importance statement of N of the form $v = \mathbf{p} : X \triangleright Y$, we define $D_v = \{Z|Z$ is a variable of N such that there is a sequence of variable importance statements in N of the form $\mathbf{p_1} : Z \triangleright X_1$, $\mathbf{p_2} : X_1 \triangleright X_2, \ldots, \mathbf{p_n} : X_{n-1} \triangleright X\}$.

The set D_s of a preference statement s on a variable X, contains the preference and variable importance statements that are defined on variables that are more important than X.

Definition 5 *Let s be a preference statement of a TCP-net N on a variable X. We define the set F_s as $F_s = \{s'|s'$ is a preference statement of N on a variable Y such that $Y \in D_s\} \cup \{v|v$ is a variable importance statement of the form $v = \mathbf{p} : Z \triangleright Y$ and $Y, Z \in D_s$ or $Z \in D_s$ and $Y = X\}$.*

Similarly, if v is a variable importance statement of N of the form $v = \mathbf{p} : X \triangleright Y$, we define $F_v = \{s|s$ is a preference statement of N on a variable Z such that $Z \in D_v\} \cup \{v'|v'$ is a variable importance statement of the form $v' = \mathbf{p} : Z \triangleright Y$ and $Y, Z \in D_v$ or $Z \in D_v$ and $Y = X\}$.

We can now define the new aggregation method of the preference and importance statements of a TCP-net. The basic idea is that preference statements on any variable Y that is more important than some variable X, overrides preference statements on X.

Definition 6 *Given a preference statement s of a TCP-net N, define the set S_s as $S_s = R_s - \{(a,b)|(b,a) \in tr(\cup_{k \in F_s} S_k)\}$. Similarly, if v is a variable importance statement of N, define S_v as $S_v = R_v - \{(a,b)|(b,a) \in tr(\cup_{k \in F_v} S_v)\}$.*

It is easy to see that $S_s = R_s - tr(\cup_{k \in F_s} S_k^{-1})$ and $S_v = R_v - tr(\cup_{k \in F_v} S_k^{-1})\}$. We now define the relation induced by a TCP-network.

Definition 7 *Let N be a TCP-net N that contains the preference statements $s_1, s_2, \ldots s_n$ and the variable importance statements $v_1, v_2, \ldots v_m$. The relation induced by N is $S_N = tr(S_{s_1} \cup S_{s_2} \cup \ldots \cup S_{s_n} \cup S_{v_1} \cup S_{v_2} \cup \ldots \cup S_{v_m})$.*

Example 5 *Consider the TCP-network N of example 4. It holds that $D_{s_1} = \{WINE\}$ and $D_{s_2} = D_{s_3} = D_v = \emptyset$. Moreover, $F_{s_1} = \{s_2, s_3, v\}$ and $F_{s_2} = F_{s_3} = F_v = \emptyset$. The relations defined by the preference statements s_2 and s_3 of N are $S_{s_2} = R_{s_2} = \{(\{meat, red\}, \{meat, white\})\}$ and $S_{s_3} = R_{s_3} = \{(\{fish, white\}, \{fish, red\})\}$. The variable importance sentence s induces the relation $S_v = R_v = \{(\{meat, red\}, \{fish, red\}), (\{fish, white\}, \{meat, white\})\}$. The relation induced by the preference statement s_1 is $S_{s_1} = R_{s_1} - tr(S_{s_2}^{-1} \cup S_{s_3}^{-1} \cup S_v^{-1}) = \{(\{meat, red\}, \{fish, red\}), (\{meat, white\}, \{fish, white\})\} - \{(\{meat, white\}, \{meat, red\}), (\{fish, red\}, \{fish, white\}), (\{fish, red\}), \{meat, red\}) (\{meat, white\}, \{fish, white\})\} = \{(\{meat, red\}, \{fish, red\})\}$. The relation induced by the network N is $S_N = S_{s_1} \cup S_{s_2} \cup S_{s_3} \cup S_v = \{\{meat, red\}, \{meat, white\}), (\{fish, white\}, \{fish, red\}), (\{meat, red\}, \{fish, red\}), (\{fish, white\}, \{meat, white\})\}$.*

The new semantics is illustrated better in the next, more complicated example.

Example 6 *Let N_1 be the TCP-network on the variables $COCKTAIL = \{rum, vodka\}$, $STARTER = \{salat, soup\}$, $FOOD = \{meat, fish\}$, $WINE = \{red, white\}$ with the following preferences*
$s_1 = : rum \succ vodka$
$s_2 = rum : salat \succ soup$ $\quad s_3 = vodka : soup \succ salat$
$s_4 = salat : meat \succ fish$ $\quad s_5 = soup : fish \succ meat$
$s_6 = salat, meat : red \succ white$
$s_7 = soup, fish : red \succ white$
$s_8 = soup, meat : white \succ red$
$s_9 = salat, fish : white \succ red$
The variable importance statements are $v_1 = STARTER \triangleright COCKTAIL$, $v_2 = FOOD \triangleright STARTER$ and $v_3 = WINE \triangleright FOOD$. As before, we represent each variable value by its initial (sa stands for salad, so for soup, ru for rum, and re for red).
The relations induced by the preference statements s_6, s_7, s_8, s_9 are $S_{s_6} = R_{s_6} = \{(\{ru, sa, m, re\}, \{ru, sa, m, w\}), (\{v, sa, m, re\}, \{v, sa, m, w\})\}$, $S_{s_7} = R_{s_7} = \{(\{ru, so, f, re\}, \{ru, so, f, w\}), (\{v, so, f, re\}, \{v, so, f, w\})\}$, $S_{s_8} = R_{s_8} = \{(\{ru, so, m, w\}, \{ru, so, m, re\}), (\{v, so, m, w\}, \{v, so, m, re\})\}$, $S_{s_9} = R_{s_9} = \{(\{ru, sa, f, w\}, \{ru, sa, f, re\}), \{v, sa, f, w\}, \{v, sa, f, re\})\}$.

The relation R_{v_3} is $S_{v_3} = R_{v_3} = \{(\{ru, sa, m, re\}, \{ru, sa, f, re\}), (\{v, sa, m, re\}, \{v, sa, f, re\}), (\{ru, sa, f, w\}, \{ru, sa, m, w\}), (\{v, sa, f, w\}, \{v, sa, m, w\}), (\{ru, so, f, re\}, \{ru, so, m, re\}), (\{v, so, f, re\}, \{v, so, m, re\}), (\{ru, so, m, w\}, \{ru, so, f, w\}), (\{v, so, m, w\}, \{v, so, f, w\})\}$.

The preference statements s_4 induces the relation $S_{s_4} = R_{s_4} - tr(S_{s_6}^{-1} \cup S_{s_7}^{-1} \cup S_{s_8}^{-1} \cup S_{s_9}^{-1} \cup S_{v_3}^{-1})$, where $R_{s_4} = \{(\{ru, sa, m, re\}, \{ru, sa, f, re\}), (\{ru, sa, m, w\}, \{ru, sa, f, w\}), (\{v, sa, m, re\}, \{v, sa, f, re\}), (\{v, sa, m, w\}, \{v, sa, f, w\}), (\{ru, so, f, re\}, \{ru, so, m, re\}), (\{ru, so, f, w\}, \{ru, so, m, w\}), (\{v, so, f, re\}, \{v, so, m, re\}), (\{v, so, f, w\}, \{v, so, m, w\})$. Therefore $S_{s_4} = R_{s_4} - \{(\{ru, sa, m, w\}, \{ru, sa, f, w\}), (\{v, sa, m, w\}, \{v, sa, f, w\}), (\{ru, so, f, w\}, \{ru, so, m, w\}), (\{v, so, f, w\}, \{v, so, m, w\})\}$.

The variable importance statement v_2 induces the relation $S_{v_2} = R_{v_2} - tr(S_{s_6}^{-1} \cup S_{s_7}^{-1} \cup S_{s_8}^{-1} \cup S_{s_9}^{-1} \cup S_{v_3}^{-1})$, where $R_{v_2} = \{(\{ru, sa, m, re\}, \{ru, so, m, re\}), \{ru, sa, m, w\}, \{ru, so, m, w\}), \{v, sa, m, re\}, \{v, so, m, re\}), \{v, sa, m, w\}, \{v, so, m, w\}), (\{ru, so, f, re\}, \{ru, sa, f, re\}), \{ru, so, f, w\}, \{ru, sa, f, w\}), \{v, so, f, re\}, \{v, sa, f, re\}), \{v, so, f, w\}, \{v, sa, f, w\})$.

The preference statement s_2 induces the relations $S_{s_2} = R_{s_2} - tr(S_{s_4}^{-1} \cup S_{s_5}^{-1} \cup S_{s_6}^{-1} \cup S_{s_7}^{-1} \cup S_{s_8}^{-1} \cup S_{s_9}^{-1} \cup S_{v_3}^{-1} \cup S_{v_2}^{-1})$, where $R_{s_2} = \{(\{ru, sa, m, re\}, \{ru, so, m, re\}), (\{ru, sa, m, w\}, \{ru, so, m, w\}), (\{ru, sa, f, re\}, \{ru, so, f, re\}), (\{ru, sa, f, w\}, \{ru, so, f, w\})\}$. Therefore, $S_{s_2} = \{(\{ru, sa, m, re\}, \{ru, so, m, re\}), (\{ru, sa, m, w\}, \{ru, so, m, w\})\}$.

The preference statement s_3 induces the relations $S_{s_3} = R_{s_3} - tr(S_{s_4}^{-1} \cup S_{s_5}^{-1} \cup S_{s_6}^{-1} \cup S_{s_7}^{-1} \cup S_{s_8}^{-1} \cup S_{s_9}^{-1} \cup S_{v_3}^{-1} \cup S_{v_2}^{-1})$, where $R_{s_3} = \{(\{v, so, m, re\}, \{v, sa, m, re\}), (\{v, so, m, w\}, \{v, sa, m, w\}), (\{v, so, f, re\}, \{v, sa, f, re\}), (\{v, so, f, w\}, \{v, sa, f, w\})\}$.

The variable importance statement v_1 induces the relation $S_{v_1} = R_{v_1} - tr(S_{s_4}^{-1} \cup S_{s_5}^{-1} \cup S_{s_6}^{-1} \cup S_{s_7}^{-1} \cup S_{s_8}^{-1} \cup S_{s_9}^{-1} \cup S_{v_3}^{-1} \cup S_{v_2}^{-1})$, where $R_{v_1} = \{(\{ru, sa, m, re\}, \{v, sa, m, re\}), (\{ru, sa, m, w\}, \{v, sa, m, w\}), (\{ru, sa, f, re\}, \{v, sa, f, re\}), (\{ru, sa, f, w\}, \{v, sa, f, w\}), (\{v, so, m, re\}, \{ru, so, m, re\}), (\{v, so, m, w\}, \{ru, so, m, w\}), (\{v, so, f, re\}, \{ru, so, f, re\}), (\{v, so, f, w\}, \{ru, so, f, w\})\}$.

Finally, the preference statement s_1 induces the relation $S_{s_1} = R_{s_1} - tr(S_{s_2}^{-1} \cup S_{s_3}^{-1} \cup S_{s_4}^{-1} \cup S_{s_5}^{-1} \cup S_{s_6}^{-1} \cup S_{s_7}^{-1} \cup S_{s_8}^{-1} \cup S_{s_9}^{-1} \cup S_{v_3}^{-1} \cup S_{v_2}^{-1} \cup S_{v_1}^{-1})$, where $R_{s_1} = \{(\{ru, sa, m, re\}, \{v, sa, m, re\}), (\{ru, sa, m, w\}, \{v, sa, m, w\}), (\{ru, sa, f, re\}, \{v, sa, f, re\}), (\{ru, sa, f, w\}, \{v, sa, f, w\}) (\{ru, so, m, re\}, \{v, so, m, re\}), (\{ru, so, m, w\}, \{v, so, m, w\}), (\{ru, so, f, re\}, \{v, so, f, re\}), (\{ru, so, f, w\}, \{v, so, f, w\})\}$. Therefore, $S_{s_1} = \{(\{ru, sa, m, re\}, \{v, sa, m, re\}), (\{ru, sa, m, w\}, \{v, sa, m, w\}), (\{ru, sa, f, re\}, \{v, sa, f, re\}), (\{ru, sa, f, w\}, \{v, sa, f, w\})\}$.

The relation defined by network N_1 is $S_{N_1} = tr(\cup_{i=1}^{9} S_{s_i} \cup S_{v_1} \cup S_{v_2} \cup S_{v_3})$. The ranking induced by this relation on the outcomes of the network is depicted in figure 1.

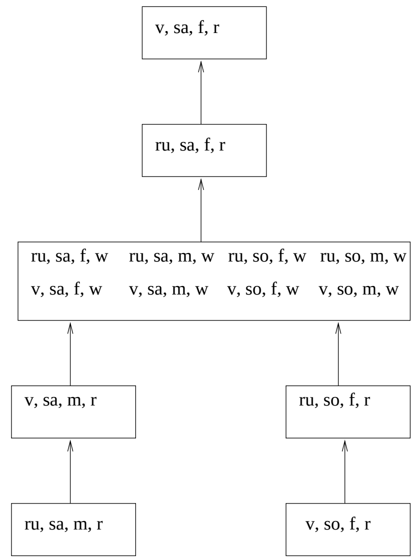

Figure 1: Ordered Outcome Classes for the example 6

4 Conclusions and discussion

In this paper we extended TCP-networks with variable importance statements that assert that a variable is more important than some of its ancestors in the network. We introduced a new semantics for such cyclic statements of length two, and a new method for preference aggregation.

The work described in this paper can be also seen as a first attempt to bring together TCP-networks and work from decision theory. When we move from TCP-nets with relatively simple structure to more complicated ones, as those presented here, there is a need for aggregation methods that are stronger than disjunctive aggregation. In decision theory there are several methods which can be used for solving complex preference aggregation problems.

One possibility is a majority based preference aggregation procedure. However, the result of such aggregation procedures is not guaranteed to be an acyclic outcomes graph [Bouyssou, 1996]. Depending on whether we are looking for a ranking or just for the best choice among the outcomes there exist several procedures which allow to find a result from such a graph [Vincke, 1992].

Clearly all such procedures satisfy some properties, but not others. Basically they all require at some point of the procedure to make some arbitrary hypothesis (for instance some require to reduce cycles into equivalence classes). Unfortunately there is no universal procedure solving this problem and there will never exist one [Vincke, 1992]. For each specific problem it is necessary to take into account [Bouyssou *et al.*, 2005]:
- the type of outcome the procedure is expected to provide;
- the properties the procedure has to satisfy or not to satisfy;
- the complexity of each such procedure wrt to the available resources;
- the intuitive correspondence between the procedure and the

client's requirements.
This will result in an ad-hoc procedure the validity of which is strictly bounded to the specific problem.

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