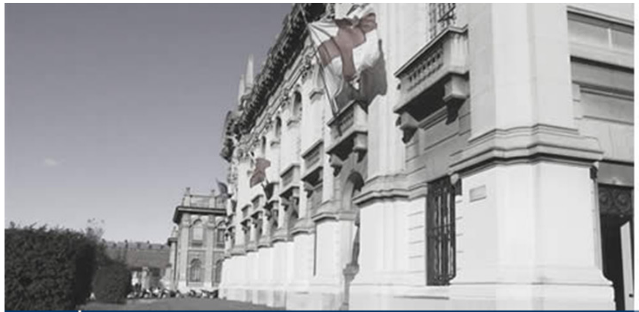

POLITECNICO DI MILANO

A framework for decision aiding *(part 1)*

Alberto Colorni – Politecnico di Milano

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A framework for decision aiding

My aims

- Recall of the general framework
- Step-by-step analysis of four cases
- Presentation of some specific tools
- *Discussion (?)*

Summary

The general framework

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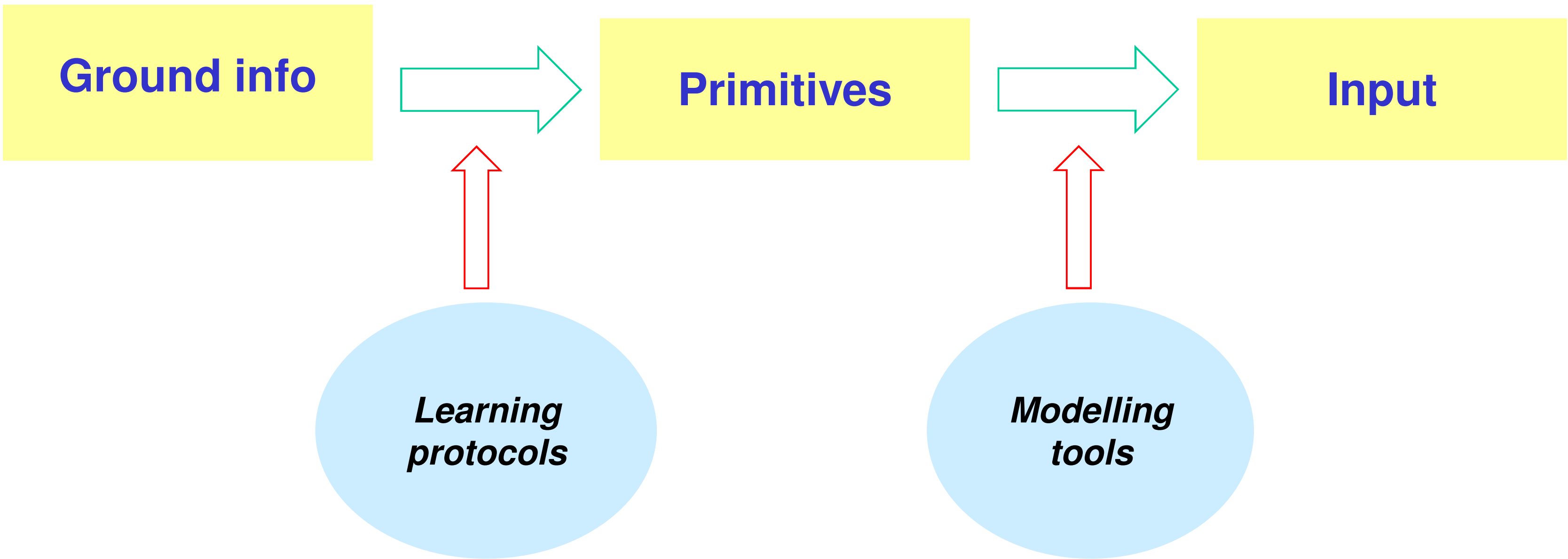
A path *(between the client and the analyst)*



Ground inform.	→	client's statements (in his natural language)
Learning prot.	→	*procedures to identify ordering relations*
Primitives	→	information (strictly) necessary to decide
Modelling tools	→	*analytic tools to define-produce the model*
Input	→	adaptation of the info to a specific algorithm

Four examples

ExA → Palio di Siena

(an italian horse race)

http://en.wikipedia.org/wiki/Palio_di_siena

ExB → St. John Hospital

(a nurse recruitment)

http://www.stjohnprovidence.org/default.aspx

ExC → Sausages

(a food classification)

http://en.wikipedia.org/wiki/Sausage

ExD → Le salaire de la peur

(a safe path problem)

http://www.youtube.com/watch?v=U3ssM6K1ldA

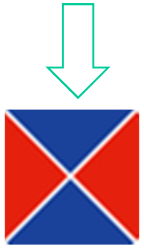
Ground information for ExA

Problem

The client is a horse race gambler; the race is Palio di Siena (10 “contrade”)
In order to assess the “value” of each possible bet he takes into account 3 different information:

- the quality of the horse,
- the quality of the jockey,
- the weather conditions.

The client wants to rank all possible bets.



Panther flag

Learning protocols

Procedures through which the analyst will try to gather the client indications.

In our case:

- which horses does he prefer ?
- with which jockey ?
- under which weather conditions ?
- etc.

Ground information for ExB

Problem

A hospital is considering the recruitment of nurses for three of their departments: General medicine (MG), Oncology (ON), Pediatrics (PE).

The selections are managed by the chief surgeon P and the vice-surgeon V.

Candidates fill an application form and go through an interview.

The result is a report where the two managers consider three information:

- the age,
- the specialisation (if any),
- the motivations of the candidate.

The managers want to assign the candidates in the best way.

Learning protocols

Procedures through which the analyst will try to gather the client indications.

In our case:

- what is a “good nurse” for a given department ?
- what specialisations with respect to the dept. requirements ?
- how the age influences the fitting of a candidate to a given dept. ?
- etc.

Ground information for ExC

Problem

The Nutrition Agency (NA) has to classify a lot of different kind of sausages. The result is a clustering of all the sausages to K categories of ”similarity” (the names will be defined after the clustering).
Each sausage is defined by a mix of m attributes.
There are some indications ”against” to be considered (if the difference between two kinds of sausages is too high, the sausages have to go to different clusters, etc.).

Learning protocols

Procedures through which the analyst will try to gather the client indications.

In our case:

- what are the attributes to be considered ?
- what is the meaning of differences ? (rate of fat ? others ?)
- how to deal with indications ”against” ?
- how to deal with continuous and discrete data ? (see the example)
- etc.

Ground information for ExD

Problem

Your company have to deliver potentially dangerous goods to a regional system of clients. You are worried about the safety of such deliveries; costs represent a secondary issue. Deliveries are done using dedicated trucks. Road network is known and you also have the annual statistics of accidents for each segment of the network. In your decision you ha
safety and costs.
The issue is how to define a set of "sufficiently safe" path
choose the most safe path within the network (for a giver

Learning protocols

The l. p. are procedures through which the analyst will try to gather

In our case:

- given a specific delivery (destination), which are t
  (it is the whole set of feasible paths connecting the origin to th
  but note that this set can be extremely large)
- given two paths x and y, when can you consider the relation "x at least as good as y" ?
  (if the overall likelihood of having an accident through path x is not superior of the corresponding through path y → you prefer less risky paths)
- what about costs ?

Modelling. We need to define in a formal way the set A of alternatives. In order to do so we represent the road network as a graph (N, A). To each couple of nodes i and j related by an arc we associate a binary variable xij . For each variable we have info:

- dij : the length of the arc,
- vij : the daily traffic,
- aij : the annual number of accidents,
- etc.

The information categories *(primitives)*

Values (related to attributes).

An alternative *x* can be described by a set of attributes; each attribute is characterized by a scale (scales can be nominal, ordinal, ratio or interval ones). This description (*x* is 10cm long; *y* is yellow, etc.) must be reinforced by further information represented by preferential statements (she prefers long tables to short ones; I do not like yellow shoes), possibly of more complex content (he prefers a train travel to Paris than a flight to Amsterdam; my preference of apples against oranges is stronger than my preference of peaches against apricots). We distinguish two types:

- *comparative sentences*, where the alternatives are compared among them (under more attributes) in order to express a preference;
- *absolute sentences*, where an alternative is directly assessed with respect to some value structure (under one ore more attributes).

Opinions (related to stakeholders).

Decisions can be affected by the judgments and opinions of many stakeholders. In this case preference statements have to be associated to opinions. It is reasonable however, to distinguish among:

- *comparative opinions* (stakeholder *i* prefers *x* to *y*), where preferences are expressed among elements of the alternative set;
- *absolute opinions* (stakeholder *i* considers *x* as worthy), where preferences are expressed under form of value assessments.

Likelihoods (related to scenarios).

Preferences often depend from uncertain conditions. If we focus on decision situations the primitives we need to consider will be preference statements of the type “under scenario j, he prefers x to y” or of the type “under scenario j, x is unworthy”.

From ground info to primitives

Ground info

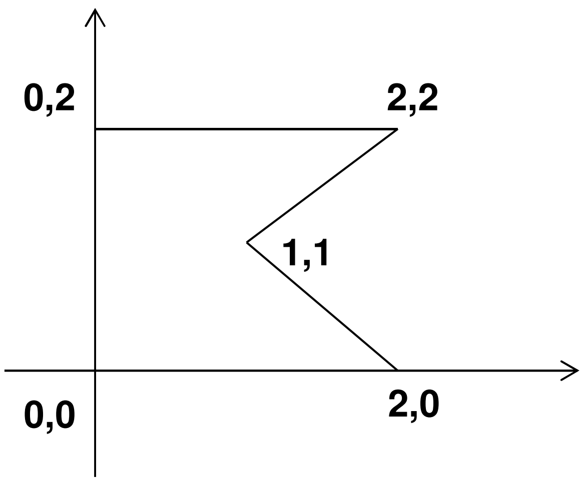
The problem has 2 variables.
The DM gives a set of couples "good" for him and other couples "not good".

For instance →

(0, 0) (2, 0) (0, 2) (2, 2) (1, 1) etc.	good	x1 neg x2 neg x1 > 2 x2 > 2 (2, 1) etc.	not good

Primitives

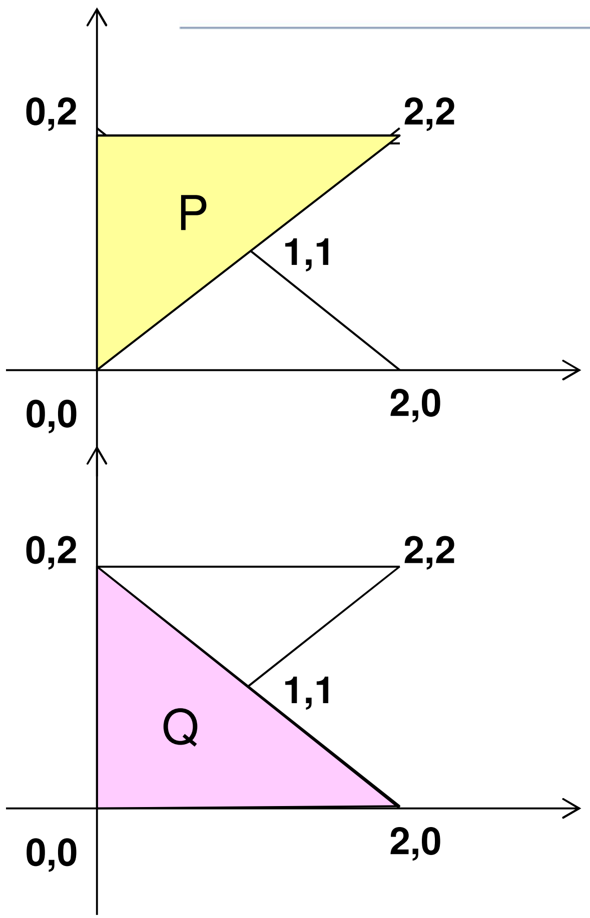
So you can define a region X of feasible solution like this …



Algorithm ?

You can use an (integer) LP,
but region **X is not convex !**

From primitives to input



Region & algorithm

The region X is not convex,
but you have → **X = P U Q (you have an OR)**

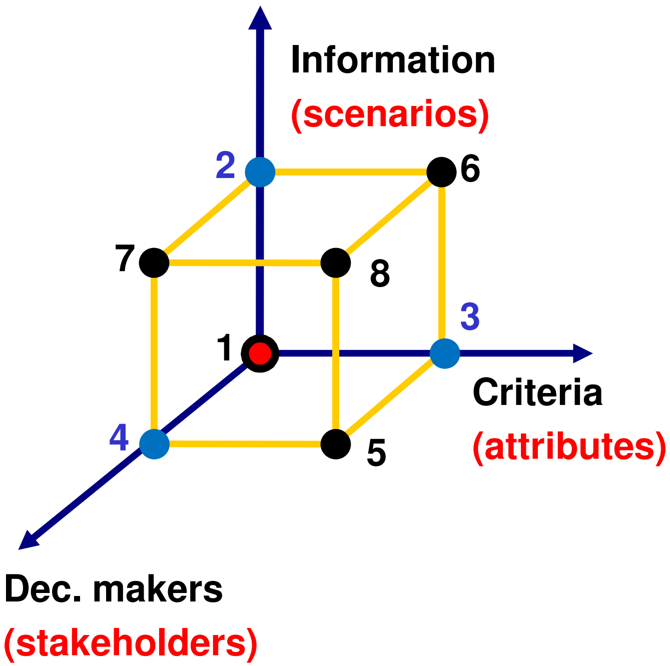
Input

(Int)LP works in convex region.
You can transform the problem:

$$0 \leq x1 \leq 2$$
$$0 \leq x2 \leq 2$$
$$x1 - x2 \leq 0 + M_P{}^*y_P$$
$$x1 + x2 \leq 2 + M_Q{}^*y_Q$$
$$y_P + y_Q \leq 1$$
$$y_P\,,\ y_Q = \{0,\ 1\}$$

This is a classical **Int.LP** , so you can solve it in the usual way !

Three elements & eight problems



Information
- complete
- **partial** → **state identif. & risk an.**

Criteria
- one
- **more** → **trade-off**

Dec. makers
- one
- **more** → **cooper. vs conflicts**

1. Math. Programming (MP)
2. **Risk analysis (RA)**
3. **Multiple criteria (MC)**
4. **Group choice (GC)**

5. 6. 7. 8. → Game theory, etc.

An ideal decision problem *(point 1)*

- **Someone who decides**

  with respect to one clear **objective**

  with a set of well defined **constraints**

  with all the suitable **information**

  in presence of a finite / infinite || **set of alternatives**

- **Examples**

  - *(1) → an ideal **discrete** case*
  - *(2) → an ideal **continuous** case*

Ideal example 1

Combinatorial optimization

Your chorus is defining the storyboard of a concert and you must choose between a set of mottetti (a “mottetto” is a choral musical composition). Each mottetto (m_1, m_2, …, m_n) has a time of execution t_j and a level of success s_j (j =1,…,n).
The total time of the exhibition is T min.

What can you do ?

If you want, consider this specific instance:
n = 4; t = (10, 22, 37, 9); s = (60, 55, 100, 15); T = 45

(i) What are the variables ?
(ii) How many solutions ?
(iii) What is the optimal choice ?

Ideal example 2

Linear programming (LP)

You must define the week production of a (small) firm that has only 2 products, A and B.
One item of A needs 4 units of the resource R1 and 2 unit of the resource R2.
One item of B needs 1 unit of the resource R1 and 3 units of the resource R2.
You have (weekly) 200 units of R1 and 480 units of R2, and you know that the maximum possible sale for B is 110 items.
The net revenue for item A is 500 €, for item B is 300 €.

What can you do ?

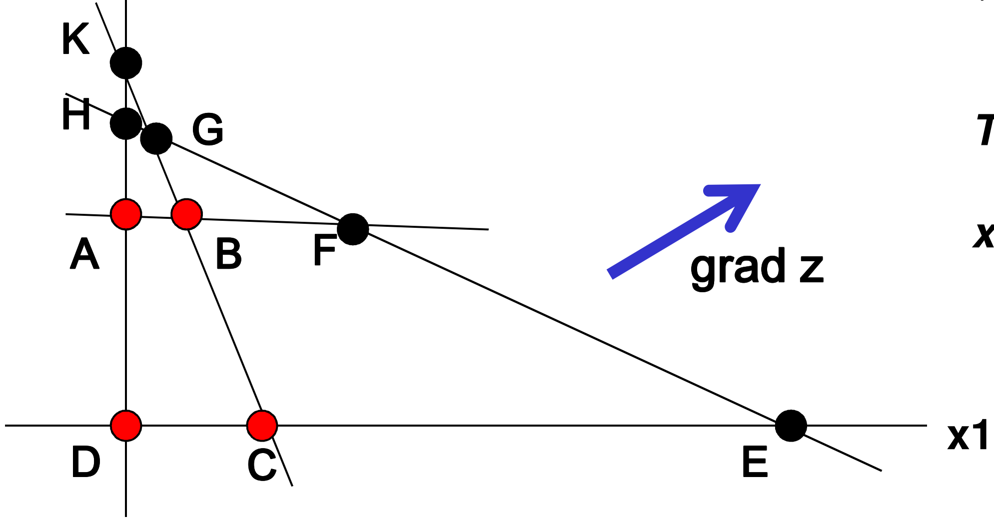
(i) What are the variables ?
(ii) How many solutions ?
(iii) What is the optimal choice ? **(solve with Excel ...)**

Ideal example 2: the model

LP properties !

- Continuity
- Proportionality
- Additivity

$z\ (max) = 500\ x1 + 300\ x2$	(objective function)
s.t.	(a set of constraints)
$4\ x1 + 1\ x2 \leq 200$	(resource R1)
$2\ x1 + 3\ x2 \leq 480$	(resource R2)
$x2 \leq 110$	(constr. x2 sale)
$x1,\ \ x2 \geq 0$	(non neg. constr.)



The optimal choice: B

x1=22.5, x2=110, z= …,

A real decision problem

- **Complexity** (problem dimension, non linearity, …)
- **Uncertainties** (non-deterministic context, data mining)
- **Several stakeholders** (distributed decision power)
- **Different rationalities** (multiple criteria and preferences)
- **Various time horizons** (often)
- **Need of simulation models** (what … if …)

↳ ***the other points (2, 3, …) of the cube***

Examples *(position in the cube)*

ExA – horse race →	*(ranking)*	**RA-MC**	→	**p. 6**
ExB – nurses →	*(assign)*	**MC-GC**	→	**p. 5**
ExC – sausages →	*(cluster)*	**MC**	→	**p. 3**
ExD – safe paths →	*(rating/ranking)*	**MC**	→	**p.3**

Five main features

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Features of a decision problem

1. Set of the alternatives
2. Problem statement
3. Independence (or not) of …
4. Differences of preferences
5. Pos./negative reasons

a quick survey

1. Set of alternatives

The set A of the alternatives:

i. A is a finite set of objects (countable enumeration)

ii. A is a subset of all possible combinations of attribute values

iii. A is the product of a set of discrete (binary) decision variables

iv. A is a vector space (all the admissible values of real variables)

Examples:

ExA (Palio) → finite set (10 contrade)

ExB (nurses) → finite set (n candidates)

ExC (sausages) → vector space (m dimensions)

ExD (paths) → product of a binary variable set (large !)

2. Problem statement *(partition of the alternative set)*

- **Different partitions**

- Classes:
  - ordered / not ordered
  - not predefined / predefined

	not pred.	predef.
ordered	RANKING	RATING (sorting)
not ord.	CLUSTER.	ASSIGNM. (recogn.)

- Two problems:

  choice → what you want (and the remaining …)
  or
  rejection → what you don't (and the remaining …)

The other features

3. Defining independence (or not) of …

There are two principal interpretations (*H* is a set of criteria):

- "*x* is at least as good as *y*, under *I*", independently on what happens to $H \setminus I$ (preferential indep.);
- "*x* is at least as good as *y*, under *I*", provided a condition holds in some $J \subseteq H \setminus I$ (conditional indep.).

These two interpretations lead to completely different problem formulations and consequently to different methods and resolution algorithms: preferential independence allows to envisage a linear (additive) model representing preferences; conditional preferences lead to more complex preference structures (non linear aggregation functions).

4. Defining differences of preferences

Let's consider the sentence "*x* is strictly better than *y* and these are both better than *z*". We can represent this sentence giving numerical values to *x, y, z* (for instance, $x = 3$, $y = 2$, $z = 1$). But we could choose the numerical representation $x = 100$, $y = 10$, $z = 1$ and it would be the same. In many cases we could either have richer information (we know for instance that *x* is twice more heavy than *y*) or we would like to have information of the type "*x* is much more better than *y*". We need to reason in terms of "differences of preferences". In other terms we need primitives of the type: "*xy* is not less than *zw*" where *xy* (*zw*) represents the difference of preference between *x* and *y* (*z* and *w*). Primitives of this type can be used in order to express ordinal preferences, while the opposite is not true. We can claim that primitives should always be considered as sentences about differences of preferences, the ordinal case being a special one

5. Defining pos./negative reasons

Consider a preference statement of the type: "I do not like *x*", or "any candidate, but not *x*".
Such statements can be considered as explicit "negative preferential statements" to be considered independently from the "positive ones" (which are the usual ones). The idea here is that there are cases where decision makers need to express negative judgments and values which are not complementary to the positive ones (such as a veto on a specific dimension).

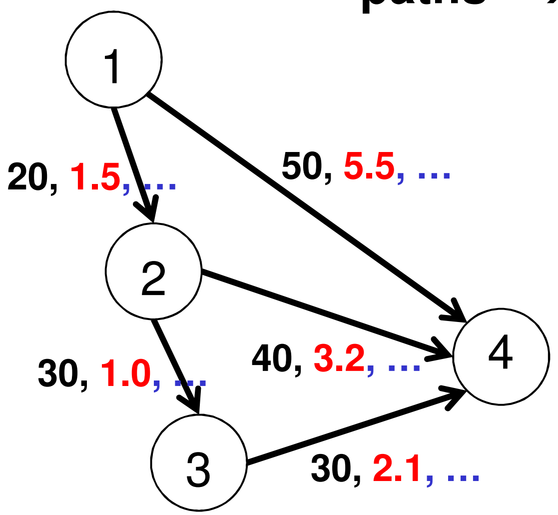
Examples *(summarizing)*

ExA – Palio	→	finite set of alternatives ranking problem RA-MC (point 6)
ExB – nurses	→	finite set of alternatives assignment MC-GC (point 5)
ExC – saus.	→	infinite alternatives (vector space) clustering MC (point 3)
ExD – paths	→	finite (but large …) set of alternatives rating ***OR*** ranking MC (point 3)

we explore this case

ExD (paths): rating *OR* ranking

Let's present a numerical instance.



paths →

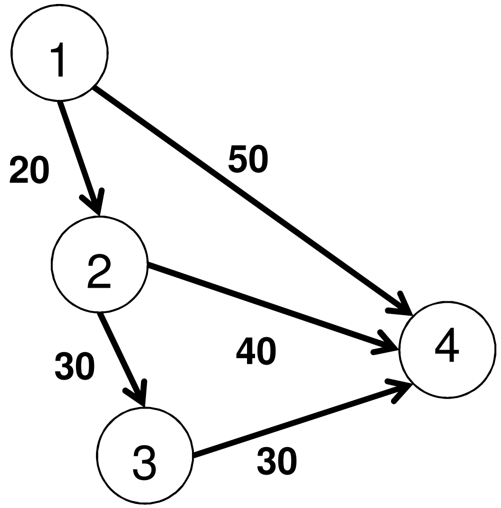
	km	acc(y)	linear.	beauty
1 – 4	50	5.5	1.5	4
1 – 2 – 4	60	4.7	1.0	6
1 – 2 – 3 – 4	80	4.6	1.2	9

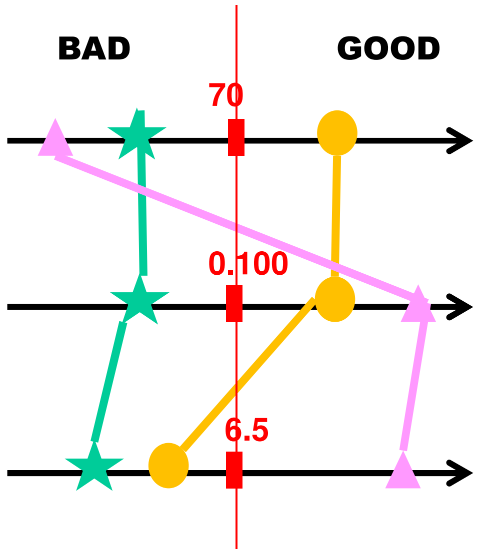
Ground info
- DM indicates as criteria: **time**, **risk**, **beauty**
- Risk is slightly more important then time
- These two are much more imp. than beauty

Rating
- Define the classes
- Define reference situations
- Put each path in its class

Ranking
- A common scale (utility)
- Compute the ranking (order.)
- Sensitivity analysis
  (w.r.t. the weights)

ExD: rating





Rating

- Define k classes (k = 2, good-bad)
- Define reference situations (for each criteria)
- Put each path in its class (compare with ref.)

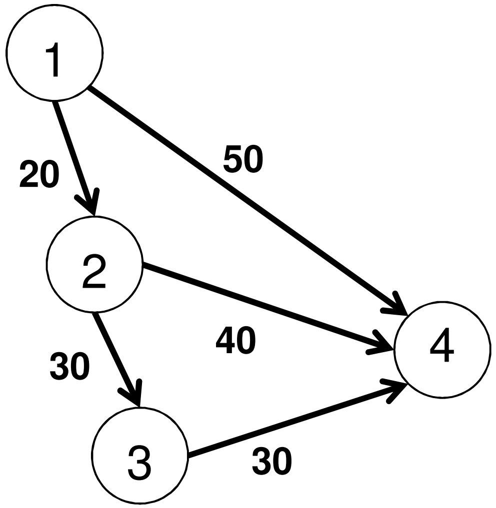
Protocols – Primitives – Ref.situations

- Define attributes related to DM indications:
  1) time = km * linear (?)
  2) risk = acc / km
  3) beauty = beauty

- Attrubutes & Ref.situations:

	time	risk	beauty
➢ **path A (14)**	75	0.110	4
➢ **path B (124)**	60	0.078	6
➢ **path C (1234)**	96	0.057	9
→ weights	**0.40**	**0.50**	**0.10**
→ ref.situations	**70 m.**	**0.100**	**6.5**

ExD: ranking



Ranking

- Define a common scale (utility)
- Compute the ranking (max utility)
- Sensitivity analysis (w.r.t. weights)

Crucial question

- What can we do when the number of paths is large ?
- What info can be associated with each single arc ?

- **Utility (a common scale)**
- Linear scales for all the attributes:
  - time → [0, 100]
  - risk → [0, 0.200]
  - beauty → [0, 10]
- Evaluation matrix:

	U(time)	U(risk)	U(beauty)
path A (14)	0.25	0.45	0.40
path B (124)	0.40	0.61	0.60
path C (1234)	0.04	0.71	0.90
Weights →	**0.40**	**0.50**	**0.10**

- **Utilities** **U(A)=0.365** **U(B)=0.525** **U(C)=0.461**

Some basic tools

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To decide …

A formal decision process needs tools:

i. **to abstract**

ii. **to analyze**

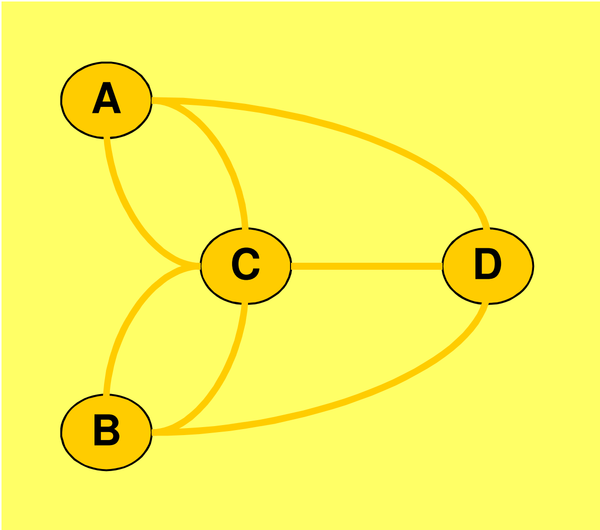
iii. **to summarize**

(and much more …)

Tools to abstract / 1

- 1736
- Konigsberg

- Euler
- Graph theory



- The 7 bridges
- A riddle

- The Euler model
- The answer (similar to …)

Tools to abstract/ 2

The death of Count Kinskij

http://pacta.org/produzioni/spettacoli/teatro-in-matematica/i-7-ponti-e-il-mistero-dei-grafi/

- The count drank poisoned water (from one of his 7 lovers)
- All 7 lovers were in the castle the day of his death
- The murderer should have come to the castle twice

  (one for…, one for…), the others only once.
- Statements of the 7 women:

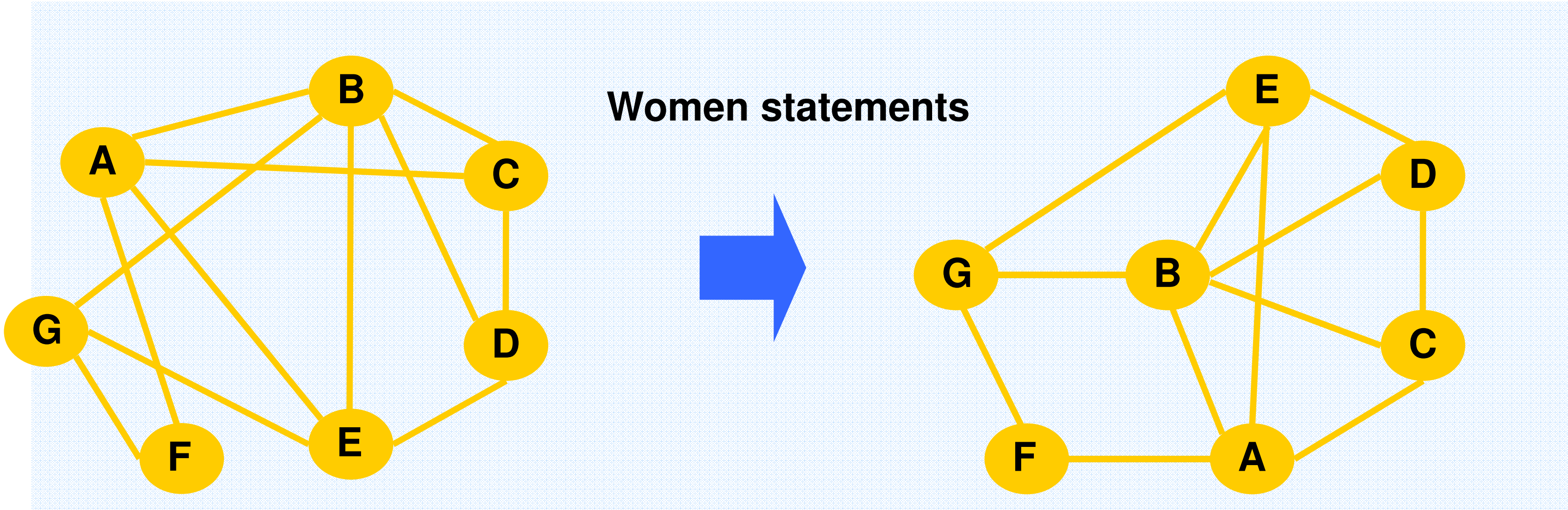
Alice saw	B C E F
Barbie saw	A C D E G
Clara saw	A B D
Diana saw	B C E
Elena saw	A B D G
Francis saw	A G
Gloria saw	B E F

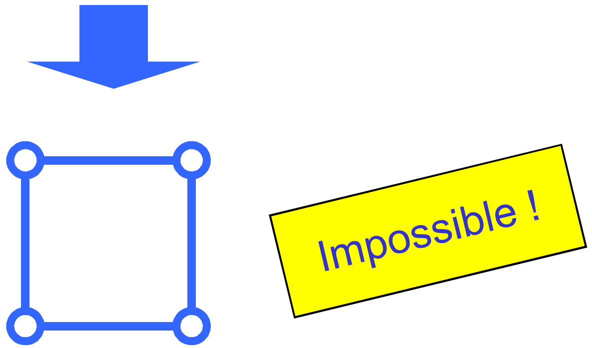
Elementary !

(the detective said)

The solution

The death of Count Kinskij





A E D C
A E G F
A B G F

} ∄ (so **A** lies)

Graph theory & decision problems

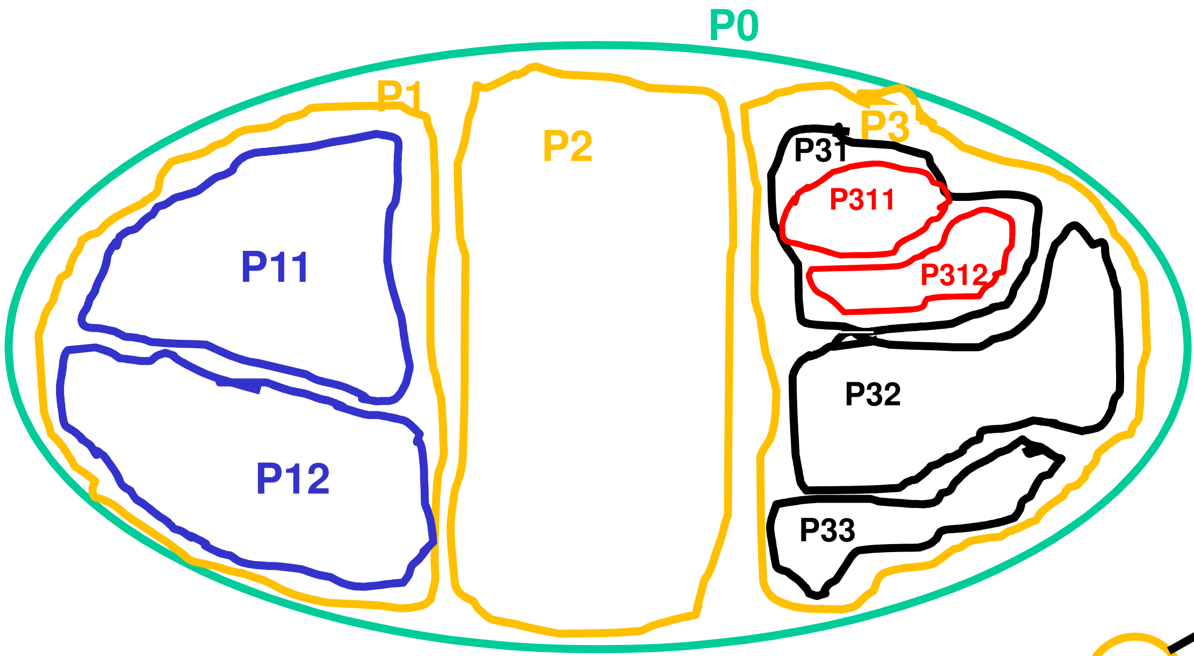
- **General reports**
  - http://en.wikipedia.org/wiki/Graph_theory
  - http://en.wikipedia.org/wiki/Route_inspection_problem
- **Applications**
  - http://www....
  - http://www....
  - http://www.ratp.info/orienter/cv/cv_en/carteparis.php (the Paris metro)

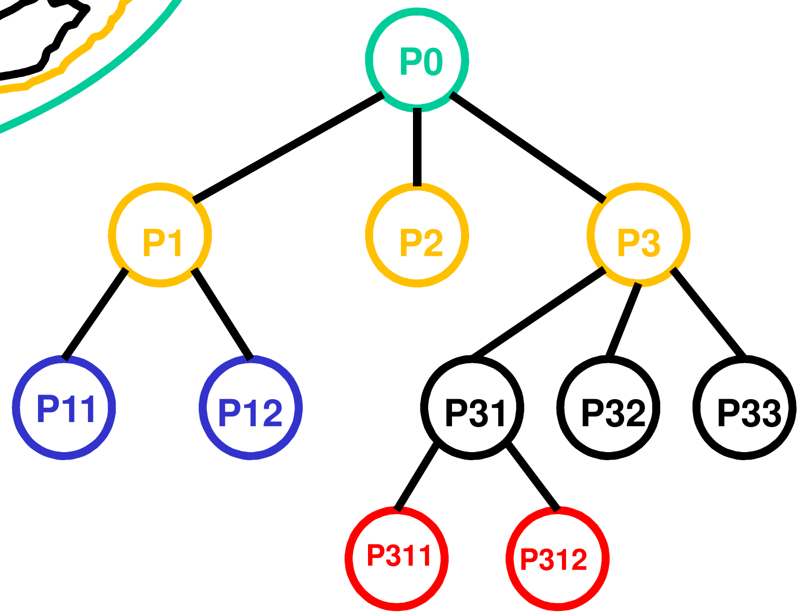
search …

- **A famous problem – TSP**
  - http://www-e.uni-magdeburg.de/mertens/TSP/index.html
  - http://www.tsp.gatech.edu/index.html
  - http://www.graphtheory.com/

Tools for analysis / 1

- Branch & Bound





- **Branching rules → build a tree**
- **A lot of (small) subproblems**
- **Bounding rules → cut the tree**
- **A lot of applications**

Tools for analysis / 2

- **Sudoku** (Corriere della Sera, 3 Sept. 2010)

		4				9		
	1	6	2		4	3	8	
	8					?	5	
4			6		2			1
3			9		8			4
	3						6	
	6	7	3		5	1	4	
		2				8		

- What number in position ? … or …
  what position for number 4 in the upper right square ?

Tools for analysis / 3

- **Sudoku** (Corriere della Sera, 3 Sept. 2010)

		4				9	1	
	1	6	2		4	3	8	7
	8	3				4	5	
4			6		2			1
3			9		8			4
	3						6	
	6	7	3		5	1	4	X
	4	2				8		

(i) undoubted positions

(ii) what in X ?

What in position X ? → 2 or 9 (and not 8 because …)

branch: (a) → X = 2 but if X = 2 …

branch: (b) → X = 9 in this case …

Tools for analysis / 4

- **Sudoku** (Corriere della Sera, 3 Sept. 2010)

		4				9	1	
	1	6	2	Y	4	3	8	7
	8	3				4	5	
4			6		2			1
3			9		8			4
	3						6	
8	6	7	3	2	5	1	4	9
	4	2				8		

(i) undoubted positions

(ii) what in **Y** ?

What in position Y ? → 5 or 9

situations to be explored are (b1) Y = 5, and (b2) Y = 9

branch: (b1) → Y = 5 (in this case 9 is not …)

Tools for analysis / 5

7	5	4	8	3	6	9	1	2
9	1	6	2	5	4	3	8	7
2	8	3	7	9	1	4	5	6
4	9	8	6	7	2	5	3	1
6	2	1	5	4	3	7	9	8
3	7	5	9	1	8	6	2	4
1	3	9	4	8	7	2	6	5
8	6	7	3	2	5	1	4	9
5	4	2	1	6	9	8	7	3

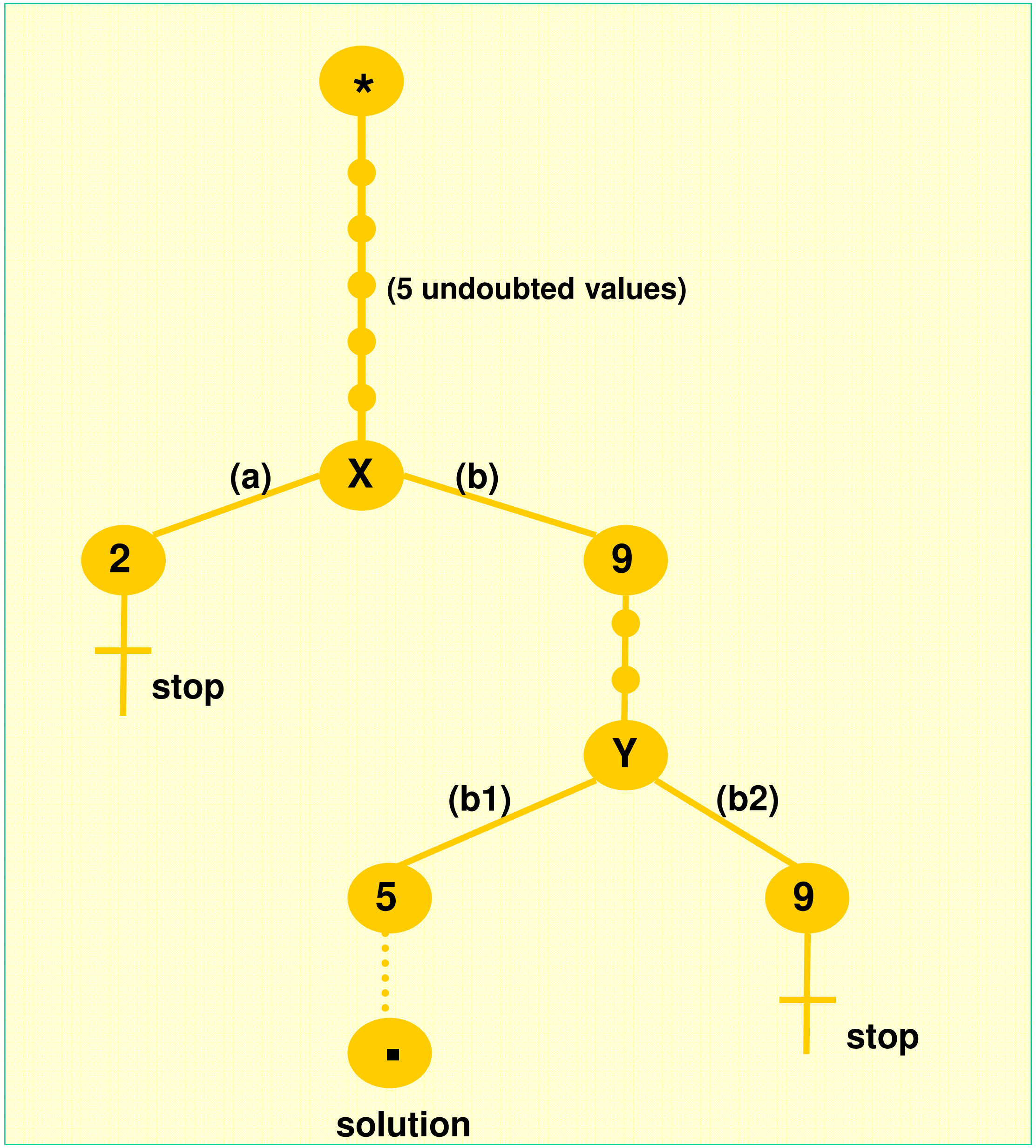
Step 53 (of b1)

Stop !

Found a unique solution, so:

no branch (b2): †

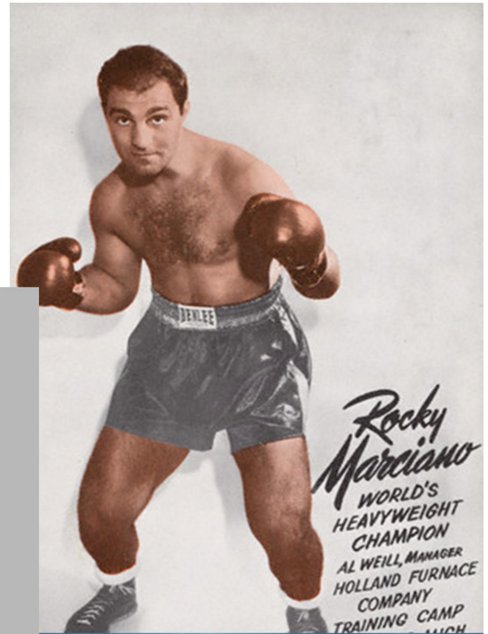
The solution (visualization)

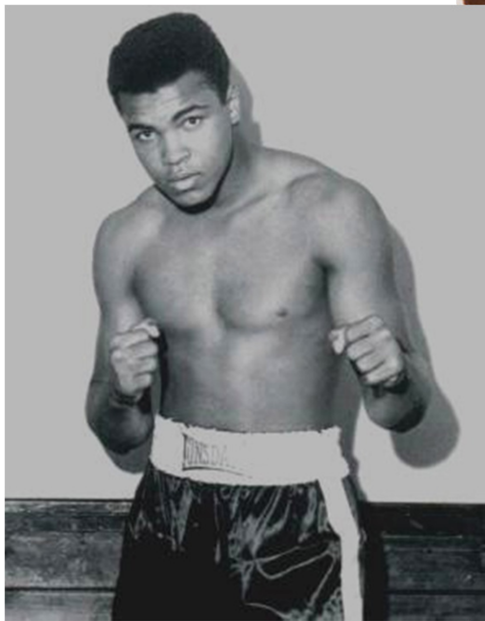


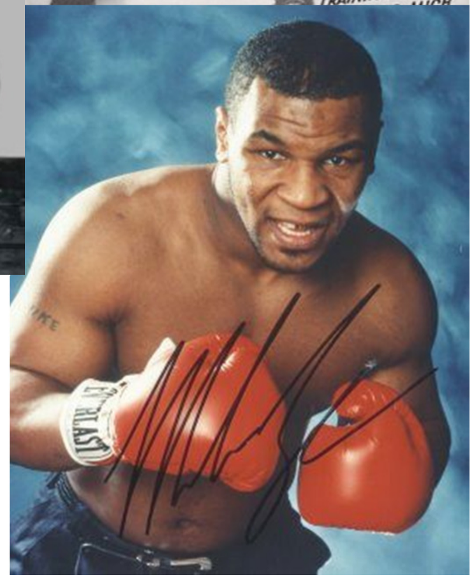
- Branching rules
- A lot of (easier) subproblems
- Ending rules

Tools to summarize / 1 (an example)

Who is the best boxeur in the world of all times?







Indicators:

- strength
- speed
- n. of victories
- years of premiership
- …

We need a common frame to compare the alternatives !

Tools to summarize / 2

All the tools helping
the paths from *k* to 1

Examples of tools

- Bayes theorem
- Experiments & dec. trees
- Pairwise comparison
- Eigenvectors
- Peer evaluation
- Linear algebra (Frobenius)
- …

Cases / models follow (in part 2)

Going to the origin

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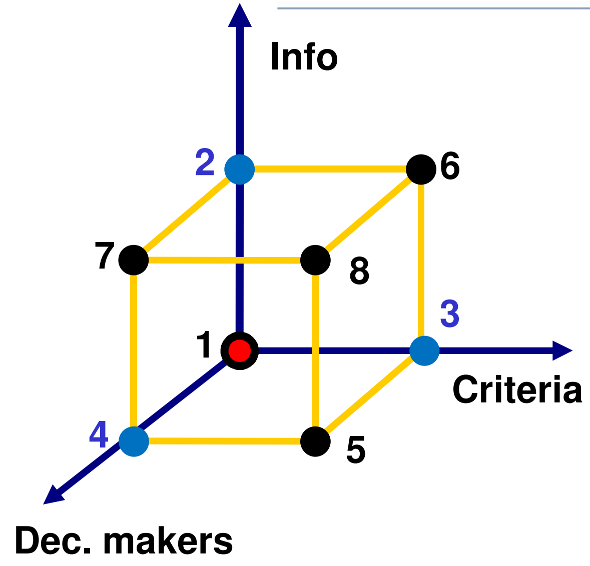
From point *k* to point 1



How ?

- *What is (are) the path(s) ?*
- *What are the the conditions ?*
- *What are the tools ?*

Let's start with:

- a numerical example (the nurse problem)
- a comparison between three situations

ExB – nurses / 1 *(assignment)*

Alternatives (candidates) → C1, C2, C3, C4, C5
Dec. makers → P (chief surgeon), V (vice-surgeon)
Destinations → dept.MG / dept.ON / dept.PE / no assumption.
Evaluation → age + form (specialisation) + interview (motivations)

Learning protocols → a set of questions for P and V (answers follow)

P: (i) more experience = better work; (ii) specialisation is an important element for working in dept.ON; (iii) experience and motivations are key factors for working in any department.

V: (i) 30 years are the ideal age to work in PE (so he is against the entry of people over 40 in that dept.); (ii) the experience and specialisation are key factors to work in MG; (iii) motivations are not significant, except for dept.ON.

Primitives → rules for modelling the problem (by learning protocols)

P: (i) his preferences are increasing with the age of the candidates; (ii) coherence specialization-dept. is important (especially for ON); (iii) the weight of first and third attribute is significant.

V: (i) in PE the ideal age is 30 years, he is contrary to ages over-40; (ii) age (as a proxy of experience) and specialization are important in MG; (iii) motivations are important in dept.ON.

Data:

	age	special.	motiv.
C1	45	general	+
C2	31	maternity	++
C3	37	orthopedics	=
C4	25	no special..	+++
C5	35	dentist	– (no)

the candidate features

ExB – nurses / 2

Primitives → rules for modelling the problem (by learning protocols)

P: (i) his preferences are increasing with the age of the candidates; (ii) coherence specialization-dept. is important (especially for ON); (iii) the weight of first and third attribute is significant.

V: (i) in PE the ideal age is 30 years, he is contrary to ages over-40; (ii) age (as a proxy of experience) and specialization are important in MG; (iii) motivations are important in dept.ON.

Data:

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C3	37	orthopedics	=
C4	25	no special..	+++
C5	35	dentist	– (no)

from candidate features
to levels of fitting (with GM)

Fitting with MG:

	age	special.	motiv.	age	special.	motiv.
C1	0.8	1.0	0.3	1.0	1.0	0.5
C2	0.3	0.6	0.7	0.3	0.8	0.5
C3	0.6	0.4	0.1	0.6	0.6	0.5
C4	0.2	0.0	0.9	0.0	0.2	0.5
C5	0.5	0.1	0.0	0.5	0.3	0.5

(for dec. maker P) **(for dec. maker V)**

Rules → see Primitives

ExB – nurses / 3

A three dimensional matrix (fitting candidates-dept.MG) and two possible paths

values V	Age	Spe	Mot
C1			0.5
C2			0.5
C3			0.5
C4			0.5
C5			0.5

values P	Age	Spe	Mot
C1	0.8	1.0	0.3
C2	0.3	0.6	0.7
C3	0.6	0.4	0.1
C4	0.2	0.0	0.9
C5	0.5	0.1	0.0

(1)

	Age	Spe	Mot
C1	0.9	1.0	0.4
C2	0.3	0.7	0.6
C3	0.6	0.5	0.3
C4	0.1	0.1	0.7
C5	0.5	0.2	0.2

(2)

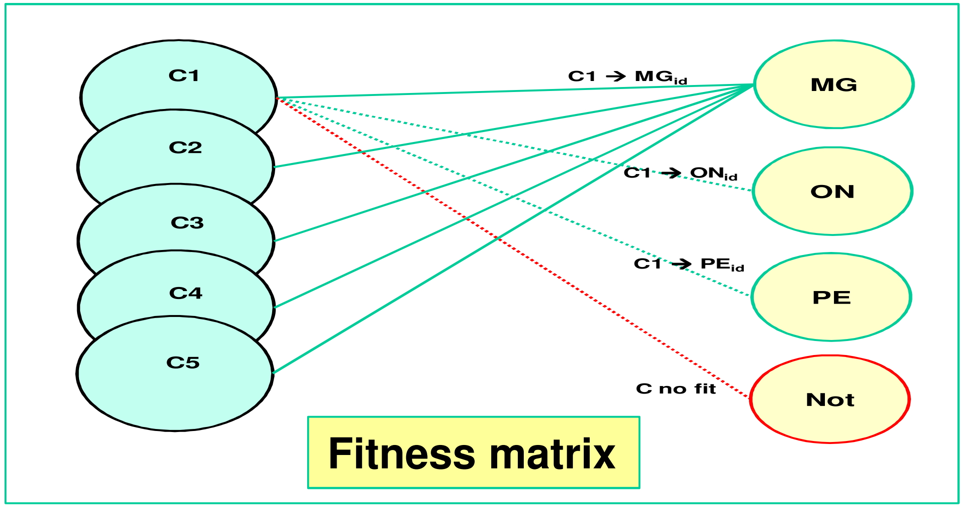
	P	V
C1	0.64	1.00
C2	0.52	0.55
C3	0.36	0.60
C4	0.44	0.10
C5	0.22	0.40

Path (1): negotiation (agreement) among decision makers to identify shared candidate-attribute values (numerical example uses the average between P and V values), thus obtaining the matrix on the left. A second phase is an aggregation via multi attribute analysis. Referring to our cube, the approach brings the problem from point 5 to point 3 and then to 1.

Path (2): each decision maker produces the vector of his fitting levels between candidates and dept MG: in the example surgeon P uses weights (0.4, 0.2, 0.4) consistent with his statements in the learning phase, while V ignores the column Mot and gives equal weight to the other two. This approach brings the problem from point 5 to point 4 and then to point 1.

Both paths produce a vector with 5 values indicating the **fitness** the other candidates with MG. With the same procedure we obtain the fitness of all the candidates with the other depts (ON and PE) or not.

The final situation is therefore an **fitness matrix** to permit the best fit between candidates and depts (or their refusal).



Comparison: a problem at point 2 of the cube

State of n.		A	B	C	D	
↓						
ω_1		90	60	50	80	
ω_2		30	80	20	20	
ω_3		10	50	90	10	

ω_i = i-th state of nature

From point 2 to point 1

State of n.		A	B	C	D		p_i
↓							
ω_1		90	60	50	80		**.50**
ω_2		30	80	20	20		**.10**
ω_3		10	50	90	10		**.40**
Exp. val.		52	58	**63**	46		

Probability

Notes:
- use of different approaches (also without prob.)
- more info on probability using experiments

Comparison: a problem at point 3 of the cube

Criteria		A	B	C	D	
↓						
c_1		90	60	50	80	
c_2		30	80	20	20	
c_3		10	50	90	10	

c_i = i-th criterion

From point 3 to point 1

Criteria		A	B	C	D		w_i
↓							
c_1		90	60	50	80		**.50**
c_2		30	80	20	20		**.10**
c_3		10	50	90	10		**.40**
Utility		52	58	**63**	46		

Weights

Notes:
- check the indipendence of criteria
- identify the DM preference structure with …

Comparison: a problem at point 4 of the cube

Dec. Mak.		A	B	C	D	
↓						
d_1		90	60	50	80	
d_2		30	80	20	20	
d_3		10	50	90	10	

d_i = i-th decision maker

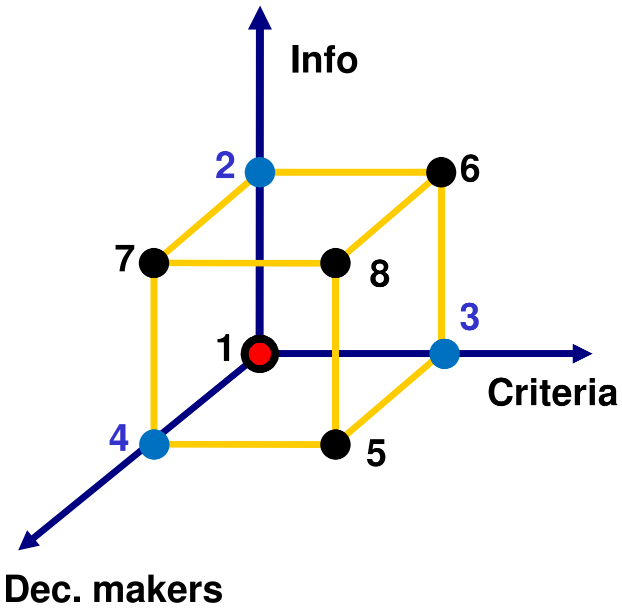
From point 4 to point 1

Dec. Mak.		A	B	C	D		π_i
↓							
d_1		90	60	50	80		.50
d_2		30	80	20	20		.10
d_3		10	50	90	10		.40
Soc. ch.		52	58	**63**	46		

Power indeces

Notes:
- a shared scale (between DM's) ?
- what method for power indeces ?

What about the mixed cases ?



From 2 to 1 → estimation of **probabilities**

From 3 to 1 → estimation of **weights**

From 4 to 1 → estimation of **powers**

⇩

TOOLS

What about the path from 5 (or …) to 1 ?

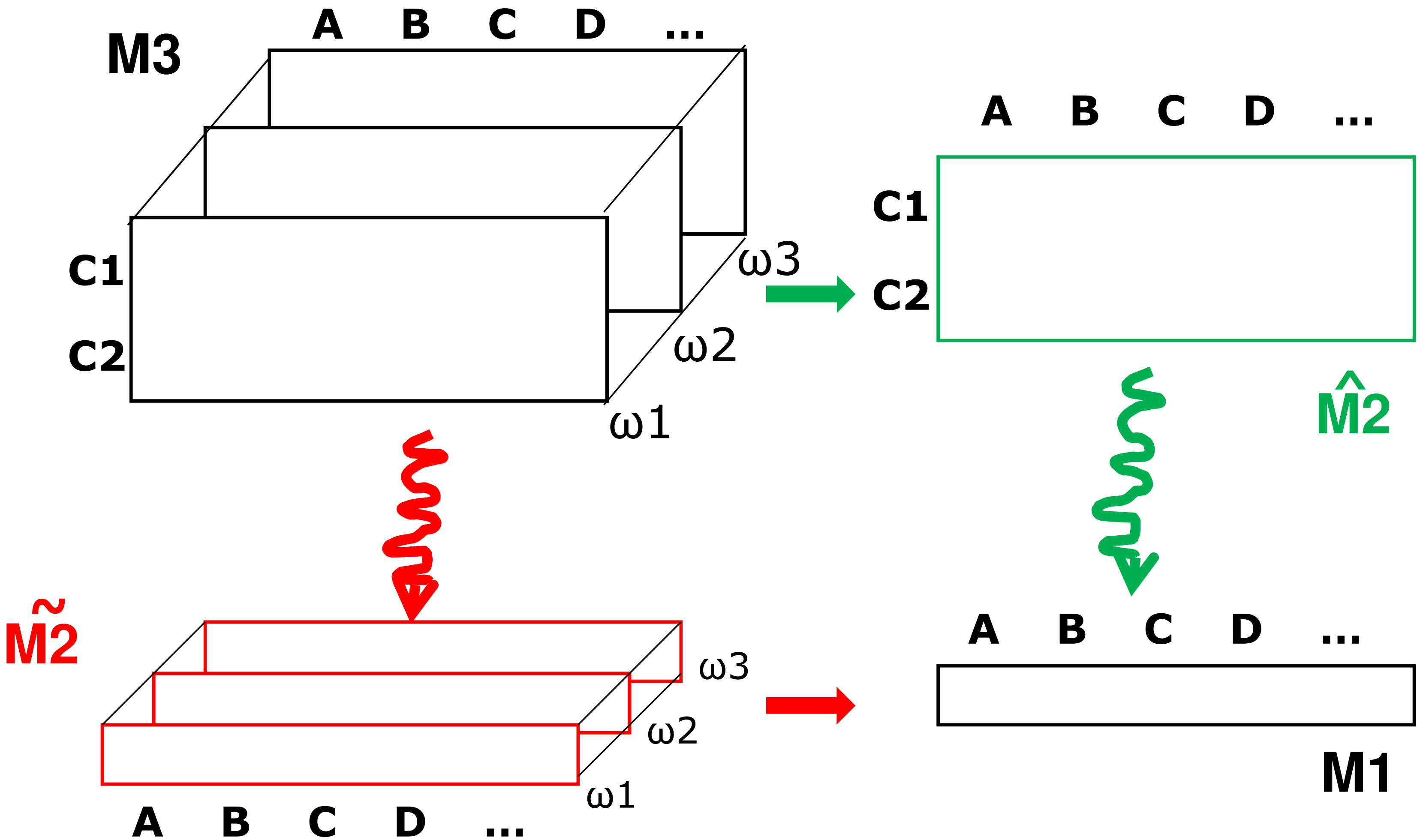
The mixed case occurs when:

i. the result depends on several criteria & DM's & st. of nature

ii. the path is not unique (see the following examples)

ExA: from M3 to M1

Matrix dimensions:
M3(ω, c, x) → M1(x)



Palio di Siena *(more)*

- RA (risk an.) → ω1 (dry), ω2 (windy), ω3 (rainy)
- MC (m. criteria) → c1 (horse), c2 (jockey)

	A	B	C	D	E	
ω1						p1 ?
c1	8	5	1	7	4	w1?
c2	4	6	9	2	5	w2?
ω2						p2 ?
c1	2	5	2	4	8	w1?
c2	4	6	7	3	5	w2?
ω3						p3 ?
c1	3	5	5	4	6	w1?
c2	5	6	7	3	8	w2?

No relation between RA and MC *(prob & weights are independent)*

- RA → p1 = 0.5, p2 = 0.1, p3 = 0.4
- MC → w1 (horse) = 0.7, w2 (jockey) = 0.3
- ***We suppose prob & weights are independent***

		p1	p2	p3		
w1		?	?	?		.70
w2		?	?	?		.30
		.50	.10	.40		

Two (symmetric) ways

- You can move from M3 = f(ω, c, x) to M1 = f(x):
  - (i) passing through $\hat{M}2$ = f(c,x) → probab. p1, p2, p3
  - (ii) passing through $\tilde{M}2$ = f(ω,x) → weights w1 e w2

wi * pj

		p1	p2	p3		
w1		.35	.07	.28		.70
w2		.15	.03	.12		.30
		.50	.10	.40		

From M3 to M1: the two ways

(i) Through $\hat{M2}$: (point 3) :

$\hat{M2}$ →

	A	B	C	D	E	wi
C1	5.4	5.0	2.7	5.5	5.2	.7
C2	4.4	6.0	8.0	2.5	6.2	.3

A	B	C	D	E
5.10	5.30	4.29	4.60	5.50

M1

(ii) Through $\tilde{M2}$ (point 2):

$\tilde{M2}$ →

	A	B	C	D	E	pi
ω1	6.8	5.3	3.4	5.5	4.3	.5
ω2	2.6	5.3	3.5	3.7	7.1	.1
ω3	3.6	5.3	5.6	3.7	6.6	.4

A	B	C	D	E
5.10	5.30	4.29	4.60	5.50

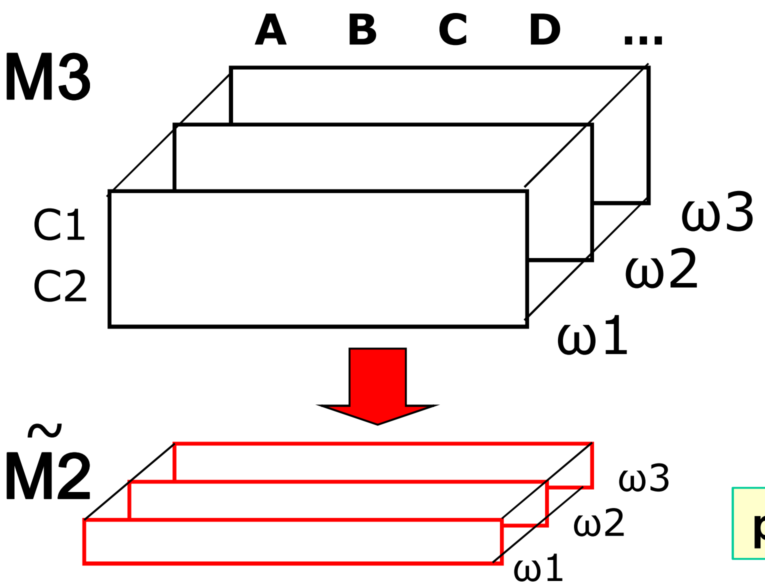
M1

Relation between RA and MC *(weights depend on probab.)*

- RA → p1 = 0.5, p2 = 0.1, p3 = 0.4
- MC → w1 and w2 different (column by column)
- ***So, in this situation the path through M2 is impôssible***

	p1	p2	p3	
w1	.7	.5	.4	?
w2	.3	.5	.6	?
	.50	**.10**	**.40**	

From M3 to M1 passing (only) through $\tilde{M}2$



point 2 of the cube

$\tilde{M}2$

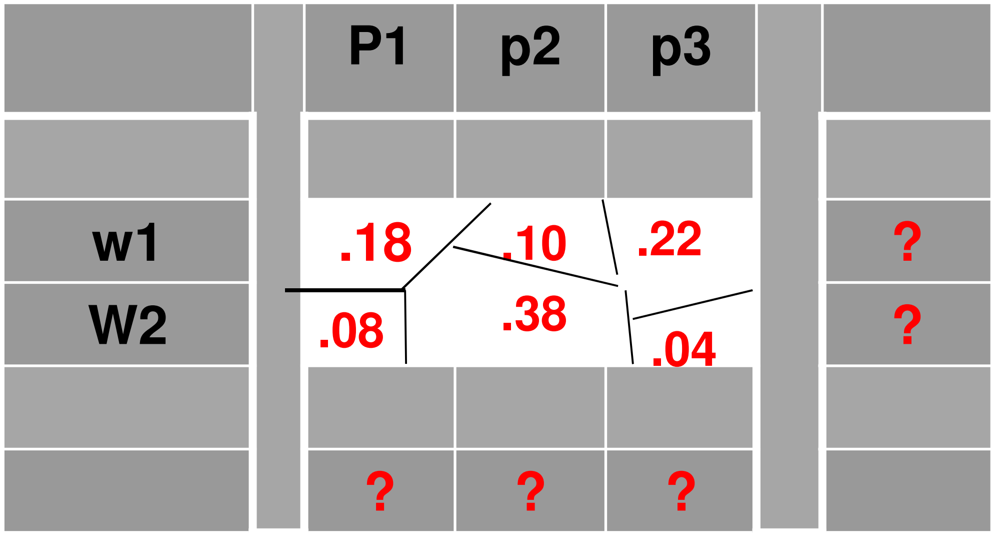
	A	B	C	D	E	ρi
ω1	6.8	5.3	3.4	5.5	4.3	.5
ω2	3.0	5.5	4.5	3.5	6.5	.1
ω3	4.2	5.6	6.2	3.4	7.2	.4

A	B	C	D	E
5.38	5.44	4.63	5.46	5.68

M1

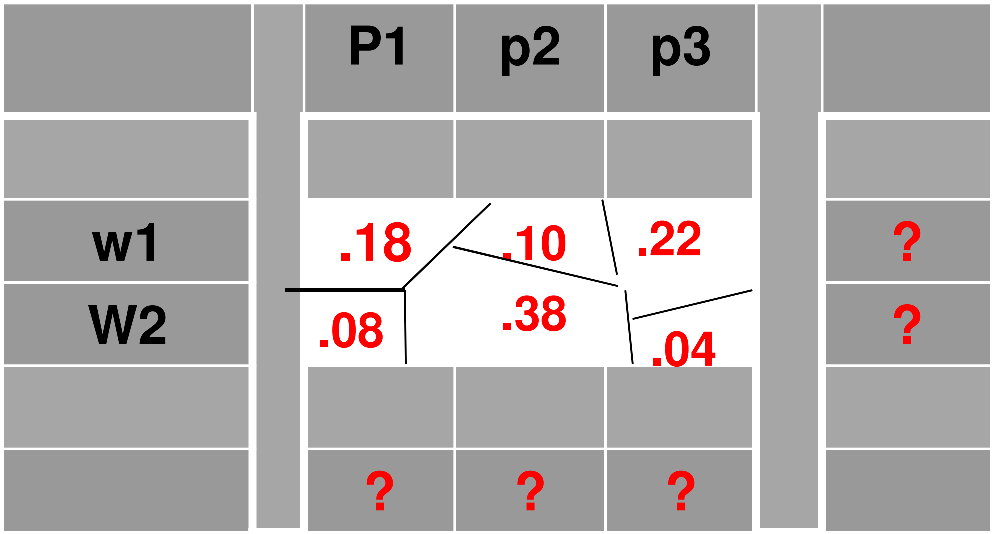
Mutual relation between RA and MC

- Correlation between probab. & weights →
  → p1, p2, p3 are connected with w1, w2
- ***This situation is displayed in the picture***



No way (using the edges)

- It is impossibile “to reduce” the problem by eliminating elements one by one (using the edges)
- ***Then, what can we do ?***



The situation *(considering, for instance, alternative A)*

		ω1	ω2	ω3		
c1		8	2	3		
c2		4	4	5		

performances

		p1	p2	p3		
w1		.18	.10	.22		
w2		. 08	.38	.04		

prob.-weights

- Total score for alternative A → 8 * 0.18 + 2 * 0.10 + …

Straight from M3 to M1 *(using the diagonal)*

- **A** → 8 * 0.18 + 2 * 0.10 + 3 * 0.22 + 4 * 0.08 + 4 * 0.38 + 5 * 0.04 = **4.34**
- **B** → 5 * 0.18 + 5 * 0.10 + 5 * 0.22 + 6 * 0.08 + 6 * 0.38 + 6 * 0.04 = **5.50**
- **C** → 1 * 0.18 + 2 * 0.10 + 5 * 0.22 + 9 * 0.08 + 7 * 0.38 + 7 * 0.04 = **5.14**
- **D** → 7 * 0.18 + 4 * 0.10 + 4 * 0.22 + 2 * 0.08 + 3 * 0.38 + 3 * 0.04 = **3.96**
- **E** → 4 * 0.18 + 8 * 0.10 + 6 * 0.22 + 5 * 0.08 + 5 * 0.38 + 8 * 0.04 = **5.46**

So we get the ranking vector M1 …

Examples *(going to the origin)*

ExA – Palio	→	different ways probably not symmetric
ExB – nurses	→	two ways symmetric (path1 & path 2)
ExC – saus.	**→**	**one way / clustering** **vector space**
ExD – paths	→	one way / ranking or rating finite/large n. of alternatives

Sausages – Data matrix

	A	B	C	D	E	F	G	un. of measure
i1	62	57	63	58	60	55	65	grammes
i2	1	9	3	8	4	2	8	% of fat
i3	*0*	*1*	*0*	*0*	*1*	*1*	*1*	*certification*
i4	*1*	*1*	*0*	*0*	*0*	*1*	*1*	*pork meat*
I5	*1*	*1*	*1*	*0*	*1*	*0*	*1*	*awards*

- i1: values between 55 and 65 → norma
- i2: values between 1 and 9 → norma
- *i3, i4, i5: binary values → Hamm*

- We are looking for K = 2 (homogeneous) grou

Variables → m = 2 continuous + 3 discrete variables, for the i-th component (i=1,…,m).
Alternative → a point x (x_1,…,x_m) in the m-dimension space (a mix of components)
Primitive → definition of the distance from x_h and x_k and checking of the "discordance conditions"

Two phases: (1) distance matrix , (2) clustering algorithm

Phase 1: from dissimilarity to distance

i1	A	B	C	D	E	F	G
A	--	.5	.1	.4	.2	.7	.3
B	.5	--	.6	.1	.3	.2	.8
C	.1	.6	--	.5	.3	.8	.2
D	.4	.1	.5	--	.2	.3	.7
E	.2	.3	.3	.2	--	.5	.5
F	.7	.2	.8	.3	.5	--	1.0
G	.3	.8	.2	.7	.5	1.0	--

i2

--	1.0	.25	.87	.37	.12	.87
1.0	--	.75	.12	.62	.87	.12
.25	.75	--	.62	.12	.12	.62
.87	.12	.62	--	.50	.75	0.0
.37	.62	.12	.50	--	.25	.50
.12	.87	.12	.75	.25	--	.75
.87	.12	.62	0.0	.50	.75	--

i3-4-5

--	*.33*	*.33*	*.67*	*.67*	*.67*	*.33*
.33	--	*.67*	*1.0*	*.33*	*.33*	*0.0*
.33	*.67*	--	*.33*	*.33*	*1.0*	*.67*
.67	*1.0*	*.33*	--	*.67*	*.67*	*1.0*
.67	*.33*	*.33*	*.67*	--	*.67*	*.33*
.67	*.33*	*1.0*	*.67*	*.67*	--	*.33*
.33	*0.0*	*.67*	*1.0*	*.33*	*.33*	--

$$d_{ij} = 0.2 * i1 + 0.4 * i2 + 0.4 * i3\text{-}4\text{-}5 = \ldots$$

d	**A**	**B**	**C**	**D**	**E**	**F**	**G**
A	**--**	**.63**	**.27**	**.70**	**.46**	**.46**	**.54**
B	**.63**	**--**	**.69**	**.47**	**.44**	**.52**	**.21**
C	**.27**	**.69**	**--**	**.48**	**.24**	**.61**	**.56**
D	**.70**	**.47**	**.48**	**--**	**.51**	**.63**	**.54**
E	**.46**	**.44**	**.24**	**.51**	**--**	**.47**	**.43**
F	**.46**	**.52**	**.61**	**.63**	**.47**	**--**	**.63**
G	**.54**	**.21**	**.56**	**.54**	**.43**	**.63**	**--**

This matrix gives a "measure" of the distance between the objects (ph. 1)

Phase 2: k-median algorithm *(with k=2)*

d	A	B	C	D	E	F	G
A	--	.63	.27	.70	.46	.46	.54
B	.63	--	.69	.47	.44	.52	.21
C	.27	.69	--	.48	.24	.61	.56
D	.70	.47	.48	--	.51	.63	.54
E	.46	.44	.24	.51	--	.47	.43
F	.46	.52	.61	.63	.47	--	.63
G	.54	.21	.56	.54	.43	.63	--

	Σ
A	3.06
B	2.96
C	2.85
D	3.33
E	2.55
F	3.32
G	2.91

Initialization → A, D

0

1

Cluster A → {A, C, E, F}
Cluster D → {B, D, G}

2

Cluster C → {A, C, E}
Cluster B → {B, D, F, G}

3

C
B

stop

Phase 2: average distance algorithm

	A	B	C	D	E	F	G
A	0	63	27	70	46	46	54
B	63	0	69	47	44	52	21
C	27	69	0	48	24	61	56
D	70	47	48	0	51	63	54
E	46	44	24	51	0	47	43
F	46	52	61	63	47	0	63
G	54	21	56	54	43	63	0

1

	A	BG	C	D	E	F
A	0	58.5	27.0	70.0	46.0	46.0
BG	58.5	**10.5**	62.5	50.5	43.5	57.5
C	27.0	62.5	0	48.0	24.0	61.0
D	70.0	50.5	48.0	0	51.0	63.0
E	46.0	43.5	24.0	51.0	0	47.0
F	46.0	57.5	61.0	63.0	47.0	0

2

	A	BG	CE	D	F
A	0	58.5	36.5	70.0	46.0
BG	58.5	**10.5**	53.0	50.5	57.5
CE	36.5	53.0	**12.0**	49.5	54.0
D	70.0	50.5	49.5	0	63.0
F	46.0	57.5	54.0	63.0	0

3

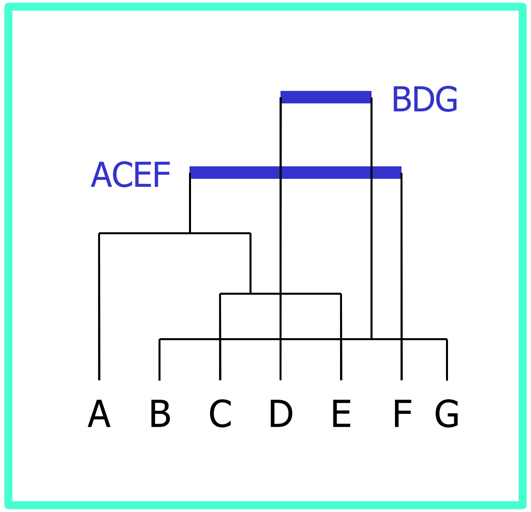
	ACE	BG	D	F
ACE	**24.2**	55.7	59.7	50.0
BG	55.7	**10.5**	50.5	57.5
D	59.7	50.5	0	63.0
F	50.0	57.5	63.0	0

4

	ACEF	BG	D
ACEF	**37.1**	56.6	61.3
BG	56.6	**10.5**	50.5
D	61.3	50.5	0

5

	ACEF	BDG
ACEF	**37.1**	58.9
BDG	58.9	**30.5**



Fase 2: minimum distance algorithm

	A	B	C	D	E	F	G
A	0	63	27	70	46	46	54
B	63	0	69	47	44	52	21
C	27	69	0	48	24	61	56
D	70	47	48	0	51	63	54
E	46	44	24	51	0	47	43
F	46	52	61	63	47	0	63
G	54	21	56	54	43	63	0

1

	A	BG	C	D	E	F
A	0	54	27	70	46	46
BG	54	0	56	47	43	52
C	27	56	0	48	24	61
D	70	47	48	0	51	63
E	46	43	24	51	0	47
F	46	52	61	63	47	0

2

	A	BG	CE	D	F
A	0	54	27	70	46
BG	54	0	43	47	52
CE	27	43	0	48	47
D	70	47	48	0	63
F	46	52	47	63	0

3

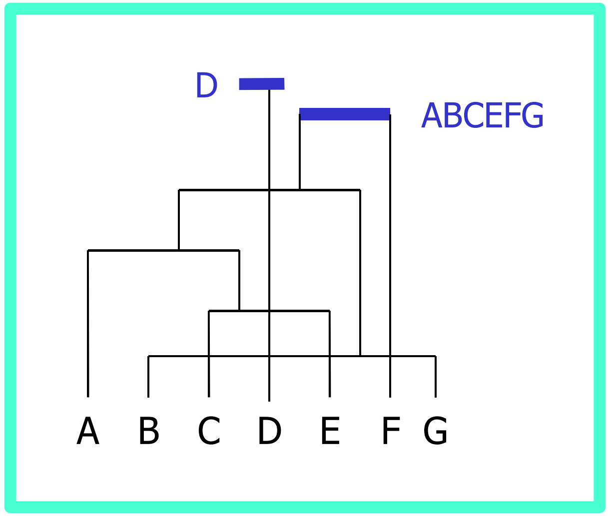
	ACE	BG	D	F
ACE	0	43	48	46
BG	43	0	47	52
D	48	47	0	63
F	46	52	63	0

4

	ABCEG	D	F
ABCEG	0	47	46
D	47	0	63
F	46	63	0

5

	AB...	D
AB...	0	47
D	47	0



Conclusions
(part 1)

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So …

- It is possible to process the elements of the “decision space" (ω, c, d) in a coordinated way.

- If the elements are independent, it is possible to eliminate them one-by-one, thus obtaining a final function or vector M1 of (continuous or discrete) decision variables .

- On the contrary, in case of dependency (i.e. criteria depend on the state of nature), the elimination follows a forced path.

- At last, in case of mutual dependency, you must proceed "along the diagonals" (by examining the behavior of the alternatives one by one).

Examples *(more)*

ExA – Palio	→	(MC-RA, ranking)	→	6 → 2 (*) → 1
ExB – nurses	→	(MC-SC, assign)	→	5 → 2 or 3 → 1
ExC – saus.	→	(MC, cluster)	→	3 → 1
ExD – paths	→	(MC, rate/ranki)	→	3 → 1

(*) the path through point 2 is more realistic …

Now let's consider specific tools