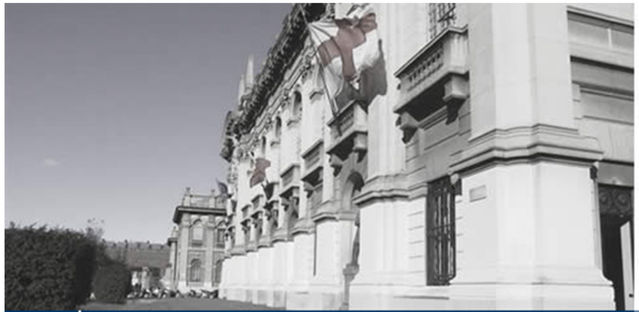


POLITECNICO DI MILANO

A framework for decision aiding *(part 2)*

Alberto Colorni – Politecnico di Milano

12-13 Feb. 2014 – Lamsade, Univ. Paris Dauphine

From part 1

- It is possible to treat the elements of the “decision space" (ω, c, d) in a coordinated way.
- If the elements are independent it is possible to eliminate them one-by-one, thus obtaining a final function or vector M1 of the (continuous or discrete) decision variables .
- On the contrary, if there is a dependence (i.e. the criteria depend on the states of nature) the elimination follows a forced path.
- Finally, if there is a mutual dependence you must proceed "along the diagonals" (by examining the behavior of the alternatives one by one).

ExA – Palio	→	(MC-RA, ranking)	→	6 – 2 (3) – 1
ExB – nurses	→	(MC-SC, assign)	→	5 – 2 or 3 – 1
ExC – saus.	→	(MC, cluster)	→	3 – 1
ExD – paths	→	(MC, rating-rank))	→	3 – 1

Now let’s consider specific tools

Tools for «point 2» problems

(i) Perception
(ii) Experiments & dec. tree

© Alberto Colorni

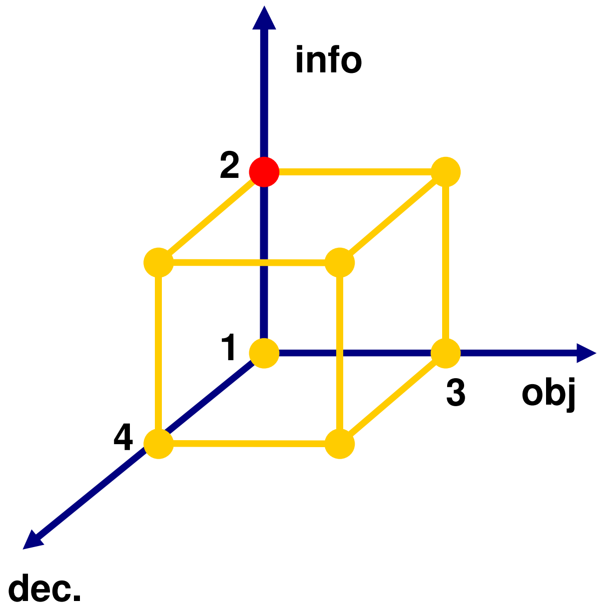
(i) A real decision process: perception

- **<u>Uncertainties</u>** (non deterministic context, …)
- Complexity (problem dimension, non linearity, …)
- Several stakeholders (distributed decision power)
- Different rationalities (criteria and preferences)
- Different time horizons (often)
- Need of simulation models

  what … if …

- The DM **perception of the problem**

Decision processes in a non-deterministic context



Information
- complete
- partial **[*]**

Objectives
- one
- more

Dec. makers
- one
- more

1. Math. programming
2. **Risk analysis**
3. Multi-objective (criteria)
4. Group choice

5, 6, 7, 8 → ….

[*] → non-deterministic context

⇩

perception & mental models

Two (opposite) theories

(a) Normative theory (prescriptive) → what the DM **should do**

(b) Cognitive theory (descriptive) → what the DM **really does** → experimental tests

When they are the same ? ➡ if the (***single***) DM has all the information (***in a deterministic way***) and has clearly in mind *the* criterion *(**one**)* of evaluation

⬇

ideal problem → point 1

N-1° Principle of INVARIANCE

→ Equivalent (from the logical point of view) versions of the same problem **must** produce the same choice

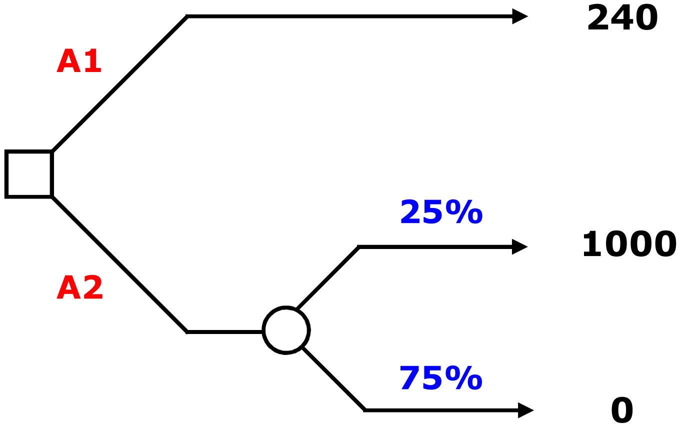
Examples

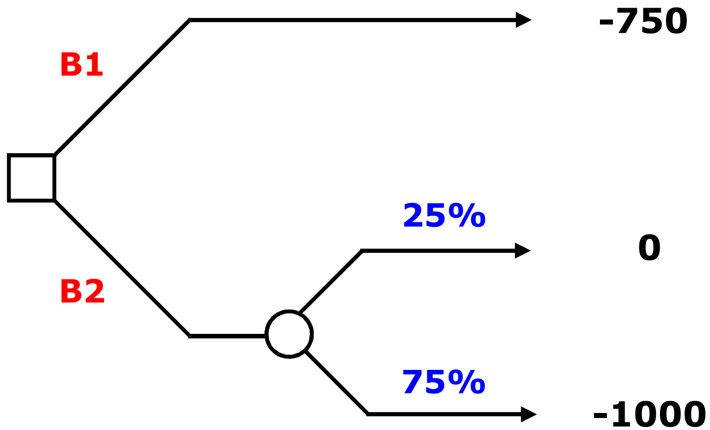
- Change names or positions for the options
- Change measure units
- Add a constant value for all the results

Counterexamples

- → Lotteries (cases A, B, C)
- → Ellsberg paradox (1961)

Lotteries (case A and case B)





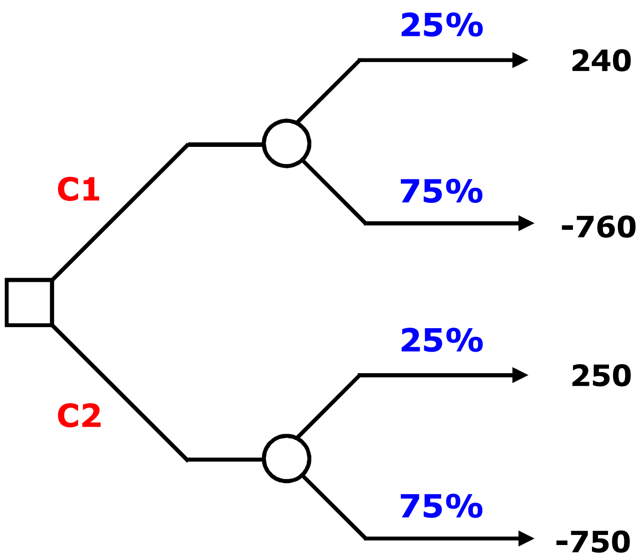
Better A1 or A2 ?

better ...

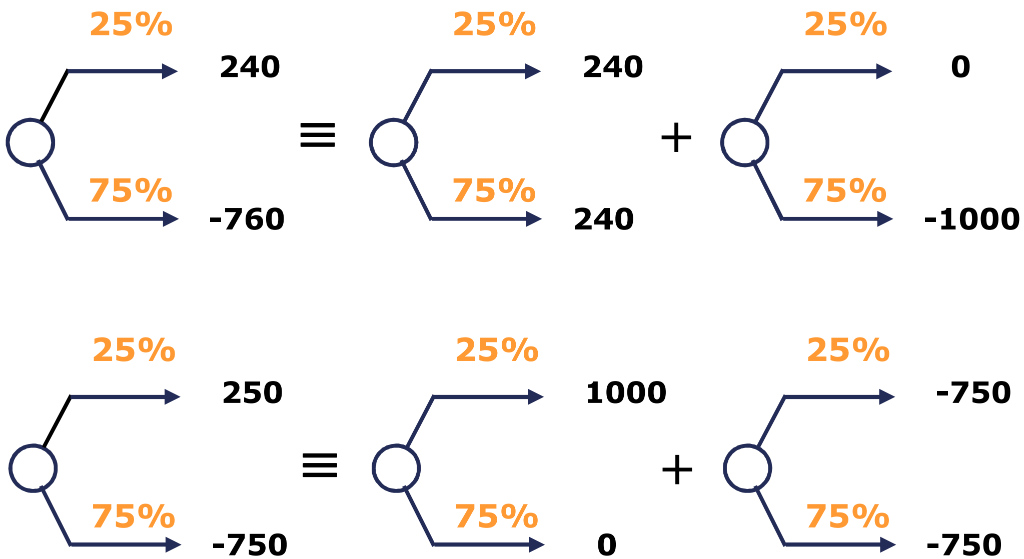
Better B1 or B2 ?

better ...

Lotteries (case C)



But notice that …



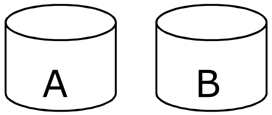
Better C1 or C2 ?

better …

C1 → sum of A1 and B2

C2 → sum of A2 and B1

Ellsberg



50 (b) **α** (b)
50 (n) **100- α** (n)

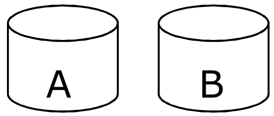
White ball win

Better to take from A or B ?

better ...

ambiguity aversion

Now you have a second chance (after the ball is re-inserted)



the same ...

Black ball win

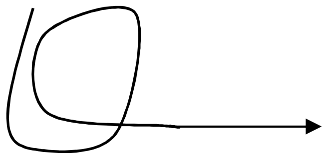
Better to take from A or B ?

better ...

ambiguity aversion ?

Cognitive theory: a first principle

C-1° **Principle of NON NEUTRALITY**



The aggregation of (decisional) options
is not a neutral operation !

Given the two preferences on A1 and B2, it is **not guaranteed**
that their aggregation (C1) is the preferred one

- **Caution: do not combine too easily the options**
- **Normally, the ambiguity is avoided, "even if this is not rational " (Ellsberg)**

N-2° Principle of DOMINANCE

If the DM prefers A with respect to B in every scenario (or context or state of nature) the choice **must be** A

Examples

- I prefer to be missionary (with respect to engineer) in peace and prefer to be missionary (...) in war
- I prefer chicken with respect to beef (when there is nothing else) and I prefer chicken … also when there is fish

so the choice … is better then the choice ...

Counterexamples (see in next lessons)

- Extraction (Tversky, Kahneman, 1986) → see the following sl.
- The possible choices in uncertainty conditions (with the DM risk attitude)

Extraction (in two conditions)

room 1

n. of balls	situation A	situation B
90 white	0	0
6 red	45	45
1 green	30	45
1 blue	-15	-10
2 yellow	-15	-15

Better A or B ?

⇩

better ...

room 2

n. of balls	situat. C	situat. D	n. of balls
90 white	0	0	90 white
6 red	45	45	7 red
1 green	30	-10	1 green
3 yellow	-15	-15	2 yellow

Better C or D ?

⇩

better ...

but C ≡ A and D ≡ B

Choice (in two conditions)

presentation 1

	w1	w2	w3	w4	w5
Invest	0	45	30	-15	-15
Build	0	45	45	-10	-15
p(w)	.90	.06	.01	.01	.02

Better < **Invest or Build** **?**

presentation 2

	w1	w2	w3	w4
Invest	0	45	30	-15
p(w)	.90	.06	.01	.03
Build	0	45	-10	-15
p(w)	.90	.07	.01	.02

Better < **Invest or Build** **?**

Cognitive theory: three more principles

C-2° Principle of **EVIDENCE**

→ The dominance among options should be **obvious**

C-3° Principle of **ASYMMETRY**

→ Possibility of **losing** K is more important than **winning** K

C-4° Principle of **COMPACTNESS**

→ An aggregated option (A) has an importance less than the sum of the importances of the single sub-options (A_1.A_2)

⬇

$$\pi(A) < \pi(A_1) + \pi(A_2)$$

N-3° **Principle of TRANSITIVITY**

If the decision prefers A over B and B over C,
then A **must** be preferred over C

Examples:

- V. Rossi is better than Stoner, and Stoner is better than Melandri, so …
- Buying emission units (Kyoto prot.) is better than cutting the production, and cutting the production is better than not respecting the emission constraints, so …

Counterexamples:

a new car + accessories

standard	10.000€
+air cond.	1.000€
+alloy rims	1.000€
+ …	

(but finally …)

	A	B	C	D
ob1	50	55	60	65
ob2	50	55	60	65
ob3	50	55	60	65
ob4	40	30	20	10

B > A
C > B
D > C

D > A ?

or rather **the options are incomparable ?**

Cognitive theory: progression vs. crash

C-5°

Principle of CRASH

The DM is (relatively) indifferent to small progressive changes,
but at some point (s)he becomes aware of the (large) gap and ...

Cognitive theory: estimation

C-6°

Principle of OVER/UNDER-ESTIMATION

There is an inclination to
- over-estimate events with small probability
- under-estimate events with high probability (except in case of certainty)

⬇

Asymmetry in dealing with subjective probability

A famous example: the frame effect

- Avian influenza (possible death)
- Group at risk: 600 people

Protocol A 200 people will survive

Protocol B
- with p=1/3 600 will survive
- with p= 2/3 nobody will survive

Better A or B ?

Protocol A 400 people will die

Protocol B
- with p=1/3 nobody will die
- with p= 2/3 600 will die

Better A or B ?

- **Aversion to the risk in case of winnings (better A)**
- **Propensity for risk in case of losses (better B)**

(ii) Experiments: axioms of probability theory

A1 - Probability **p(e)** of an event (e): value between 0 (impossible) / 1 (certain)

A2 - Complementary probability (the event does not occur): **1-p(e)**

A3 – For events (e_1, e_2, …, e_k) *that are mutually exclusive* : $\mathbf{p(e_1 \text{ OR } \ldots \text{ OR } e_k) = p(e_1) + \ldots + p(e_k)}$

A4 – For 2 independent events (e_1, e_2) : $\mathbf{p(e_1 \text{ AND } e_2) = p(e_1) * p(e_2)}$

A5 – For 2 non-indipendent events (e_1, e_2): → **conditional probability (Bayes, 1763)**

$$p(e1/e2) = \frac{p(e1 \text{ AND } e2)}{p(e_2)} = \frac{p(e_2/e_1) * p(e_1)}{p(e_2)}$$

Example follows

Probabilities before and after the experiment (Bayes)

ω1 = good weather

ω2 = bad weather

p(ω) — before

ω 1	ω2
.80	.20

p(y)

y1	.55
y2	.25
y3	.20

p(ω,y)

	ω 1	ω2
y1	.50	.05
y2	.20	.05
y3	.10	.10

p(ω/y) — after

	ω 1	ω2
y1	.91	.09
y2	.80	.20
y3	.50	.50

y1 = clear

y2 = variable

y3 = rain

p(y/ω)

	ω 1	ω2
y1	.63	.25
y2	.25	.25
y3	.12	.50

p(y/ω) → this case does not make much sense

Uncertainty: the expected value

- If the probability distribution of ω is available …
- … consider the logic of the expected value (to be maximized)
- In the example → $p(\omega_1) = 0.3$, $p(\omega_2) = 0.7$

	x_1	x_2	x_3	x_4
$\omega 1$	5	8	1	10
$\omega 2$	6	3	9	2
L_5	5.7	4.5	6.6	4.4

$\bar{f}(x_j)$ = the expected value is calculated multiplying the values $f(x_j, \omega_i)$ by the probabilities $p(\omega_i)$ and then summing them.

- Sometimes also variance is considered (to be minimized)

$$\sigma_j^2 = \sum_i \left[f(\varpi_i, x_j) - \bar{f}(x_j)\right]^2 \cdot p(\varpi_i)$$

i : state of nature

j : alternative

$\bar{f}(x_j)$: expected value of alternative j

0.21	5.25	13.44	13.44

The expected value logic removes the dependence from the state of nature (if probabilities are available)

An example: oil extraction

- A potentially rich area
- States of nature → ω1: oil; ω2= no oil
- Possible actions → x1: buy taking full advantage
  x2: rent for 50 years
  x3: rent for 10 years
  x4: do nothing

	x1	x2	x3	x4
ω_1	100	80	20	0
ω_2	-30	-6	-4	0

Experiment (a sample drill):

- y1 = probably there is oil
- y2 = analysis not clear
- y3 = probably there is no oil

$p(\omega)$=

ω_1	0.50
ω_2	0.50

←

p(ω,y)

	y_1	y_2	y_3
ω_1	0.18	0.24	0.08
ω_2	0.02	0.16	0.32

→

p(ω/y)

0.90	0.60	0.20
0.10	0.40	0.80

↓

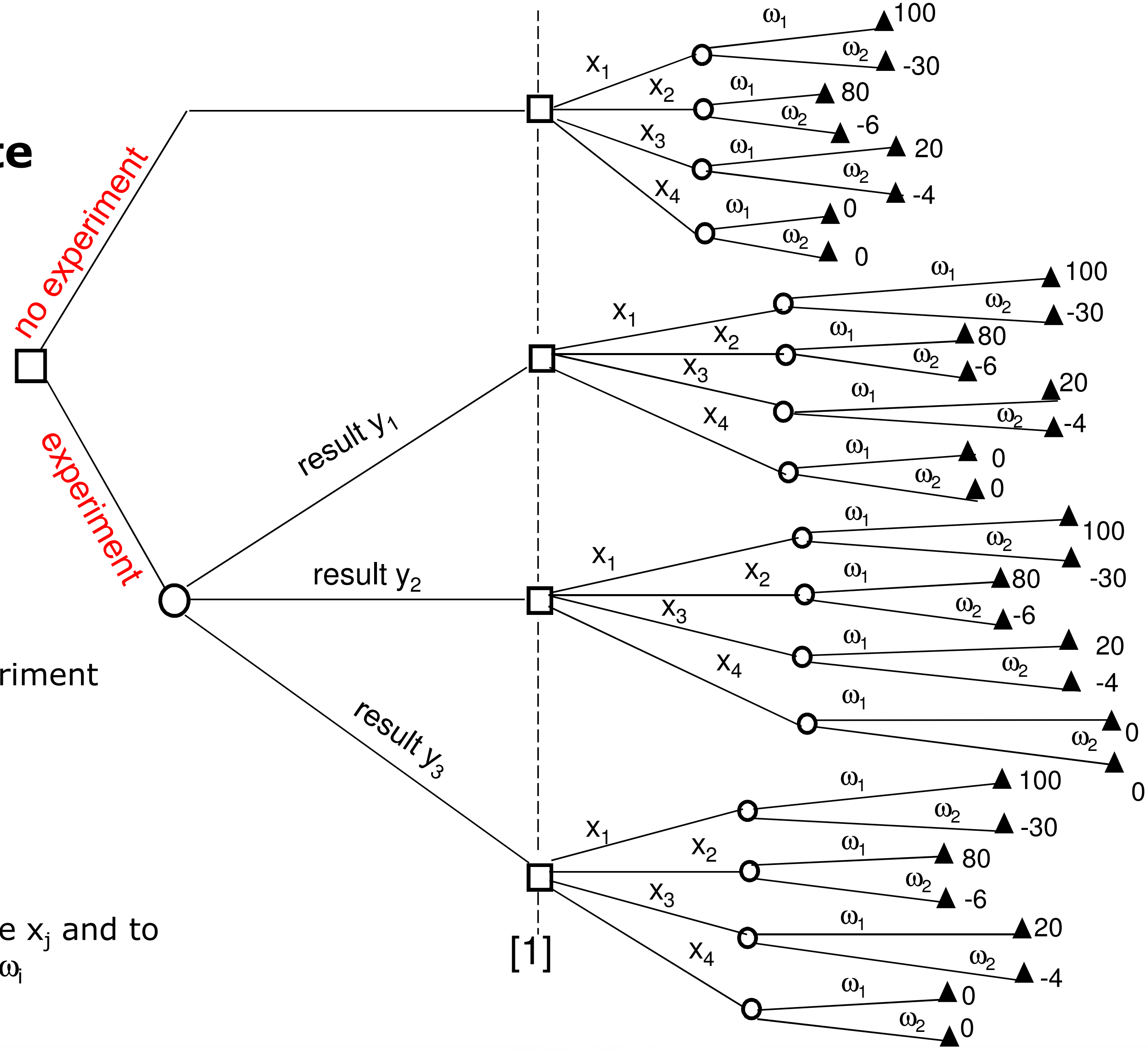
p(y)=

0.20	0.40	0.40

different w.r.t. p(ω)

Decision tree: construction

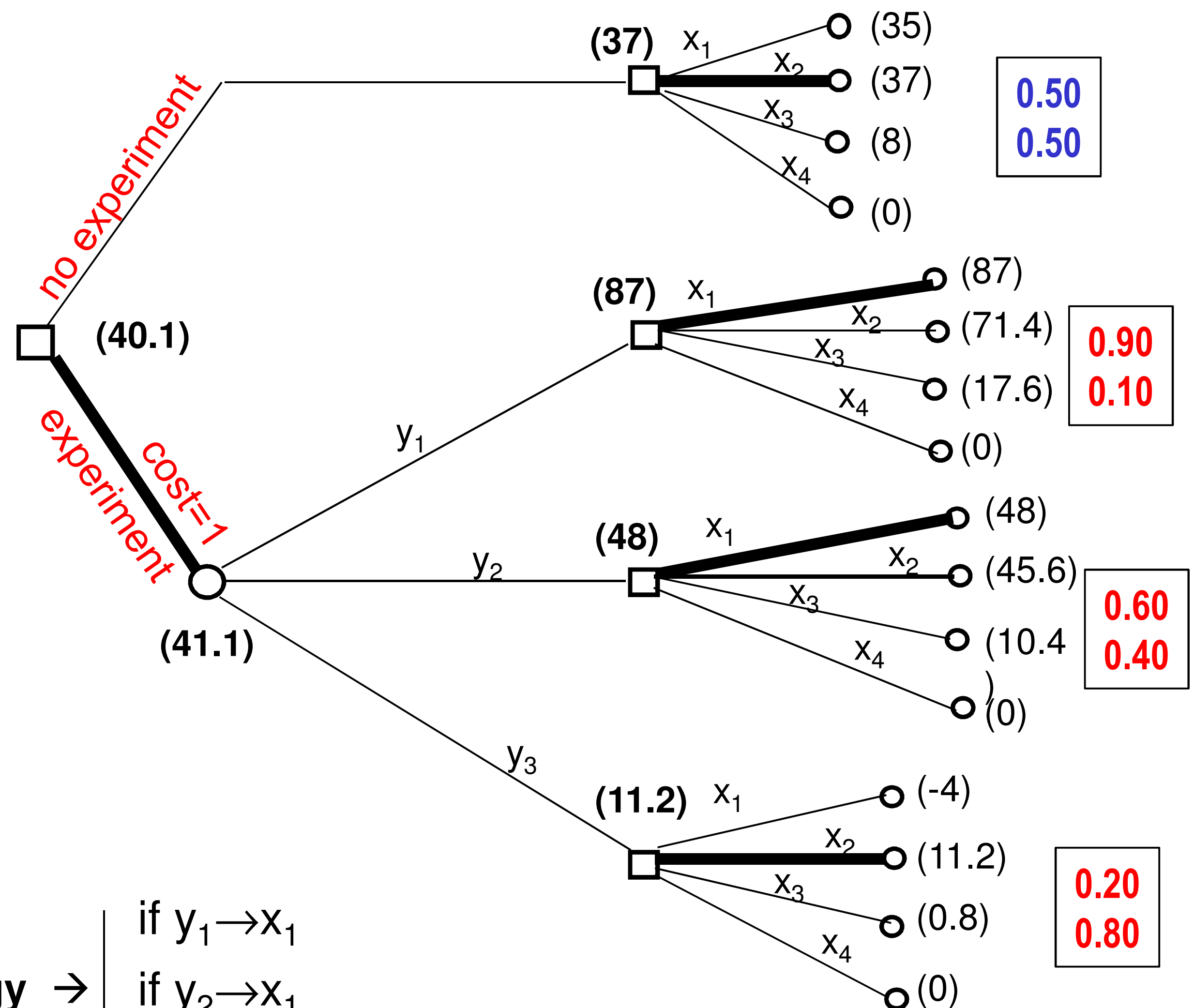
- **The complete tree**



- **Left part**
  (before [1])
  related to the experiment and to its outcome

- **Right part**
  (after [1])
  related to the choice x_j and to the state of nature ω_i

The final outcome: a strategy



Node labels

○ → expected value

□ → best option

Conclusion:

- do the experiment
- select the following **strategy** →

if $y_1 \rightarrow x_1$

if $y_2 \rightarrow x_1$

If $y_3 \rightarrow x_2$

Tools for «point 3» problems

(i) Pairwise comparison
(ii) Choquet integral

© Alberto Colorni

(i) Pairwise comparison

To obtain the vector w of the weights it is possible to do a set of pairwise comparisons, thus obtaining a matrix A

Example

	w1	w2	w3
w1	1	1/3	2/3
w2	3	1	2
w3	3/2	1/2	1

w2 is 3 times more important than w1

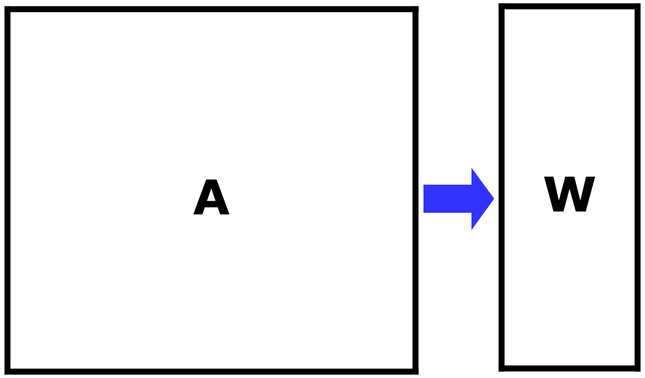
w2 is 2 times more important than w3

... (etc.)

Matrix A is:

- positive → $a_{ij} > 0$
- reciprocal → $a_{ij} = 1/a_{ji}$
- **consistent** → $a_{ik} = a_{ij} \cdot a_{jk}$

From matrix A to vector w



If A is consistent ➡ $a_{ij} = a_{ik} / a_{jk}$

by definition is:

$a_{ij} = w_i / w_j$ ➡ $a_{ik} / a_{jk} = w_i / w_j$

	w1	w2	w3
w1	1	1/3	2/3
w2	3	1	2
w3	3/2	1/2	1

➡

	w
w1	2
w2	6
w3	3

➡

	w
w1	2/11
w2	6/11
w3	3/11

All the columns represent (with a coeff. of proportionality) the vector w ➡ easy case !

Eigenvalues

- **Matrix A <u>consistent</u>**

  ↓

  - columns proportional

  ↓

  - rank of the matrix = 1

  ↳ only one eigenvalue $\lambda_{max} \neq 0$

- **Matrix A positive**

  ↓

  - trace = sum of eigenvalues

- **Elements of the diagonal = 1 (all)**

  ↓

  - trace = n

$$\lambda_{max} = n$$

unique (λ_{max}) — **trace** (n)

Vector of the weights

$$A \cdot x$$

with $a_{ij}=a_{ik}/a_{jk}=x_i/x_j$ (it is independent by k)

$$\forall i \quad \sum_{j=1}^{n} a_{ij}x_j = \sum_{j=1}^{n} \frac{x_i}{x_j} x_j = n x_i \quad \Rightarrow \quad A \cdot x = n\, x$$

x is the main eigenvector

each column is proportional to the eigenvector

vector w of the weights:

w is the main eigenvector normalized (sum = 1)

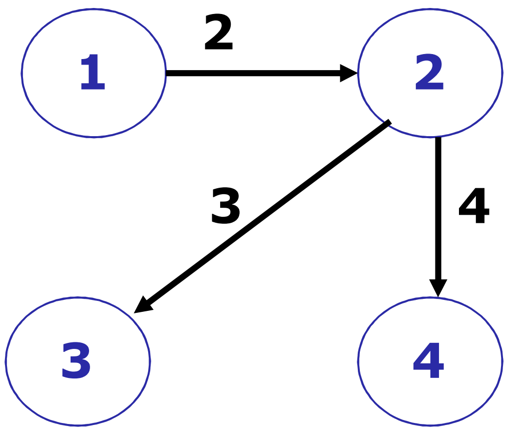
Supporting (spanning) tree

The minimum number of pairwise comparisons is n-1
but only if they are «spanning» the graph

	1	2	3	4
1	1	2	6	8
2		1	3	4
3			1	4/3
4				1

reciprocal

(selfloops) = 1



No isolated nodes

$$a_{13} = a_{12} \cdot a_{23}$$

$$a_{14} = a_{12} \cdot a_{24}$$

$$a_{34} = a_{32} \cdot a_{24} = (1/\ a_{23}) \cdot a_{24}$$

If matrix A is not consistent ?

Inconsistencies ➡ $a_{ik} \neq a_{ij} \cdot a_{jk}$

Example

1	2	6
1/2	1	3
1/6	1/3	1

Matrix is consistent ➡

0.6
0.3
0.1

1	2	4
1/2	1	3
1/4	1/3	1

Matrix is <u>not</u> consistent ➡ **?**

We must estimate the main eigenvector (and the error)

If the consistency error is "small" OK (if no ...)

What about the cons. error μ ?

$A*x = \lambda_{max}*x$ $\quad \forall i \quad \sum_{j=1}^{n} a_{ij}x_j = \lambda_{max}x_i$ (row i-th of the matrix)

$$\forall i \quad \lambda_{max} = \sum_{j=1}^{n} a_{ij}\frac{x_j}{x_i} \quad \left(a_{ij}\frac{x_j}{x_i} = \sigma_{ij}\right) \Rightarrow \lambda_{max} = \sum_{j=1}^{n} \sigma_{ij}$$

Sum of the rows →

$$\sum_{i=1}^{n} \lambda_{max} = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij}$$

$$n\lambda_{max} = n + \sum_{1 \le i < j \le n}^{n} \left(\sigma_{ij} + \frac{1}{\sigma_{ij}}\right)$$

$$\sum_{1 \le i < j \le n}^{n} \left(\sigma_{ij} + \frac{1}{\sigma_{ij}}\right) = n(\lambda_{max} - 1)$$

Divide the result by n(n-1) and subtract 1

$$\mu \Rightarrow \frac{\sum_{1 \le i < j \le n}^{n} \left(\sigma_{ij} + \frac{1}{\sigma_{ij}}\right)}{n(n-1)} - 1 = \frac{n(\lambda_{max} - 1)}{n(n-1)} - 1 \Rightarrow \boxed{\mu = \frac{\lambda_{max} - n}{n - 1}}$$

If A consistent: $\lambda_{max} = n \rightarrow \mu = 0$

(ii) Going back to the MAUT ... Choquet

What happens if the attributes (objectives or criteria) are not mutually independent ?

OR

if it is not possible to demonstrate their independence ?

The Choquet integral

Palio di Siena (ExA)

You have to help a Palio bookmaker. His evaluation concerning the contrada's chance to win are based on two attributes: values (utilities) of horse and jockey. The situation (utilities) of the four contrada are in the following table.

Bookmaker perception:

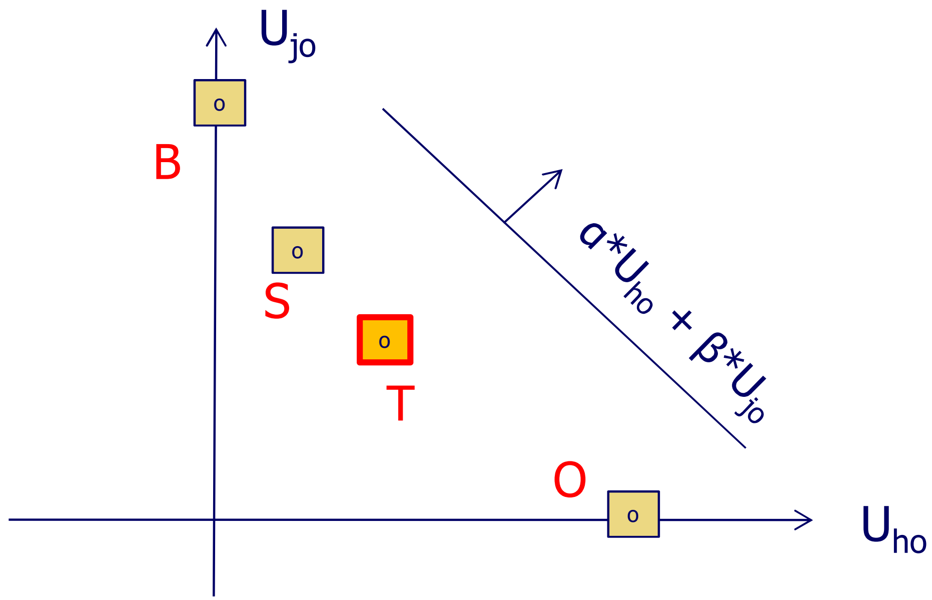
a. same weight

b. contrada Torre is the favourite

	Onda	Bruco	Torre	Selva
horse	100	0	45	30
jockey	0	100	45	65
average	*50*	*50*	*45*	*47.5*

Which weight is it possible to assign to the two attributes ?

In the utility space ...



- No couple of weights (α,β) determines the victory of Torre, the contrada indicated by the bookmaker as the best one.
- It's necessary to change the model ...

MAUT modifications

- Association of a unique value U (utility) to each alternative (among the n, finite or infinite, possible alternatives):

  U expresses the overall satisfaction with respect to the m attributes t_1, t_2, ... t_m considered.

- It is necessary to obtain the utility function U on the base of the utilities of each attribute.

- Both comments are true, but it is necessary to take care of:

  (i) synergies,

  (ii) redundance

Example: a student grant

You have to help the commission for an Erasmus grant.
The evaluation is based on three attributes, the results of the student in **M** (mathematics), **F** (physics), **L** (literature).
The situation is the following.

	Colorni	Luè	Noce	Lia
M – mathematics →	9	5	7	8
F – physics →	8	6	7	5
L – literature →	5	10	7.5	8
Average	7.33	7	7.16	7
Minimum	5	5	7	5
Maximum	9	10	7.5	8

The commission (decision maker) says that:

1. criteria **M** and **F** have the same importance (weight)
2. criteria **M** and **F** are more relevant than **L** (1.5 time)
3. criteria **M** and **F** are redundant (a student good in **M** is also …)
4. students are favorite if they are balanced (synergy **M**-**L** and **F**-**L**)

Case of criteria not mutually independent

- It is based on the definition of **two elements**:
  - a capacity (fuzzy measure)
  - a sum (Choquet integral)

- **Capacity**:
  - if M = {1, …, m} is the attributes (criteria) set
  - **capacity** is a function $\mu : 2^M \rightarrow [0, 1]$ such that

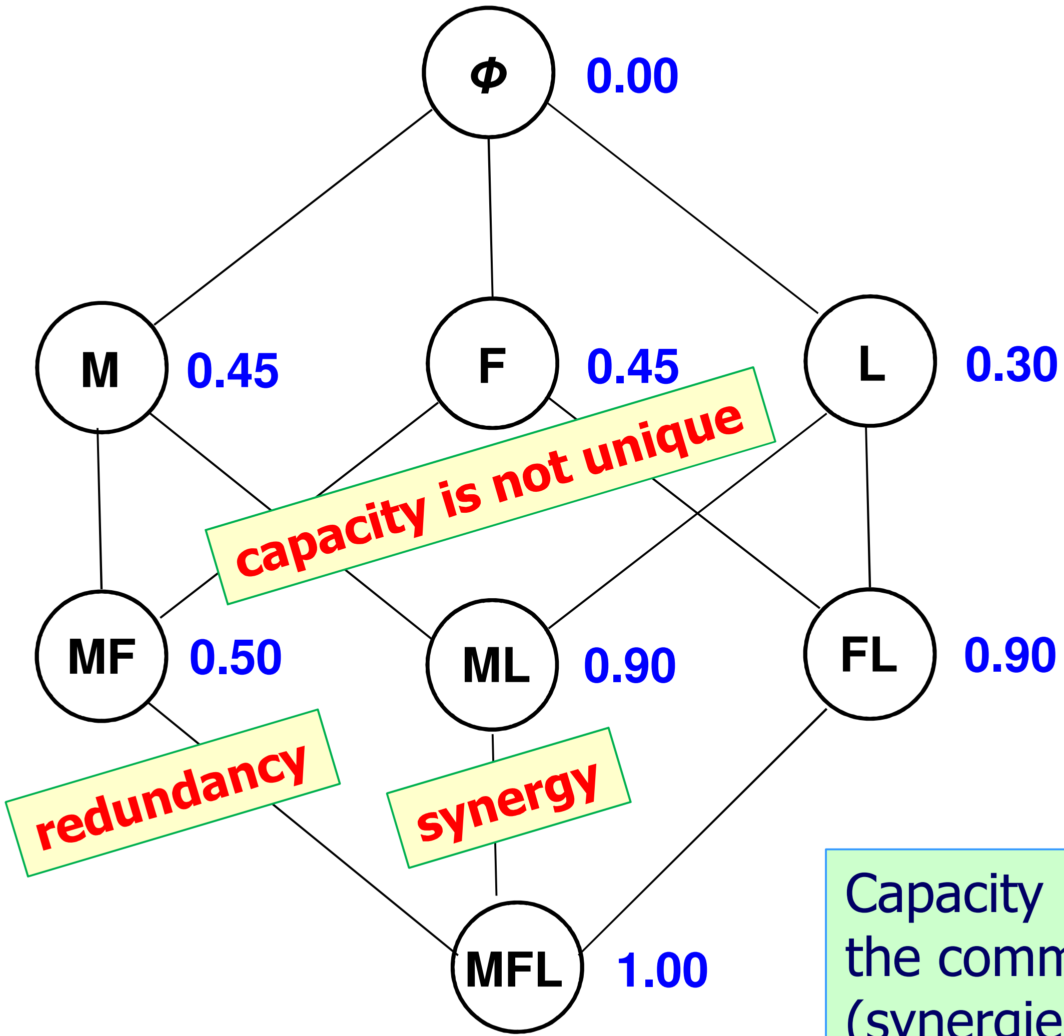
  $\mu(\Phi) = 0 ; \mu(M) = 1 ; \mu(A) \leq \mu(B)$ ***if A is included in B***

- **Choquet integral**:
  - μ is a capacity M = {1, …, m} and f is the function that represents the results (utility) of the alternatives among the different criteria
  - the Choquet integral C_μ is the sum (with i=1,…,m)

    $C_\mu = [f(\sigma_1) - f(\sigma_0)]*\mu(A_1) + \ldots + [f(\sigma_m) - f(\sigma_{m-1})]*\mu(A_m)$

    with $A_i=\{\sigma_i, \sigma_{i+1}, \ldots, \sigma_m\}$ and σ_i permutation with $f(\sigma_i)$ ascending

Representation (lattice)



Capacity:

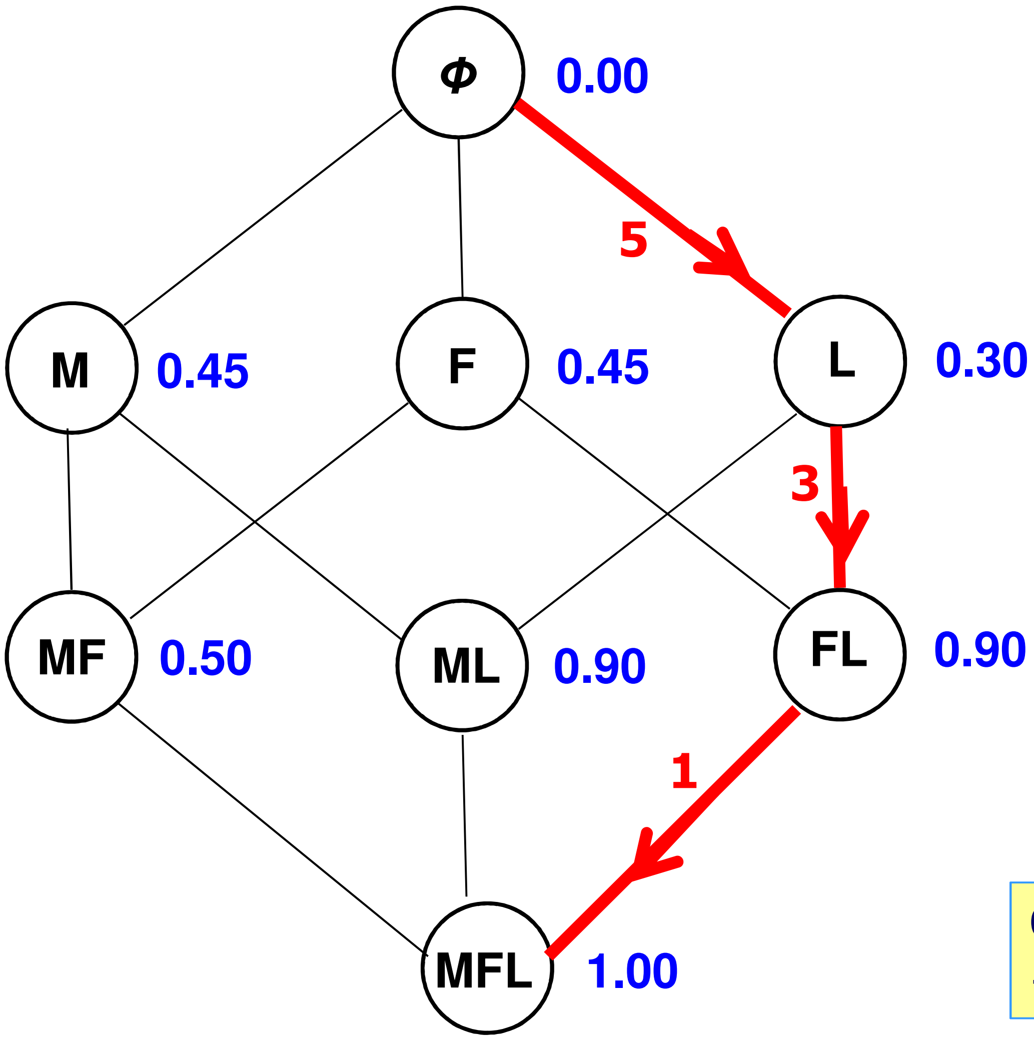
$\mu(\Phi) =$ 0.00
$\mu(M) =$ 0.45
$\mu(F) =$ 0.45
$\mu(L) =$ 0.30
$\mu(MF) =$ 0.50
$\mu(ML) =$ 0.90
$\mu(FL) =$ 0.90
$\mu(MFL) =$ 1.00

Capacity $\boldsymbol{\mu(A_i)}$ takes into account the commission indications ? (synergies and redundancies)

Results

- There are n candidates *(n=4: Colorni, Luè, Noce, Lia)*
- For each it is necessary to calculate C_μ *(Choquet integral)*
- For each it should be necessary to define a permutation
- It is better to use a graphic scheme *(see next slide)*
- Each candidate has an ascending order of results
- It is possible to represent it as a path between Φ and MFL
- To each node an increment Δ is associated *(added value)*
- To each node a weight is associated *(weight is the capacity)*
- C_μ value is calculated with a weighed sum

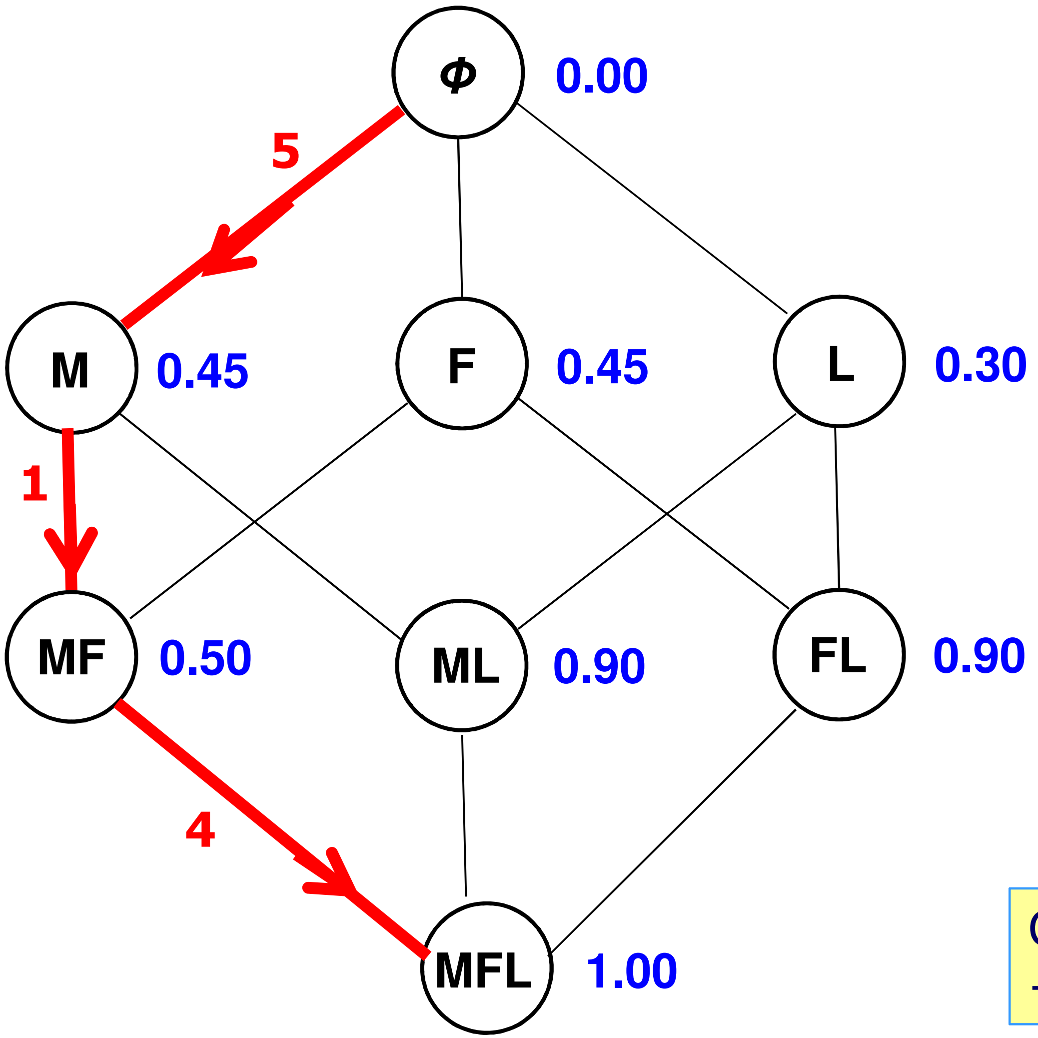
Student Colorni



M → 9
F → 8
L → 5

C_μ (**Colorni**) = 5 * 1.0 +
+ 3 * 0.5 + 1 * 0.45 = **6.95**

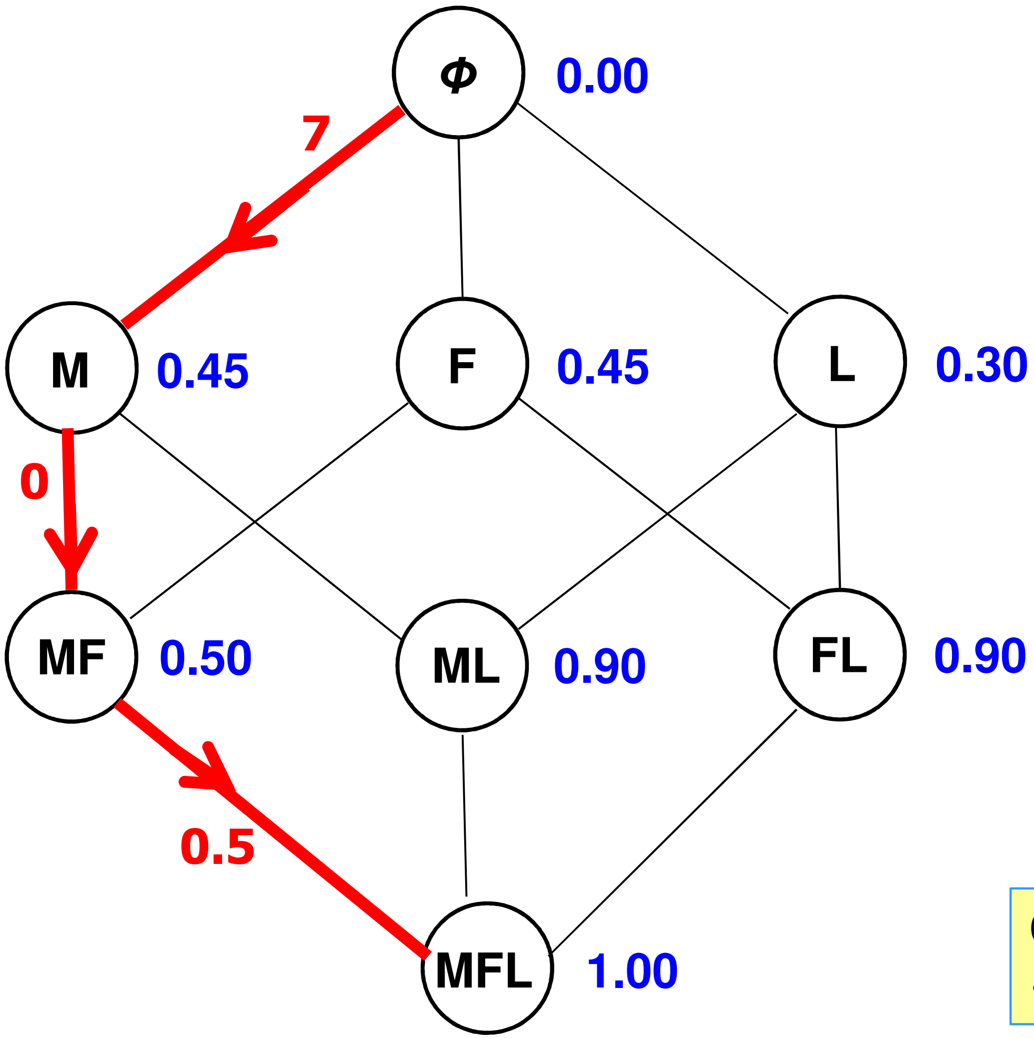
Student Luè



M → 5
F → 6
L → 10

C_μ (**Luè**) = 5 * 1.0 +
+ 1 * 0.9 + 4 * 0.30 = **7.10**

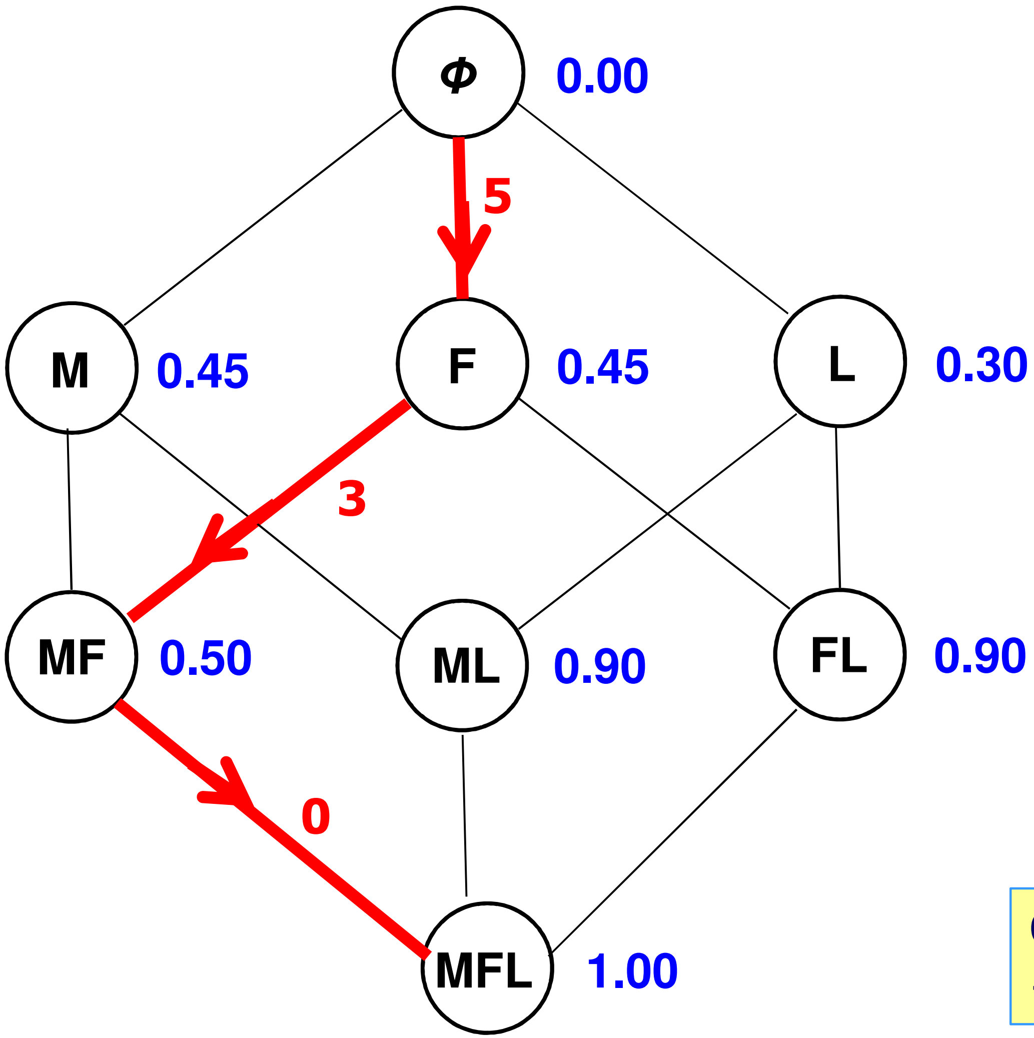
Student Noce



M → 7
F → 7
L → 7.5

C_μ (**Noce**) = 7 * 1.0 + 0 * 0.9 + 0.5 * 0.3 = **7.15**

Student Lia



M → 8
F → 5
L → 8

C_μ (**Lia**) = 5 * 1.0
+ 3 * 0.9 + 0 * 0.3 = **7.7**

Final result

- Colorni seemed to be the best candidate
- But we weren't considering the redundancies
- The best one is Lia, thanks to the synergy
- A graph is used for calculating (lattice 2^M)
- Increment represents the added value

In this way it is possible to take into account:

- **synergies** → given $\mu_{ij} > \mu_i + \mu_j$
- **redundancies** → given $\mu_{ij} < \mu_i + \mu_j$

Palio di Siena *(more)*

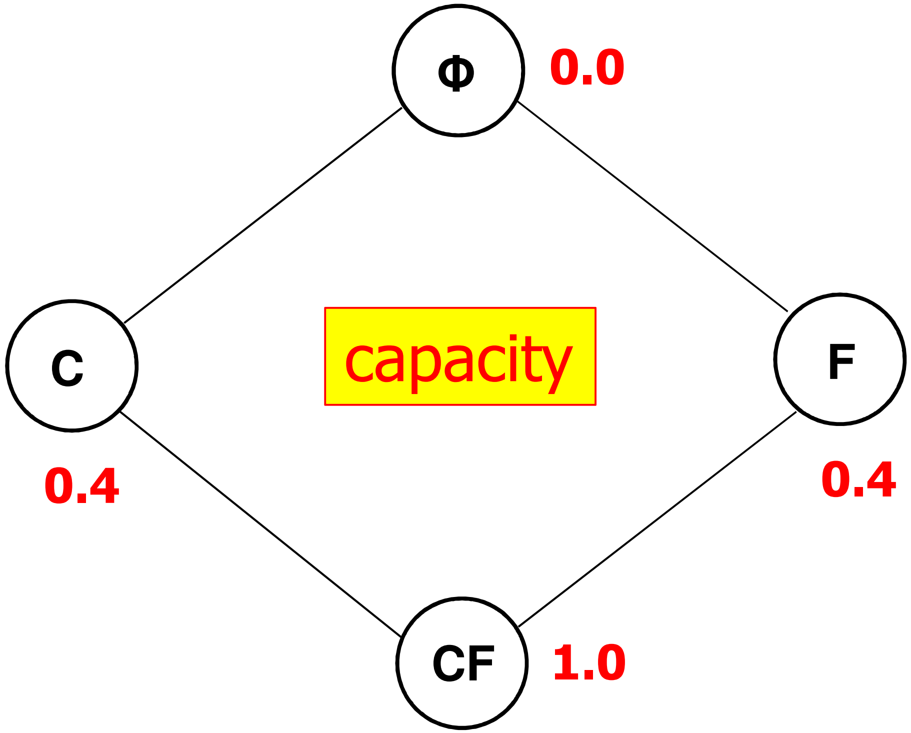
Bookmaker's perception:
(i) same weights to the attributes
(ii) Torre is the favourite

The couple horse-jockey makes contrada Torre the favorite for the bookmaker. The weights have to be given: to the 2 attributes and to the combination of these → **how ?**

	Onda	Bruco	Torre	Selva
Horse	100	0	45	30
Jockey	0	100	45	65
μ_i = …				
μ_j = …				
μ_{ij} = …				

Synergy → $\mu_i + \mu_j < \mu_{ij}$

The "horse/jockey" factor



Onda
C = 100
F = 0

Bruco
C = 0
F = 100

Torre
C = 45
F = 45

Selva
C = 30
F = 65

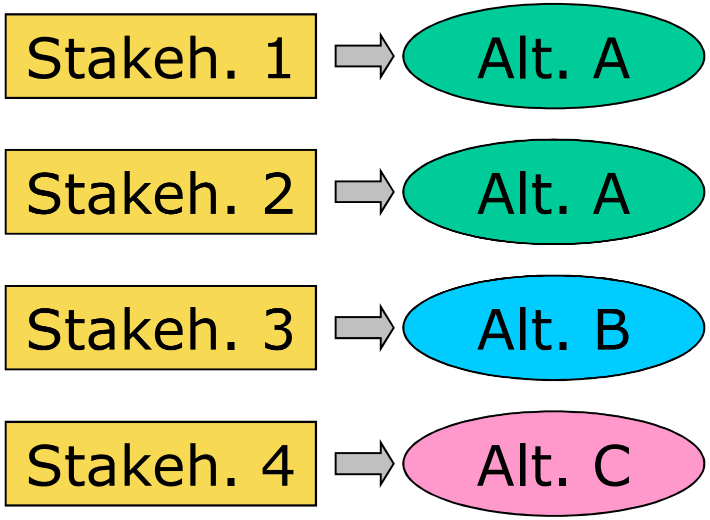
C_μ (**O**) = …, C_μ (**B**) = …, C_μ (**T**) = …, C_μ (**S**) = …

Tools for «point 4» problems

(i) Two approaches
(ii) Peer evaluation

© Alberto Colorni

(i) The two approaches to group decision



Possible conflict …

How to manage it ?

- Research of the critical points
- Proposing new/mitigating/compensative measures
  (from “dividing” to “enlarging the cake”)
- Do "win-win» solutions exist?
  (game can be not a zero-sum game)

Create information / 1

Analytic support: calculation of the indices of conflict,
based on the distances between decision makers.

- Impacts *(numbers of impacts may not coincide)*:
  - o distance of each player from the average value of each impact
- Utility funct. → examination of those which do not coincide
- **Weights**: construction of distance D matrix

$$D = [d_{ij}], \qquad \text{with } d_{ij} \geq 0 \quad \text{(symmetric ?)}$$

↓

Distance matrix among [weights vectors of] decision makers

Create information / 2

	utente	commer...	albergator...	residente...
utente	–	0.602	0.452	0.427
commerciante...	–	–	0.354	0.758
albergatore_si...			–	0.608
residente_sirm...	–	–	–	–

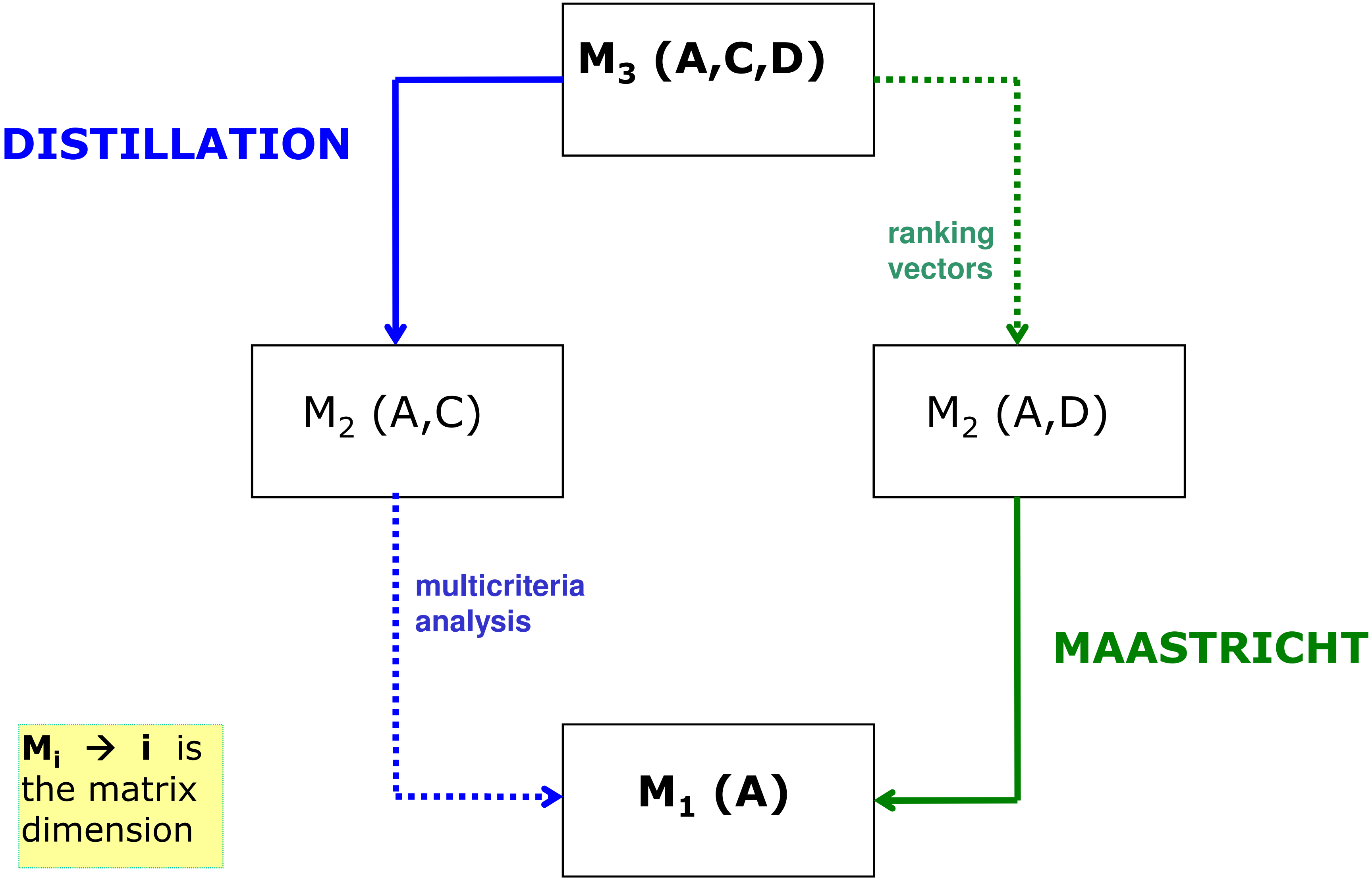
distance matrix

- Decisore più vicino
- Decisore più lontano
- Coppia di decisori più lontani

- Individual indeces of conflict:
  - sum for rows = distance of the row player from the others
  - sum for columns = distance of others from the column player
- Global indeces of conflict:
  - number $d_{ij} \neq 0$ → number of different vectors of weights
  - average d_{ij} → average distance among weight vectors
  - max d_{ij} → maximum level of conflict among two players
- Barycentric solution:
  - vector at the minimum distance from the vectors of the others

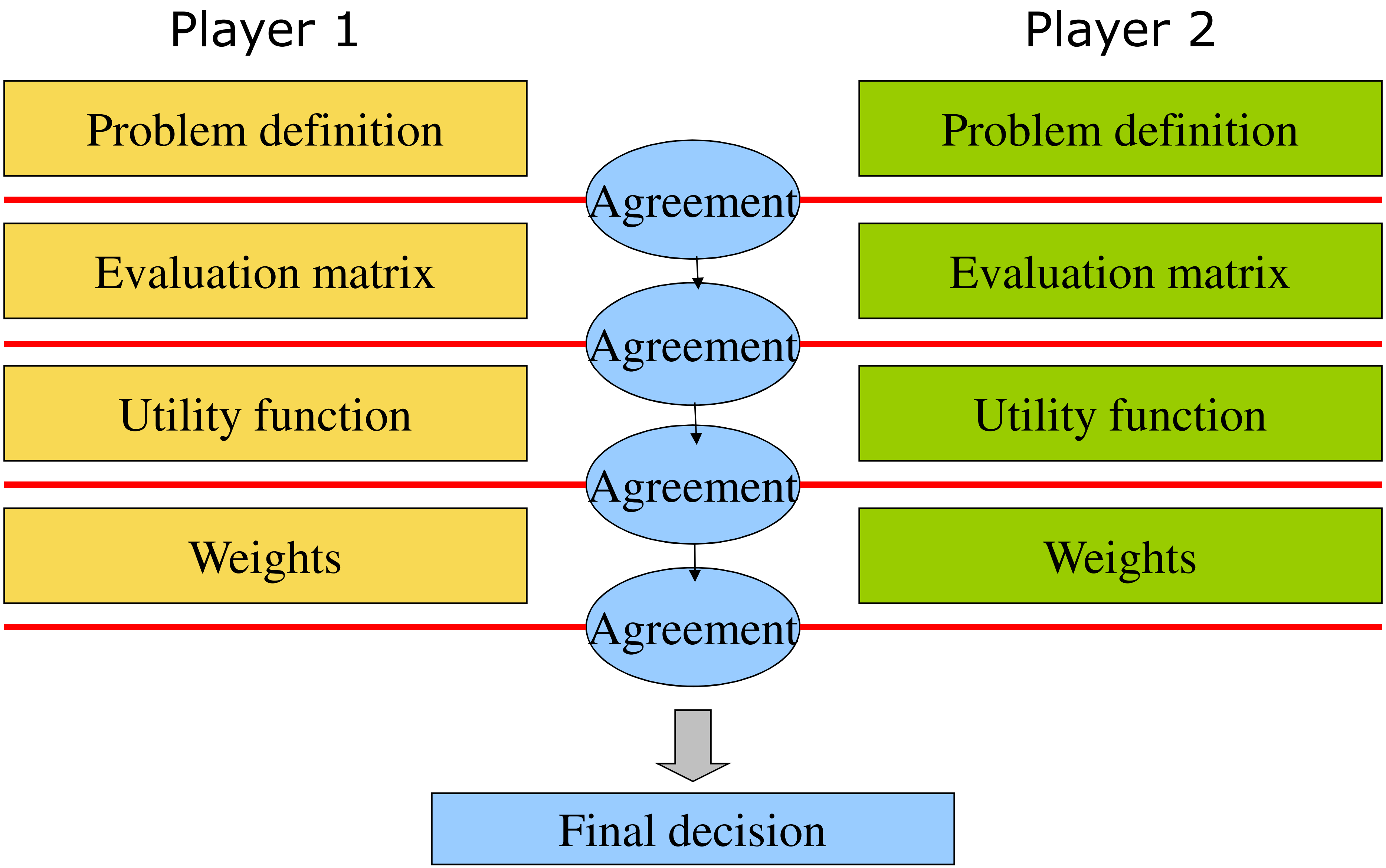
The two approaches

A = alternatives
C = criteria
D = dec. makers



M_i → **i** is the matrix dimension

Distillation



Distillation: compromise research

- Cooperative approach: trust building

- Decision makers move to barycentric position
  - synchronous method → together
  - a-synchronous method → the first is the most critical decision maker

- For each step:
  - information about global conflict (global conflict index)
  - information about the most critical decision maker (individual conflict index)

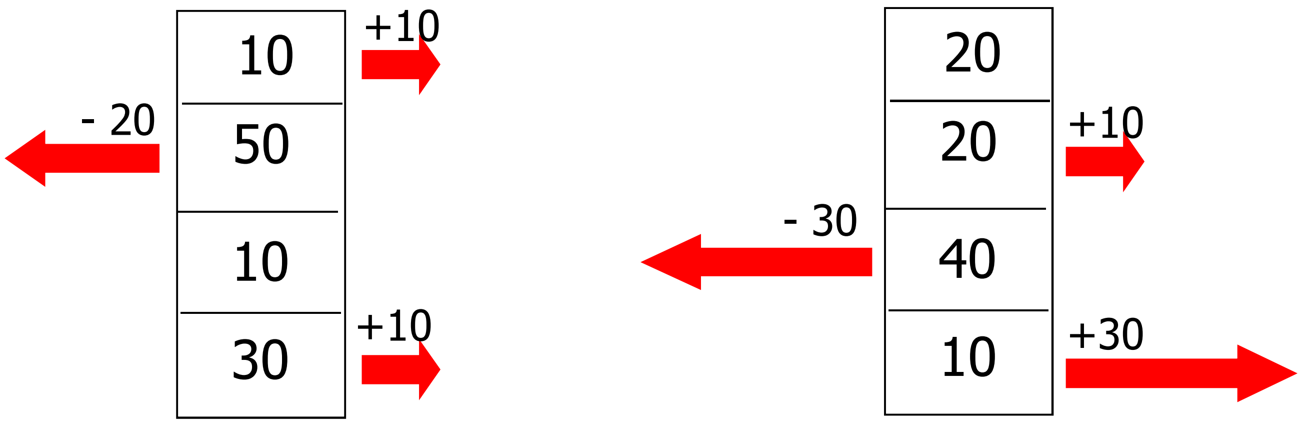
Distillation: to the barycenter

Weighted barycentric vector

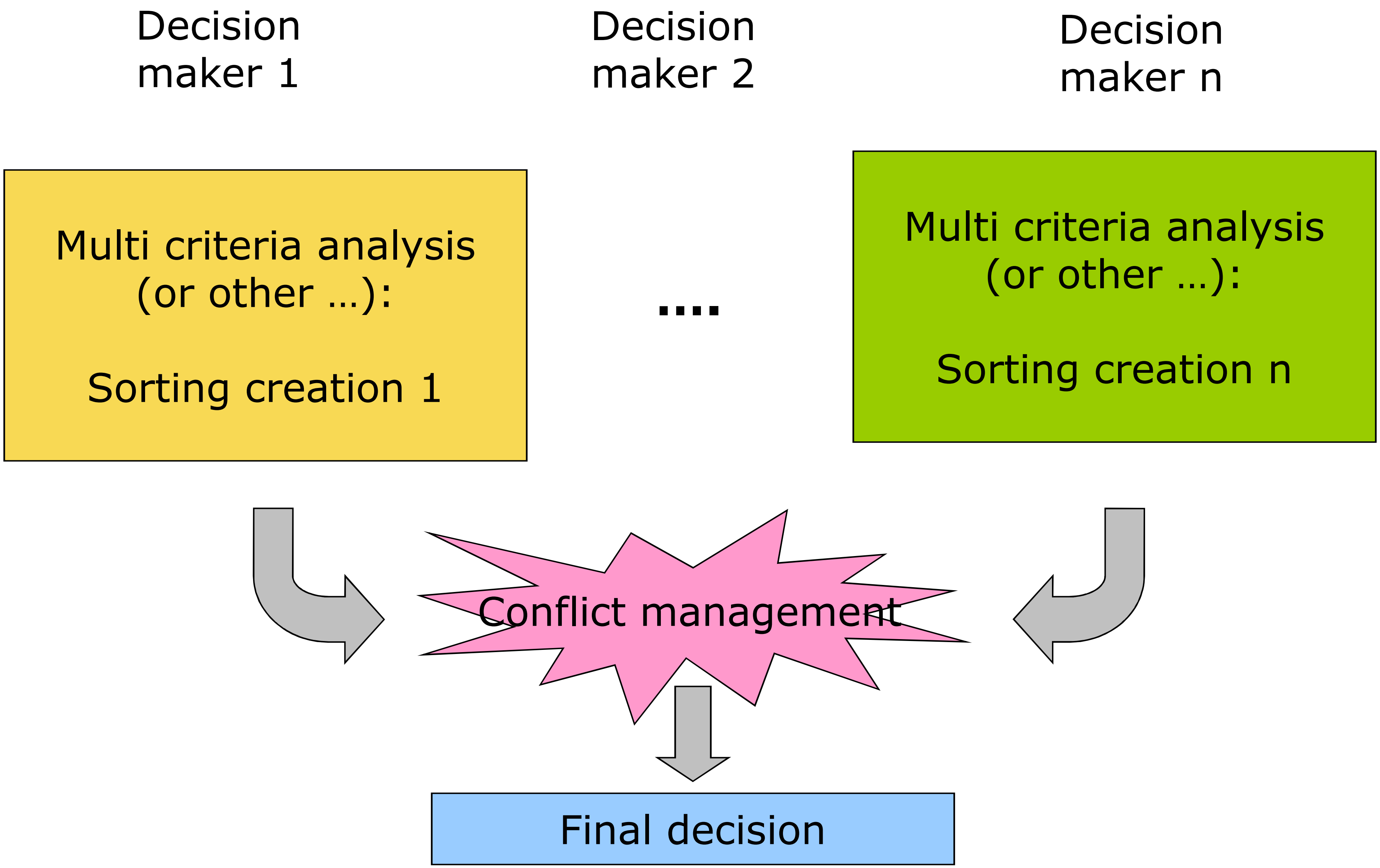
20
30
10
40

We calculate the distances between the components of the vectors of the weights of each decision maker and the components of the weighted barycentric vector

Weights dec. maker 1 Weights dec. maker 2 **...** (others)



Maastricht


Decision maker 1
Decision maker 2
Decision maker n
Multi criteria analysis (or other …):
Sorting creation 1
….
Multi criteria analysis (or other …):
Sorting creation n
Conflict management
Final decision

(ii) Peer evaluation

Have all the decision makers the same importance ?

- **Weights determined a priori:**
  - a meta-decision maker exists;
  - he has a weight proportional to the number of people that he represents.

- **Weights determinated by the group itself →**
  → cross-check, peer evaluation:
  - (1) average method
  - (2) eigenvector method
    - ✓ *player can assign a weight to himself,*
    - ✓ *player must assign weights just to other players.*

1. Average method

Player **Di** can vote to himself

	D1	D2	D3
D1	0.80	0.10	0.10
D2	0.05	0.70	0.30
D3	0.15	0.20	0.60

w
0.333
0.350
0.317

Vectors of the weights expressed by each player (column sum = 1)

Average vectors of the weights of the players

Inconsistent → players have the same relevance !

2. Eigenvector method

Player **Di** can vote to himself

	D1	D2	D3
D1	0.80	0.10	0.10
D2	0.05	0.70	0.30
D3	0.15	0.20	0.60

Vectors of the weights expressed by each player (column sum = 1)

Sw = w

w
0.330
0.363
0.307

Vectors of the weights of the players

$Sw = \lambda w$ (principal eigenvalue = 1)

A paradox

Rule (now) → player **Di** can not give a weight to himself

http://en.wikipedia.org/wiki/Perron-Frobenius_theorem

L'assioma del cazzone

	D1	D2	D3	D4
D1	0	0	0	0
D2	0	0	0	1
D3	0	0	0	0
D4	1	1	1	0

w
0
0.5
0
0.5

D1 and D3 have no **preferences** (> 0), so D1 and D3 p. of view are **not relevant** !!!

Possible solutions:

- **each player has to vote at least for two players;**
- **0 can not be used (weights ≥ predeterminated ε).**

Veto → United Nations Security Council

15 members, 5 can veto
(USA, Russia, China, France, Great Britain)

Rule → a resolution is approved if:
(i) gets at least 9 votes,
(ii) there is no veto (from 1 of the 5).

How to determinate the weight of the members (USA, Russia, China, … , D1, D2, … , D9, D10) and coalition threshold in order to simulate UN Security Council working process ???

Service design

© Alberto Colorni

Green Move

Objective: design & test an **electric car-sharing system** in Milan

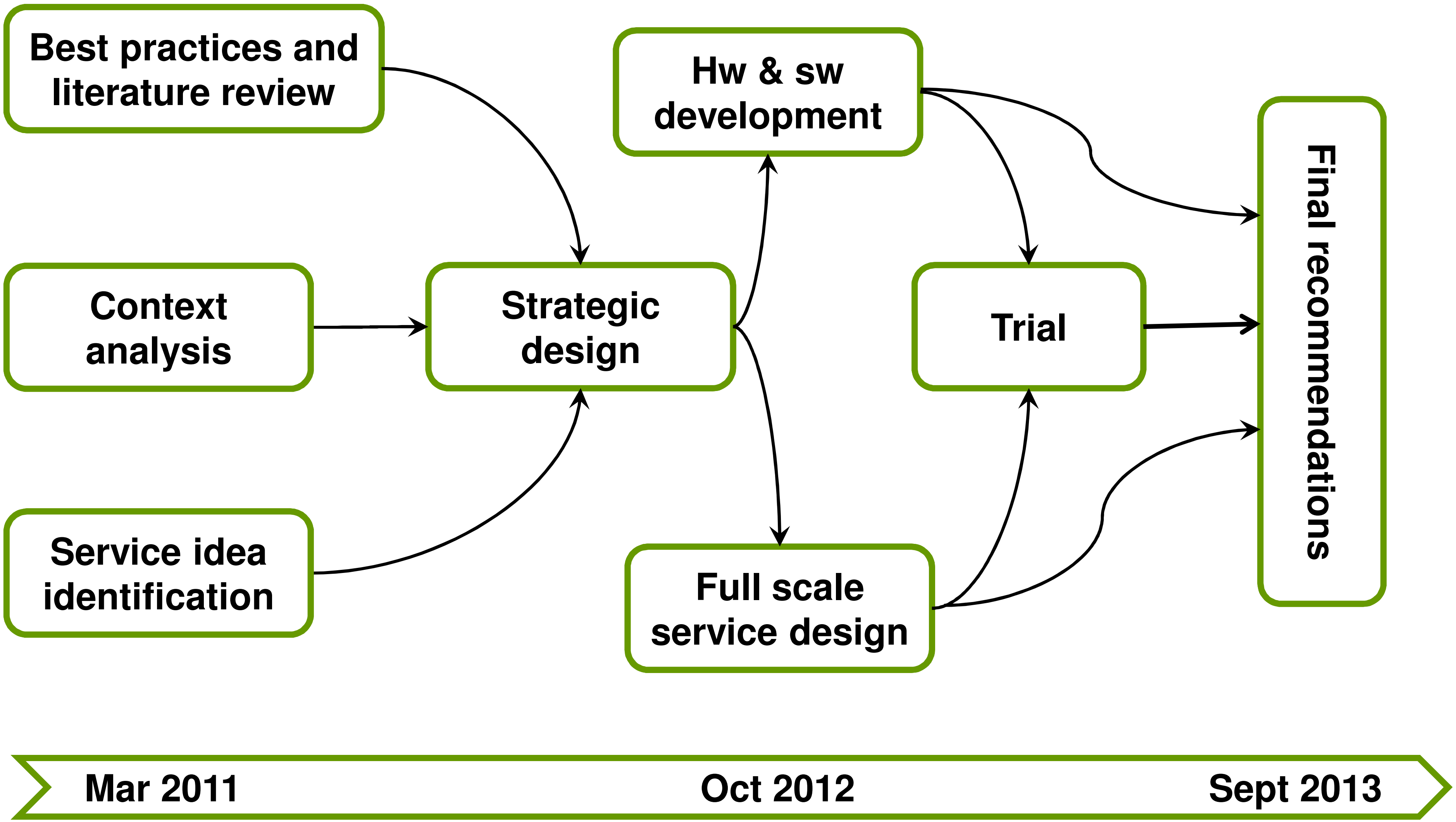
Coordinating a 2½ years project financed by the Lombardia Region (5 millions €), **involving 8 research centers** of Politecnico di Milano

Outcome:

- the design of a **full scale** service
- a **trial** with a limited number of docking stations in Milan

Switch from "buy a vehicle" paradigm to **"buy mobility services"**

The scheme


Best practices and literature review
Context analysis
Service idea identification
Strategic design
Hw & sw development
Full scale service design
Trial
Final recommendations
Mar 2011
Oct 2012
Sept 2013

How to design/formalize the service ?

Problem characteristics:

- **different actors** and stratification of governance levels,
  e.g. public administration (municipality, region), associations, …
- **uncertainty** in the definition of the variables,
  e.g. future policies for urban mobility, travel demand estimation for a non-existing service
- **conflicting criteria**,
  e.g. costs vs territorial coverage (such as in BikeMI)
- structuring the problem itself is an issue,
  e.g. definition of the configuration options to be evaluated is a key issue *(Hull and Tricker, 2005; Kelly et al., 2008; Jones et al., 2009)*

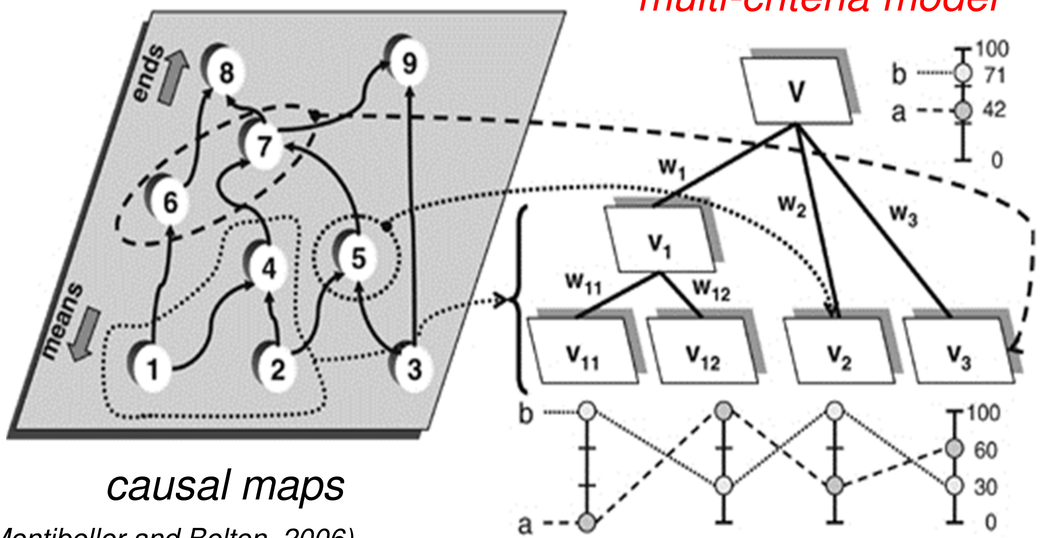
How to formalize the complexity ?

Casual maps & MCDA

<u>Integration of causal maps with multi-criteria analysis</u>:

- a powerful way of capturing decision-makers' views
- widely used in problem-structuring
  (Rosenhead and Mingers, 2004)
- model the **effects of a link**
  (qualitative or quantitative methods)
- definition of
  aggregation rules
  qualitative → experts
  quantitative
  models (e.g.
  demand analysis)

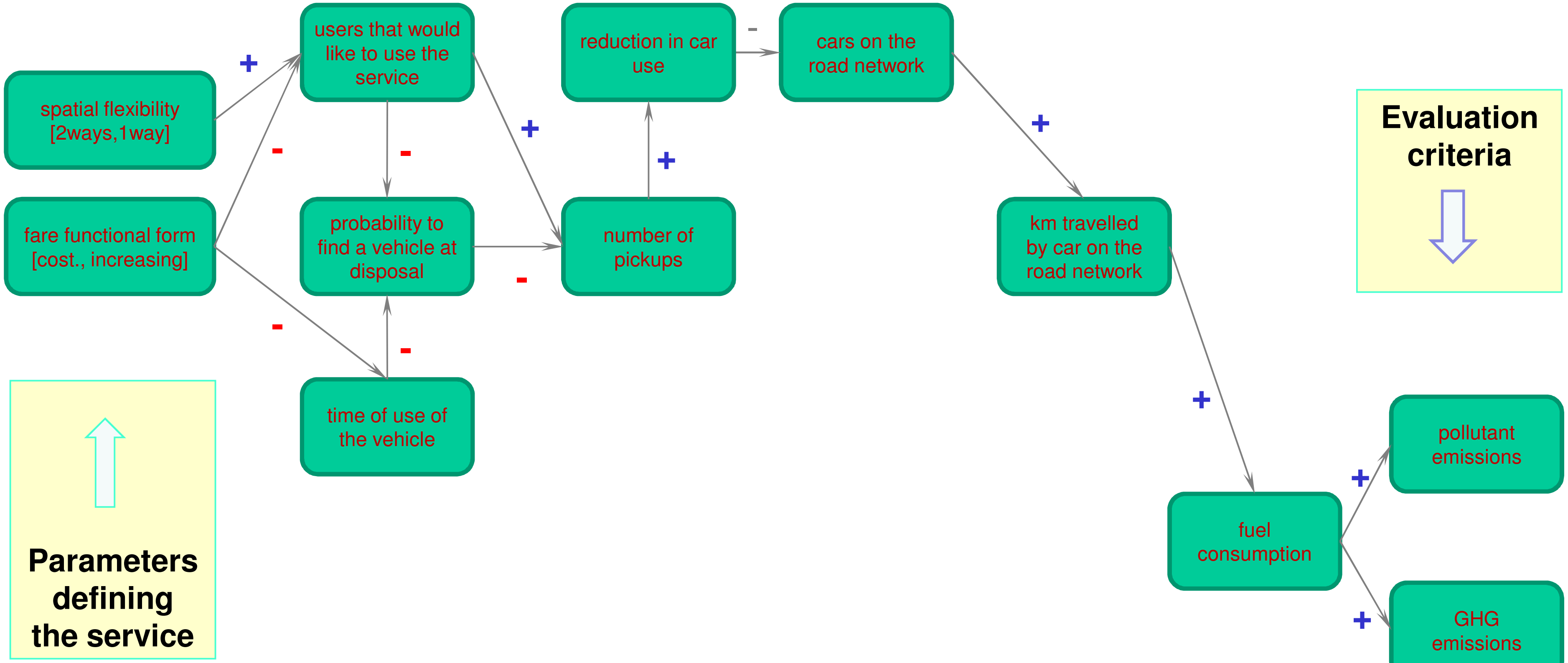
multi-criteria model



causal maps

(Montibeller and Belton, 2006)

A (partial) map for GM



Partial map
for the design of a vehicle sharing service

Parameters

1. Type of vehicle (EV, hybrid, ICE)
2. Service area
3. Capillarity and intermodality
4. Spatial flexibility → 1w-2w
5. Flexibility of service → temporal flexibility of booking
6. Fare:
    6.1. modes (hourly, km)
    6.2. function type (concave, convex, linear)
    6.3. level (high, medium, low)
7. Economic incentives (parking, congestion tax, LPT)
8. Incentives for service (areas with traffic restrictions, lanes ...)
9. Re-allocation model
10. Mechanisms for promotion and marketing

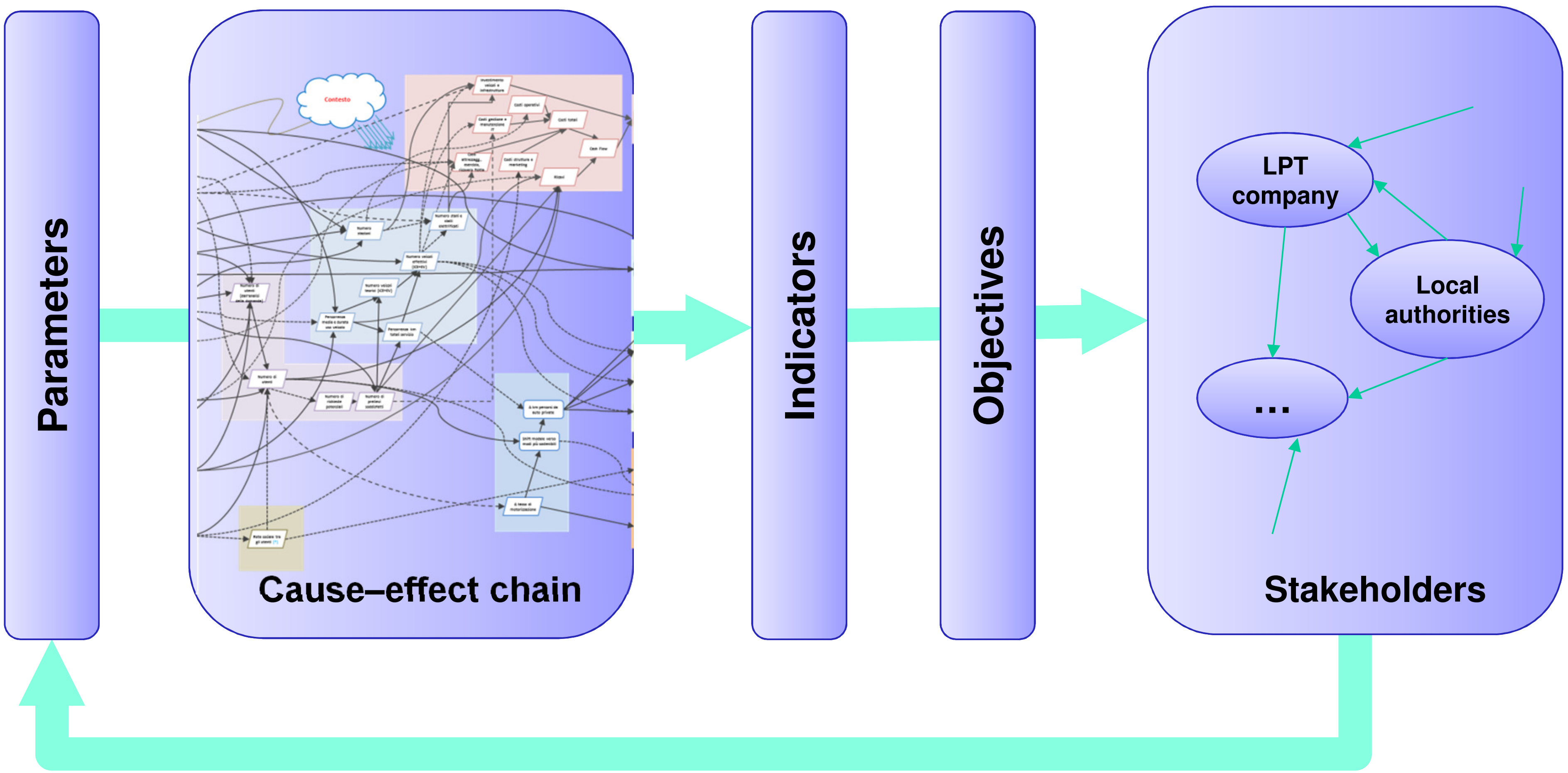
Indicators

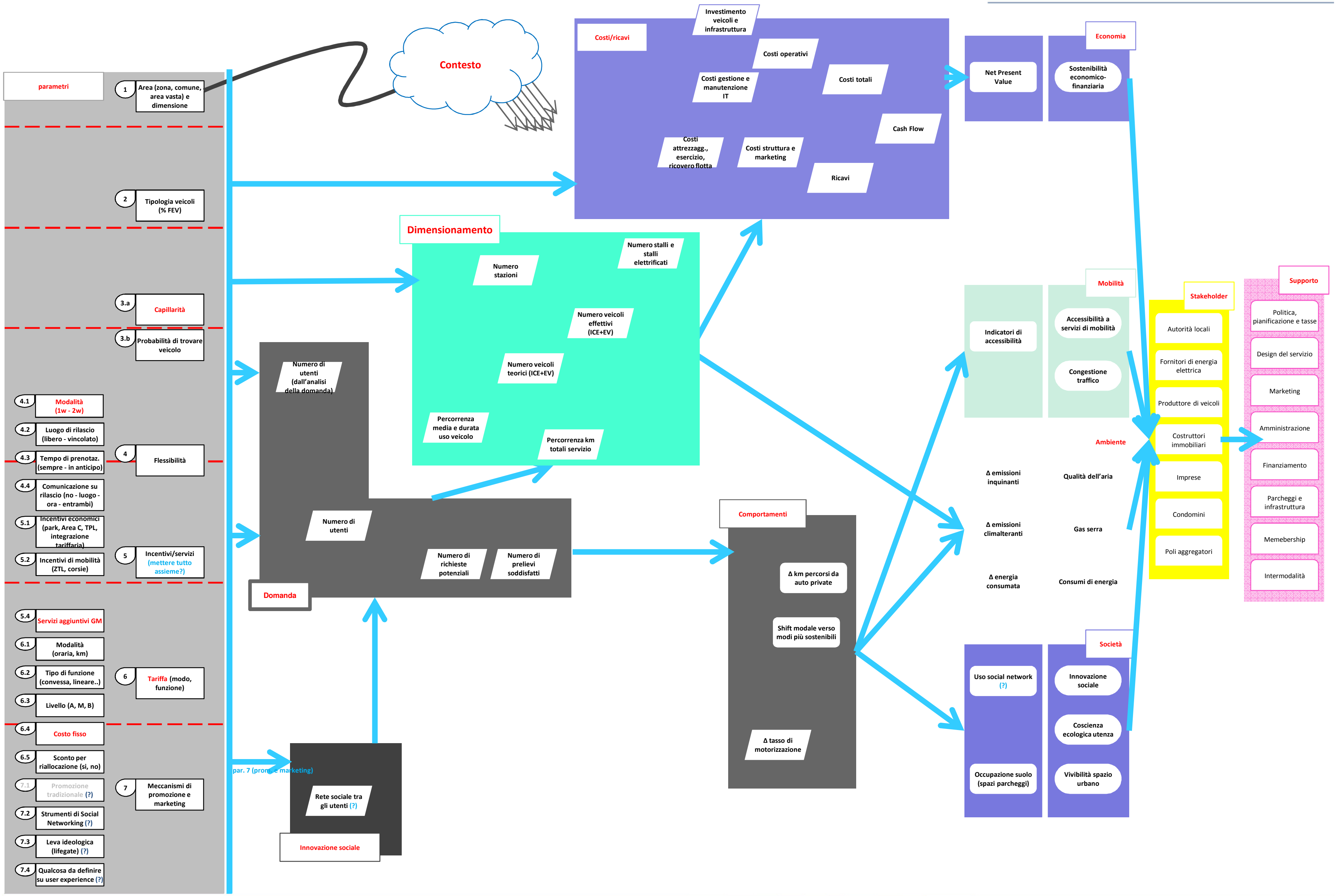
After the setting up of different options (alternatives), they can be evaluated and compared thanks to adequate indicators.

Evaluation and performance indicators (to measure the achievement of the objectives of stakeholders):

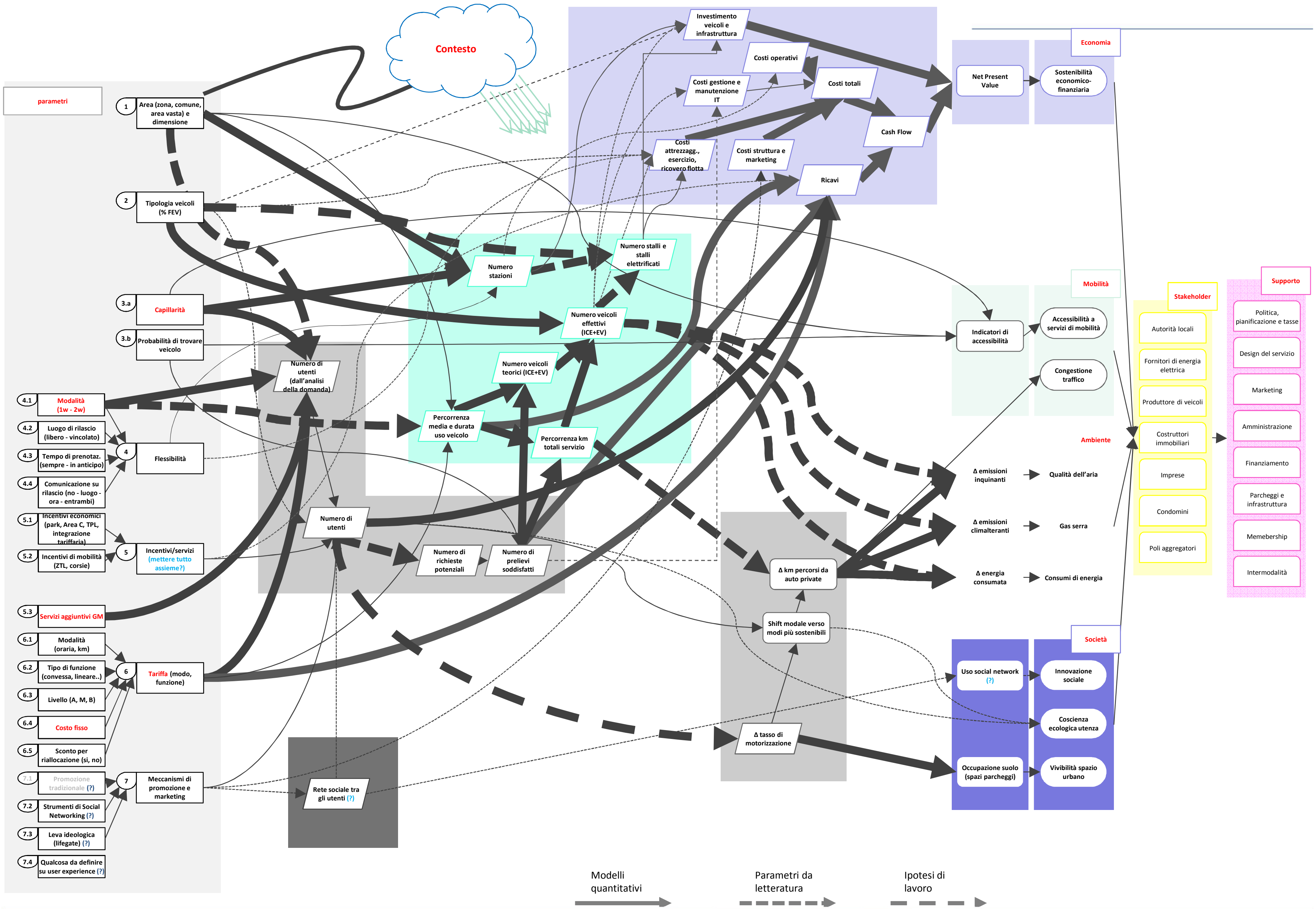
- Net Present Value
- Δ km traveled on the network
- Δ greenhouses gases
- Δ polluting emission
- Modal shift to sustainable mobility
- Number of users
- Connection in the social network
- Space occupied by parking
- Accessibility indicators
- ….

Flow chart


Parameters
Cause–effect chain
Indicators
Objectives
LPT company
Local authorities
...
Stakeholders



Contesto
parametri
1
Area (zona, comune, area vasta) e dimensione
2
Tipologia veicoli (% FEV)
3.a
Capillarità
3.b
Probabilità di trovare veicolo
4.1
Modalità (1w - 2w)
4.2
Luogo di rilascio (libero - vincolato)
4.3
Tempo di prenotaz. (sempre - in anticipo)
4
Flessibilità
4.4
Comunicazione su rilascio (no - luogo - ora - entrambi)
5.1
Incentivi economici (park, Area C, TPL, integrazione tariffaria)
5
Incentivi/servizi (mettere tutto assieme?)
5.2
Incentivi di mobilità (ZTL, corsie)
5.4
Servizi aggiuntivi GM
6.1
Modalità (oraria, km)
6.2
Tipo di funzione (convessa, lineare..)
6
Tariffa (modo, funzione)
6.3
Livello (A, M, B)
6.4
Costo fisso
6.5
Sconto per riallocazione (si, no)
7.1
Promozione tradizionale (?)
7
Meccanismi di promozione e marketing
7.2
Strumenti di Social Networking (?)
7.3
Leva ideologica (lifegate) (?)
7.4
Qualcosa da definire su user experience (?)
par. 7 (promo e marketing)
Costi/ricavi
Investimento veicoli e infrastruttura
Costi operativi
Costi gestione e manutenzione IT
Costi totali
Cash Flow
Costi attrezzagg., esercizio, ricovero flotta
Costi struttura e marketing
Ricavi
Net Present Value
Economia
Sostenibilità economico-finanziaria
Dimensionamento
Numero stazioni
Numero stalli e stalli elettrificati
Numero veicoli effettivi (ICE+EV)
Numero veicoli teorici (ICE+EV)
Percorrenza media e durata uso veicolo
Percorrenza km totali servizio
Numero di utenti (dall'analisi della domanda)
Numero di utenti
Numero di richieste potenziali
Numero di prelievi soddisfatti
Domanda
Rete sociale tra gli utenti (?)
Innovazione sociale
Comportamenti
Δ km percorsi da auto private
Shift modale verso modi più sostenibili
Δ tasso di motorizzazione
Mobilità
Indicatori di accessibilità
Accessibilità a servizi di mobilità
Congestione traffico
Ambiente
Δ emissioni inquinanti
Qualità dell'aria
Δ emissioni climalteranti
Gas serra
Δ energia consumata
Consumi di energia
Società
Uso social network (?)
Innovazione sociale
Coscienza ecologica utenza
Occupazione suolo (spazi parcheggi)
Vivibilità spazio urbano
Stakeholder
Autorità locali
Fornitori di energia elettrica
Produttore di veicoli
Costruttori immobiliari
Imprese
Condomini
Poli aggregatori
Supporto
Politica, pianificazione e tasse
Design del servizio
Marketing
Amministrazione
Finanziamento
Parcheggi e infrastruttura
Memebership
Intermodalità



Contesto
parametri
1
Area (zona, comune, area vasta) e dimensione
2
Tipologia veicoli (% FEV)
3.a
Capillarità
3.b
Probabilità di trovare veicolo
4.1
Modalità (1w - 2w)
4.2
Luogo di rilascio (libero - vincolato)
4.3
Tempo di prenotaz. (sempre - in anticipo)
4.4
Comunicazione su rilascio (no - luogo - ora - entrambi)
4
Flessibilità
5.1
Incentivi economici (park, Area C, TPL, integrazione tariffaria)
5.2
Incentivi di mobilità (ZTL, corsie)
5
Incentivi/servizi (mettere tutto assieme?)
5.3
Servizi aggiuntivi GM
6.1
Modalità (oraria, km)
6.2
Tipo di funzione (convessa, lineare..)
6.3
Livello (A, M, B)
6.4
Costo fisso
6.5
Sconto per riallocazione (si, no)
6
Tariffa (modo, funzione)
7.1
Promozione tradizionale (?)
7.2
Strumenti di Social Networking (?)
7.3
Leva ideologica (lifegate) (?)
7.4
Qualcosa da definire su user experience (?)
7
Meccanismi di promozione e marketing
Numero di utenti (dall'analisi della domanda)
Numero di utenti
Numero di richieste potenziali
Numero di prelievi soddisfatti
Rete sociale tra gli utenti (?)
Numero stazioni
Numero stalli e stalli elettrificati
Numero veicoli effettivi (ICE+EV)
Numero veicoli teorici (ICE+EV)
Percorrenza media e durata uso veicolo
Percorrenza km totali servizio
Investimento veicoli e infrastruttura
Costi operativi
Costi gestione e manutenzione IT
Costi totali
Costi attrezzagg., esercizio, ricovero flotta
Costi struttura e marketing
Cash Flow
Ricavi
Net Present Value
Economia
Sostenibilità economico-finanziaria
Mobilità
Indicatori di accessibilità
Accessibilità a servizi di mobilità
Congestione traffico
Ambiente
Δ emissioni inquinanti
Qualità dell'aria
Δ emissioni climalteranti
Gas serra
Δ energia consumata
Consumi di energia
Δ km percorsi da auto private
Shift modale verso modi più sostenibili
Δ tasso di motorizzazione
Società
Uso social network (?)
Innovazione sociale
Coscienza ecologica utenza
Occupazione suolo (spazi parcheggi)
Vivibilità spazio urbano
Stakeholder
Autorità locali
Fornitori di energia elettrica
Produttore di veicoli
Costruttori immobiliari
Imprese
Condomini
Poli aggregatori
Supporto
Politica, pianificazione e tasse
Design del servizio
Marketing
Amministrazione
Finanziamento
Parcheggi e infrastruttura
Memebership
Intermodalità
Modelli quantitativi
Parametri da letteratura
Ipotesi di lavoro

Conclusions
(part 2)

© Alberto Colorni

Alberto Colorni

alberto.colorni@polimi.it

www.poliedra.polimi.it

Thank you

© Alberto Colorni