What is a Decision Problem?
Designing Alternatives

Alberto Colorni[1] and Alexis Tsoukiàs[2]

[1] Design Department, Politecnico di Milano
[2] LAMSADE-CNRS, Université Paris Dauphine

Abstract. This paper presents a general framework for the design of alternatives in decision problems. The paper addresses both the issue of how to design alternatives within "known decision spaces" and on how to perform the same action within "partially known or unknown decision spaces". The paper aims at providing archetypes for the design of algorithms supporting the generation of alternatives.

1 Introduction

Most scholar articles in decision analysis and operational research, when introducing the problem formulation they talk about, start with a claim of the type "given a set A of alternatives". Both researchers and practitioners know that in reality the set A is never "'given" ... It is actually constructed during the decision aiding process and most of the times defined several times during that same process.

Surprisingly enough this topic is almost ignored in the specialised literature. With the notable exception of Keeney ([14]) who stated the principle that decision making should start considering "values" (in the sense of attributes) and not "alternatives" the latters derived from the formers, (see also [15], [16]) very few contributions are available: some early attempts include [19] and [20], while other contributions were mainly focussed on how to structure the decision problem suggesting alternatives generation algorithmes (see [1], [3], [6], [11]). To our knowledge the topic has been partially considered in behavioural and cognitive sciences studies analysing how real decision makers handle alternatives construction (see [18]).

This is remarkably strange. Practically the mainstream decision analysis literature focus on how to "choose" an alternative without considering where these alternatives come from and how they can be established. On the other hand it should be obvious: if all the alternatives in the considered set are "bad" we are going to choose a bad option even if it is the best one ... On the other hand who and how decides which are "good" options to include in the set of alternatives?

This paper is far from being a survey. We want to construct a general framework allowing handling this topic in a formal way. The topic results as part of the research in conducting decision aiding processes (see [29]). We recall that within that framework we will always make the hypothesis that the information used within such a process is the result of the interaction of at least two agents: the client and the analyst. This attempt follows our recent work on defining what a decision problem is (see [4]) and should include both known procedures which are actually used in order to generate alternatives

as well as to give the basis for defining new procedures of more general validity. Our objective is two-fold:
- show that constructing a set of alternatives is a decision problem itself;
- show which are the conceptual and algorithmic challenges in developing a general theory about alternatives construction, a key topic in conducting decision aiding processes (see [29]).

The paper is organised as follows. In Section 2 we present the general framework (what is a decision problem) within which we consider the problem of generating alternatives. In Section 3 we show that this problem is a decision problem itself. Section 4 discusses how existing methods handle the issue of generating "known" alternatives. In Section 5 we show instead how to handle the issue of generating "unknown" alternatives when the set of available ones is unsatisfactory. Section 6 discusses related literature.

2 Concepts and Notation

This work follows our previous contribution about "What is a Decision Problem" ([4]) where we introduced a general framework aiming to characterise decision problems on the basis of the information the client in a decision situation can provide. Indeed our framework is independent of any method characterisation: it should instead help defining a decision problem (and thus choosing or constructing any new method) from some minimal information which we call the primitives. Within such a framework a decision problem is *"the partitioning of a set A satisfying some properties and preferential information"*. The primitives then are:
- the set A described along a set of attributes satisfying separability, in other terms these attributes are the minimal descriptors necessary to make a decision and each one considered alone is sufficient to make a decision;
- the problem statement Π establishing the type of partitioning to perform;
- the preference statements $\mathcal{H}$ provided by the client, to be modelled through appropriate structures and languages.
Let's present these topics with more details.

1. The set A of alternatives can be essentially of three types:
   - a subset of a vector space, where alternatives are described as points (vectors) of an n-dimensional "feasible" decision space (often each dimension being associated to a "decision variable"), $A \subseteq \mathbb{R}^n$;
   - a subset of a combinatorial structure , where alternatives are described as combinations of decision variables having a finite and discrete number of possible values (possibly binary), $A \subseteq \prod_j X_j$ where $\forall j X_j = \{x_{1j}, \cdots x_{nj}\}$, X_j being ordered;
   - an explicit enumeration of objects, possibly described by one or more features or attributes.
2. The problem statement Π can be:
   - a ranking: construct a partition of ordered equivalence classes which are not defined a-priori;
   - a rating: construct a partition of ordered equivalence classes which are defined a-priori;

- a clustering: construct a partition of unordered equivalence classes which are not defined a-priori;
- an assignment: construct a partition of unordered equivalence classes which are defined a-priori.

3. The preference statements $\mathcal{H}$ (the reader should note that we use the term of preference in a very general way: any ordering relation can be considered as a preference relation, see [21], [26], including similarity and equivalence relations) can be of different types:
   - single or multi-attribute ones;
   - relative (comparing elements of A among them) or absolute (comparing elements of A to some external norm);
   - simple (comparing single elements of A) or extended (comparing whole subsets of A);
   - ordinal or more than ordinal (expressing some notion of difference of preference);
   - positive or negative (negative preference statements should not be considered as the complement of positive ones);
   - first order or higher (preferences about preferences).
4. Let's recall finally that in order to choose or to construct a "resolution" method what we strictly need is the set A (minimally described), a problem statement Π and enough preference statements where we need to check (wrt to $\mathcal{H}$):
   - how differences of preferences are considered on each single dimension/attribute;
   - how differences of preferences are considered among the different dimensions or attributes;
   - whether preferences are conditional/dependent from other preferences;
   - whether negative preferences should be considered explicitly or not.

   It is important to note that the concept of "preference" applies to all three principal reasons for which decisions are variable: values, opinions and scenarios.

3 Constructing A as a recursion

Proposition 1. *Constructing the set A is itself a decision problem.*

Proof. Suppose a decision situation where any option is possible. In other terms a situation where we do not really have a well established set, but only hypotheses of what this should be. We can represent this situation representing this ill defined set $\mathcal{A}$ as follows:
$\mathcal{A} \subseteq \mathbb{R}^n \vee \prod_j X_j$ admitting that n is unknown and that equally exist unknown X_j. That is, the set $\mathcal{A}$ is only partially known (possibly totally unknown).

On the other hand let's recall that in order to establish a decision problem we need at least a set A, a problem statement Π and some preference statements $\mathcal{H}$ (at least of the type $x \succeq y$ or $x \succeq k$ where x and y are members of A and k an external norm not necessarily member of A). Finally the description of set A needs to satisfy separability. With these elements in mind we can establish a fix point:
A decision problem exists iff
- $\exists X_j$ such that X_j is known and

- $\forall X_j$ such that X_j is unknown these are not separable.
In other terms applying our minimality requirements either there is no decision problem or if there is one then there is at least one known descriptive dimension of the set $\mathcal{A}$, any other potential, but unknown dimension, being not separable and thus irrelevant. Let's call this the set $\hat{A}$.
We can now establish a recursion constructing the set A:
- $A_1 = \hat{A}$
- $A_n = \bigcup_i [A_{n-1}]_i$ where $[A_{n-1}]$ are some of the equivalence classes constructed for a decision problem defined at step $n-1$ and thus upon the set A_{n-1}.

□

Let's explain better our proposition. Despite the fact that the set A is not given, there is always a starting point for constructing it. It can be large and ill defined, but there always exist a set to start with (otherwise there is no problem ... to work with). The construction of the set A is a recursion where at each step we construct a set as a result of the partition of the set defined at the previous step. The ending condition of this process is subjective. It is the client of the decision aiding process that declares that the present version of set A satisfies his/her requirements. In the following we provide three small examples in order to show the generality of our model.

Example 1. Consider the problem of constructing the feasible set of some linear programming problem. We can start establishing $\hat{A} = \mathbb{R}^n$, n being the known decision variables (at least one should be known). Then:
- $A_1 = \hat{A}$
- $A_2 = [A_1 : x_1 \geq 0]$
- $\cdots$
- $A_m = [A_{m-1} : x_m \geq 0]$
- establishing thus a first feasible set this being the non negative reals; then:
- $A_{m+1} = [A_m : f(x_1, \cdots x_m) \geq 0]$, introducing a first linear constraint
- and then introducing all known constraints.

The reader should note that each time we solve a rating decision problem with two possible equivalence classes (the feasible and the unfeasible solutions) defined by an externa norm (the rhs of each constraint). It should also note our implicit preference statements (feasible solutions are better than the unfeasible ones) and that the preferences upon each variable and then upon bundles of variables (the constraints) are independent (thus allowing to establish a linear, additive, model).

Example 2. Consider the case of a company aiming to offer promotional tickets to the population for some advertising purpose. Then if Ω is the target population, $\hat{A}$ will be the subset of Ω for which some information is known (sex, age, education, income etc.).

A clustering decision problem would generate n equivalence classes (unknown at the beginning) $[A_1], \cdots [A_n]$ each being an homogeneous advertising target (ie. young, female, not-single, no-children, low income). Each of such equivalence classes could then become the set A_1 for some ranking decision problems identifying the recipients of the promotional tickets.

Example 3. Consider the case of a national park administrator who needs to apply preservation policies for the park's animals. The starting point will be to consider the whole animal population Ω of the park. Then through an assignment decision problem she will identify the species within the park (let's say mammals, birds and reptiles, $A_1 = [A_1]_m \cup [A_1]_b \cup [A_1]_r$). Then a rating decision problem may distinguish between endangered and not endangered animals (A_2). A clustering decision problem will identify "geographical communities" of animals within the park ((A_3). Further on an assignment procedure may distinguish between local and imported animals (A_4). Finally a ranking procedure may order the animals on the basis of their attractiveness for the visitors (A_5). Why these sets may be generated? The client (the park administrator) first realises that different species need different policies (she thus introduces the attributes characterising species), then she realises that endangered animals may be a priority (using new attributes describing animals' threats), then she decides to consider the differences which might be necessary for the different locations in the park (using now spatial attributes), she decides to separate local from imported animals since this is imposed by bio-diversity considerations and finally considering cost (and revenue) issues she decides to rank animals by attractiveness. Different intersections (and unions of intersections) of the above partitionings will produce now the input for further decision problems. For instance, given a group of animals being described by their relevant attributes: "local endangered mammals breading around X", cluster preservation actions into policies. In this case the starting set will be a universe of potential preservation actions (known in the literature), but the separable attributes are the ones relevant for that specific group of animals, resulting to an initial set of relevant preservation actions for that group.

4 Generating known alternatives

All existing methods in operational research, decision analysis and artificial intelligence implicitly follow the general procedure shown in the previous section, generating sets of alternatives as part of the resolution algorithm they implement. Alternatives are implicitly known and only explicitly shown when they happen to be a solution for the algorithm within the method (most of the times an optimisation one).

The reason for this is that alternatives are almost never explicitly enumerated (most of the times the whole set could be impossible to describe explicitly or even be infinite). They are described as combination of variables. Humans also, in order to handle their limited computing capability, tend to use the same approach: either reduce the number of variables (thus reducing the number of alternatives) or just focus to a limited set of "interesting alternatives" (most of the times resulting from some screening process).

Let's start with some simple human heuristics. These are always based on two simple ideas: screening and choosing (see also [30]) and/or fixing the value of one or more variables and exploring the reduced set of combinations (possibly applying the method recursively). However let's consider the following simple example (borrowed from [25]):

Example 4. Consider the transportation problem shown in Table 1, implying 3 production units (p1, p2 and p3) and 3 warehouses (w1, w2 and w3; the figures in the cells representing the costs).

	w1	w2	w3	prd. capacity
p1	0	4	1	300
p2	1	6	3	600
p3	3	7	6	500
wrh capacity	600	300	500	

Table 1. A simple 3×3 transportation problem

Most experienced managers, when trying to solve intuitively the problem, try to maximise the amount of shipping corresponding to variable x_1 (from p_1 to w_1, cost 0, the lowest), keeping at 0 the shipping corresponding to variable x_8 (p_3 to w_2, cost 7, the highest). This gives a relatively reasonable solution, but far from the optimal one which is $\langle x_1 = 0, x_2 = 0, x_3 = 300, x_4 = 400, x_5 = 0, x_6 = 200, x_7 = 200, x_8 = 300, x_9 = 0\rangle$. The reason for failing to see intuitively the optimal solution is due to the fact that without a model and an algorithm is difficult to consider a counterintuitive choice (ship nothing from p_1 to w_1). For a more general and interesting discussion about these topics the reader can see [9].

The use of a formal model and some exploring algorithm certainly improves the situation. However, we know that due to algorithmic complexity most exact resolution algorithms are of little practical interest since in the worst case they require inconceivable amount of computing resources or time. Most of the times we end using heuristics (see [2], [22]).

The use of heuristics does not really change the problem. Consider the well known "knapsack" problem and the use of the equally well known simple heuristic consisting in choosing the variables (the objects to put in the knapsack) following the magnitude of the ratio between the value (the coefficient of the objective function) and the weight (the coefficient of the constraint). This procedure produces rapidly good results, but can easily miss the best solution since this may not necessarily respect this reasonable order. Heuristics generate sets of alternatives biased by the specific resolution procedure they use and in doing so they tend to eliminate alternatives which could be "interesting".

Finally let us consider the case where efficient exact algorithms are available for the problem at hand. In this case we are sure to be able to explore the whole set of potential alternatives although not explicitly enumerating them. The problem here is that despite this algorithm will provide a solution (most of the times denoted optimal), this might not be satisfactory for the client. The reason most of the times is that we are using the "wrong" set of alternatives. We should bear in mind that clients have a limited knowledge of the technical details of algorithms and more generally of problem solving methods. An initial description of a decision problem using a set of separable attributes (variables) might not be immediately perceived as partial. Usually it is when we present the results to the client that they realise that this first description of their problem does not really fit what they have in mind: all suggested solutions are perceived as unsatisfactory.

Let us summarise: generating alternatives only through resolution oriented procedures does not allow to conduct neither efficiently nor creatively a decision aiding process. We need to be able to generate further "unknown" alternatives and we need specific procedures to do so.

5 Generating unknown alternatives

Let's start with three examples where the known alternatives might be unsatisfactory for the decision maker.

Example 5. Ahmed, is a young man going to an appointment with his recent new girlfriend. Crossing a flowers' shop he suspects it might be her birthday. To buy or not to buy the flowers? That's the dilemma ... However these two options appear to be equally unsatisfactory. If he buys the flowers and is not the birthday (actually the most likely scenario) there will be interminable discussions on why he did that. If he does not buy the flowers and it happens to be the birthday then it is just a tragedy. Ahmed needs more options before deciding.

Example 6. Aisha is a young French PhD student having the opportunity to visit Sydney for a conference (if her paper is accepted and conditional to the finances of the lab). Aisha's boyfriend is considering joining her. Tickets for Sydney sell presently as low as 1000 €, but they are expected to rise very soon. The problem is that Aisha will know if she will make the travel only one month before the conference, while today we are four months before the conference. Once again the available options are unsatisfactory: either low price tickets combined to high risk of losing the money in case Aisha does not make the travel, or being sure about the travel combined to a high risk of not being able to pay for the ticket. Aisha and her boyfriend would like to have more alternatives before deciding.

Example 7. Aisha and Ahmed are celebrating 10 years of living together and they look for a one week holiday package. The problem is that what they get are either expensive resorts in attractive locations or cheap resorts located in unattractive locations ... Aisha and Ahmed need to expand the set of alternatives they are looking for.

The three examples are inspired from the decision analysis literature (see [7], [14], [27]). Indeed there already exist suggestions on how to handle such decision situations expanding appropriately the set of alternatives. These include "decision trees", "real options theory" and "valued focussed thinking".

1. A well known strategy in decision under uncertainty consists in asking for more information (an action called an "oracle" given the limited trust to the information provided). Under such a perspective the two options b (buy) and $\neg b$ (not buy) can be expanded to ib (get information and then buy), $i\neg b$ (get information and then not buy), $\neg ib$ and $\neg i\neg b$ (same as before, deciding to buy or not without any further information). The reader will note that until information is not a separable characteristic of the decision to take, this variable simply does not exist (consistently with

our hypothesis that not separable variables are not relevant). The new expanded set results thanks to information becoming a separable dimension (influencing our decision).

2. In real options theory the idea is to add "'time" as a separable explicit dimension among the attributes. The unsatisfactory nature of the alternatives is due to the fact that we need to decide today for something expected to occur after a certain time. Introducing time as a further dimension we could introduce alternatives which realisation has a a shortest time horizon but not preclude realising the original options. For instance airlines offer today the possibility to pay a non refundable fee fixing the price of a ticket at today's price for a certain amount of time. Instead having the two options b_0 (buy today) and $\neg b_0$ (not buy today) we get the expanded set ob_1 (pay the fee and then buy one month later), $o\neg b_1$ (pay the fee, but then not buy), $\neg ob_0$ and $\neg o\neg b_0$ (same as before, deciding to buy today or not without paying any fee). This set can be further expanded if we introduce options with different time horizons. Once again we note that is the explicit separation of time as a relevant decision dimension that allows to expand the set of alternatives.
3. In valued focussed thinking Keeney suggests to consider principally the values behind any decision questioning instead fixing the set of alternatives. In the vacation example we can relax the "one week" constraint allowing getting more interesting offers (for instance two weeks packages could be more valuable than the one weeks ones, although relatively more expensive). However, we can do more than that. After all, why celebrating 10 years of common life should be done through a holiday? What about buying 10 concert tickets or booking 10 famous restaurants or 10 tickets for recent Broadway productions? Keeney's suggestion to distinguish between core objectives (celebrating) and mean objectives (buy a holiday) allows identifying dimensions with which we can compose more alternatives from the ones initially considered. An approach more likely to generate satisfying alternatives to assess.

Let's make a first summary of what we knew about the generating algorithms problem.

Claim 1 *From a decision aiding process perspective (implying some time extension), generating further sets of alternatives is related to some non satisfactory assessment of the present set of alternatives.*

Claim 2 *Generating unknown alternatives is always related to some expansion (or more generally revision) of the separable attributes describing the existing set.*

Let's focus on claim 2 and see what happens in a combinatorial optimisation case.

Example 8. Consider a client formulating a problem where a city (organised in n districts) should be covered by shops belonging to the client's brand, under the hypothesis that a shop opened in a certain district "covers" also the adjacent ones. The client asks to do the minimum necessary.

This is a well known location problem formulated as follows:

$$\min \sum_j x_j$$
$$\text{st}$$
$$\mathcal{D}\mathbf{x} \geq \mathbf{1}$$
$$x_j \in \{0, 1\}$$

where $j = 1 \cdots n$ are the districts;
x_j are binary variables representing the opening in a certain district;
$\mathcal{D}$ is the adjacency matrix;
the meaning of the set of contraints being to satisfy covering the whole city.
Once the problem solved, the client realises that the minimum openings necessary to cover the whole city cannot be inferior of k (the minimum value of $\sum_j x_j$). At this point he realises that this goes beyond his budget capacity. How the problem formulation should evolve? A new version of the problem will be the following one:

$$\max \sum_j w_j y_j$$
$$\text{st}$$
$$\mathcal{D}\mathbf{x} \geq \mathbf{y}$$
$$\sum_j c_j x_j \leq C$$
$$x_j, y_j \in \{0, 1\}$$

where $j = 1 \cdots n$ are the districts;
x_j are binary variables representing the opening in a certain district;
y_j are binary variables representing the covering of a certain district;
$\mathcal{D}$ is the adjacency matrix;
w_j representing the importance of each district;
and c_j representing the cost of each opening, C being the available budget;
the meaning of the set of contraints being to satisfy the logical relations between opening and covering as well as the budget availability provided by the client.
The reader should note that the problem could also be formulated as a bi-objective optimisation one:

$$\max \sum_j w_j y_j$$
$$\min \sum_j c_j x_j$$
$$\text{st}$$
$$\mathcal{D}\mathbf{x} \geq \mathbf{y}$$
$$x_j, y_j \in \{0, 1\}$$

Discussion For the time the problem being formulated under the constraint of covering the whole city, the covering dimension characterising potential alternatives is not separable (since all covering variables are implicitly equal to 1). The set A is established considering only combinations of the variables x_j. The unsatisfactory result obliges us to expand this set using the covering variables (since now we allow some of these to be 0: some districts might not be covered). To put it on a formal basis, using our general decision problem framework the decision aiding process will be described as follows:

1. The starting set A_1 is defined by all combinations of the variables x_j (openings).
2. The constraints $\mathcal{A}\mathbf{x} \geq \mathbf{1}$ defines a rating decision problem resulting to a new set A_2 to be used in the next step.
3. The objective function $\min \sum_j x_j$ defines a ranking decision problem resulting to a minimum of k openings. This information qualifies the whole set A_2 as unsatisfactory since k openings are practically impossible (but we only discover it at this stage of the process).
4. A_2 being unsatisfactory we backtrack to the initial set A_1 and we create a new starting set, let's call it B_1 as combinations of all opening and covering variables. This is possible relaxing the constraint obliging to cover the whole city, resulting in making the covering variables separable (relevant for the client's decisions).
5. The constraints $\mathcal{A}\mathbf{x} \geq \mathbf{y}$ and $\sum_j c_j x_j \leq C$ establish a new rating decision problem resulting to a new feasible set B_2.
6. The objective function $\max \sum_j w_j y_j$ establishes a new ranking problem which hopefully will provide a satisfactory solution to the client.

Can we generalise what we described until now? Yes! Let's go back to the procedure used in order to prove Proposition 1. Introducing at each step a generalised rating decision problem (is the resulting set A_i satisfying?) we are able to control the process of generating subsequent As. Further on we need to add two more possible actions (remember that A is always described by separable attributes):
- backtrack at any point of the recursion and open a new branch;
- revise the set of separable variables describing the set A in order to generate alternatives not considered until this moment (unknown alternatives).
What do we get?

Claim 3 *Generating unknown alternatives is possible allowing within the recursion constructing A two actions: backtracking and revising the set of separable variables.*

6 Discussion

What we are presenting here are not necessarily completely new ideas, although we are sure that they have never been discussed as in this paper and combined under our perspective.

Expanding the set of variables describing a set of objects is borrowed from C-K theory ([12], [13]). This is the only formal theory of design we are aware of and is very powerful although essentially simple. The theory addresses the problem of designing

new "objects" (products or services) identifying two spaces:
- the knowledge one, where objects are completely described on finite set of known attributes (a house, a car ...);
- the concept one, where objects are only partially known, the list of attributes describing them being only partially defined;
The design process is then described as a sequence of variables transformation between the two spaces allowing the exchange of attributes between knowledge and concepts such that "new objects" can appear: a house which is also a car; a camping car.

We are firmly convinced that there are many more important links between C-K theory and our suggestion about the process of constructing alternatives. These links are yet to be explored.

Algorithms controlling the execution of algorithms and allowing intelligent backtracking are as old as TMS (see [5]) and are regularly used in planning and automated reasoning devices ([23]). We can certainly see our alternatives generation procedure under such a perspective although the information conducting the process is provided on-line (during the decision aiding process) and not as an input (as it happens is most of the existing literature, for an exception see [24]).

The whole idea of revising the conclusion of a process as a result of new information is central in Non Monotonic Reasoning (NMR) formalisms ([10], [17]). In [28] it has already been suggested a relation between model revision in NMR and preference modelling. Our idea about generating alternatives shows several relations to this literature:
- a decision aiding process is naturally subject to updates (new information becoming available and existing information becoming obsolete and/or inconsistent) and revisions (of values, opinions and scenarios) two notions central in the study of NMR (see [8]);
- expanding the set of conclusions derivable from given knowledge, adding defeasible reasoning is a suitable logical framework for our suggestion about the alternatives generation process starting from a partially described decision space. Since this space is only partially known we can proceed to multiple expansions which could (and actually are) defeasible, as soon as the client assess their satisfiability.
It is less obvious to us how the dimension of "creative" construction of alternatives can be considered within this framework, but this is only a special case of the more general problem; in most cases the dimensions which could be added, revised or updated are already implicitly considered in the problem formulation, but not yet explicitly considered due to the separability condition.

Concluding we are not afraid to state that is likely that other relations exist between our proposal and other artificial intelligence areas including argumentation theory, learning and knowledge discovery. But these are yet to be explored.

7 Conclusions

The paper presents a problem often neglected and/or underestimated in decision analysis: how the set of alternatives on which a decision support method/algorithm applies is constructed. Our effort to discuss this topic is part of a long term project aiming at establishing a characterisation of decision problems independent from methods and only relying on simple primitives, the set A of alternatives being one of these ones.

In the paper we have been able to show two results. The first consists in showing that the construction of A is itself a decision problem (allowing a recursion of decision problems) and thus, that it can be studied within our general framework. The second consists in showing that the crucial problem in constructing A is the generation of "unknown" alternatives, when the set presently available is considered to be unsatisfactory. Under such a perspective we have been able to show that generating such alternatives is practically possible through backtracking the recursion which generated the present set A and expanding/revising the set of separable dimensions describing the set.

We concluded showing that our research is strongly related to existing fields of research in design theory and artificial intelligence. Given the low interest of this topic in the mainstream literature, it is not surprising that most of these links are yet to be explored. We hope our contribution may motivate more efforts in this promising (for us) direction.

References

1. B.W Baetz, E.I Pas, and A.W Neebe. Generating alternative solutions for dynamic programming-based planning problems. *Socio-Economic Planning Sciences*, 24:27 – 34, 1990.
2. M.O Ball. Heuristics based on mathematical programming. *Surveys in Operations Research and Management Science*, 16:21–38, 2011.
3. S. Chakhar and V. Mousseau. Generation of spatial alternatives based on a planar subdivision of the study area. In *Proceedings of the 2006 ACM/IEEE International Conference on Signal-Image Technology and Internet-Based Systems, Hammamet, Tunisie*, pages 78–87, 2006.
4. A. Colorni and A. Tsoukiàs. What is a decision problem? preliminary statements. In *Proceedings of ADT 2013, LNAI 8176*, pages 139 – 153. Springer Verlag, Berlin, 2013.
5. J. Doyle. A truth maintenance system. *Artificial Intelligence*, 12:231–272, 1979.
6. P.H. Farquhar and A.R. Pratkanis. Decision structuring with phantom alternatives. *Management Science*, 39(10):1214–1226, October 1993.
7. S. French. *Decision theory - An introduction to the mathematics of rationality*. Ellis Horwood, Chichester, 1988.
8. P. Gärdenfors. *Knowledge in flux*. MIT Press, Cambridge, 1988.
9. T. Gilovich, D. Griffin, and D. Kahneman. *Heuristics and Biases: The Psychology of Intuitive Judgment*. Cambridge University Press, Cambrighe, 2002.
10. M.L Ginsberg. *Readings in nonmonotonic reasoning*. Morgan Kaufmann, San Francisco, 1987.
11. A. Guimarães Pereira, G. Munda, and M. Paruccini. Generating alternatives for siting retail and service facilities using genetic algorithms and multiple criteria decision techniques. *Journal of Retailing and Consumer Services*, 1:40 – 47, 1994.
12. A. Hatchuel. Towards design theory and expandable rationality: The unfinished program of Herbert Simon. *Journal of Management and Governance*, 5:260–273, 2001.

13. A. Hatchuel and B. Weil. C-K design theory: An advanced formulation. *Research in Engineering Design*, 19:181 – 192, 2009.
14. R.L. Keeney. *Value-Focused Thinking. A Path to Creative Decision Making*. Harvard University Press, Cambridge, 1992.
15. R.L Keeney. Creativity in decision-making with value-focused thinking. *Sloan Management Review*, 35(4):33–41, 1994.
16. O.G. Leon. Valued-focused thinking versus alternative-focused thinking: Effects on generation of objectives. *Organizational Behavior and Human Decision Processes*, 80(3):213–227, December 1999.
17. D. Makinson. Bridges between classical and nonmonotonic logic. *Logic Journal of IGPL*, 11:69 – 96, 2003.
18. S.E. Newstead, V.A. Thompson, and S.J. Handley. Generating alternatives: A key component in human reasoning? *Memory and Cognition*, 30:129–137, 2002.
19. M.F. Norese and A. Ostanello. Identification and development of alternatives: introduction to the recognition of process typologies. In A.G. Lockett and G. Islei, editors, *Improving Decision Making in Organisations*, LNEMS 335, pages 112–123. Springer-Verlag, Berlin, 1989.
20. V.M. Ozernoy. Generating alternatives in multiple criteria decision making problems: a survey. In Haimes Y.Y. Chankong, V., editor, *Decision making with multiple objectives: proceedings of the sixth international conference on multiple-criteria decision making*, pages 322–330. Springer-Verlag, Berlin, 1985.
21. M. Oztürk, A. Tsoukiàs, and Ph. Vincke. Preference modelling. In J. Figueira, S. Greco, and M. Ehrgott, editors, *Multiple Criteria Decision Analysis: State of the Art Surveys*, pages 27–72. Springer Verlag, Boston, Dordrecht, London, 2005.
22. J. Pearl. *Heuristics: Intelligent Search Strategies for Computer Problem Solving*. Addison-Wesley, New York, 1984.
23. J.L Pollock. OSCAR - a general purpose defeasible reasoner. *Journal of Applied Non-Classical Logics*, 6:89 – 113, 1996.
24. J.L Pollock. Against optimality: Logical foundations for decision-theoretic planning in autonomous agents. *Computational Intelligence*, 22:1 – 25, 2006.
25. P. Rivett. *The craft of decision modelling*. J. Wiley, New York, 1994.
26. M. Roubens and Ph. Vincke. *Preference Modeling*. LNEMS 250, Springer Verlag, Berlin, 1985.
27. H. T. J. Smit and L. Trigeorgis. *Strategic Investment: Real Options and Games*. Princeton University Press, Princeton, New Jersey, 2004.
28. A. Tsoukiàs. Preference modelling as a reasoning process: a new way to face uncertainty in multiple criteria decision support systems. *European Journal of Operational Research*, 55:309–318, 1991.
29. A. Tsoukiàs. On the concept of decision aiding process. *Annals of Operations Research*, 154:3 – 27, 2007.
30. A. Tversky. Elimination by aspects: A theory of choice. *Psychological Review*, 79:281–299, 1972.