

RUBIS : a bipolar-valued outranking method for the choice method

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2nd Decision Deck workshop, Paris, February 22, 2007

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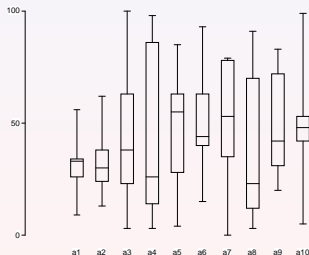
Introductory example

Decision problem: *Choose the best from a set of ten alternatives evaluated on 7 criteria as shown below.*

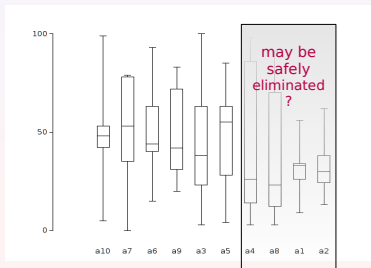
criterion	weight	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
g_1	7	33	13	3	14	48	44	18	47	31	98
g_2	7	9	30	23	86	63	40	79	3	83	48
g_3	5	34	38	63	16	85	53	78	91	47	42
g_4	5	53	24	38	3	28	93	35	12	72	5
g_5	5	26	44	60	98	62	15	53	23	37	44
g_6	4	26	29	100	36	4	63	54	70	24	53
g_7	1	56	62	33	36	21	49	0	13	20	99

- The performance scale on each criteria is 0 – 100 pts, with a weak preference threshold of 10 points, a preference threshold of 20 pts, and a veto threshold of 80 pts.
- We assume that the criteria are not commensurable.

Introductory example: Boxplots of the performances



Introductory example: Boxplots of the performances



Introductory example: Ranking the performances?

Criterion	a10	a7	a6	a9	a3	a5	a4	a8	a1	a2
"g1"	98	18	44	31	3	48	14	47	33	13
"g2"	48	79	40	83	23	63	86	3	9	30
"g3"	42	78	53	47	63	85	16	91	34	38
"g4"	5	35	93	72	38	28	3	12	53	24
"g5"	44	53	15	37	60	62	98	23	26	44
"g6"	53	54	63	24	100	4	36	70	26	29
"g7"	99	0	49	20	33	21	36	13	56	62

Introductory example: Pairwise comparisons

Is a_{10} globally at least as good as a_7 ?

Outranking thresholds: weak preference (≥ 10), preference (≥ 20), veto (≤ -80).

criterion	w_i	a_{10}	a_7	$\Delta_i(10,7)$	balance	veto ?
g_1	7	98	18	80	+7	no
g_2	7	48	79	-31	-7	no
g_3	5	42	78	-36	-5	no
g_4	5	5	35	-30	-5	no
g_5	5	44	53	-9	+5	no
g_6	4	53	54	-1	+4	no
g_7	1	99	0	99	+1	no

total balance 0

We observe a **balanced** situation.
No conclusion can be drawn.

Introductory example: Pairwise comparisons (continued)

Is a_7 globally at least as good as a_{10} ?

Outranking thresholds: weak preference (≥ 10), preference (≥ 20), veto (≤ -80).

criterion	w_i	a_7	a_{10}	$\Delta_i(10,7)$	balance	veto ?
g_1	7	18	98	-80	-7	yes
g_2	7	79	48	+31	+7	no
g_3	5	78	42	+36	+5	no
g_4	5	35	5	+30	+5	no
g_5	5	53	44	+9	+5	no
g_6	4	54	53	+1	+4	no
g_7	1	0	99	-99	-1	yes

total balance +18-34

We observe a **veto** situation on criteria g_1 and g_7 .
 a_7 is **clearly not** globally at least as good as a_{10} ? !

Is a_{10} (resp. a_6) globally at least as good as a_6 (resp. a_{10}) ?

g_i	w_i	a_{10}	a_6	$\Delta_i(10,6)$	balance	veto?	$\Delta_i(6,10)$	balance	veto?
g_1	7	98	44	54	+7	no	-54	-7	no
g_2	7	48	40	8	+7	no	-8	+7	no
g_3	5	42	53	-11	-5	no	11	+5	no
g_4	5	5	93	-88	-5	yes	88	+5	no
g_5	5	44	15	29	+5	no	-29	-5	no
g_6	4	53	63	-10	0	no	10	+4	no
g_7	1	99	49	50	+1	no	-50	-1	no
				total balance	+10	-34	total balance	+8	

- a_{10} is **clearly not** globally at least as good as a_6 (veto (-88) on criterion g_4)!
- Note the **weak preference** situation on criterion g_6 !
- a_6 is globally at least as good as a_{10} (balance of +8 in favour).

Is a_6 globally at least as good as a_7 ?

criteria	weight	a_7	a_6	$\Delta_i(7,6)$	balance	veto ?
g_1	7	44	18	26	+7	no
g_2	7	40	79	-39	-7	no
g_3	5	53	78	-25	-5	no
g_4	5	93	35	58	+5	no
g_5	5	15	53	-38	-5	no
g_6	4	63	54	9	+4	no
g_7	1	49	0	49	+1	no
				total balance	0	

We observe again a **balanced** situation.
No conclusion can be drawn.

Introductory example: Global outranking relation

\tilde{S}	a_{10}	a_7	a_6	a_9	a_3	a_5	a_4	a_8	a_1	a_2
a_{10}	-	0	-34	10	1	2	10	20	24	29
a_7	-34	-	8	15	24	18	22	10	20	32
a_6	8	0	-	10	11	0	-34	24	29	23
a_9	10	11	7	-	10	7	19	9	32	32
a_3	-34	8	2	-4	-	-4	3	10	13	25
a_5	10	19	14	2	-34	-	1	26	14	24
a_4	-34	10	-34	7	6	0	-	2	10	12
a_8	-34	0	-34	-34	-10	5	-34	-	22	3
a_1	-9	-8	-10	5	-1	-7	6	9	-	15
a_2	-34	-3	-10	3	6	-9	10	2	10	-

Content

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○Backbone of RUBIS : \tilde{S}

- Let X be a finite set of p alternatives.
- Let N be a finite set of $n > 1$ criteria.
- Let m be the total significance of the criteria.
- Let x and y be two alternatives from X .
- Let x_i be the value taken by x on criterion g_i

Definition (The outranking situation)

- x outranks y ($x \tilde{S} y$) if there is a significant majority of criteria which support an **at least as good** statement and there is **no** criterion which raises a **veto** against it.
- The bipolar valued relation $\tilde{S} \in [-m, m]$ expresses the credibility of the **validation** or the **non-validation** of the outranking relation S .

Definition (The bipolar valued outranking situation)

$$\tilde{S}(x, y) = \min \left\{ \left(\sum_{i \in N} w_i \cdot C_i(x, y) \right), \min_{i \in N} (-V_i(x, y)) \cdot m \right\}$$

$$C_i(x, y) = \begin{cases} 1 & \text{if } x_i + q_i > y_i; \\ -1 & \text{if } x_i + p_i \leq y_i \\ 0 & \text{otherwise} \end{cases}$$

$$-V_i(x, y) = \begin{cases} 1 & \text{if } x_i + wv_i > y_i; \\ -1 & \text{if } x_i + v_i \leq y_i \\ 0 & \text{otherwise} \end{cases}$$

where q_i, p_i represent the **weak preference**, resp. the **preference**, and wv_i, v_i , the **weak veto**, resp. the **veto**, threshold on criterion g_i .

\tilde{S} is defined on a bipolar-valued credibility scale $\mathcal{L} = [-m, m]$ supporting the following demantics denotation:

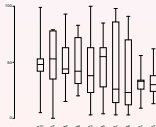
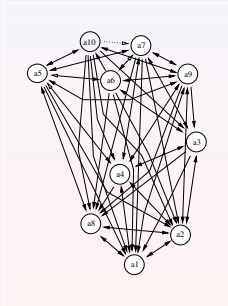
- $\tilde{S}(x, y) = +m$ means that assertion $x S y$ is **clearly validated**.
- $\tilde{S}(x, y) = -m$ means that assertion $x S y$ is **clearly non-validated**.
- $\tilde{S}(x, y) > 0$ means that assertion $x S y$ is **more validated than non-validated**.
- $\tilde{S}(x, y) < 0$ means that assertion $x S y$ is **more non-validated than validated**.
- $\tilde{S}(x, y) = 0$ means that assertion $x S y$ is **undetermined**.

Backbone of RUBIS : $\tilde{G}(X, \tilde{S})$

Definition (The bipolar valued outranking digraph)

- We denote $\tilde{G}(X, \tilde{S})$ the **bipolar-valued outranking digraph** modelled via \tilde{S} on $X \times X$.
- The associated crisp outranking relation S may be recovered from \tilde{S} as the set of pairs (x, y) such that $\tilde{S} > 0$.
- $\tilde{G}(X, \tilde{S})$ is called the **crisp** outranking digraph associated with $\tilde{G}(X, \tilde{S})$.

Introductory example: The crisp outranking digraph



RUBIS decision aiding approach

- A choice problem traditionally consists in the search for a **single best** alternative.
- We adopt a **progressive** decision analysis process which allows to uncover the best single choice via possible **intermediate recommendations**.
- These intermediate choice recommendations, the case given, have to be **refined at some further stages** of the decision analysis.

Pragmatic choice recommendation (CR) principles

- \mathcal{P}_1 : Non-retainment for **well motivated reasons**.
all eliminated alternative must be considered worse as at least one recommended alternative.
- \mathcal{P}_2 : **Minimal** size.
the CR should be as limited as possible.
- \mathcal{P}_3 : **Efficient** and **informative**.
each CR must deliver a stable recommendation.
- \mathcal{P}_4 : Effectively **better**.
the CR should not correspond simultaneously to a choice and an elimination recommendation.
- \mathcal{P}_5 : **Maximally** credible.
the CR must be as credible as possible wrt the preferential knowledge modelled via \tilde{S} .

Useful choice qualifications in $\tilde{G}(X, \tilde{S})$

Let Y be a non-empty subset of X , called a **choice** in \tilde{G} .

- Y is said to be **outranking** (resp. **outranked**) iff $x \notin X \Rightarrow \exists y \in Y : \tilde{S}(x, y) > 0$.
- Y is said to be **independent** iff for all $x \neq y$ in Y we have $X \tilde{S}(x, y) \leq 0$.
- Y is called an **outranking kernel** (resp. **outranked kernel**) iff it is an outranking (resp. outranked) and independent choice.
- Y is called an outranking **hyperkernel** (resp. outranked hyperkernel) iff it is an outranking (resp. outranked) choice which consists of **independent chordless circuits** of odd order $p \geq 1$.

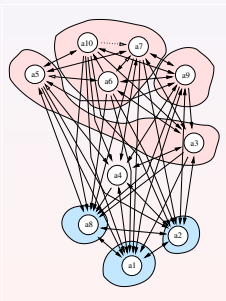
Translating CR principles into choice qualifications

- \mathcal{P}_1 : Non-retainment for well motivated reasons.
A CR is an **outranking choice**.
- \mathcal{P}_{2+3} : Minimal size & stable.
A CR is a **hyperkernel**.
- \mathcal{P}_4 : Effectivity.
A CR is a **strictly more outranking than outranked** choice.
- \mathcal{P}_5 : Maximal credibility.
A CR has **maximal determinateness**.

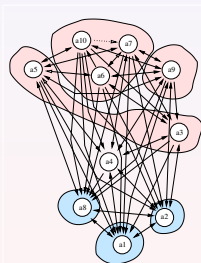
Theorem

Any bipolar outranking digraph contains at least one outranking and one outranked hyperkernel.

Introductory example: All outranking and outranked hyperkernels



Introductory example: all kernels and hyperkernels



outranking choices:

$\{a_{10}, a_7, a_6\}$

$\{a_9\}$

$\{a_3, a_5\}$

outranked choices:

$\{a_8\}$

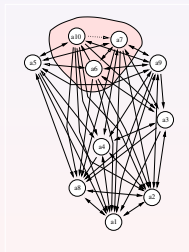
$\{a_2\}$

$\{a_1\}$

The RUBIS choice recommendation (RCR)

- A RCR **verifies** the five CR principles.
- A **maximally determined strict outranking hyperkernel**, if it exists in \bar{G} , gives a RCR.
- A RCR is a **provisional** subset of alternatives, most certainly containing the best alternative, if it exists !.
- A RCR must not be confused with the ultimate best choice of the decision maker.
- The **RUBIS** choice method is only convenient in a progressive decision aiding approach.

Introductory example: The RUBIS choice recommendation



choice : $\{a_{10}, a_7, a_6\}$

(chordless 3-circuit!)

determinateness : 72%

(weighted majority of criterion)

irredundancy : 100%

independence : 100%

outrankingness : 72%

outrankedness : 38%

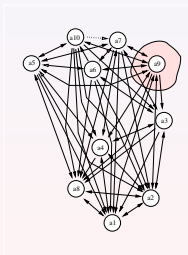
characteristic vector = [

$\{a_{10}, a_7, a_6\}$: 72%, a_1 : 28%,

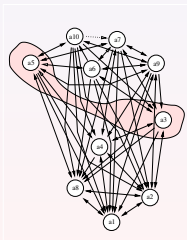
a_2 : 28%, a_3 : 28%, a_4 : 28%,

a_5 : 28%, a_6 : 28%, a_7 : 28%, a_8 :

28%, a_9 : 28%, a_{10} : 28%]



choice : $\{a_9\}$
 determinateness : 60%
 (weighted majority of criterion)
 irredundancy : 100%
 independence : 100%
 outrankingness : 60%
 outrankedness : 0%
 characteristic vector = [a_9 :
 60%, a_1 : 40%, a_2 : 40%, a_3 :
 40%, a_4 : 40%, a_5 : 40%, a_6 :
 40%, a_7 : 40%, a_8 : 40%, a_{10} :
 40%, $\{a_{10}, a_7, a_6\}$: 40%]



choice : $\{a_3, a_5\}$
 determinateness : 53%
 (weighted majority of criterion)
 irredundancy : 65%
 independence : 56%
 outrankingness : 53%
 outrankedness : 48.5%
 characteristic vector = [
 a_3 : 53%, a_5 : 53%, a_1 : 47%,
 a_2 : 47%, a_4 : 47%, a_6 : 47%,
 a_7 : 47%, a_8 : 47%, a_{10} : 47%,
 $\{a_{10}, a_7, a_6\}$: 47%]

Concluding remarks

Properties of the RUBIS choice recommendation:

- Progressiveness:** intermediate solutions are proposed to the decision maker;
- Existence:** A RCR always exists in a non-symmetrical bipolar-valued outranking digraph;
- Multiplicity:** In case multiple RCR coexist, their union gives a suitable intermediate choice recommendation;
- Missing values:** They are treated as information which is not available at a given stage of the decision analysis; which might be determined later on;
- Efficient decision aiding:** Strongly motivated conclusions can nevertheless be drawn.

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