UTA^{GMS}/GRIP plugin to the Decision Deck platform

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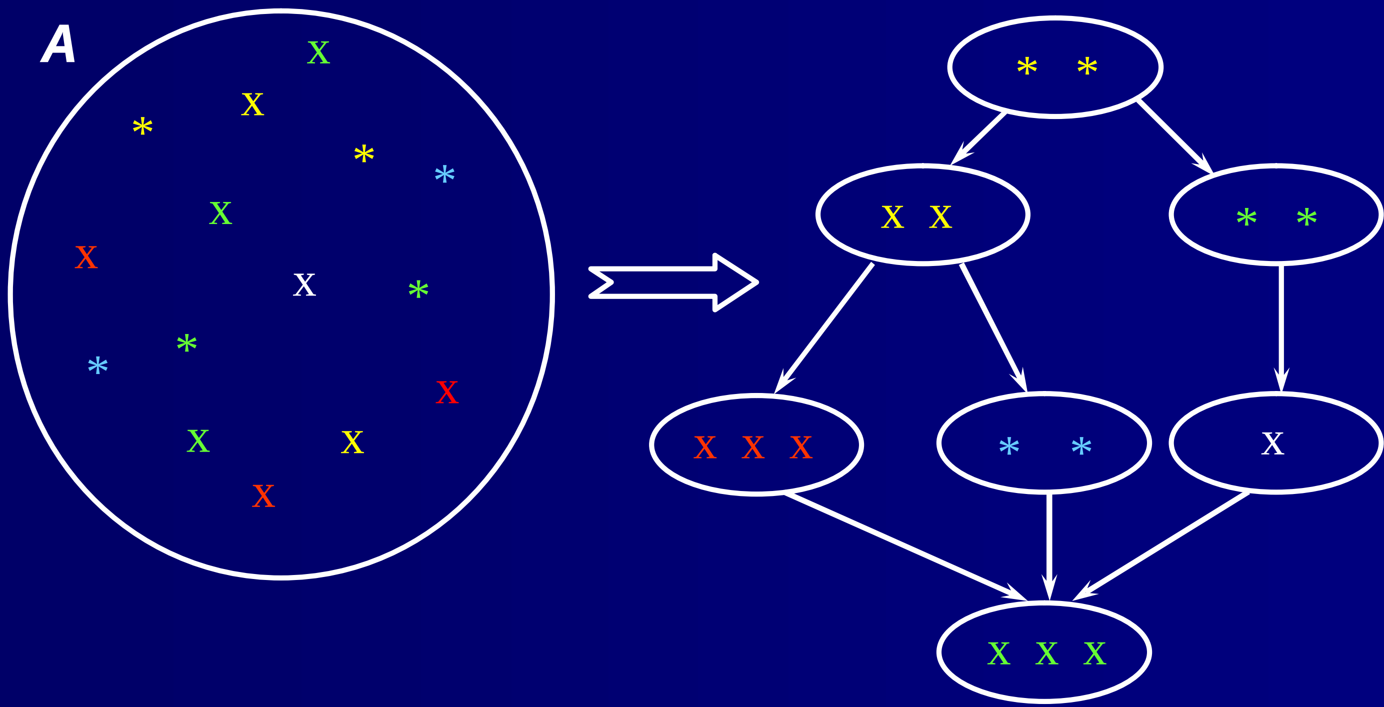
Problem statement

- Consider a finite set *A* of alternatives (actions) evaluated by *m* criteria from a consistent family $F = \{g_1,...,g_m\}$
- Taking into account preferences of a Decision Maker (DM), rank all the actions of set *A* from the best to the worst



Preference model

- To solve a multicriteria decision problem one needs a preference model, i.e. criteria aggregation model

- Traditional aggregation paradigm:

  The preference model is first constructed and then applied on set *A* to get information about the comprehensive preference

- Disaggregation-aggregation (ordinal regression) paradigm:

  The comprehensive preference on a subset $A^R \subseteq A$ is known a priori, and a consistent preference model is inferred from this information to be applied on set *A*

Disaggregation-aggregation (regression) approach

The preference model is

a set of additive utility functions

compatible with a non-complete set

of pairwise comparisons of some reference actions

and information about comprehensive and partial

intensities of preference

- The additive utility function is defined on *A* as follows:

$$U(x) = \Sigma u_i(x_i), \quad i \in \mathrm{I} = \{1, ..., m\}$$

The UTA^{GMS} method (Greco, Mousseau & Słowiński 2004)

- $B^R \subseteq A^R$ x A^R is the set of pairs of reference actions compared by the DM
- The preference information is a partial preorder $\underline{\mathbf{f}}$ on a subset of reference actions $A^R \subseteq A$
- $\underline{\mathbf{f}}$ – weak preference (outranking) relation

  for each pair $(x, y) \in B^R$

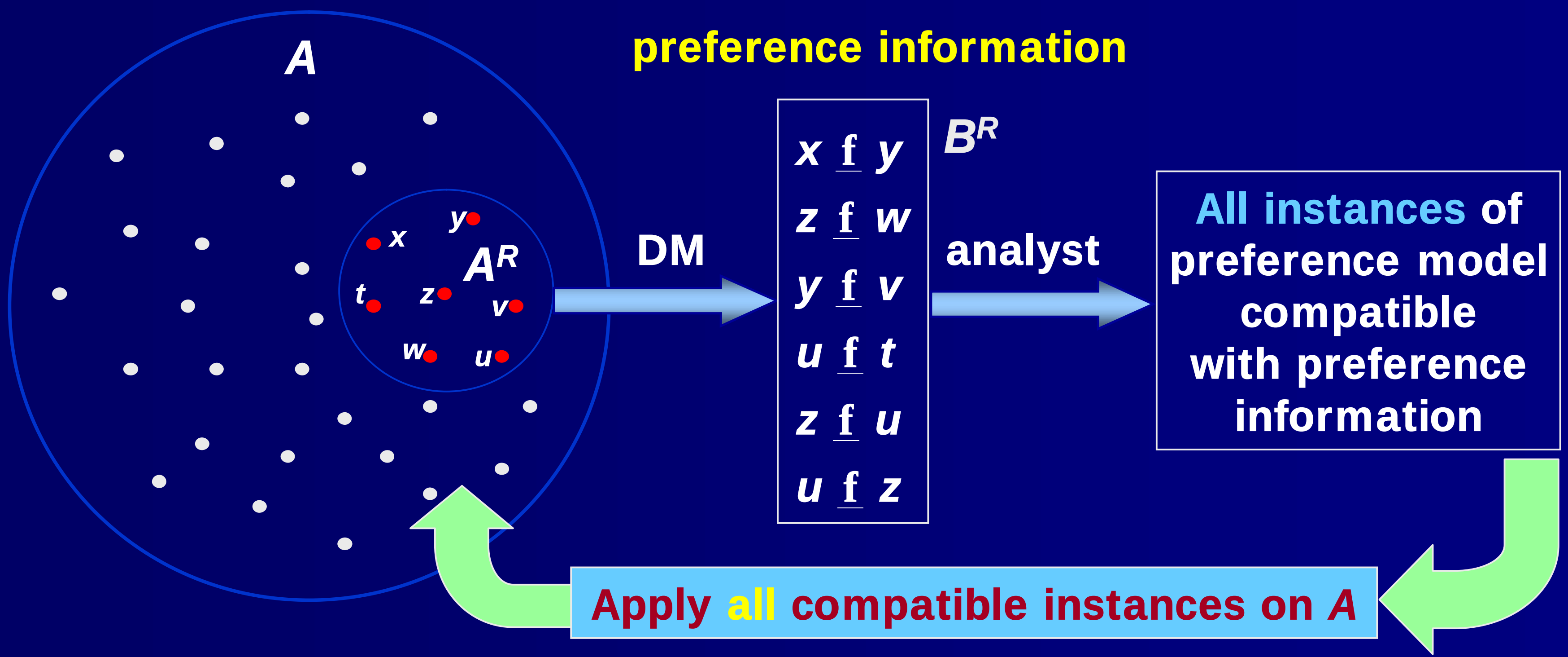
  $x \ \underline{\mathbf{f}} \ y \Leftrightarrow$ „x is at least as good as y"

  $x \ \mathbf{f} \ y \equiv [x \ \underline{\mathbf{f}} \ y$ and not $y \ \underline{\mathbf{f}} \ x] \Leftrightarrow$ „x is preferred to y"

  $x \sim y \equiv [x \ \underline{\mathbf{f}} \ y$ and $y \ \underline{\mathbf{f}} \ x] \Leftrightarrow$ „x is indifferent to y"

- A utility function is called compatible if it is able to restore all pairwise comparisons from B^R (i.e. partial preorder) on A^R

The UTA^{GMS} method (Greco, Mousseau & Słowiński 2004)



- Questions:
  - Are any two actions *x*, *y* ∈ *A* ordered in the same way by all compatible utility functions?
  - Is there at least one compatible utility function ordering *x* at least as good as *y* (or *y* at least as good as *x*)?

The UTA^{GMS} method (Greco, Mousseau & Słowiński 2004)

- Having answers to these questions for all pair of actions $(x, y) \in A \times A$, one gets:
  - necessary weak preference relation $\succsim^N$, whose semantics is $U(x) \geq U(y)$ <u>for all</u> compatible utility functions
  - possible weak preference relation $\succsim^P$, whose semantics is $U(x) \geq U(y)$ <u>for at least one</u> compatible utility function

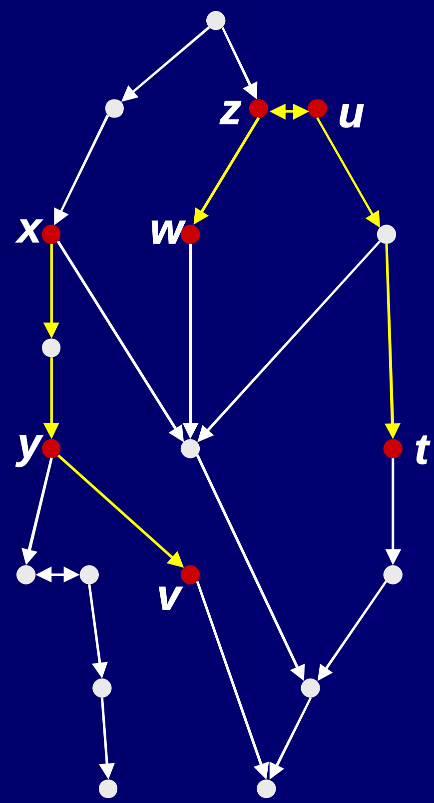
- The necessary and possible weak preference relations are exploited such that one finally obtains two rankings in the set of actions:
  - necessary ranking (partial preorder)
  - possible ranking (complete and negatively transitive binary relation)

The UTA^{GMS} method (Greco, Mousseau & Słowiński 2004)

- Two rankings result: necessary and possible

preference information

x f *y*
z f *w*
y f *v*
u f *t*
z f *u*
u f *z*



necessary ranking

Includes

necessary ranking

and

does not include

the complement of

necessary ranking

possible ranking

The UTA^{GMS} method (Greco, Mousseau & Słowiński 2004)

- For any pair of actions $(x, y) \in A$, and for available preference information represented by B^R, preference of x over y is determined by compatible utility functions U verifying set $E(x, y)$ of constraints:

$$\left.\begin{array}{l} \left.\begin{array}{l} U'(x) \geq U'(y) + \varepsilon \Leftrightarrow x \ \mathbf{f} \ y \\ U'(x) = U'(y) \Leftrightarrow x \sim y \end{array}\right\} \ \forall (x, y) \in B^R \\ u_i(x_i^j) - u_i(x_i^{j-1}) \geq 0, \ \ i = 1, \ldots, m, \quad j = 1, \ldots, \omega + 1 \\ u_i(x_i^0) = 0, \ \ i = 1, \ldots, m \\ \Sigma u_i(x_i^{\omega+1}) = 1, \ \ i = 1, \ldots, m \end{array}\right\} E(x, y)$$

where ε is a small positive constant, and $\omega = m + 2 - |A^R \cap \{x, y\}|$

The UTA^{GMS} method (Greco, Mousseau & Słowiński 2004)

- Given a pair of actions $x, y \in A$

$$x \succsim^{N} y \iff d(x, y) \geq 0$$

where

$$d(x, y) = \min_{\text{s.t. } E(x, y)} \{U(x) - U(y)\}$$

- $d(x, y) \geq 0$ means that for all compatible utility functions x is at least as good as y
- For any $(x, y) \in B^R$:

$$x \succsim y \Rightarrow x \succsim^{N} y$$

The UTA^{GMS} method (Greco, Mousseau & Słowiński 2004)

- Given a pair of actions $x, y \in A$

$$x \succsim^{P} y \;\Leftrightarrow\; D(x, y) \geq 0$$

where

$$D(x, y) = \underset{\text{s.t. } E(x,\, y)}{\text{Max}} \{U(x) - U(y)\}$$

- $d(x, y) \geq 0$ means that for at least one compatible utility functions x is at least as good as y
- For any $(x, y) \in B^R$:

$$x \succsim y \Rightarrow x \succsim^{P} y$$

The GRIP method (Figueira, Greco & Słowiński 2006)

- GRIP (*Generalized Regression with Intensities of Preference*) extends UTA^{GMS} method by adopting all features of UTA^{GMS} and by taking into account additional preference information:
  - comprehensive comparisons of intensities of preference between some pairs of reference actions, e.g. „*x* is preferred to *y* at least as much as *w* is preferred to *z*"
  - partial comparisons of intensities of preference between some pairs of reference actions on particular criteria, e.g. „*x* is preferred to *y* at least as much as *w* is preferred to *z*, on criterion $g_i \in F$"

The GRIP method (Figueira, Greco & Słowiński 2006)

- DM is supposed to provide the following preference information:
  - a partial preorder $\underline{\mathbf{f}}$ on A^R, such that $\forall x, y \in A^R$

    $x \ \underline{\mathbf{f}}\ y \Leftrightarrow$ „x is at least as good as y"

    $\mathbf{f} = \underline{\mathbf{f}} \cap non\ \underline{\mathbf{f}}^{-1}, \quad \sim\ = \underline{\mathbf{f}} \cap \underline{\mathbf{f}}^{-1}$
  - a partial preorder $\underline{\mathbf{f}}^*$ on $A^R \times A^R$, such that $\forall x, y, w, z \in A^R$

    $(x, y)\ \underline{\mathbf{f}}^*\ (w, z) \Leftrightarrow$ „x is preferred to y at least as much as w is preferred to z"

    $\mathbf{f}^* = \underline{\mathbf{f}}^* \cap non\ \underline{\mathbf{f}}^{*-1}, \quad \sim^* = \underline{\mathbf{f}}^* \cap \underline{\mathbf{f}}^{*-1}$
  - a partial preorder $\underline{\mathbf{f}}_i^*$ on $A^R \times A^R$, $i = 1, \ldots, m$, such that $\forall x, y, w, z \in A^R$

    $(x, y)\ \underline{\mathbf{f}}_i^*\ (w, z) \Leftrightarrow$ „x is preferred to y at least as much as w is preferred to z, on criterion $g_i \in F$"

    $\mathbf{f}_i^* = \underline{\mathbf{f}}_i^* \cap non\ \underline{\mathbf{f}}_i^{*-1}, \quad \sim_i^* = \underline{\mathbf{f}}_i^* \cap \underline{\mathbf{f}}_i^{*-1}$

The GRIP method (Figueira, Greco & Słowiński 2006)

- A utility function U is called compatible if it satisfies the constraints corresponding to DM's preference information:

  a) $U(x) \geq U(y)$ iff $x \; \underline{\mathbf{f}} \; y$

  b) $U(x) > U(y)$ iff $x \; \mathbf{f} \; y$

  c) $U(x) = U(y)$ iff $x \sim y$

  d) $U(x) - U(y) \geq U(w) - U(z)$ iff $(x, y) \; \underline{\mathbf{f}}^* \; (w, z)$

  e) $U(x) - U(y) > U(w) - U(z)$ iff $(x, y) \; \mathbf{f}^* \; (w, z)$

  f) $U(x) - U(y) = U(w) - U(z)$ iff $(x, y) \sim^* (w, z)$

  g) $u_i(x) \geq u_i(y)$ iff $x \; \underline{\mathbf{f}}_i \; y$, $i \in I$

  h) $u_i(x) - u_i(y) \geq u_i(w) - u_i(z)$ iff $(x, y) \; \underline{\mathbf{f}}_i^* \; (w, z)$, $i \in I$

  i) $u_i(x) - u_i(y) > u_i(w) - u_i(z)$ iff $(x, y) \; \mathbf{f}_i^* \; (w, z)$, $i \in I$

  j) $u_i(x) - u_i(y) = u_i(w) - u_i(z)$ iff $(x, y) \sim_i^* (w, z)$, $i \in I$

The GRIP method (Figueira, Greco & Słowiński 2006)

- Moreover, the following normalization constraints should also be taken into account:

  k) $u_i(x_i^*) = 0,\ i \in I$

  where x_i^* is such that $x_i^* = \min\{g_i(x): x \in A\}$

  l) $\Sigma\, u_i(y_i^*) = 1,\ i \in I$

  where y_i^* is such that $y_i^* = \max\{g_i(y): x \in A\}$

- Let as remark that like in UTA^{GMS} method, constraints b), e) and i) should be written as:

  b') $U(x) \geq U(y) + \varepsilon$

  e') $U(x) - U(y) \geq U(w) - U(z) + \varepsilon$

  i') $u_i(x) - u_i(y) \geq u_i(w) - u_i(z) + \varepsilon$

  where ε is a small positive constant

The GRIP method (Figueira, Greco & Słowiński 2006)

- If constraints *a) – l)* are consistent, then we get two weak preference relations $\succsim^N$ and $\succsim^P$, and two binary relations comparing intensity of preference $\succsim^{*N}$ and $\succsim^{*P}$:

  1. for all $x, y \in A$, a necessary weak preference relation

     $$x \succsim^N y \Leftrightarrow \min \{U(x) - U(y)\} \geq 0$$

  2. for all $x, y \in A$, a possible weak preference relation

     $$x \succsim^P y \Leftrightarrow \max \{U(x) - U(y)\} \geq 0$$

  3. for all $x, y, w, z \in A$, a necessary relation of preference intensity

     $$(x, y) \succsim^{*N} (w, z) \Leftrightarrow \min \{[U(x) - U(y)] - [U(w) - U(z)]\} \geq 0$$

  4. for all $x, y, w, z \in A$, a possible relation of preference intensity

     $$(x, y) \succsim^{*P} (w, z) \Leftrightarrow \max \{[U(x) - U(y)] - [U(w) - U(z)]\} \geq 0$$

where „min” and „max” are calculated over all utility functions satisfying *a) – l)*

The GRIP method (Figueira, Greco & Słowiński 2006)

- In order to conclude the truth or falsity of necessary and possible weak preference relations $\underline{\mathbf{f}}^N$, $\underline{\mathbf{f}}^P$ and $\underline{\mathbf{f}}^{*N}$, $\underline{\mathbf{f}}^{*P}$, one can use LP

- To obtain the result which is independent on the value of ε, one should:

$$\text{Max} \rightarrow \varepsilon$$

  subject to constraints a) – l), with b), e), i) written as b'), e'), i')

- If maximal $\varepsilon^* > 0$, the set of compatible utility functions is not empty

The GRIP method (Figueira, Greco & Słowiński 2006)

- Then, to verify the truth or falsity of $x \succsim^{P} y$, for any $x, y \in A$, one should:

$$\text{Max} \rightarrow \varepsilon$$

subject to constraints *a) – l)*, with *b)*, *e)*, *i)* written as *b')*, *e')*, *i')*

and $U(x) \geq U(y)$

- Maximal $\varepsilon^* > 0 \Leftrightarrow x \succsim^{P} y$

  This means that there exists at least one compatible utility function satisfying the hypothesis $U(x) \geq U(y)$

The GRIP method (Figueira, Greco & Słowiński 2006)

- In order to verify the truth or falsity of $x \succsim^N y$, rather than to check directly that for each compatible utility function $U(x) \geq U(y)$, we make sure that among the compatible utility functions there is no one such that $U(x) < U(y)$:

$$\text{Max} \rightarrow \varepsilon$$

subject to constraints *a) – l)*, with *b)*, *e)*, *i)* written as *b')*, *e')*, *i')*

and $U(y) \geq U(x) + \varepsilon$

- Maximal $\varepsilon^* \leq 0 \Leftrightarrow x \succsim^N y$

The GRIP method (Figueira, Greco & Słowiński 2006)

- Analogously, in order to verify the truth or falsity of $(x, y) \underline{\mathbf{f}}^{*P} (w, z)$ for any $x, y, w, z \in A$, one should:

$$\text{Max} \rightarrow \varepsilon$$

subject to constraints *a) – l)*, with *b)*, *e)*, *i)* written as *b')*, *e')*, *i')*

and $U(x) - U(y) \geq U(w) - U(z)$

- Maximal $\varepsilon^* > 0 \Leftrightarrow (x, y) \underline{\mathbf{f}}^{*P} (w, z)$

The GRIP method (Figueira, Greco & Słowiński 2006)

- Analogously, in order to verify the truth or falsity of $(x, y) \succsim^{*N} (w, z)$ for any $x, y, w, z \in A$, one should:

$$\text{Max} \rightarrow \varepsilon$$

subject to constraints *a) – l)*, with *b)*, *e)*, *i)* written as *b')*, *e')*, *i')*

and $U(w) - U(z) \geq U(x) - U(y) + \varepsilon$

- Maximal $\varepsilon^* \leq 0 \Leftrightarrow (x, y) \succsim^{*P} (w, z)$
- The value of ε^* is not meaningful – the result does not depend on it

UTA^{GMS}/GRIP plugin overview

- Current implementation of UTA^{GMS}/GRIP plugin works on the first version of Decision Deck platform (1.0.2)
- To verify the truth or falsity of preference relations it uses GLKP linear solver which is the part of D2 platform (GLPK plugin)
- To visualize rankings of alternatives in the form of graph it uses the JGraph library implemented as additional plugin
- UTA^{GMS}/GRIP plugin main features:
  - add/remove alternatives to/from reference set
  - add/remove/edit preference information (partial preorder, comprehensive and/or partial intensities of preferences)
  - shows comparison of alternatives
  - view necessary ranking of alternatives

UTA^{GMS}/GRIP plugin demonstration

- Illustrative example

  Car ranking problem

x_1 : Skoda Octavia Wagon1;

x_2 : Opel Astra 1.9d SW;

x_3 : Ford Mondeo SW;

x_4 : CitroenC5 2.0 HDi:

x_5 : Seat Toledo 2.0 TDi;

x_6 : VW Golf 4motion.

g_1: Price [min] $(Euros)$;

g_2: Speed [max] (Km/h);

g_3: Space [max] (m^3);

g_4: Fuel consumption [min] $(liters/100Km)$;

g_5: Acceleration [min] (seconds needed to reach the $100Km/h$).

UTA[GMS]/GRIP plugin demonstration

- Illustrative example – Car ranking problem

  Alternatives:

  x_1 : Skoda Octavia Wagon1;

  x_2 : Opel Astra 1.9d SW;

  x_3 : Ford Mondeo SW;

  x_4 : CitroenC5 2.0 HDi:

  x_5 : Seat Toledo 2.0 TDi;

  x_6 : VW Golf 4motion.

  Criteria:

  g_1: Price [min] ($Euros$);

  g_2: Speed [max] (Km/h);

  g_3: Space [max] (m^3);

  g_4: Fuel consumption [min] ($liters/100Km$);

  g_5: Acceleration [min] (seconds needed to reach the $100Km/h$).

UTA^{GMS}/GRIP plugin demonstration

Performance matrix:

		Price	Speed	Space	Fuel_cons.	Acceleration
		$g_1(x_k)$	$g_2(x_k)$	$g_3(x_k)$	$g_4(x_k)$	$g_5(x_k)$
Skoda	x_1	17 351	188	8,0889	7,4	12,4
Opel	x_2	21 061	190	8,0908	5,9	10,8
Ford	x_3	22 201	200	8,6880	7,8	11,2
Citroen	x_4	25 401	200	8,6152	6,0	10,0
Seat	x_5	24 071	201	7,8942	5,8	10,0
VW	x_6	22 679	185	7,3920	5,8	12,1

Conclusions and future works

- The preference information used in GRIP does not need to be complete: the DM can compare only those pairs of reference alternatives on particular criteria for which his/her judgment is sufficiently certain
- Distinguishing necessary and possible consequences of preference information, GRIP answers questions of robustness analysis using all utility functions instead of a single „best-fit” utility function
- Plugin future works:
  - visualization of possible ranking of alternatives
  - resolving inconsistency in preference information
  - visualization of necessary and possible relations of preference intensity for the pair of alternatives
  - manage preference information using „classes of attractiveness”