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Research Article

GIS-based multicriteria spatial modeling generic framework

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Multicriteria analysis is a set of mathematical tools and methods allowing the comparison of different alternatives according to many criteria, often conflicting, to guide the decision maker towards a judicious choice. Multicriteria methods are used in spatial context to evaluate and compare spatial decision alternatives, often modeled through constraint-based suitability analysis and represented by point, line, and polygon features or their combination, and evaluated on several space-related criteria, to select a restricted subset for implementation. Outranking methods, a family of multicriteria methods, may be useful in spatial decision problems, especially when ordinal evaluation criteria are implied. However, it is recognized that these methods, except those devoted to multicriteria classification problems, are subject to computational limitations with respect to the number of alternatives. This paper proposes a framework to facilitate the incorporation and use of outranking methods in geographical information systems (GIS). The framework is composed of two phases. The first phase allows producing a planar subdivision of the study area obtained by combining a set of criteria maps; each represents a particular vision of the decision problem. The result is a set of non-overlapping spatial units. The second phase allows constructing decision alternatives by combining the spatial units. Point, line and polygon feature-based decision alternatives are then constructed as an individual, a grouping of linearly adjacent or a grouping of contiguous spatial units. This permits us to reduce considerably the number of alternatives, enabling the use of outranking methods. The framework is illustrated through the development of a prototype and through a step-by-step application to a corridor identification problem. This paper includes also a discussion of some conceptual and technical issues related to the framework.

Keywords: GIS; Spatial multicriteria modeling; Outranking methods; Spatial decision alternatives

1. Introduction

Multicriteria analysis has experienced very successful applications in different domains. Since the early 1990s, multicriteria analysis has been coupled with geographical information systems (GIS) for an enhanced spatial multicriteria decision making (*see* Malczewski (2006) for a survey of the literature). Multicriteria methods are often categorized into discrete or continuous, depending on the set of alternatives. In this paper we focus on the first category. There are two main families of methods within discrete category: utility function-based methods and outranking

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relation-based methods. Most multicriteria based-GIS tools use utility function-based methods (*see*, e.g. Dai *et al.* 2001, Araujo and Mecedo 2002). These methods still dominate today and only a few works (*see*, e.g. Joerin *et al.* 2001, Pereira 2001, Martin *et al.* 2003, Marinoni 2005, 2006) use methods based on outranking relations. However, outranking methods may be useful in spatial decision problems since they:

- permit us to consider qualitative evaluation criteria for which preference interval ratios have no sense,
- permit us to consider evaluation criteria with heterogenous scales,
- limit the compensation between evaluation criteria, and
- require, in general, limited information from the decision maker.

The three first points are cited by Roy and Bouyssou (1993). The last one is by Malczewski (1999). Joerin *et al.* (2001) add that outranking methods are more appropriate to territory management problems in which decision alternatives are often incomparable because they are fundamentally different. But the major drawback (from the practical and technical points of view) of outranking methods – except those devoted to multicriteria classification problems since they consider only intrinsic aspects of alternatives to assign them to a set of pre-defined categories – is that they are unsuitable to problems implying a high number of decision alternatives. Indeed, several authors (*see*, e.g. Eastman *et al.* 1993, Pereira and Duckstein 1993, Jankowski 1995, Tkach and Simonovic 1997, Joerin *et al.* 2001, Marinoni 2005, 2006) argue that outranking methods may face difficulties since they have serious computational limitations in respect to the number of decision alternatives (Marinoni 2006). It is, however, fruitful to mention that the application of outranking methods as such is not really put into question. The problem of computational limitations results from the fact that these methods require a pairwise comparison across all the decision alternatives. This explains why, in comparaison with utility-based methods, outranking methods have received little attention in the GIS and multicriteria analysis integration works.

This paper proposes a framework to facilitate the incorporation and use of outranking methods in GIS. The framework is organized in two phases. The first one allows producing a *decision map* (Note: the term 'decision map' has been used in the past in other contexts. To avoid ambiguity, a clarification will be given at the end of this section). The decision map concept, proposed in Chakhar (2006) (*see* also Chakhar *et al.* 2005), is a planar subdivision of the study area obtained by combining a set of criteria maps; each represents a particular vision of the decision problem. The result is a set of non-overlapping spatial units. As the spatial units result from the overlay of different criteria maps, each spatial unit is characterized with several partial evaluations – one for each criterion map. These spatial units are then assigned, using a multicriteria classification method, into different pre-defined categories.

The second phase permits us to generate decision alternatives by combining the spatial units. Indeed, using the concept of decision map, point, line and polygon (or area) feature-based decision alternatives, conventionally used in spatial multicriteria decision making, can be constructed as an individual, a grouping of linearly adjacent or a grouping of contiguous spatial units. This permits us to reduce considerably the number of alternatives and enables the use of outranking methods. The framework is illustrated through the development of a prototype and its application to a corridor identification problem using real data relative to Ile-de-France (Paris and its suburbs) region in France.

The description of territory as a set of homogenous zones is not new in territory-related problems (Joerin *et al.* 2001). In conventional cartographic modeling and when only social criteria are implied, these zones may be defined through census tracts or by political/administrative boundaries (*see*, e.g. Can 1992). However, when other such criteria as environmental ones are involved, the spatial units can be defined by a natural reserve, a watershed, a viewshed, a landscape classification, etc. Further, we think, as Joerin *et al.* (2001), that the subdivision of territory into homogenous zones should take into account and respect the natural (e.g. forest, river) or artificial (e.g. highways, parks) boundaries. We also think that it is important to incorporate in the definition of homogenous zones the economic, social, financial, historic, environmental, etc., aspects of the decision problem. This is well handled in our decision map concept and in all the approaches based on the use of multicriteria analysis.

There are some other similar proposals in the literature. Hall *et al.* (1992) and Wang (1994) make use of the overlay procedure to define homogenous zones and estimate the convenience level to different agriculture types. Hall *et al.* (1992) have explored crisp and fuzzy classification techniques. They conclude that classification based on fuzzy logic is more appropriate since the boundaries of the zones cannot be defined crisply. Wang (1994) has used artificial neural networks-based classification. These two works are different from our proposal in the sense that the main objective of Hall *et al.* (1992) and Wang (1994) is not to apply a multicriteria method, which we think is one important limitation to these proposals.

The work reported in Joerin *et al.* (2001) is conducted to facilitate the incorporation of outranking methods in GIS. Joerin *et al.* (2001) use a homogeneity index permitting us to compute the similarity between each element of the map (i.e. raster cell) and the average characteristics of the zone to subdivide the study area into homogenous zones. To compute the similarity degree, Joerin *et al.* (2001) apply a rough set-based function proposed by Slowinski and Stefanowski (1994). The generated zones represent the potential alternatives which are then classified, by ELECTRE TRI (*see* Figueira *et al.* 2005b), into three categories: favorable, uncertain and unfavorable. Joerin *et al.* (2001) have identified two limitations to their work: (i) the obtained maps are very sensitive to the spatial division used; and (ii) since the number of zones is limited, the description of the territory is rather imprecise, leading to a substantial loss of information. However, Joerin *et al.* (2001) remark that this dilemma may be 'solved' when a homogenous zone is seen both as a part of the physical space and as a particular solution to the problem. We fully share this last point of view, which still works in the proposed framework.

Aerts and Heuvelink (2002) affirm that all multicriteria methods (discrete as well as continuous) are subject to the computational limitations problem. To overcome this problem, Aerts and Heuvelink (2002) use the simulated annealing technique to reduce the number of alternatives to be evaluated. They have applied their approach to a mining zone restoration problem in the region of Galicia (Spain). We think that the work of Aerts and Heuvelink (2002) and also those of Hall *et al.* (1992) and Wang (1994), presented earlier, do not take into account the multicriteria aspects of the decision problem at the time of the subdivision of the study area into homogenous zones, which is an important distinction from our approach.

Marinoni (2005) discusses the integration of the PROMETHEE (*see* Figueira *et al.* 2005a) method into a GIS, which is, according to Marinoni (2005), the most attractive outranking method and that is due to its mathematical simplicity and its transparency for the decision makers. In Marinoni (2005), the decision alternatives take the form of regular or irregular zones of raster cells. Two versions (a standard version and a stochastic version) of PROMETHEE have been incorporated into ArcGIS and used to select a parcel for habitat construction.

Marinoni (2006) discusses first the problem of computational limitations of outranking methods. Then, Marinoni (2006) proposes an iterative procedure where location alternatives are defined as a spatial raster aggregation based on geometric constraints, and where raster cells are aggregated within a geometrically defined neighborhood. By comparing the obtained evaluation to an existing evaluation reached through the application of the AHP method of Saaty (1980), Marinoni (2006) concludes that outranking methods behave rather well for problems with a high number of decision alternatives.

In comparison to the proposals of Joerin *et al.* (2001) and Marinoni (2005, 2006), the proposed framework has two main merits: (i) in both works, the decision alternatives are directly assimilated to the homogenous zones. In our proposal, alternatives may be constructed as groupings of several spatial units; and (ii) the frameworks of Joerin *et al.* (2001) and Marinoni (2005, 2006) apply essentially to location and territory management problems. In particular, Joerin *et al.* (2001) and Marinoni (2005, 2006) do not propose solutions to generate composed alternatives or even linear ones. In our framework, different types of alternatives are considered atomic as well as composed ones. Furthermore, we think that the proposed framework is generic enough and it seems possible to situate in this framework all methodological propositions, combining GIS and multicriteria analysis. It covers raster and vector GIS and utility-based functions or outranking relation-based multicriteria analysis methods as well. There are still, however, some conceptual and technical aspects that should be improved. They will be discussed at the end of the present paper in Section 6.

As mentioned earlier, the term 'decision map' has been used in the past in other contexts by different authors such as Haimes *et al.* (1990), Lotov *et al.* (1997) and Jankowski *et al.* (1999). The following is a useful clarification. Haimes *et al.* (1990) employed the term 'decision map' to design the 'mathematical' decision space in multi-objective decision making. Lotov *et al.* (1997) employ the term 'interactive decision map' to design graphical representation of the decision space, enabling the decision maker(s) to visually appreciate how the feasibility frontiers and criteria trade-offs evolve when one or several decision parameters change. It is based on the use of modern interactive visualization techniques such as animation. The work of Jankowski *et al.* (1999) is an extension of Lotov *et al.* (1997)'s work to spatial context. Accordingly, Haimes *et al.* (1990)'s decision map concept, Lotov *et al.* (1997) and Jankowski *et al.* (1999)'s interactive decision map tool are, in terms of construction, different from the concept of decision map presented in this paper.

Andrienko and Andrienko (1999) and Jankowski *et al.* (2001) employ the term 'interactive map' to design a map-based spatial decision support system. In these works, the role of maps is enhanced to go beyond the mere display of geographic decision space and multicriteria evaluation results towards an effective decision support system by including 'visual indexes' through which the user orders decision options, assigns priorities to decision criteria, and augments the criterion outcome space by map-derived heuristic knowledge. Interactive map tool is further enhanced by recent technologies for exploratory spatial data analysis, data mining, and dynamic maps. Interactive map tool is different from our decision map concept in the sense that the objective of Andrienko and Andrienko (1999) and Jankowski *et al.* (2001) is to support interactive spatial decision making, which is different from our objective, i.e. construction of spatial decision alternatives. We, however, believe that interactivity undoubtedly adds value to our decision map by providing a more user-friendly communication support and permitting a better what-if analysis.

The rest of this paper is organized as follows. Section 2 introduces some basic concepts in spatial multicriteria modeling while Section 3 presents the concept of decision map. Section 4 details the proposed framework. Two phases are distinguished at this level. The first phase, detailed in Section 4.1, is devoted to construct a decision map. The second one, detailed in Section 4.2, seeks to use this decision map to construct spatial decision alternatives and to multicriteria evaluation. Section 5 illustrates the framework through a step-by-step application to a corridor identification problem. Section 6 discusses some conceptual and technical aspects of the framework. Section 7 concludes this paper and outlines some future research directions.

2. Spatial multicriteria modeling

2.1 *Multicriteria analysis*

In multicriteria analysis the decision maker has to choose among several options, called *alternatives*, on the basis of a set of, often conflicting, evaluation *criteria*. The multicriteria methods are categorized on the basis of the set of alternatives A into *discrete* and *continuous*. In this paper, we are concerned with the first category. Let $A=\{a_1, a_2, \ldots, a_n\}$ denote a set of n alternatives. The evaluation criteria are factors on which alternatives are evaluated and compared. A criterion is a function g, defined on A, taking its values in an ordered set, and representing the decision maker's preferences according to some points of view. The evaluation of an alternative a according to criterion g is written $g(a)$. Let $G=\{g_1, g_2, \ldots, g_m\}$ be the set of m evaluation criteria and $F=\{1, 2, \ldots, m\}$ be the set of their indices.

To compare alternatives in A, it is necessary to aggregate the partial evaluations (i.e. in respect to each criterion) into overall preferences by using a given *decision rule* (or *aggregation procedure*). There are two main families of decision rules within multicriteria discrete methods: (i) *utility function-based decision rules*; and (ii) *outranking relation-based decision rules*. The basic principle of the first family is that the decision maker looks to maximize a utility function $U(a)=U(g_1(a), g_2(a), \ldots, g_m(a))$ summarizing the different partial evaluations. The most simple and also most used utility function is the additive form: $U(a)=\Sigma_j u_j(g_j(a))$, where u_j ($j=1, \ldots, m$) are the marginal utility functions. With the second family, criteria are aggregated into a binary relation S, such that aSb means that 'a is at least as good as b'. The binary relation S is called *outranking relation*. There is a relatively large number of outranking methods in the literature (*see* Figueira *et al.* 2005a). They differ essentially in the way the outranking relation is constructed and exploited. More information concerning decision rules in multicriteria analysis is available in Chakhar and Mousseau (2007b).

2.2 *Spatial multicriteria decision making*

Spatial decision problems may be roughly defined as those problems in which the decision implies the selection among several potential alternatives that are associated with some specific locations in space. Examples of spatial problems include: facility location, health care planning, forest management, vehicle routing, administrative redistricting, etc. Spatial multicriteria decision making refers to the application of multicriteria analysis to deal with spatial decision problems. In the examples cited above, potential alternatives are characterized at least by their geographic positions and the selection of the appropriate one(s) will depend on the satisfaction of some space-dependent constraints and the 'optimization' of one or several space-related evaluation criteria. In the rest of this sub-section, we will focus only on the spatial dimensions of decision alternatives and evaluation criteria since they constitute the basic ingredients of any multicriteria decision problem. More details concerning spatial multicriteria decision making are provided in works of Malczewski (1999), Chakhar (2006) and Chakhar and Mousseau (2007b).

A spatial decision alternative consists of at least two elements (Malczewski 1999): *action* (what to do?) and *location* (where to do it?). The cardinality of the set of spatial decision alternatives is an important characteristic permitting us to distinguish if this set is discrete or continuous. In the first case, the problem involves a discrete set of (pre-defined) decision alternatives. Spatial alternatives are modeled through a constraint-based suitability analysis and represented by point, line, and polygon features or their combination. The second case corresponds to a high or infinite number of decision alternatives, often defined in terms of constraints. For practical reasons, the set of decision alternatives in this case is often represented in a discrete form where each raster cell represents an alternative. Alternatives may also be constructed by groupings of raster cells (e.g. clumps or zone).

The computational limitations of outranking methods are particularly clear with reaster cell-based alternatives. Indeed, with a high number of raster cells (and hence alternatives) say $n=200$, a utility function-based method requires us to perform n global evaluations while at least $n(n-1)=39{,}800$ pairwise comparisons are necessary with an outranking method.

Spatial evaluation criteria are associated with geographical entities and relationships between entities and therefore can be represented in the form of maps (Malczewski 1999). A criterion map is composed of a set of spatial units; each of which is characterized with one value relative to the concept modeled. Formally, a criterion map is defined as follows:

Definition 2.1: a *criterion map* $\mathbf{c_j}$ is the set $\{(s, g_j(s)) : s \in S_j\}$ where S_j is a set of spatial mapping units and g_j is a mono-valued criterion function defined as follows (E is a measurement scale):

$$\begin{array}{cccc} g_j & : & S_j & \rightarrow & E \\ & & s & \rightarrow & g_j(s) \end{array}$$

3. Concept of decision map

3.1 *Background*

We consider only simple polygon (or area), line and point features of $\mathbf{R}^2$. In the rest of this paper, the letters P, L, and Q are used to indicate point, line, and polygon features, defined as follows: (i) A polygon feature Q is a two-dimensional open point-set of $\mathbf{R}^2$ with simply connected interior Q° (i.e. with no hole) and simply connected boundary ∂Q; (ii) A line feature L is a closed connected one-dimensional point-set in $\mathbf{R}^2$ with no self-intersections and with two end-points. The boundary ∂L of L is a set containing its two end-points and its interior L° is the set of the other points; and (iii) A point feature P is a zero-dimensional set consisting of only one element of $\mathbf{R}^2$. The interior P° of a point feature P is the point itself and its boundary is empty (i.e. $\partial P = \emptyset$).

Below, the symbol γ may represent anyone of the three feature types. Figure 1 illustrates graphically these definitions.

There are several proposals for classifying topological spatial relationships (*see* Clementini and Felice (1995) for a comparative study of some classification methods). These classifications are based on the intersection of boundaries, interiors and exteriors of features. Clementini *et al.* (1993) introduced the calculus-based method (CBM) based on object calculus that takes into account the *dimension* of the intersections. They provided formal definitions of five relationships (which are: touch, in, cross, overlap, and disjoint) and for boundary operators. Clementini *et al.* (1993) also proved that these operators are mutually exclusive and constitute a full covering of all topological situations. In the following, we recall the definitions of the CBM. We mention that several other methods are available in the literature (*see*, e.g. Randell *et al.* 1992, Li and Ying 2004, Schneider and Behr 2005). The CBM method is used here for its simplicity and because it is sufficient to formalize the concept of decision map and the proposed solutions for the construction of spatial decision alternatives.

Definition 3.1: the touch relationship applies to all groups except point/point one:

$$(\gamma_1, \text{touch}, \gamma_2) \Leftrightarrow \left(r_1^\circ \cap \gamma_2^\circ = \emptyset\right) \Lambda (\gamma_1 \cap \gamma_2 \neq \emptyset)$$

Definition 3.2: the in relationship applies to every group:

$$(\gamma_1, \text{in}, \gamma_2) \Leftrightarrow (\gamma_1 \cap \gamma_2 = \gamma_1) \Lambda \left(\gamma_1^\circ \cap \gamma_2^\circ \neq \emptyset\right)$$

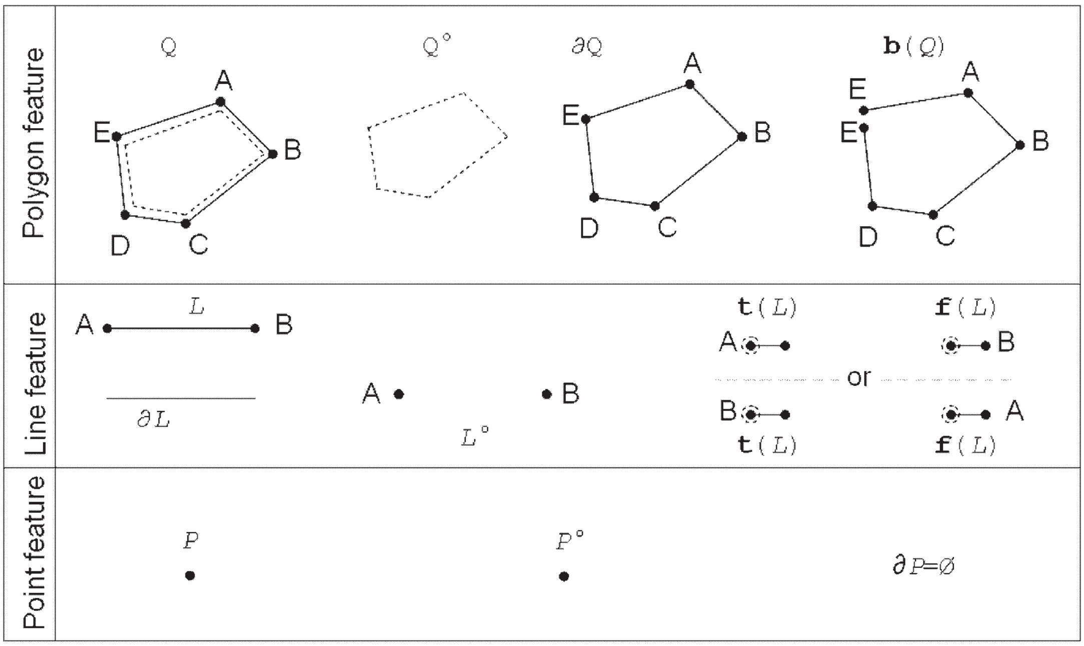


Figure 1. Polygon, line and point features in $\mathbf{R}^2$.

Definition 3.3: the cross relationship applies to line/line and line/polygon groups:

$$(L_1, \text{cross}, L_2) \Leftrightarrow (L_1 \cap L_2 \neq \varnothing) \Lambda (dim(L_1 \cap L_2) = 0)$$
$$(L, \text{cross}, Q) \Leftrightarrow (L \cap Q \neq \varnothing) \Lambda (L \cap Q \neq L)$$

Definition 3.4: the overlap relationship applies to polygon/polygon and line/line groups:

$$(\gamma_1, \text{overlap}, \gamma_2) \Leftrightarrow \left(dim\left(\gamma_1^\circ\right) = dim\left(\gamma_2^\circ\right) = dim\left(\gamma_1^\circ \cap \gamma_2^\circ\right)\right)$$
$$\Lambda(\gamma_1 \cap \gamma_2 \neq \gamma_1) \Lambda (\gamma_1 \cap \gamma_2 \neq \gamma_2)$$

Definition 3.5: the disjoint relationship applies to every group:

$$(\gamma_1, \text{disjoint}, \gamma_2) \Leftrightarrow (\gamma_1 \cap \gamma_2 = \varnothing))$$

To enhance the use of the above relationships, Clementini *et al.* (1993) defined operators permitting us to extract boundaries from polygon and line features. The boundary operator b for a polygon feature Q returns the circular line of ∂Q. The boundary operators f and t for a line feature return the two end-point features corresponding to the set ∂L. We notice that line features are just point-sets, and we do not consider an orientation on the line. Therefore, the two operators f, t are used symmetrically to avoid a distinction on which of the two end-points is called f and which is called t. Boundaries operators are also shown in figure 1.

3.2 *Definition of decision map concept*

Let Ψ be the study area and $\mathbf{G}=\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_m\}$ a set of m polygonal criteria maps defined on Ψ. Let U be a set of spatial units resulting from the overlay of the criteria maps in $\mathbf{G}$. Concretely, U is a planar subdivision of the study area Ψ composed of homogenous and disjoint spatial units resulting from the intersection of the boundaries of spatial objects in the criteria maps. Let E be an ordinal measurement scale whose echelons define a set of p categories $C_1, C_2, \ldots, C_p$. We call e_i the evaluation of category C_i on E. We suppose that $e_i \succ e_j$, $\forall i > j$ $(i, j=1, \ldots, p)$. Let $\Gamma_w : U \rightarrow E$ be a multicriteria classification method permitting to assign each spatial unit $u \in U$ to one and only one category on E. We write $\Gamma_w(u)=e_i$ to indicate that spatial unit u is assigned to category C_i. The subscript w is a set of preference parameters required to apply Γ. The map obtained by the application of Γ_w on U can be described mathematically by the set $\{(u, \Gamma_w(u)) : u \in U\}$. Formally, a decision map is defined as follows:

Definition 3.6: a *decision map* $\mathbf{M}$ is the set $\{(u, \Gamma_w(u)) : u \in U\}$, where U is a set of homogeneous spatial units and Γ_w is a multicriteria classification method defined as follows:

$$\begin{array}{rcccl} \Gamma_w & : & U & \rightarrow & E \\ & & u & \rightarrow & \Gamma_w[g_1(u), \cdots, g_m(u)] \end{array}$$

where $g_j(u)$ $(j=1, \ldots, m)$ is the evaluation (or performance) of spatial unit u in respect to criterion $g_j \in G$ associated with criterion map $\mathbf{c}_j \in \mathbf{G}$.

3.3 *Construction hypothesis*

To be useful, a decision map **M** should be composed of non-overlapping spatial units, which together constitute **M**. Suppose that each spatial unit $u_i \in U$ is uniquely identified by its subscript i. Let $I=\{1, 2, \ldots, r\}$ be the set of subscripts (i.e. identifiers) of all spatial units composing **M**. To ensure that the partition is total, the following conditions need to be verified:

$$\mathbf{M}=\bigcup_{i\in I} u_i, \quad \text{and} \quad u_i^{\circ} \cap u_j^{\circ}=\varnothing, \forall i, j\in I \wedge i\neq j.$$

Furthermore, we mention that criteria maps must be polygonal ones. However, input datasets may be sample data points, raster maps, contour lines, digital terrain models, etc. We need to transform all non-polygonal input datasets into polygonal ones. For example, a set of sample points may be transformed into a TIN by computing the triangulation having vertices at data points, a contours map may be transformed into a polygonal map by a constrained Delaunay triangulation, or a digital terrain model may be transformed into a thematic map through thematic slicing (*see*, e.g. Câmara *et al.* 1994, de Floriani *et al.* 1999, Chrisman 2002, for more details concerning map transformation operations). Criteria map conversion may lead to information loss problem. This point will be briefly discussed in Section 6.7.

4. Proposed framework

The proposed framework is composed of two phases. The first phase is devoted to the construction of a decision map. The second one looks to use this decision map to construct spatial decision alternatives and to multicriteria evaluation.

4.1 *Phase I: construction of a decision map*

This phase is composed of four steps:

(i) construction of criteria maps;
(ii) construction of an intermediate map;
(iii) multicriteria classification; and
(iv) grouping of adjacent spatial units having the same evaluation.

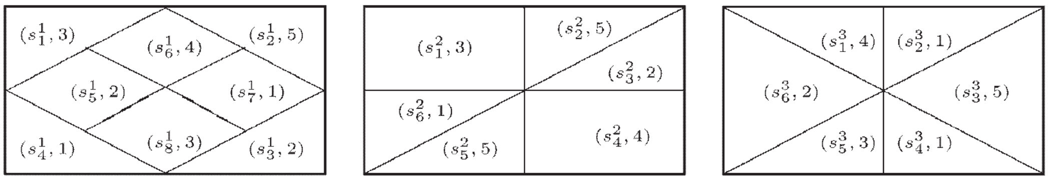
The last step is optional. For convenience and better illustration, a schematic representation of the different operations implied in this phase is depicted in figure 2. Clearly, maps shown in this figure are not realistic. They were deliberately simplified for better illustration.

4.1.1 Construction of criteria maps. The first step consists in the construction of the criteria maps. A criterion map $\mathbf{c_j}$ was defined earlier (*see* Section 2.2) as a set $\{(s, g_j(s)) : s \in S_j\}$ where S_j is a set of spatial units and g_j is the criterion function associated with $\mathbf{c_j}$.

Example 4.1 For instance, figure 2(a) contains three criteria maps $\mathbf{c_1}$, $\mathbf{c_2}$ and $\mathbf{c_3}$. All of them are evaluated on a cardinal scale of five levels. Criterion map $\mathbf{c_1}$, for instance, is composed of eight spatial units and can be represented by the set $\{(s_1^1, 3), (s_2^1, 5), (s_3^1, 2), (s_4^1, 1), (s_5^1, 2), (s_6^1, 4), (s_7^1, 1), (s_8^1, 3)\}$. Each spatial unit in this map is associated with only one value representing its performance in respect to

a

Measurement scale: $E: 1 \rightarrow 5$ | Measurement scale: $E: 1 \rightarrow 5$ | Measurement scale: $E: 1 \rightarrow 5$



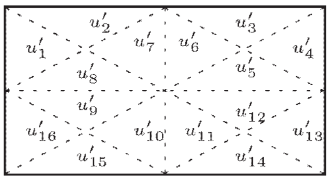
$\mathbf{c}_1$ $\mathbf{c}_2$ $\mathbf{c}_3$

Criteria maps

Step 1: *Construction of criteria maps.* Three criteria maps $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are first constructed. All of them are evaluated on a cardinal scale of five levels. Criterion map $\mathbf{c}_1$, for instance, is composed of eight spatial units and can be represented by the set $\{(s_1^1, 3), (s_2^1, 5), (s_3^1, 2), (s_4^1, 1), (s_5^1, 2), (s_6^1, 4), (s_7^1, 1), (s_8^1, 3)\}$. Each spatial unit in this map is associated with only one value representing its performance in respect to criterion g_1. For example, the performance of spatial unit s_2^1 is 5.

b

u'_i	$\mathbf{g}(u'_i)$
u'_1	(3, 3, 2)
u'_2	(3, 3, 4)
u'_3	(5, 5, 1)
u'_4	(5, 2, 5)
u'_5	(1, 2, 5)
u'_6	(4, 5, 1)
u'_7	(4, 3, 4)
u'_8	(2, 3, 2)



Intermediate map (**I**)

u'_i	$\mathbf{g}(u'_i)$
u'_9	(2, 1, 2)
u'_{10}	(3, 5, 3)
u'_{11}	(3, 4, 1)
u'_{12}	(1, 4, 5)
u'_{13}	(2, 4, 5)
u'_{14}	(2, 4, 1)
u'_{15}	(1, 5, 3)
u'_{16}	(1, 1, 2)

Step 2: *Construction of an intermediate map.* The intermediate map **I** resulting from the overlay of criteria maps $\mathbf{c}_1$, $\mathbf{c}_2$ and $\mathbf{c}_3$ is composed of 16 new spatial units: $\{(u'_1, (3,3,2)), (u'_2, (3,3,4)), \cdots, (u'_{16}, (1,1,2))\}$. For instance, spatial unit u'_7 is obtained by an intersection of spatial units s_6^1, s_2^1, and s_3^1, while spatial unit u'_{14} results from the intersection of spatial units s_3^1, s_4^2, and s_4^3. The performance vector of u'_7 is $\mathbf{g}(u'_7) = (4, 3, 4)$.

c

Γ_w: ELECTRE TRI

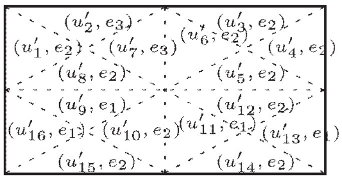
Profile limits:

	g_1	g_2	g_3
$\mathbf{g}(b_3)$	5	4	5
$\mathbf{g}(b_2)$	3	3	4
$\mathbf{g}(b_1)$	1	2	1

Thresholds:

$q_j = p_j = v_j = 0 \quad (j = 1, 2, 3)$

Cutting level: $\lambda = 0.85$



Decision map (**M'**) (*before grouping*)

Weights:

$k_1 = 0.75$
$k_2 = 0.5$
$k_3 = 0.5$

Measurement scale:
E: $e_1 \prec e_2 \prec e_3 \prec e_4$
$(\Gamma_w(u_i) = e_h \Leftrightarrow u_i \in C_h)$
Assignment rule:
$u_i \in C_h$ iif $u_i S b_{h-1}$
$\Leftrightarrow \sigma(u_i, b_{h-1}) \geq \lambda$

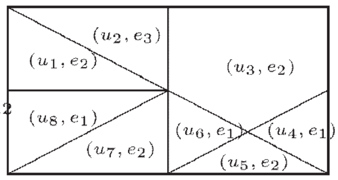
Step 3: *Multicriteria classification.* The map **M'** is obtained by the application of ELECTRE TRI method on the intermediate map **I**. Four categories C_1, C_2, C_3 and C_4 have been considered in this example. The performances of the three profile limits b_1, b_2 and b_3 of these categories are depicted on the left side of **M'**. For instance, the performance vector of b_2 is $\mathbf{g}(b_2) = (3, 3, 4)$. The weights associated with the different criteria maps are $k_1 = 0.75$, $k_2 = 0.5$ and $k_3 = 0.5$. The cutting level is $\lambda = 0.85$. The indifference, preference and veto thresholds are supposed to be equal to zero in this example. To be assigned to category C_3, a spatial unit u must have a performance vector $(g_1(u), g_2(u), g_3(u))$ that outranks the one of b_2 (i.e. (3,3,1)) with a credibility indice equal or greater to $\lambda = 0.85$, i.e., $\sigma(u, b_2) \geq 0.85$.

d

Aggregation operators:
Θ_1 : Arithmetic average
Θ_2 : Majority rule†
Θ_3 : Sum

Examples:
$g_1(u_3) = (5 + 5 + 1 + 4 + 1)/5 = 3.2$
$g_2(u_7) = 5$

† That is most frequent value.



Decision map (**M**) (*after grouping*)

Partial evaluations:

u_i	$g_1(u_i)$	$g_2(u_i)$	$g_3(u_i)$
u_1	2.5	3	4
u_2	3.5	3	8
u_3	3.2	2	17
u_4	5	2	5
u_5	1	2	5
u_6	3	5	1
u_7	2	5	6
u_8	1.5	1	4

Step 4: *Grouping of adjacent spatial units.* The final decision map **M** is obtained by applying algorithm `GROUPING` on map **M'** to group all adjacent spatial units that are assigned to the same category. For instance, spatial unit u_3 is obtained by grouping spatial units u'_3, u'_4, u'_5, u'_6 and u'_{12}. In turn, spatial unit u_5 is issued directly from spatial unit u'_{14} since the adjacent spatial units (u'_{11} and u'_{13}) have evaluations different from the one of u'_{14}. In addition, `GROUPING` should assign a new performance vector to the newly constructed spatial units using aggregation operators Θ_j $(j = 1, \cdots, m)$. For instance, the performance vector of spatial u_3 obtained by grouping spatial units u'_3, u'_4, u'_5, u'_6 and u'_{12} is (3.2,2,17). The number 3.2, for instance, corresponds to the evaluation of u_3 on g_1 obtained by applying the arithmetic average (Θ_1) on the evaluations of u'_3, u'_4, u'_5, u'_6 and u'_{12} on g_1: $g_1(u_3) = (5 + 5 + 1 + 4 + 1)/5 = 3.2$.

Figure 2. Schematic representation of decision map construction process.

criterion g_1. For example, the performance of spatial unit s_2^1 is 5 while the one of spatial unit s_7^1 is 1.

Geographical information systems technology is particularly useful to the construction of criteria maps (Cowen and Shirley 1991, Pereira and Duckstein 1993, Laaribi 2000). Criteria maps construction process is often represented through flowcharts where nodes correspond to geographic maps and arcs correspond to different map algebra operations. Details can be found in Malczewski (1999) and Chakhar (2006).

4.1.2 Construction of an intermediate map. The construction of a decision map needs first the superposition of a set of criteria maps. The result is an intermediate map composed of a new set of spatial units that result from the intersection of the boundaries of the features in the criteria maps:

$$\begin{array}{cclcl} \oplus & : & \mathbf{G} & \rightarrow & U \\ & & \mathbf{c}_1 \times \mathbf{c}_2 \times \cdots \times \mathbf{c}_m & \rightarrow & \vee_{i=1}^{m} S_i \end{array}$$

where $\oplus$ is the union variant of GIS overlay operation that yields a new map by combining all involved features in the input criteria maps $\mathbf{c}_1$, $\mathbf{c}_2$, …, $\mathbf{c}_m$; and $\vee$ returns the set of spatial units resulting from the intersection of the boundaries of the spatial objects contained in S_1, S_2, …, S_m. Recall that S_j (j=1, …, m) is the set of spatial units composing criterion map $\mathbf{c}_j$. The map obtained by the application of $\oplus$ on $\mathbf{G}=\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_m\}$ may be described by the set $\{(u, \mathbf{g}(u)) : u \in U\}$ with $\mathbf{g}(u)=(g_1(u), \ldots, g_m(u))$. This last vector represents the evaluations of spatial unit u in respect to evaluation criteria g_1, g_2, …, g_m associated with the criteria maps in $\mathbf{G}$. An intermediate map is then defined as follows:

Definition 4.2: an *intermediate map* $\mathbf{I}$ is the set $\{(u, \mathbf{g}(u)) : u \in U\}$ where $\mathbf{g}(u)=(g_1(u), g_2(u), \ldots, g_m(u))$. That is, a map where each spatial unit is associated with a vector of m evaluations relative to m evaluation criteria.

Example 4.3 The intermediate map in figure 2(b) obtained by the overlay of criteria maps $\mathbf{c_1}$, $\mathbf{c_2}$ and $\mathbf{c_3}$ is composed of 16 new spatial units: $\left\{\left(u_1', (3, 3, 2)\right), \left(u_2', (3, 3, 4)\right), \cdots, \left(u_{16}', (1, 1, 2)\right)\right\}$. For instance, spatial unit u_7' is obtained by an intersection of spatial units s_6^1, s_2^1 and s_3^1 while spatial unit u_{14}' results from the intersection of spatial units s_3^1, s_4^2 and s_4^3. The performance vector of u_7' is $\mathbf{g}\left(u_7'\right)=(4, 3, 4)$ and the one of u_{14}' is $\mathbf{g}\left(u_{14}'\right)=(2, 4, 1)$. The performance vectors of the other spatial units are shown in figure 2(b).

The overlay operation may generate *silver polygons* which should be removed. The problem of silver polygons will be discussed in Section 6.6.

4.1.3 Multicriteria classification. The first version $\mathbf{M}'$ of $\mathbf{M}$ is obtained by applying the multicriteria classification method Γ_ω to associate each spatial unit u in $\mathbf{I}$ to a category in E:

$$\begin{array}{cclcl} \mathbf{M}' & : & \mathbf{I} & \rightarrow & E \\ & & u & \rightarrow & \Gamma_\omega(u) \end{array}$$

The multicriteria classification method incorporated in the framework is ELECTRE TRI (Yu 1992, Figueira *et al.* 2005b). In ELECTRE TRI, the levels of the measurement scale E represent the evaluations of p ordered categories. These categories are defined in terms of $p-1$ profile limits. Let $B=\{b_1, b_2, \ldots, b_{p-1}\}$ be the set of the indices of these profiles, b_h is the higher profile limit of category C_h and the lowest profile limit of category C_{h+1}, $h=1, 2, \ldots, p$. Each profile b_h is characterized by its performances in respect to the evaluation criteria ($g_j(b_h)$; $j=1, \ldots, m$) and different preference parameters including indifference, preference, veto thresholds. An additional parameter, called cutting level, is also needed to apply ELECTRE TRI. An overview of ELECTRE TRI method including the definitions of these concepts is given in Appendix A.

Example 4.4 For illustration, figure 2(c) shows the result of the application of ELECTRE TRI method on the intermediate map of figure 2(b). Four categories C_1, C_2, C_3 and C_4 have been considered in this example. The performances of the three profile limits b_1, b_2 and b_3 of these categories are depicted in figure 2(c). For instance, the performance vector of b_2 is $\mathbf{g}(b_2)=(3, 3, 4)$. The weights associated with the different criteria maps are $k_1=0.75$, $k_2=0.5$ and $k_3=0.5$. The cutting level is $\lambda=0.85$. The indifference, preference and veto thresholds are supposed to be equal to zero in this example.

ELECTRE TRI has two assignment algorithms: pessimist and optimist. Algorithm ASSIGNMENT below corresponds to the pessimist version. The algorithm compares each spatial unit u to each of the profile limits starting from the highest one and assigns u to the first category for which u outranks its lower profile limit. This is implemented by the test 'SIGMA(g(u), g(b_h), w)$\geqslant\lambda$' in algorithm ASSIGNMENT. The boolean variable assigned is used in algorithm ASSIGNMENT to avoid unnecessary loops. The algorithm SIGMA, given in Appendix B, permits us to compute the credibility indexes $\sigma(u, b_h)$ measuring the level to which spatial unit u outranks the profile b_h:uSb_h. Recall that the parameter $\lambda \in [0.5, 1]$ used in this algorithm is the *cutting level* representing the lowest value for the credibility indexes $\sigma(u, b_h)$ to validate the outranking situation of u upon b_h. As far as this paper is concerned, it is sufficient to know that SIGMA returns $\sigma(u, b_h)$ as mentioned above and that the complexity of SIGMA is $O(m)$, where m is the number of evaluation criteria. The algorithm ASSIGNMENT runs in $O(r \times p \times m)$, where r is the number of spatial units in $\mathbf{I}$, p the number of profile limits and m the number of evaluation criteria.

```
Algorithm ASSIGNMENT
INPUT: I, w
OUTPUT: M′
BEGIN
FOR each u ∈ I
h←p
g(u)←(g_1(u), ..., g_m(u))
assigned←False
WHILE h≥0 and ¬(assigned)
g(b_h)←(g_1(b_h), ..., g_m(b_h))
IF SIGMA(g(u), g(b_h), w)≥λ THEN
Γ_ω(u)←e_{h+1}
assigned←True
END-IF
h←h−1
END–WHILE
END–FOR
END
```

Example 4.5 Consider again the map in figure 2(c). The assignment rule of ELECTRE TRI is depicted on the right side of this figure. For instance, to be assigned to category C_3, a spatial unit u must have a performance vector ($g_1(u)$, $g_2(u)$, $g_3(u)$) that outranks the one of b_2 (i.e. (3, 3, 1)) with a credibility indices equal or greater to λ=0.85, i.e. $\sigma(u, b_2) \geqslant 0.85$.

Notice that other multicriteria classification methods (e.g. PROAFTN (Belacel 2000), MHDIS (Zopounidis and Doumpos 2000), UTADIS (Doumpos and Zopounidis 2002)) or any kind of approach that produces homogeneous spatial units could be applied. A brief description of the assignment rules in some multicriteria classification methods is given in Appendix C.

4.1.4 Grouping of adjacent spatial units. First, we mention that this step is optional. Its objective is to obtain a final decision map in such a way that all adjacent spatial units (i.e. share at least one segment) are assigned to different categories. Indeed, the map $\mathbf{M}'$ obtained in the end of multicriteria classification step may contain adjacent spatial units that are assigned to the same category. These spatial units should be merged together to constitute a new spatial unit.

Example 4.6 For instance, spatial unit u_3 in figure 2(d) is obtained by grouping the spatial units u'_3, u'_4, u'_5, u'_6 and u'_{12} in figure 2(c). It is easy to see that these spatial units are adjacent and have the same evaluation (e_2). In turn, spatial unit u_5 in figure 2(d) is issued directly from spatial unit u'_{14} in figure 2(c) since the adjacent spatial units (u'_{11} and u'_{13}) have evaluations (e_1 for both) different from the one of u'_{14} (which is e_2).

The subsequent algorithm GROUPING, denoted Λ, permits us to group the spatial units having the same evaluations:

$$\begin{aligned} \Lambda \quad : \quad \mathbf{M}' \quad &\rightarrow \quad E \\ u \quad &\rightarrow \quad \Gamma_\omega(u) \end{aligned}$$

Algorithm GROUPING (Λ)
<u>INPUT</u>: $\mathbf{M}'$, Θ_j (j=1, ..., m)
<u>OUTPUT</u>: $\mathbf{M}$
<u>BEGIN</u>
$u \leftarrow u_1$
$Z \leftarrow \varnothing$
<u>WHILE</u> $\exists u \in \mathbf{I} \wedge u \notin Z$
<u>FOR</u> each $s \in v(u)$
<u>IF</u> $\Gamma_\omega(s) = \Gamma_\omega(u)$ <u>THEN</u>
$u \leftarrow$MERGE(u, s)
<u>END_IF</u>
<u>END_FOR</u>
$Z \leftarrow Z \cup \{u\}$
<u>END_WHILE</u>
<u>FOR</u> each $u \in \mathbf{M}$
<u>FOR</u> each $j \in G$
$g_j(u_i) = \Theta_j\left[g_j\left(u'_i\right)\right]_{u'_i \in u'_i}$
<u>END_FOR</u>
<u>END_FOR</u>
<u>END</u>

The symbol $v(u)$ in algorithm Λ represents the set of spatial units which are 'neighbors' to u. There are different ways to define the concept of 'neighbors'. Here, two spatial units u_i and u_j are considered as neighbors if and only if they share at least one segment: $(\partial u_i \cap \partial u_j)$=true. This model is known as the *model of Rook*. Other models of 'neighbors' may also be applied as the *Queen model* which considers also diagonal 'neighbors' or the *Knight model* which considers 'neighbors' of more than one level (*see*, e.g. Goodchild 1977, Xu and Lathrop 1995). The operator MERGE in Λ permits to combine two or several spatial units. The last 'FOR…END_FOR' clauses in Λ permit us to compute the partial evaluations of the newly grouped spatial units. To compute the partial evaluations, we need to specify a set of m aggregation operators Θ_1, Θ_2, …, Θ_m associated with the different evaluation criteria. For a spatial unit u, the partial evaluations of u on g_j are:

$$g_j(u) = \Theta_j\left[g_j\left(u'_i\right)\right] u'_i \in u.$$

The following example explains this operation better.

Example 4.7 Illustrating the computing of partial evaluations of spatial units u_1, u_2 and u_8 in decision map of figure 2(d) in respect to criteria g_1, g_2 and g_3. According to figure 2(c) and 2(d), it is easy to see that spatial unit u_1 is a grouping of u'_1 and u'_8; spatial unit u_2 is a grouping of u'_2 and u'_7; and spatial unit u_8 is a grouping of u'_9 and u'_{16}. Suppose now that Θ_1 is the arithmetic average operator, then the partial evaluations of u_1, u_2 and u_8 on criterion g_1 are:

- $g_1(u_1) = \Theta_1\left[g_1\left(u'_1\right), g_1\left(u'_8\right)\right] = \left(g_1\left(u'_1\right) + g_1\left(u'_8\right)\right)/2 = (3+2)/2 = 2.5,$
- $g_1(u_2) = \Theta_1\left[g_1\left(u'_2\right), g_1\left(u'_7\right)\right] = \left(g_1\left(u'_2\right) + g_1\left(u'_7\right)\right)/2 = (3+4)/2 = 3.5,$
- $g_1(u_8) = \Theta_1\left[g_1\left(u'_9\right), g_1\left(u'_{16}\right)\right] = \left(g_1\left(u'_9\right) + g_1\left(u'_{16}\right)\right)/2 = (2+1)/2 = 1.5$

Now suppose that Θ_2 is the majority rule (i.e. most frequent value) and that Θ_3 is the sum operator (Note: when several values are possible for the majority rule, then the least one is used). Using the same reasoning as above and by applying Θ_2 and Θ_3, we obtain: $g_2(u_1)=3$, $g_2(u_2)=3$, $g_2(u_8)=1$, $g_3(u_1)=4$, $g_3(u_2)=8$, and $g_3(u_8)=4$. The partial evaluations for all the spatial units of the decision map shown in figure 2(d) in respect to criteria g_1, g_2 and g_3 are summed up in the right side table of figure 2(d).

It is important to mention that the selection of the aggregation operator to apply at this step is fully arbitrary. It depends essentially on the nature of available data (e.g. it is not possible to apply the sum operator on ordinal data) and on the application domain. In the example above, we have used three different types of aggregation mechanisms, namely arithmetic average, majority rule, and sum operators but other mechanisms may also apply such as the min, max or mode operators.

The algorithm Λ runs in $O(r^2+mn)$, where r is the number of spatial units initially in **I**, m the number of evaluation criteria and n the number of spatial units after grouping. The operator MERGE acts directly at the database level and its complexity is not included in the complexity of Λ.

4.2 *Phase II: exploitation*

The exploitation phase contains three steps:

(i) construction of decision alternatives;
(ii) multicriteria evaluation; and

(iii) construction of prescription and recommendation.

4.2.1 Construction of decision alternatives. As introduced earlier, spatial decision alternatives are modeled through a constraint-based suitability analysis and represented by point, line and polygon features or their combination. Inside a raster GIS, these alternatives may be modeled as a raster cell, a grouping of linearly adjacent raster cells, or a grouping of contiguous raster cells. The modeling of alternatives based on basic spatial primitives may generate a high number of decision alternatives, which penalizes mainly the outranking methods. One possible idea to avoid this problem consists in 'emulating' point, line and polygon feature-based alternatives through one or several spatial units with some additional topological relationships. More specifically, decision alternatives are constructed as follows:

- *Point feature-based alternatives*: each decision alternative is modeled through an individual spatial unit.
- *Line feature-based alternatives*: each decision alternative is modeled as a grouping of linearly adjacent spatial units.
- *Polygon feature-based alternatives*: each decision alternative is modeled as a grouping of contiguous spatial units.

Formal solutions to construct different types of alternatives are provided in the following paragraphs. A detailed description of these solutions can be found in Chakhar (2006) and Chakhar and Mousseau (2006). Two cases are distinguished: (i) construction of atomic spatial decision alternatives; and (ii) construction of composed spatial decision alternatives, that is, alternatives that are obtained by a combination of atomic spatial decision alternatives.

4.2.1.1 Construction of atomic spatial decision alternatives.

(i) Construction of point feature-based alternatives. This type of alternatives applies essentially to location problems. They may be modeled as individual spatial units. Theoretically, any spatial unit may serve as an alternative. However, in practice the decision maker may wish to exclude some specific spatial units from consideration. Let $\mathfrak{E}\subset U$ be the set of excluded spatial units: $\mathfrak{E}=\{u'_i : u'_i\in U$ and that the decision maker states that $u'_i\notin A\}$.

Definition 4.8: a *point feature-based alternative p* is a spatial unit $u \in U\backslash\mathfrak{E}$. The set of potential alternatives is then defined as $A=\{u{:}u \in U\backslash\mathfrak{E}\}$.

Example 4.9 Consider for instance the decision map in figure 2(d) and suppose that the decision maker specifies that $\mathfrak{E}=\{u_3, u_6\}$, then the set of spatial decision alternatives in this case is $A=\{u_1, u_2, u_4, u_5, u_7, u_8\}$.

In the rest of this paragraph, we compare the number of point feature-based alternatives generated using the decision map concept and those generated by using classical modeling (i.e. based on point features). In a raster map, each raster cell may be considered as a decision alternative. Thus, with a map with m rows and p columns, an outranking method requires the establishment of $mp(mp-1)$ pairwise comparaisons. The use of a decision map reduces considerably this number to $r^2(r^2-1)$ where r is the number of spatial units in $\mathbf{M}$ which is generally very small in comparaison with mp, i.e. $r^2(r^2-1)\ll mp(mp-1)$. For instance, if we suppose that each spatial unit is a grouping of four raster cells, the number of the pairwise comparaisons is reduced to about 50%. This still applies to a vector map. Indeed, in this case each geographic point is considered as a decision alternative. With a decision map, this number is reduced largely since each spatial unit contains a high number of point features.

(ii) Construction of line feature-based alternatives. Line feature-based alternatives are often used to model linear infrastructures as highways, pipelines, etc. They may be modeled as a grouping of linearly adjacent spatial units. These alternatives are constructed on the basis of the connectivity graph resulting from the decision map. The connectivity graph $\mathfrak{C}=(U, V)$ is defined such that: $U=\{u: u \in \mathbf{M}\}$ and $V=\left\{(u_i, u_j) : u_i, u_j \in U \wedge \partial u_i \cap \partial u_j \neq \emptyset \wedge u_i^{\circ} \cap u_j^{\circ} = \emptyset\right\}$. Each vertex in $\mathfrak{C}$ is associated with the evaluation $v_E(u)$ of the spatial unit u it represents. Figure 3 presents the connectivity graph issued from the decision map in figure 2(d).

In practice, the decision maker may claim that the linear alternative t must pass through some spatial units or avoid some other ones. Let $Y=\{u \in U: (t \cap u = u) \wedge (t^{\circ} \cap u^{\circ} \neq \emptyset)\}$ be the set of spatial units that should be included and $X=\{u \in U: (t \cap u = \emptyset)\}$ be the set of spatial units to be avoided. The conditions in the definition of set Y signify that (u, in, t) is true and the one in the definition of X means that (u, disjoint, t) is true. Let also $\mathsf{f}(t)$ and $\mathsf{t}(t)$ denote the start and end spatial units for an alternative t. A line feature-based alternative t is defined as follows:

Definition 4.10: a *line feature-based alternative* t is a grouping of spatial units:

$$t=\left\{u_1, \cdots, u_q : u_i \in U \backslash X, i=1..q\right\}$$

with:

- $\mathsf{f}(t)=u_1$ and $\mathsf{t}(t)=u_q$,
- $\left(\partial u_i \cap \partial u_{j+1}\right) \neq \emptyset \wedge u_i^{\circ} \cap u_{j+1}^{\circ} = \emptyset, \forall i=1, \cdots, q-1$,
- $t \supseteq Y$ (or $t \cap Y = Y$).

The first condition sets the origin and the destination. The second condition ensures that spatial units in t are linearly adjacent. The last one ensures that all spatial units in set Y are included in t. Alternatives are then constructed based on $\mathfrak{G}'=(U \backslash X, V')$ with the condition that these alternatives should pass through spatial units in Y. Then, a possible solution to construct decision alternatives is to apply one of the well-established graph algorithms.

Example 4.11 For instance, suppose that $\mathsf{f}(t)=u_1$ and $\mathsf{t}(t)=u_5$, and $X=Y=\emptyset$, then four possible linear alternatives can be identified in the connectivity graph of figure 3: $t_1=\{u_1, u_2, u_3, u_4, u_5\}$, $t_2=\{u_1, u_2, u_3, u_6\ u_5\}$, $t_3=\{u_1, u_8, u_7, u_6, u_5\}$ and $t_4=\{u_1, u_8, u_7, u_6, u_3, u_4, u_5\}$.

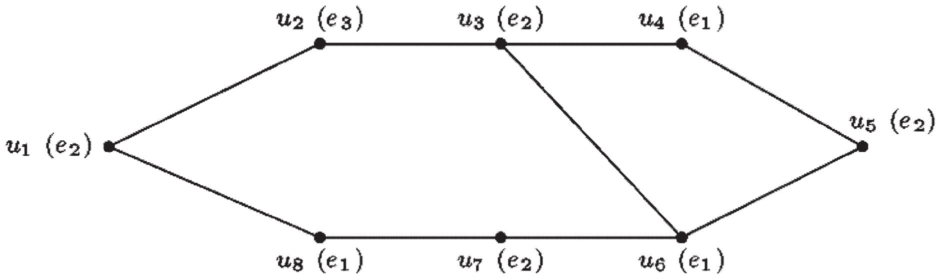


Figure 3. Connectivity graph issued from decision map of figure 2(d).

Line feature-based alternatives may also be constructed using the following idea. Let ($v_E(u_1)$, $v_E(u_2)$, ..., $v_E(u_q)$) be the set of the evaluations of all spatial units composing t. Then, to evaluate an alternative t we need to map a vector of q evaluations to a vector of k evaluations using a transformation rule φ:

$$\varphi : E^q \rightarrow E'$$
$$\left(v_E(u_1), v_E(u_2), \cdots, v_E(u_q)\right) \rightarrow \left(e'_1, e'_2, \cdots, e'_k\right)$$

where $E' : \left[e'_1, e'_2, \cdots, e'_k\right]$ is an ordinal evaluation scale with $\left(e'_1 \prec e'_2 \prec \cdots \prec e'_k\right)$ (E' can be the same one used in decision map construction process). The level e'_i may be the number of nodes x_j (i.e. spatial units) such that $v_E(x_j)=e_i$, the area of spatial units $u_j \in t$ and evaluated e_i, or any other spatial criterion. The global evaluation of an alternative t is $v(t)=\Phi(r_1, r_2, ..., r_k)$ where Φ is an aggregation mechanism. Before performing the global evaluation, dominated alternatives need to be eliminated from consideration. The *dominance relation* Δ cannot be defined directly on the initial evaluation vector ($v_E(x_1)$, ..., $v_E(x_q)$) since alternatives may have different lengths (in terms of the number of spatial units). It can be expressed on the transformed evaluation vector as follows. Let t and t' be two linear alternatives with transformed evaluation vectors (r_1, r_2, ...,r_k) and ($r'_1, r'_2, \cdots, r'_k$), respectively. Then t dominates t', denoted $t\Delta t'$, holds only and only if:$r_i \succeq r'_i, \forall i=1, \cdots, k$ with at least one strict inequality.

(iii) Construction of polygon-feature based alternatives. In several problems, alternatives are modeled as a grouping of contiguous spatial units. A polygon feature-based alternative is defined as follows:

Definition 4.12: a *polygon-feature based alternative y* is a collection of spatial units:

$$y=\{u_1, \cdots, u_s : u_i \in U, i=1, \cdots, s\}$$

with:

- $\sqcup_{u_i \in y} u_i = y$,
- $\forall u_i, u_j \in y, \left(u_i^{\circ} \cap u_j^{\circ}\right) = \varnothing$.

These conditions ensure the contiguousness of y and the absence of holes.

Example 4.13 Two examples of polygonal alternatives taken from the decision map in figure 2(d) are $y_1=\{u_8, u_1, u_2\}$ and $y_2=\{u_4, u_5\}$. In turn, the collection $\{u_7, u_5\}$ does not tally much with the definition above since spatial units u_7 and u_5 are not contiguous. Equally, the collection $\{u_3, u_4, u_5, u_7\}$ does not much with this definition since it contains a hole (corresponding to spatial unit u_6).

To construct this type of alternatives, we use the following idea. Let

$$T^{\alpha} = \left\{u_j \in U : v_E(u_j) = \alpha\right\}$$

be the set of spatial units in U with level α (i.e. $\Gamma_w(u_j)=\alpha$); $\alpha=1$, ..., k. Let

$$T_i^{\beta} = \left\{u_j \in U : \partial u_i \cap \partial u_j \neq \varnothing \wedge v_E(u_j) = \max_{l \in E \wedge l < \beta} l\right\}$$

be the set of spatial units that are contiguous to u_i and having the best evaluation strictly inferior to β. Next, we construct a tree T as follows. To each spatial unit u_i in T^{α} associate spatial units in T_i^{α} as sons. If $T_i^{\alpha}=\varnothing$, then use set $T_i^{\alpha-1}$; and if $T_i^{\alpha-1}=\varnothing$, then use $T_i^{\alpha-2}$, and so on. Next, add a hypothetical node r having as sons the first spatial units in the different constructed branches. Node r is evaluated by e_0. Note that if there is only one branch, then node r is omitted and the first node in this branch is used as the root of T. Then, an areal alternative may be constructed as an elementary path starting in r and continues until some conditions (concerning, e.g. the total surface of spatial units in the path) are verified.

Example 4.14 Considering again the decision map in figure 2(d). Then, according to the definitions above, we have:

- $T^3=\{u_2\}$
- $T^2=\{u_1, u_3, u_5, u_7\}$
- $T^1=\{u_4, u_6, u_8\}$
- $T_2^2=\{u_1, u_3\}$
- $T_1^1=\{u_8\}$
- $T_3^1=\{u_4, u_6\}$
- $T_5^1=\{u_6, u_4\}$
- $T_7^1=\{u_6, u_8\}$

Using the construction rules above, we obtain the tree shown in figure 4. For instance, the set $T^3=\{u_2\}$ means that node u_2 has as sons the spatial units in $T_2^{3-1}=T_2^2$, i.e. u_1 and u_3. Notice in particular that since there are more then one initial node (u_2, u_5 and u_7), a hypothetical node r is added. Notice also that any path in T produces a contiguous polygonal alternative.

As for linear alternatives, dominated areal alternatives should be eliminated from consideration. Let a and a' be two areal alternatives constructed as mentioned above, then $a\Delta a'$ holds if and only if a' is included in a, i.e.

$$u_i \in a' \Rightarrow u_i \in a, \ \forall u_i \in a'.$$

The definition of polygonal alternatives as an elementary path in T may lead to overlapping alternatives since the same spatial unit may be the son of several spatial

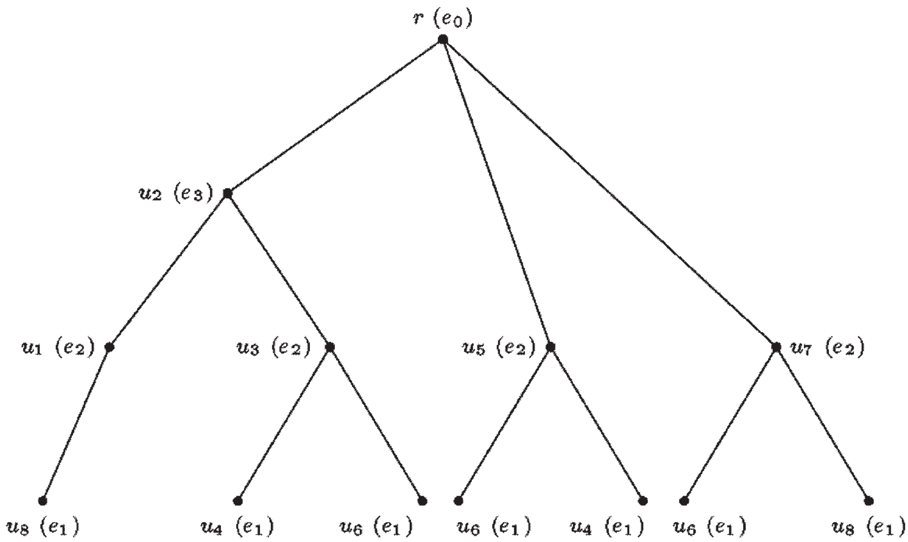


Figure 4. Schematic representation of tree T.

units. This situation holds when a spatial unit is adjacent to several other spatial units having global evaluations strictly less to the evaluation of this spatial unit.

Example 4.15 For instance, spatial unit u_6 in figure 4 is the son of u_5 and u_3. Thus, it is included, for instance, in areal alternatives $y_1=\{u_2, u_3, u_6\}$ and $y_2=\{u_5, u_6\}$. This leads to $y_1 \cap y_2 \neq \emptyset$.

However, in some problems it is required to construct totally disjoint polygonal decision alternatives as for example in zoning problems (*see*, e.g. Pereira *et al.* 2007). In this case, we may use the connectivity graph and apply one of the well-known graph algorithms (e.g. graph partitioning, spanning tree).

4.2.1.2 Construction of composed spatial decision alternatives. In practice, spatial decision alternatives may be modeled as a combination of two or several atomic decision alternatives. For example, a school partitioning problem may be represented by a set of 'point-polygon' composed decision alternatives where points schematize schools while polygons represent zones to serve, a set of 'point-point' composed alternatives may represent potential paths in a corridor identification problem, and a set of 'polygon-polygon' composed alternatives may be used in hierarchical administrative zoning problem where districts, departments, etc. take the form of hierarchical polygons. Other examples of composed spatial decision alternatives may be found in Malczewski (1999, pp. 142–144) and Chakhar and Mousseau (2004).

Spatial problems implying composed alternatives may be addressed as follows. The idea consists in decomposing the problem into a series of subproblems each of which implies only one type of atomic decision alternatives. Considering, for instance, that the initial problem involves composed decision alternatives which are constituted of two atomic alternatives of type γ_1 and γ_2 that should verify a set of spatial relationships R. This problem can be dealt with in two steps (*see* figure 5): (i) in the first step we resolve a first sub-problem involving alternatives of type γ_1; and (ii) in the second step we resolve another sub-problem involving alternatives of type γ_2 taking into account the spatial relations in R. The spatial relationships in R may be topological, (e.g. (γ_1, disjoint, γ_2)=true), metrical (e.g. distance(γ_1, γ_2) is less to a given value) or directional (e.g. γ_1 is besides γ_2). The resolution of the first sub-problem permits us to define the new decision space(s) over which the second sub-problem is defined and resolved. The resolution of the second sub-problem should take into account spatial relations in R. We notice that the process illustrated in figure 5 is an iterative one. This enables the decision maker (i) to modify the input data; and (ii) to repeat the process as many times as necessary.

Example 4.16 To better illustrate this idea, consider the school partitioning problem mentioned earlier and illustrated in figure 6. In this example we seek to locate n schools in a given region. This problem may be decomposed into two sub-problems as follows. The first sub-problem is a partitioning one, which aims to subdivide the study region into several homogenous zones using the solution previously proposed to construct polygon feature-based alternatives. The resolution of this problem permits us to define the decision spaces of n location sub-problems which can be handled as n point feature-based alternative problems and solved based on the solution described earlier. In this example, the spatial relations in R may be the appurtenance (i.e. each school belongs necessarily to the zone that it services) and the proximity to houses relations.

Table 1 presents the general schema of algorithms that may be used to construct composed decision alternatives. In this table the symbols P and P'; L and L'; and Q

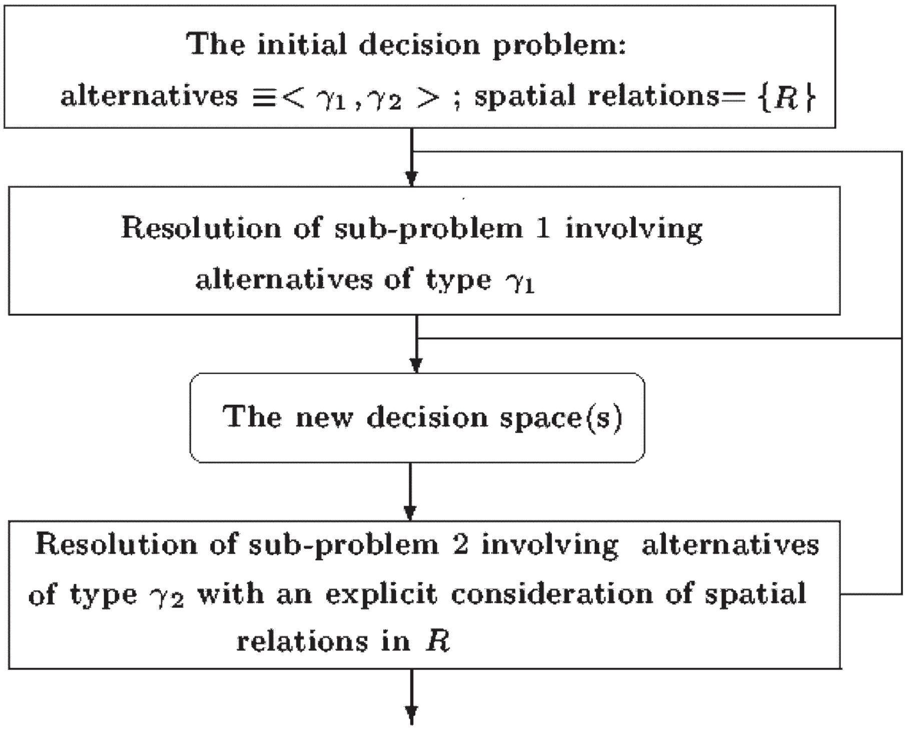


Figure 5. A process for handling problems implying composed decision alternatives.

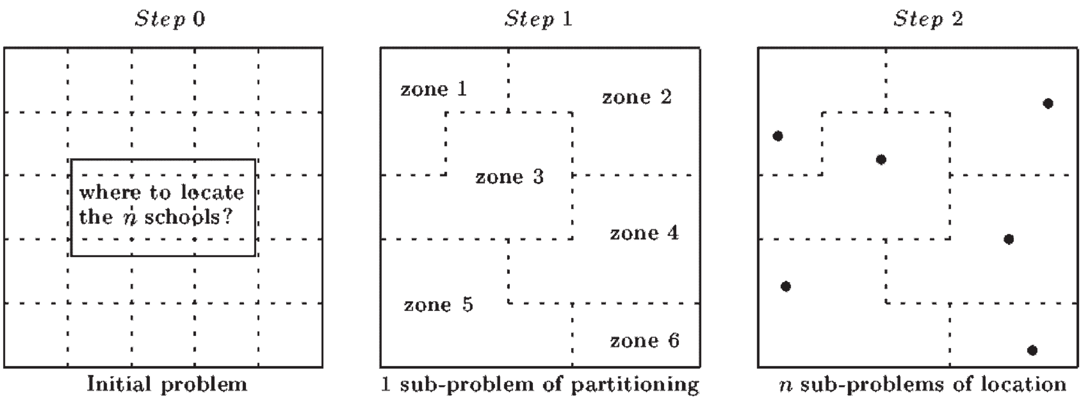


Figure 6. School partitioning problem.

Table 1. Algorithms for generating composed alternatives.

	P'	L'	Q'
P	Find t with f(t)=P and t(t)=P'	Find L' with $P \in Y$	Find Q' with (P, in, Q')=true
L	Find P' with (P', in, L)=true	Find L and L' with, e.g. $L \cap L' \neq \emptyset$, and f(L)=t(L′)	Find L and Q' with, e.g. $L \cap Q' \neq \emptyset$
Q	Find Q and P' with, e.g. $\mathbf{M}=Q$	Find Q and L' with, e.g. (L', cross, Q)=true	Find Q and Q' with, e.g. (Q, overlap, Q')=false

and Q' denote point, line and area feature-based decision alternatives, respectively. The symbol t denotes a linear alternative. These algorithms use the solutions proposed previously with some topological relationships. We notice that generally in a problem implying two types of alternatives γ_1 and γ_2, we may either start by generating γ_1 then γ_2, or by generating γ_2 then γ_1. In Example 4.16, for instance, it is also possible to start by first locating the n schools (point feature-based alternatives) and then defining the different zones (polygon feature-based alternatives). However, generating alternatives of the first type may restrict the solution space for the second type of alternatives.

The algorithms in table 1 are straightforward. Here we just comment briefly on some of them. Consider for example the P/P' case. This type of problem can be simply considered as line feature-based alternatives and may be resolved as detailed above with the additional conditions $\mathsf{f}(\mathsf{t})=P$ and $\mathsf{t}(\mathsf{t})=P'$ (or the inverse). The Q/Q' case involves two areal decision alternatives that can apply for the location of two facilities. The school partitioning problem mentioned above corresponds to Q/P' case.

4.2.2 Multicriteria evaluation. The multicriteria evaluation step requires the definition and use of different multicriteria evaluation functions. These functions have been identified in Chakhar and Martel (2003) based on the general schema of multicriteria methods. A formal definition of these functions is detailed in Chakhar (2006). Additional information is also available in Malczewski (1999) and Chakhar and Mousseau (2007b). For the sake of brevity, we only discuss two important conceptual issues rising from the way the potential decision alternatives are constructed and evaluated. The first point concerns the evaluation criteria to be used in this step. The second point concerns the way the partial evaluations of decision alternatives are computed.

Focusing on the first point, it is fruitful to mention that the evaluation criteria used in this step may be different from the ones used for the construction of the decision map. This option has two advantages. First, it offers the possibility to maintain in the first phase only highly technical evaluation criteria having explicit spatial dimensions and let the other, economic and social, evaluation criteria apply for the second phase. Second, it offers the possibility to apply effectively discriminative evaluation criteria in the multicriteria evaluation step, facilitating sensitivity analysis and elaboration of a final recommendation.

The second point concerns the definition of partial evaluations that should be associated with linear or polygonal decision alternatives. According to the solutions introduced above, these alternatives are composed of several spatial units, each of which has its 'own' partial evaluations that apply only to it. Thus, to be able to compare the different decision alternatives, it is necessary to aggregate, for each alternative, the partial evaluations associated with the different spatial units composing it to obtain partial evaluations that correspond to this alternative taken as a whole.

Indeed, each decision map can be represented as a hierarchical structure with three levels. The first level corresponds to the decision map **M** itself. The second level corresponds to the groupings of the spatial units as defined in the map **M**′. The third level corresponds to the individual spatial units as defined in the intermediate map **I**. The spatial decision alternatives are constructed on the basis of the elements of the first level whereas partial evaluations are associated with the third level.

From a practical point of view, we need to define, for each evaluation criterion g_j, an aggregation mechanism Υ_j that permits us to calculate, for an alternative a, the partial evaluation of a on criterion g_j:

$$g_j(a) = \Upsilon_j\left[g_j(u_i)\right]_{u_i \in a}.$$

Example 4.17 Consider the decision map of figure 2(d) and suppose that a polygon feature-based alternative a is composed of spatial units u_1, u_2 and u_8. Suppose now that Υ_1 is the sum operator, then the partial evaluation of a on criterion g_1 is:

$$\begin{aligned} g_1(a) &= \Upsilon_1[g_1(u_1), g_1(u_2), g_1(u_8)] \\ &= g_1(u_1) + g_1(u_2) + g_1(u_8). \end{aligned}$$

The values of $g_1(u_1)$, $g_1(u_2)$, $g_1(u_8)$ have been computed in Example 4.7: $g_1(u_1)=2.5$, $g_1(u_2)=3.5$, and $g_1(u_8)=1.5$. This leads to $g_1(a)=7.5$. Using the same reasoning and supposing that Υ_2 and Υ_3 are the sum and arithmetic average operators, respectively, then we obtain: $g_2(a)=23$ and $g_3(a)=10$.

Here again, it is important to mention that the selection of the aggregation operator to use is fully arbitrary and depends essentially on the nature of available data and on the application domain.

4.2.3 Prescription construction and recommendation. The input for this step is the global evaluations obtained by the application of a decision rule on the set of decision alternatives. The objective of this step is to exploit the output of the multicriteria evaluation step to help the decision maker make a final recommendation. The final recommendation in multicriteria analysis may take different forms, according to the manner in which a problem is stated. Roy (1996) identifies three main types of recommendation:

(i) *choice*: selecting a restricted set of alternatives;
(ii) *ranking*: classifying alternatives from best to worst with eventually equal positions; and
(iii) *sorting*: assigning alternatives to different pre-defined categories.

Within decision rules based on utility function, the prescription is directly deduced from the multicriteria evaluation step. When the preference model is based on an outranking relation, an exploitation phase is required to establish the prescription. Different possible solutions to construct prescriptions are detailed in Chakhar (2006). Most of these solutions, which are largely inspired from Vanderpooten (1990), are based on the use of algorithms issued from graph theory. Bearing in the mind the objective of this paper, these solutions are not reproduced here.

5. Implementation and application

A prototype has been developed to illustrate the proposed framework. We have used the GIS ArcGIS of ESRI – more precisely its ArcMap component – as the main software and VBA as the development language. In the rest of this section, we provide a step-by-step application of the prototype to a hypothetical corridor identification problem using real data relative to the Ile-de France (Paris and its suburbs) region in France. The available data are essentially of socio-economic nature relative to the districts of this region. The objective of the problem is to identify a corridor relating two different districts.

First, it is important to notify that the problem that will be presented in the rest of this section is simply a didactic example and is quite far from the reality of spatial decision problems. Different acronyms like NIMBY (Not In My Back Yard), LULUs (Locally Unwanted Land Uses), NOPE (Not On Planted Earth), or CAVE (Citizens Against Virtually Everything) (*see*, e.g. Heiman (1990), Dear (1992), Coucelelis and Monmonnier (1995), for more information, signification and practical implication of these terms) illustrate the difficulty and complexity of spatial decision problems. Our objective here is simply to show the feasibility of the framework.

The first step to deal with the corridor identification problem is to construct the criteria maps. Three criteria maps called DemographicDensity, EmploymentLevel, and AvailableLand have been constructed on the basis of the following three equations:

$$g_1(u_i) = \frac{\text{Total\_Population}(u_i)}{\text{Total\_Surface}(u_i)} \tag{1}$$

$$g_2(u_i) = \frac{\text{Active\_Population}(u_i)}{\text{Total\_Population}(u_i)} \tag{2}$$

$$g_3(u_i) = \text{Total\_Surface}(u_i) - \text{Surface\_of\_Used\_Land}(u_i) \tag{3}$$

To construct a criterion map, the user should first select a basic map layer and then specify a map algebra procedure whose operands are the attributes of the basic map. The criterion map given in figure 7 measures the employment level for the districts of the study area, which has been constructed on the basis of equation (2).

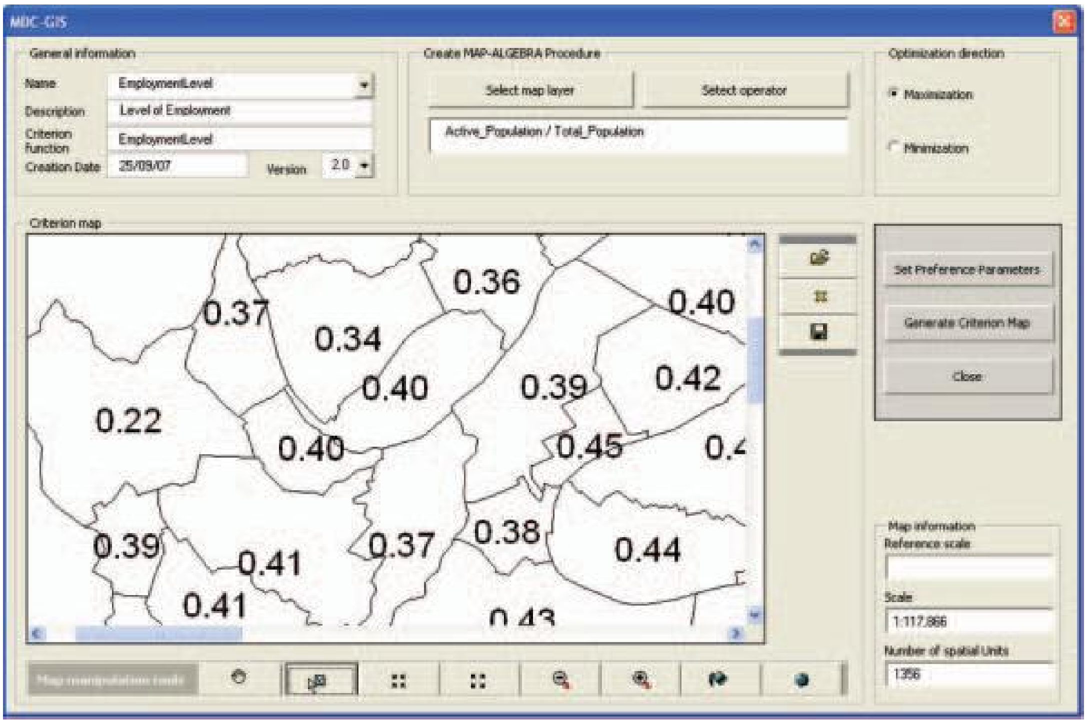


Figure 7. Criterion map 'EmploymentLevel'.

Each spatial unit (i.e. district here) of this map is associated with one value relative to employment level relative to this spatial unit. This is shown in the criterion map in figure 7. Criteria maps DemographicDensity and AvailableLand have been constructed similarly on the basis of equations (1) and (3), respectively.

The next step consists in combing the three criteria maps obtained in terms of the previous step to construct an intermediate map. It is important to note that the criteria maps in this example have all the same topology. Thus, the application of the union operator $\oplus$ implies only the descriptive attributes of these maps. The result is a new intermediate map where each spatial unit is characterized by three evaluations, one for each evaluation criterion.

The next step corresponds to multicriteria classification. In our example, we have applied the multicriteria classification method ELECTRE TRI. To apply ELECTRE TRI, we need to define a set of preference parameters and specify the profile limits of the categories. Four categories have been defined. The profile limits associated with the categories are given in table 2.

In practice, the definition of the preference parameters needs an important cognitive effort from the decision maker. To reduce this effort, an inference procedure, not presented in this paper, permitting us to infer the preference parameters from a set of holistic information provided by the decision maker is integrated in decision map construction process. We provide only a brief description of the inference procedure. This procedure, initially proposed in Mousseau and Slowinski (1998), proceeds as follows. Let $U^* \subset U$ denote a subset of spatial units that the decision maker intuitively assigns to a category or a range of categories on scale E. We define two sets $S^+ = \{(u, b_h) \in U^* \times B$: that the decision maker states that $uSb_h\}$ and $S^- = \{(u, b_h) \in U^* \times B$: that the decision maker states that $\neg(uSb_h)\}$. We recall that S is the outranking binary relation defined such that uSb_h means that the evaluation of spatial unit u upon all criteria is at least as good as the evaluation of the profile b_h (lower limit of category C_{h+1}). The idea of the inference procedure consists in searching a set of preference parameters w that permit us to re-do the assignment examples provided by the decision maker. This can be obtained by resolving the following system:

$$\begin{cases} \sigma(u, b_h) \geq \lambda, & \forall (u, b_h) \in S^+ \\ \sigma(u, b_h) < \lambda, & \forall (u, b_h) \in S^- \\ \lambda \in [0.5, 1] & \\ q_j(b_h) \leq p_j(b_h) \leq v_j(b_h), & \forall (j, h) \in F \times B \\ \sum_{j=1}^{m} k_j = 1; \; k_j \geq 0, & \forall j \in F \end{cases}$$

This system can be expressed through a mathematical program having as variables the parameters to infer. Then, the values for these parameters are obtained by maximizing the minimum slack for this system of constraints (*see* Mousseau and Slowinski (1998) and Mousseau (2005) for more details). The inferred parameters are then used to apply ELECTRE TRI. The list of assignment examples used in this application is given in table 3. Columns cMin(u_i) and cMax(u_i) are, respectively, the lowest and highest categories to which spatial unit u_i should be assigned. Table 3 also includes the partial evaluations of the different spatial units. Spatial units for which the lowest and highest categories are equal correspond to exact assignments, i.e. only one category is possible. The other assignment examples correspond to the

Table 2. Parameters of profile limits.

g_j	$g(b_3)$	q_3	p_3	$g(b_2)$	q_2	p_2	$g(b_1)$	q_1	p_1
g_1	15,000	1,000	2,000	10,000	1,000	2,000	3,000	1,000	2,000
g_2	0.7	0.05	0.09	0.5	0.05	0.09	0.3	0.05	0.09
g_3	15	1	1.5	10	1	1.5	5	1	1.5

case where a range of assignments is possible. For instance, spatial unit u_{1111} in table 3 must be assigned to C_2 while spatial unit u_{974} may be assigned to C_3 or C_4.

The computational complexity for solving the mathematical program varies in the sense that the mathematical program to resolve may be linear or non-linear. Specifically, with an outranking relation, obtaining a global optimum is not obvious and requires the resolution of a non-linear mathematical program. Two approaches are possible to overcome this difficulty: first, it is possible to specify a method based on a meta-heuristic; second, if the value of some preference parameters can be considered as known, 'partial' inference procedures can be designed. A partial inference is useful in situations in which the value of some parameters can reasonably be set. If not, it is possible to partition the parameters in sets, and proceed through a sequence of partial inference procedures in which the value of some parameters is fixed.

In our example, the inference procedure was used to infer the cutting level and the weights of the different criteria maps. The result is given in table 4. The other (fixed) parameters are provided in table 5.

These preference parameters are used to apply ELECTRE TRI on the intermediate map. The initial version of the decision map obtained is given in figure 8(a). It contains 1,356 spatial units.

It is easy to see that in the map of figure 8(a), several adjacent spatial units are assigned to the same category. It is necessary to group all of them by applying the algorithm GROUPING using the parameters given in table 6. These parameters correspond to the aggregation operators Θ_j and Υ_j introduced in Sections 4.1.4 and 4.2.2, respectively. Note, however, that operators Θ_j ($j=1, 2, 3$) are not used in this specific example since the criteria maps have the same topology (i.e. each spatial unit in the initial version ($\mathbf{M}'$) of the decision map is composed of only one unit). The

Table 3. Assignment examples.

u_i	Name	$g_1(u_i)$	$g_2(u_i)$	$g_3(u_i)$	cMin(u_i)	cMax(u_i)
u_{811}	Breviaires	52	0.3	10.5152	C_1	C_2
u_{934}	Limours	4,530	0.44	7.5578	C_2	C_3
u_{1067}	Dourdan	17,310	0.7	16.324	C_3	C_4
u_{1172}	Morigny-Champigny	126	0.45	16.4618	C_1	C_2
u_{1111}	Champcueil	5,002	0.44	8.745	C_2	C_2
u_{1142}	Fontainebleau	220	0.57	31.535	C_1	C_2
u_{984}	Bretigny/Orge	1,479	0.45	7.7592	C_2	C_2
u_{1023}	Vert-St-Denis	462	0.43	8.5966	C_1	C_3
u_{848}	Athis-Mons	16,089	0.52	16.5633	C_3	C_4
u_{785}	Tournan-En-Brie	486	0.46	8.2362	C_1	C_2
u_{927}	Marcoussis	427	0.47	8.9623	C_1	C_1
u_{974}	Sonchamp	9,032	0.43	24.6026	C_3	C_4

Table 4. Inferred preference parameters.

Parameter	λ	k_1	k_2	k_3
Inferred value	0.693	0.4	0.2	0.4

application of GROUPING algorithm on the map of figure 8(a) permits us to obtain the final decision map given in figure 8(b).

The last step consists in applying the solution proposed in Section 4.2.1.1 for the construction of line feature-based alternatives. The result of the application of this solution is given in figure 9. The generated corridors represent the input for the multicriteria evaluation step. As it is not the aim of this paper, this step is not described since it is common to different GIS-based multicriteria evaluation frameworks. More details can be found in Chakhar (2006).

6. Discussion of some conceptual and technical problems

In this section, we discuss some conceptual and technical aspects of the framework. Some possible solutions to deal with these problems will be enumerated. However, further investigations are still required. These will be the concern of our future research.

6.1 *Applicability of the framework in real world*

A first point to address is related to the applicability of the framework in real world problems. The proposed approach is highly soliciting the decision maker for determining the different preference parameters. However, when applying decision support in spatial decision problems, the decision makers are often hardly available. The multicriteria classification step is probably the most demanding in terms of preference parameters. The set of required parameters in this step may be reduced largely. In fact, the profile limits may be defined transparently to the user. The idea consists in subdividing the different criteria measurement scales into different intervals which are then used as profile limits.

Adding to that, stakeholders usually are not very familiar with multicriteria analysis and its concepts and it might be difficult to explain the framework to a decision maker unfamiliar with multicriteria tools. The objective of the inference procedure used in multicriteria classification step is in fact to reduce the cognitive effort of the decision maker. Indeed, the only required information is a set of assignment examples, which are then used to determine, indirectly, the preference parameters. We think that decision makers are more cooperative in producing assignment examples than giving exact values for the different preference parameters.

6.2 *Spatial problems difficult to handle*

The framework may be difficult or inappropriate to be applied in some spatial decision problems. There are two types of problems, in particular, for which the framework is

Table 5. Values of known preference parameters.

	g_1	g_2	g_3
q_j	1,000	0.05	1.0
p_j	2,000	0.09	1.5
v_j	3.000	1.0	2.0

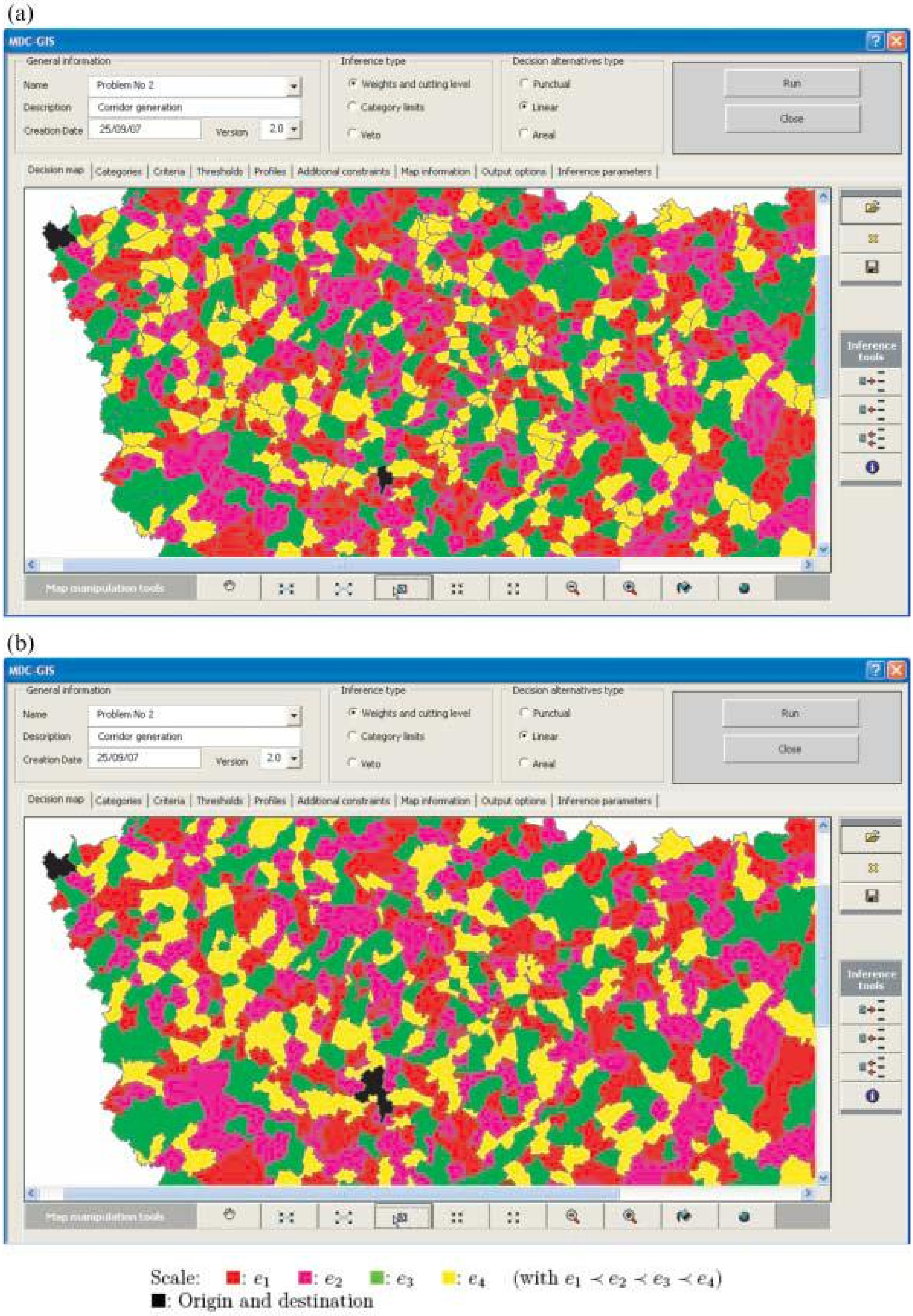


Figure 8. Decision map (a) before and (b) after grouping.

unsuitable. The first one has to do with problems where decision alternatives are represented as map structures (*see* Chakhar and Mousseau 2004). Map structures are relevant mainly in spatial problems that are related to the control of spatial phenomena (*see*, e.g. Janssen and van Herwijnen 1998, Sharifi *et al.* 2002). In Janssen and van

Table 6. Aggregation operators Θ_j and Υ_j.

Criterion map ($\mathbf{c}_j$)	Θ_j	Υ_j
$\mathbf{c}_1$: DemographicDensity	Arithmetic average	Arithmetic average
$\mathbf{c}_2$: EmploymentLevel	Arithmetic average	Arithmetic average
$\mathbf{c}_3$: AvailableLand	Sum	Sum

Herwijnen (1998), for instance, the authors have proposed several transformation and aggregation methods to represent the performance of different policies of antipollution fights in the 'Green Heart' region of the Netherlands. The results of the transformation and aggregation operations have been presented in performance maps, which represent the relative quality of the different policies along with their spatial patterns. These maps are then used as input for the multicriteria evaluation step.

The second type of problems includes those which are based on network structures. These latter intervene in a variety of real-world problems such as shortest path, minimal spanning tree, maximal flow, travelling salesman, airline scheduling, telecommunication, transportation and commodity flow problems. The specificity of network alternatives in comparison with point or line feature-based alternatives is that they have inherent spatial information about connectivity which is relevant essentially in road and transportation or drainage network analysis. Network alternatives intervene also in a variety of socio-economic applications such as public transportation, automatic route finding in car and truck navigation, commodity flow problems, etc. The specificity of this type of problem is that there is not an

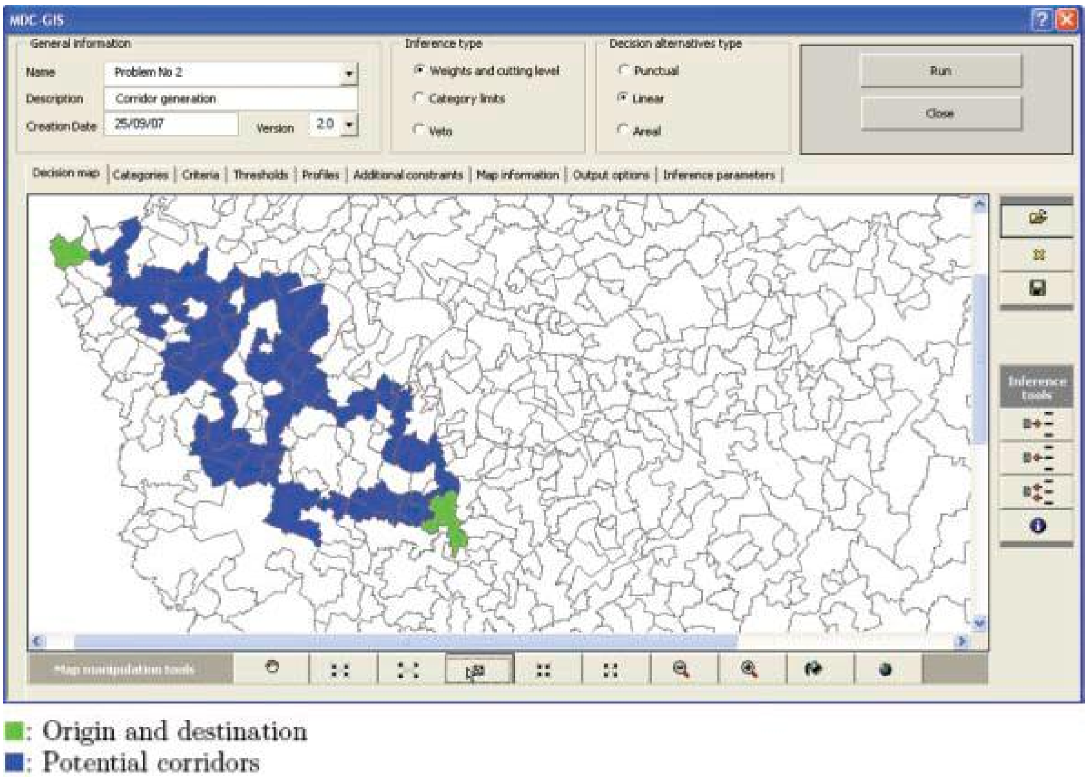


Figure 9. Potential corridors.

explicit selection of alternatives but a single one which is dynamically constructed. Hence, a dynamic formulation will apply better.

6.3 *Definition of spatial units*

In some problems, the boundaries of some spatial units may be also natural entities/ phenomena (e.g. river) or artificial infrastructures (e.g. highways). In this case, the decision maker may wish to consider that two spatial units with the same evaluation on both sides of the same frontier as two different ones. This necessitates the intervention of the decision maker. One possible solution to handle this operation is to propose the list of spatial units with common frontiers and equivalent evaluations and ask her/him to identify the ones to exclude from the grouping operation. Alternatively, the user may provide the list of natural or artificial objects that must appear in the final decision map and which should not be dissolved during the grouping operation. Another possible practical situation holds when the decision maker wishes to combine two adjacent spatial units having different evaluations or even to subdivide an individual spatial unit into two or more different ones. In this case, the decision maker should split/dissolve 'manually' the specified spatial unit(s) using GIS basic operations.

6.4 *Construction of spatial decision alternatives*

As detailed above, decision alternatives are constructed by combining different spatial units. The combination of spatial units could sometimes badly represent the spatial shape of alternatives. Indeed, the size of the decision alternatives generated through the solutions introduced earlier may be larger than the effective space required for implementing these alternatives. For instance, in the same spatial unit representing a point feature-based decision alternative, different geographic positions may be used in a location problem. Similarly, a large set of layouts may be identified in a grouping of linearly adjacent spatial units representing a line feature-based decision alternative.

We think that there is not any conceptual problem since, in general, in territory-related decision problems, one starts by identifying 'coarse' decision alternatives and then s/he looks to 'refine' them. Additionally, one possible solution to deal with this problem is to apply a successive refinement of the decision alternatives as follows. Initially, the decision map construction process is applied to the whole study area Ψ. Then, a new decision map is generated considering only the best spatial units identified initially. The process can be repeated as many times as necessary. At the end, relatively small spatial units are obtained, which permits us to obtain more accurate decision alternatives. Note, however, that this idea requires the availability of data with better resolution and this could be really time consuming. For instance, for linear alternatives, we may apply one of the polygonal path algorithms developed essentially in computational geometry (*see*, e.g. Agarwal and Varadarajan (2000)) to resolve this problem. We note that most of the proposed algorithms consider only one polygon. The extension to the whole corridor is obvious and needs only the addition of constraints ensuring the linearity of the path. Equally, point feature-based alternatives may be formulated as a set-cover, a *p*-center or a *p*-median problem or any other formulation issued from location theory (*see*, e.g. Hamacher and Drezner (2002)). In this case, the decision space is reduced to a unique spatial unit.

6.5 *Complexity of overlay operation*

The construction of a decision map requires the overlay of a set of criteria maps and yields a new map by combining all involved features in these maps. This operation may lead to several problems. First, criteria maps may have different geographical scales. Second, the overlay operation could produce a huge number of spatial units; most of these units would in fact have probably no real meaning because they just result from the difference in data sources. Lastly, with a relatively important number of criteria, it may become technically difficult to realize the overlay operation. One possible solution to this last problem consists in subdividing the criteria into different groups. Then, for each group we generate a partial decision map by using the same procedure given in the first phase of the framework. Finally, the different partial decision maps are considered as the input maps and a final decision map is obtained by combining the different partial decision maps. This solution may be time consuming and may lead to information loss. However, it permits us to reduce the number of polygons since during the producing of partial decision maps the adjacent polygons need to be grouped. The problem of different scales can be resolved through map transformation operations, which, however, may lead to information loss.

Apart from these simple solutions, further investigations concerning overlay operation need to be performed.

6.6 *Silver polygons problem*

The overlay operation may also generate silver polygons which should be removed. The generally applied solution consists in incorporating the silver polygon in an adjacent polygon having the most similar evaluation or the highest area. With this solution, the resulting polygon is not necessarily homogeneous and additional aggregations (of the partial evaluations of the silver polygon and the adjacent polygon) may be required. More generally, management of silver polygons raises several questions such as how to detect these polygons. One possible idea is to consider all polygons having a surface that does not exceed a minimal value as silver polygons.

Management of silver polygons is a relatively challenging problem and its utility goes beyond our framework. Initially, we intend to implement the solutions mentioned above. However, an in-depth analysis of this problem is required.

6.7 *Information loss problem*

As all aggregation processes, the framework involves a certain loss of information. Indeed, during the definition of criteria maps, it is necessary to transform all non-polygonal criteria maps into polygonal ones. This conversion operation might bring about loss of information. This will influence the accuracy of the result and potential errors might be introduced. For instance, the conversion of point sets to a TIN implies an interpolation between the available points in space, and raster conversion to polygons might lead to 'strangely' looking polygons. The loss of information problem may also result from raster-vector and point-polygon conversions. Moreover, information loss may take place when cardinal data are transformed into ordinal data.

Different tools may be used to deal with the information loss problem, including statistical techniques and fuzzy set theory.

7. Concluding remarks and future work

In the present paper, we have introduced a framework to facilitate the incorporation and use of outranking methods in GIS. The proposed framework uses a planar subdivision of the study area, called decision map, composed of a set of non-overlapping spatial units. The latter are then used to 'emulate' point, line and polygon feature-based decision alternatives, often used to model decision alternatives in conventional multicriteria spatial decision making. This way of modeling permits us to reduce significantly the number of alternatives to be evaluated, hence enabling outranking methods to be used.

The framework has been implemented on ArcGIS 9.1 and applied to a hypothetical corridor identification problem using real data relative to Ile-de-France region in France. It is clear that the problem described is quite far from the reality of spatial decision problems. It should be born in mind, however, that the framework is under application in a real-world problem relative to management of nuclear pollution risk in the south of France. This permits us to refine and enhance the framework. Currently, the computational behavior of the proposed solutions with larger datasets is under investigation. The use of other multicriteria classification methods is also envisaged.

We think that the most innovative part of the framework is decision alternatives construction solutions. Conventionally, alternatives are modeled by constraint-based suitability analysis and represented by point, line, and polygon features or their combination. In this paper, alternatives are constructed by combining different spatial units. The main advantage of this idea is that it facilitates the incorporation and use of outranking methods in the GIS by producing 'a handleable set of decision alternatives' (Hall *et al.* 1992, Wang 1994, Joerin *et al.* 2001). Besides, these solutions are different from manual or semi-automated merge and other transformation operations well established in modern GIS software in the sense that they (i) permit us to take into account different multicriteria aspects of the decision problem; (ii) imply the decision maker; (iii) use well-established graph algorithms; and (iv) are based on more than just simple-constrained solutions.

Although the initial motivation for this research is to facilitate the integration of outranking methods in GIS, the proposed framework is generic enough and supports outranking relation-based as well as utility-based multicriteria methods. In addition, we think that the framework goes beyond (or complete) the proposition of Joerin *et al.* (2001) or Marinoni (2006). However, even if it constitutes a step forward, this proposition has still to be improved. Indeed, several conceptual and technical problems that should be addressed have been discussed in the previous section. They concern the applicability of the framework in real-world problems, difficulty of handling some spatial problems, definition of spatial units, construction of spatial decision alternatives, complexity of overlay operation, the silver polygons problem and the information loss problem. Some possible solutions to these problems have been briefly discussed in Section 6 but further investigations are still required. These will be the concern of our future research.

In addition to these technical problems, we also intend to enrich the framework by adding capabilities for multicriteria spatial modeling. Indeed, the use of multicriteria analysis in the GIS is complicated by the lack of an appropriate multicriteria spatial modeling environment. A possible solution is to develop a script-like programming language supporting the different multicriteria evaluation functions. *Decision map algebra*, DMA, proposed in Chakhar and Mousseau (2007) and inspired from Tomlin (1990)'s map algebra seems to be a good starting point.

Another important topic is related to the support of collaborative and communicative spatial decision making. Indeed, the framework as described in this paper implies only one decision maker (or a homogenous group, i.e. a group of persons having the same value system). More specifically the decision map concept can easily be extended to support participation in spatial decision making. The idea consists in generating a composite decision map that gathers the preference information of all the participants. Two approaches for constructing the composite decision map, along with the level where the global aggregation is performed, have been briefly investigated in Chakhar *et al.* (2005) and Procaccini *et al.* (2007). In the first approach, aggregation is performed at the input level, i.e. at the level of criteria maps definition. Operationally, this approach starts by defining the composite criteria maps. Next, these composite criteria maps are aggregated so as to obtain a composite intermediate map. Then, the process is similar to the one used for the construction of a single decision map presented in this paper. In the second approach, the global aggregation is performed at the output level. Operationally, each group first generates an individual decision map. Then, the obtained individual decision maps are aggregated to construct a composite decision map.

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Appendix A. Overview of ELECTRE TRI method

ELECTRE TRI is a multicriteria sorting method used to assign alternatives to predefined ordered categories. The assignment of an alternative a results from the comparison of a with the profiles defining the limits of the categories. Let A denote the set of alternatives to be assigned. Let F denote the set of the indices of the criteria $g_1, g_2, \ldots, g_m$ (F=1, 2, …, m), k_j the importance coefficient of the criterion g_j, B the set of indices of the profiles defining $p+1$ categories (B=1, 2, …, p), b_h the upper limit of category C_h and the lower limit of category C_{h+1}, h=1, 2, …, p.

ELECTRE TRI builds a fuzzy outranking relation S whose meaning is 'at least as good as'. Preferences on each criterion are defined through pseudo-criteria. The threshold $q_j(b_h)$ represents the largest difference $g_j(a)-g_j(b_h)$ preserving an indifference between a and b_h in respect to criterion g_j. The threshold $p_j(b_h)$ represents the smallest difference $g_j(a)-g_j(b_h)$ compatible with a preference in favor of a in respect to criterion g_j. Thus, the limits of categories are defined in terms of profiles b_h, $h \in B$; each one is delimited by two imprecision zones.

To validate the proposition aSb_h, two conditions must hold:

- *Concordance*: an outranking aSb_h is accepted only if a 'sufficient' majority of criteria are in favor of this proposition.
- *Non-discordance*: when the concordance holds, none of the minority of criteria shows an 'important' opposition to aSb_h.

Beside the intra-criterion preferential information, represented by the indifference and preference thresholds, $q_j(b_h)$ and $p_j(b_h)$, the construction of S also makes use of two types of inter-criterion preferential information:

- The set of weight-importance coefficients (k_1, k_2,..., k_m) is used in the concordance test when computing the relative importance of the coalitions of criteria being in favor of the assertion aSb_h.
- The set of veto thresholds ($v_1(b_h)$, $v_2(b_h)$, ..., $v_m(b_h)$); $h \in B$, is used in the discordance test; $v_j(b_h)$ represents the smallest difference $g_j(b_h)-g_j(a)$ incompatible with the assertion aSb_h.

As the assignment of alternatives to categories does not result directly from the relation S, an exploitation phase is necessary; it requires the relation S to be 'defuzzyfied' using a so-called λ-cut: the assertion aSb_h is considered to be valid if the credibility index of the fuzzy outranking relation is greater than a 'cutting level' λ such that λ [0.5, 1]. ELECTRE TRI constructs an indices $\sigma(a, b_h) \in [0, 1]$ representing the credibility of the proposition aSb_h, $a \in A$, $h \in B$. The proposition aSb_h holds if $\sigma(a, b_h) \geqslant \lambda$. The indices ($a$, b_h) is defined using algorithm SIGMA given in Appendix B.

Two assignment procedures are available: optimistic and pessimistic. Their role is to analyze the way in which an alternative a compares to the profiles so as to determine the category to which a should be assigned. The result of these two assignment procedures differs when the alternative a is incomparable (*see* Figueira *et al.* (2005b)) with at least one profile b_h:

- *Pessimistic procedure*:

(i) compare a successively to b_i, $i=p,p-1$, ..., 0;
(ii) let b_h the first profile such that aSb_h, then assign a to category C_{h+1}.

- *Optimistic procedure*:

(i) compare a successively to b_i, $i=1$, 2, ..., p;
(ii) let b_h the first profile such that b_hSa, then assign a to category C_h.

Appendix B. Algorithm SIGMA

Algorithm SIGMA
INPUT: g(a)=($g_1(a)$, ..., $g_m(a)$): performance of alternative a
g(b_h)=($g_1(b_h)$, ..., $g_m(b_h)$): performance of profile b_h
w=(q,p,v,k,λ):
$(q_l(b_h),\ldots,q_m(b_h))$ indifference thresholds for profile b_h
$(p_l(b_h),\ldots,p_m(b_h))$ preference thresholds for profile b_h
$(v_l(b_h),\ldots,v_m(b_h))$ veto thresholds for profile b_h
$k=(k_l,\ldots,k_m)$ evaluation criteria weights
λ: cutting level
OUTPUT: $\sigma(a, b_h)$: credibility indices of assertion aSb_h
BEGIN
1. Compute partial concordance indices $S_j(a, b_h)$, $\forall_j \in F$:
$S_j(a,b_h)=\begin{cases}0, & \text{if } g_j(b_h)-g_j(a)\geq p_j(b_h);\\ 1, & \text{if } g_j(b_h)-g_j(a)\leq q_j(b_h);\\ \dfrac{p_j(b_h)-g_j(b_h)+g_j(a)}{p_j(b_h)-q_j(b_h)}, & \text{otherwise.}\end{cases}$
2. Compute global concordance indice $S(a, b_h)$:
$S(a,b_h)=\sum_{j\in F} k_j\cdot S_j(a,b_h)$
3. Compute partial discordance indices $ND_j(a, b_h)$, $\forall_j \in F$:
$ND_j(a,b_h)=\begin{cases}0, & \text{if } g_j(a)\leq g_j(b_h)+p_j(b_h);\\ 1, & \text{if } g_j(a)>g_j(b_h)+v_j(bh);\\ \dfrac{v_j(b_h)-g_j(a)+g_j(b_h)}{v_j(b_h)-p_j(b_h)}, & \text{otherwise.}\end{cases}$
4. Compute the global discordance indice $ND(a, b_h)$:
$ND(a,b_h)=\prod_{j\in F'}\frac{1-ND_j(a,b_h)}{1-S(a,b_h)}$
with $F'=\{j \in F{:}ND_j(a, b_h)>S(a, b_h)\}$
5. Compute credibility indice $\sigma(a, b_h)$:
$\sigma(a, b_h)=S(a, b_h)\cdot ND(a, b_h)(5)$
END

Appendix C. Assignment rules of some classification methods

This appendix briefly presents the assignment rules of three multicriteria classification methods, namely PROAFTN, UTADIS and MHDIS. More information can be found in the given references.

C.1: *assignment rule in PROAFTN*

PROAFTN (Belacel 2000) is a fuzzy classification method. The assignment of alternatives to categories is based on a fuzzy belonging degree. The fuzzy belonging degree of an alternative a to category C^h ($h=1, \ldots, k$), denoted $d(a, C^h)$, is defined by a set of prototypes (i.e. reference alternatives) B^h ($h=1, \ldots, k$), and it is measured by the indifference degrees between a and its nearest neighbor in B^h according to fuzzy indifference relations $I(a, b_i^h)(i=1, \cdots, L_h)$. The fuzzy indifference relations are based on extended concordance and non-discordance concepts. The value of $I(a, b_i^h)$ permits us to compute the degree to which a is inside an imprecision interval around the profile limits of the prototypes. Then, the fuzzy belonging degree is computed as follows:

$$d(a, C_h) = \max\left\{I(a, b_1^h), I(a, b_2^h), \cdots, I\left(a, b_{L_h}^h\right)\right\}$$

The belonging degrees $d(a, C_h)$ ($h=1, \ldots, k$) is used to assign alternatives. For each alternative a, a crisp assignment using the following rule is made:

$$a \in C_h \Leftrightarrow d(a, C_h) = \max\{d(a, C_i) : i \in 1, \cdots, k\}$$

C.2: *assignment rule in MHDIS*

The MHDIS (Zoopounidis and Doumpos 2000) method distinguishes the groups progressively, starting by discriminating the first group from all the others, and then proceeds to the discrimination between the alternatives belonging into the other groups. To accomplish this task, two addivtive utility functions are developed in each step of the $q-1$ steps; q is the number of groups. The first function $U_k(a)$ describes the alternatives of group C_1, while the second function $U_{\sim k}(a)$ describes the remaining alternatives that are classified in lower groups $C_{k+1}, \ldots, C_q$. Then, the classification rule has the following form:

$$\text{If } U_1(a) \geq U_{\sim 1}(a) \text{ then } a \in C_1$$
$$\text{Else } if U_1(a) \leq U_{\sim 1} \text{ then } a \in C_2$$

C.3: *assignment rule in UTADIS*

The UTADIS (Doumpos and Zopounidis 2002) method implies the development of an additive utility function that is used to score the different alternatives and decide upon their classification. The general form of the utility function is $U(a) = \sum_{j=1}^{m} w_j u_j(g_j)$ where w_j is a normalized weight of criterion g_j and $u_j(g_j)$ is the marginal utility function normalized between 0 and 1. The developed utility function provides an aggregate score $U(a)$ of each alternative along all criteria. The classification rules will be defined in terms of cut-off utility points on the global utility scale. For instance, for two categories C_1 and C_2, a possible classification rule is (v is a cut-off utility point):

$$U(a) \geq v \Rightarrow a \in C_1$$
$$U(a) < v \Rightarrow a \in C_2$$