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INFERRING AN ELECTRE TRI MODEL FROM ASSIGNMENT EXAMPLES

V. Mousseau [1] , R. Slowinski [2]

Abstract

Given a finite set of alternatives, the sorting problem consists in the assignment of each alternative to one of the pre-defined categories. In this paper, we are interested in multiple criteria sorting problems and, more precisely, in the existing method ELECTRE TRI. This method requires the elicitation of parameters (weights, thresholds, category limits,...) in order to construct the DM's preference model. A direct elicitation of these parameters being rather difficult, we proceed to solve this problem in a way that requires from the DM much less cognitive effort. We elicit these parameters indirectly using hollistic information given by the DM through assignment examples.

We propose an interactive approach that infers the parameters of an ELECTRE TRI model from assignment examples. The determination of an ELECTRE TRI model that best restitutes the assignement examples is formulated through an optimization problem. The interactive aspect of this approach lies in the possibility given to the DM to revise his/her assignment examples and/or to give additional information before the optimization phase restarts.

Keywords : Multiple criteria decision aid, Sorting problem, ELECTRE TRI method, Parameters' elicitation, Inference procedure, Optimization.

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1 Introduction

When modeling a real world decision problem using multiple criteria decision aid, several problematics (or problem formulations) can be considered. [15] distinguishes three basic problematics: choice, sorting and ranking (see also [1]).

Given a set A of alternatives (or actions), the choice (or selection) problematic (see Figure 1) consists in a choice of a subset $A' \subset A$, as small as possible, composed of alternatives being judged as the most satisficing. Optimisation problems are particular cases of a choice problematic where A' is restricted to one alternative.

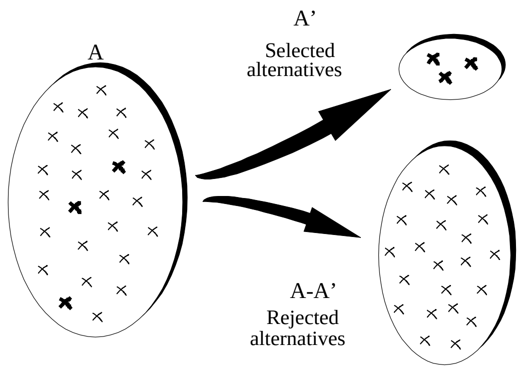

Figure 1: Choice problematic

The sorting problematic (see Figure 2) consists in formulating the decision problem in terms of a classification so as to assign each alternative from A to one of the predefined categories. The assignment of an alternative a to the appropriate category should rely on the intrinsic value of a (and not on the comparison of a to other alternatives from A).

The ranking problematic (see Figure 3) consists in establishing a preference pre-order (either partial or complete) in the set of alternatives A.

In this paper, we are interested in the multiple criteria sorting problematic and, more precisely, in an existing method called ELECTRE TRI (see [24], [25] and [17]). When using this method, the analyst must determine values of several parameters (profiles that define the limits between the categories, weights, discrimination thresholds, ...). These parameters are used to construct a preference model of the decision maker (DM). Apart from some very specific cases, it is not realistic to assume that the DM would be able to give explicitly the values of these parameters. They are far different from the natural terms in which the DM usually expresses his/her preferences and expertise. Our aim is to infer the model parameters of ELECTRE TRI through an analysis of assignment examples given by the DM, i.e., from hollistic judgments. This approach represents the paradigm of disaggregation of preferences (see [6]) which aims at extracting implicit information contained in hollistic statements given by a DM.

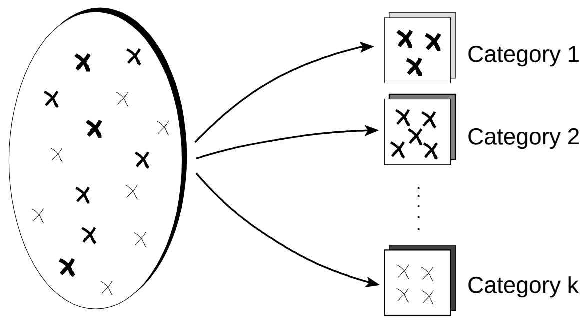

Figure 2: Sorting problematic

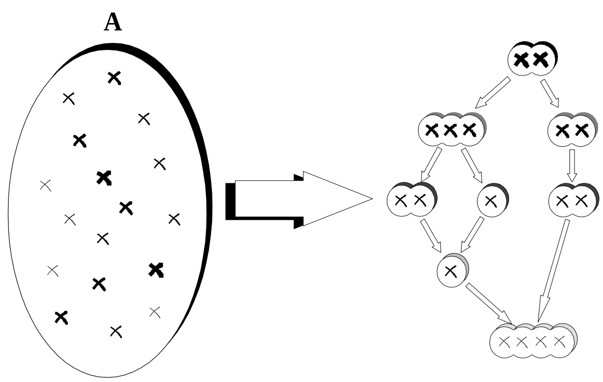

Figure 3: Ranking problematic

In our case the statements to be disaggregated are assignment examples.

The paper is organized as follows. In the next section, we characterize the general objectives of our approach. In section 3, we recall the main steps of the ELECTRE TRI method and then we pass, in section 4, to the description of our inference procedure from assignment examples. In section 5, we are considering the choice of an optimization technique for our inference procedure and we provide, in section 6, an illustrative example. A final section groups conclusions.

2 General objectives

2.1 Scheme of the proposed approach

The general scheme of the inference procedure using the paradigm of disaggregation is presented in Figure 4. Its aim is to find an ELECTRE TRI model as compatible as possible with the assignment examples given by the DM. The assignment examples concern a subset $A^* \subset A$ of alternatives for which the DM has clear preferences, i.e., alternatives that the DM can easily assign to a category, taking into account their evaluation on all criteria. The compatibility between the ELECTRE TRI model and the assignment examples is understood as an ability of the ELECTRE TRI method using this model to reassign the alternatives from A^* in the same way as the DM did. To get a representative model, the subset A^* must be defined such that the numbers of alternatives assigned to the categories are almost equal and sufficiently large to "contain enough information".

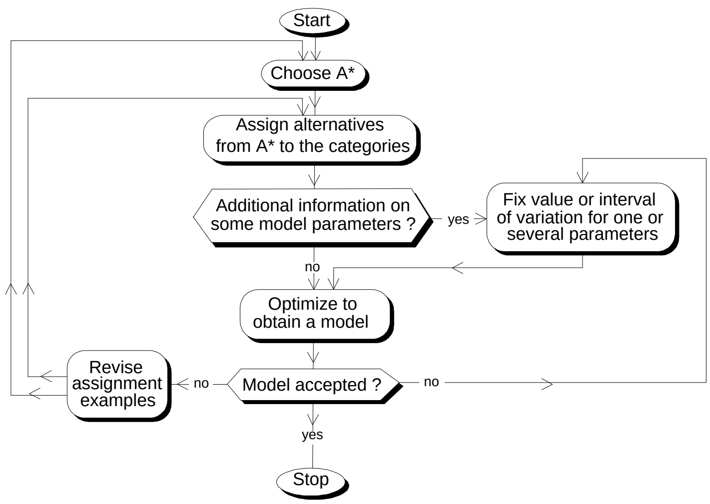

Figure 4: General scheme of the inference procedure

In order to minimize the differences between the assignments made by ELECTRE TRI and the assignments made by the DM, an optimization procedure is used. The resulting ELECTRE TRI model is denoted by M_π. The DM can tune up the model in the course of an interactive procedure. He/she may either revise the assignment examples or fix values (or intervals of variation) for some model parameters. In the former case, the DM may:

- remove and/or add some alternatives from/to A^*,
- change the assignment of some alternatives from A^*.

In the latter case, the DM can give additional information on the range of variation of some model parameters basing on his/her own intuition. For example, he/she may specify:

- ordinal information on the importance of criteria,
- noticeable differences on the scales of criteria,
- incomplete definition of some profiles defining the limits between categories.

When the model is not perfectly compatible with the assignement examples, the procedure should be able to detect all "hard cases", i.e., the alternatives for which the assignment computed by the model strongly differs from the DM's assignment. The DM could then be asked to reconsider his/her judgment.

2.2 Interest of the approach

One of the main difficulties that an analyst must face when interacting with a DM in order to build a decision aid procedure is the elicitation of various parameters of the DM's preference model. In the ELECTRE TRI method, the analyst should assign values to profiles, weights and thresholds (see section 3). Even if these parameters can be interpreted, it is difficult to fix directly their values and to have a clear global understanding of the implications of these values in terms of the output of the model.

Our approach to the construction of an ELECTRE TRI model aims at substituting assignment examples for direct elicitation of the model parameters. The values of the parameters will be inferred through a certain form of regression on assignment examples.

Inferring a form of knowledge from examples of expert's decisions is a typical approach of artificial intelligence. Induction of rules or decision trees from examples in machine learning (see [9], [14]), knowledge acquisition based on rough sets (see [3], [13], [21]), supervised learning of neural nets (see [2], [22]) are well-known representatives of this approach. The appeal of this approach is that the experts are typically more confident exercising their decisions than explaining them.

In Multiple Criteria Decision Analysis, this approach is concordant with the principle of posterior rationality (see [7]) and with the aggregation-disaggregation paradigm used for the construction of a preference model in UTA-like procedures (see [6], [19], [23], [5], [4], [12], [20]). It has been also applied for the elicitation of weights used for the construction of an outranking relation in the DIVAPIME method (see [11] and [10]).

Moreover, such an approach may be used within a different context from the one it was initially intended for: construction of ordinal criteria. When a criterion should take into account several dimensions related to specific aspects of the decision, it is sometimes difficult to define directly a satisfactory index that "measures" the performance of alternatives relatively to this criterion. A way to overcome this difficulty is to proceed as follows:

- define, for the considered criterion, an ordinal scale made of several impact levels using linguistic terms,

- specify several prototypes of alternatives that meet these impact levels,
- consider the impact levels as categories and prototypes as assignment examples and infer the corresponding ELECTRE TRI model using the proposed approach,
- use this ELECTRE TRI model to define the evaluation of any other alternative on the considered criterion.

3 Presentation of the ELECTRE TRI method

ELECTRE TRI is a multiple criteria sorting method, i.e., a method that assigns alternatives to predefined categories. The assignment of an alternative a results from the comparison of a with the profiles defining the limits of the categories. Let F denote the set of the indices of the criteria $g_1, g_2, ..., g_m$ (F=$\{1, 2, ..., m\}$) and B the set of indices of the profiles defining p+1 categories (B=$\{1, 2, ..., p\}$), b_h being the upper limit of category C_h and the lower limit of category C_{h+1}, h=1, 2, ...,p (see Figure 5). In what follows, we will assume, without any loss of generality, that preferences increase with the value on each criterion.

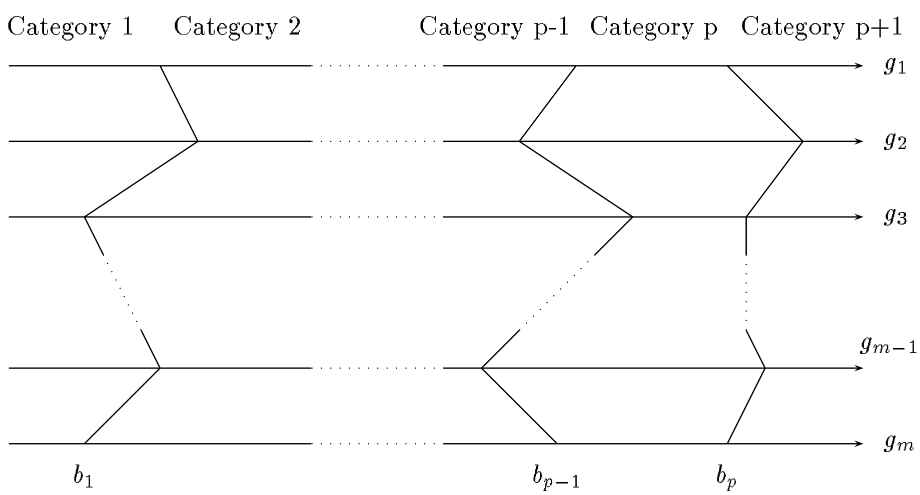

Figure 5: Definition of categories using limit profiles

ELECTRE TRI builds an outranking relation S, i.e., validates or invalidates the assertion aSb_h (and b_hSa), whose meaning is "a is at least as good as b_h". Preferences restricted to the significance axis of each criterion are defined through pseudo-criteria (see [18] for details on this double-threshold preference representation). The indifference and preference thresholds ($q_j(b_h)$ and $p_j(b_h)$)

constitute the intra-criterion preferential information. They account for the imprecise nature of the evaluations g_j(a) (see [16]). $q_j(b_h)$ specifies the largest difference $g_j(a) - g_j(b_h)$ that preserves indifference between a and b_h on criterion g_j; $p_j(b_h)$ represents the smallest difference $g_j(a) - g_j(b_h)$ compatible with a preference in favor of a on criterion g_j.

At the comprehensive level of preferences, in order to validate the assertion aSb_h (or b_hSa), two conditions should be verified:

- concordance: for an outranking aSb_h (or b_hSa) to be accepted, a "sufficient" majority of criteria should be in favor of this assertion,
- non-discordance: when the concordance condition holds, none of the criteria in the minority should oppose to the assertion aSb_h (or b_hSa) in a "too strong way".

Two types of inter-criteria preference parameters intervene in the construction of S:

- the set of weight-importance coefficients $(k_1, k_2, ..., k_m)$ is used in the concordance test when computing the relative importance of the coalitions of criteria being in favor of the assertion aSb_h,
- the set of veto thresholds $(v_1(b_h), v_2(b_h), ..., v_m(b_h))$ is used in the discordance test. $v_j(b_h)$ represents the smallest difference $g_j(b_h) - g_j(a)$ incompatible with the assertion aSb_h.

ELECTRE TRI builds an index $\sigma(a, b_h) \in [0, 1]$ ($\sigma(b_h, a)$, resp.) that represents the degree of credibility of the assertion aSb_h (b_hSa, resp.), $\forall a \in A, \forall h \in B$. The assertion aSb_h (b_hSa, resp.) is considered to be valid if $\sigma(a, b_h) \geq \lambda$ ($\sigma(b_h, a) \geq \lambda$, resp.), λ being a "cutting level" such that $\lambda \in [0.5, 1]$.

Determining $\sigma(a, b_h)$ consists of the following steps (the value of $\sigma(b_h, a)$ is computed analogously):

1 - compute the partial concordance index $c_j(a, b_h), \forall j \in F$:

$$c_j(a, b_h) = \begin{cases} 0 \text{ if } g_j(b_h) - g_j(a) \geq p_j(b_h) \\ 1 \text{ if } g_j(b_h) - g_j(a) \leq q_j(b_h) \\ \frac{p_j(b_h)+g_j(a)-g_j(b_h)}{p_j(b_h)-q_j(b_h)} \text{ otherwise} \end{cases} \quad (1)$$

2 - compute the comprehensive concordance index c(a,b_h):

$$c(a, b_h) = \frac{\sum_{j \in F} k_j c_j(a, b_h)}{\sum_{j \in F} k_j} \quad (2)$$

3 - compute the discordance indices $d_j(a, b_h), \forall j \in F$:

$$d_j(a, b_h) = \begin{cases} 0 \text{ if } g_j(a) \leq g_j(b_h) + p_j(b_h) \\ 1 \text{ if } g_j(a) > g_j(b_h) + v_j(b_h) \\ \in [0, 1] \text{ otherwise} \end{cases}$$

4 - compute the credibility index $\sigma(a, b_h)$ of the outranking relation:

$$\sigma(a, b_h) = c(a, b_h) \prod_{j \in \overline{F}} \frac{1 - d_j(a, b_h)}{1 - c(a, b_h)}, \quad \text{where} \quad \overline{F} = \{j \in F : d_j(a, b_h) > c(a, b_h)\} \tag{3}$$

The values of $\sigma(a, b_h)$, $\sigma(b_h, a)$ and λ determine the preference situation between a and b$_h$:

- $\sigma(a, b_h) \geq \lambda$ and $\sigma(b_h, a) \geq \lambda \Rightarrow aSb_h$ and $b_hSa \Rightarrow aIb_h$, i.e., a is indifferent to b_h,
- $\sigma(a, b_h) \geq \lambda$ and $\sigma(b_h, a) < \lambda \Rightarrow aSb_h$ and not $b_hSa \Rightarrow a \succ b_h$, i.e., a is preferred to b_h (weakly or strongly),
- $\sigma(a, b_h) < \lambda$ and $\sigma(b_h, a) \geq \lambda \Rightarrow$not aSb_h and $b_hSa \Rightarrow b_h \succ a$, i.e., b_h is preferred to a (weakly or strongly),
- $\sigma(a, b_h) < \lambda$ and $\sigma(b_h, a) < \lambda \Rightarrow$not aSb_h and not $b_hSa \Rightarrow aRb_h$, i.e., a is incomparable to b_h.

Two assignment procedures are then available:

Pessimistic procedure :

a) compare a successively to b_i, for i=p,p-1, ..., 0,
b) b_h being the first profile such that aSb_h,
assign a to category C_{h+1} ($a \rightarrow C_{h+1}$).

Optimistic procedure :

a) compare a successively to b_i, i=1, 2, ..., p,
b) b_h being the first profile such that $b_h \succ a$,
assign a to category C_h ($a \rightarrow C_h$).

4 Inferring an ELECTRE TRI model from assignment examples

An ELECTRE TRI model M_π is composed of:

- the profiles defined by their evaluations $g_j(b_h)$, $\forall j \in F$, $\forall h \in B$,
- the importance coefficients k_j, $\forall j \in F$,
- the indifference and preference thresholds $q_j(b_h)$, $p_j(b_h)$, $\forall j \in F$, $\forall h \in B$,
- the veto thresholds $v_j(b_h)$, $\forall j \in F$, $\forall h \in B$,
- a selected assignment procedure (either pessimistic or optimistic).

In what follows, we will confine our analysis to the case were the pessimistic assignment procedure is used and where the inferred parameters are the profiles, the importance coefficients and preference and indifference thresholds. As for veto thresholds, they are not inferred from the assignment examples because of computational complexity. However, they can be introduced directly by the DM in the Electre Tri model.

So as to determine a model M_π that best matches the assignment examples given by the DM, one should formulate an appropriate optimization problem, i.e., define the variables, an accuracy criterion and the constraints.

4.1 Variables of the problem

In ELECTRE TRI pessimistic assignment procedure, an alternative a_k is assigned to category C_h (b_{h-1} and b_h being the lower and upper profiles of C_h, respectively) iff $\sigma_\pi(a_k, b_{h-1}) \geq \lambda$ and $\sigma_\pi(a_k, b_h) < \lambda$.
Let us suppose that the DM has assigned the alternative $a_k \in A^*$ to category C_{h_k} ($a_k \rightarrow C_{h_k}$). Let us define the slack variables x_k and y_k unrestricted in sign such that $\sigma_\pi(a_k, b_{h_k-1}) - x_k = \lambda$ and $\sigma_\pi(a_k, b_{h_k}) + y_k = \lambda$.

The optimization problem will include the following variables:

$x_k,\ y_k,\ \forall a_k \in A^*$	slack variables (2n)
λ	cutting level (1)
$k_j,\ \forall j \in F$	importance coefficients (m)
$g_j(b_h),\ \forall j \in F,\ \forall h \in B$	profile evaluations (mp)
$q_j(b_h),\ \forall j \in F,\ \forall h \in B$	indifference thresholds (mp)
$p_j(b_h),\ \forall j \in F,\ \forall h \in B$	preference thresholds (mp)

4.2 An accuracy criterion

If the values of the slack variables x_k and y_k are both positive, then ELECTRE TRI pessimistic assignment procedure will assign alternative a_k to the "correct" category. If, however, one or both of these values are negative, the ELECTRE TRI pessimistic assignment procedure will assign alternative a_k to a "wrong" category. The lower the minimum of these two values, the less adapted is the model M_π to give an account of the assignment of a_k made by the DM. Moreover, if x_k and y_k are both positive, then a_k is assigned consistently with the DM's statement, for all $\lambda' \in [\lambda - y_k, \lambda + x_k]$.
Let us consider now the set of alternatives $A^* = \{a_1, a_2, ..., a_k, ..., a_n\}$ and suppose that the DM has assigned the alternative a_k to the category C_{h_k}, $\forall a_k \in A^*$. The model M_π will be consistent with the DM's assignments iff $x_k \geq 0$ and $y_k \geq 0$, $\forall a_k \in A^*$.

Consistently with the preceding argument, an accuracy criterion to be maximized can be defined as:

$$\min_{a_k \in A^*} (x_k, y_k) \quad \rightarrow \ max \tag{4}$$

If the accuracy criterion takes a non-negative value, then all alternatives contained in A^* are "correctly" assigned, for all $\lambda' \in [\lambda - min_{a_k \in A^*}(y_k), \lambda + min_{a_k \in A^*}(x_k)]$.

This criterion, however, takes into account the "worst case" only, i.e., the alternative for which the model M_π gives the most different assignment from the DM. An accuracy criterion should be able to take into account an average information concerning the accuracy of the model, i.e., its overall ability to assign the alternatives from A^* to the "correct" category. Hence, we propose to replace criterion (4) by the following one:

$$\min_{a_k \in A^*}(x_k, y_k) + \epsilon \sum_{a_k \in A^*}(x_k + y_k) \quad \rightarrow \; max \tag{5}$$

where ϵ is a small positive value. (5) can be rewritten as :

$$\begin{aligned} (\alpha + \quad & \epsilon \textstyle\sum_{a_k \in A^*}(x_k + y_k)) \quad \rightarrow \; max & (6)\\ s.t. \quad & \alpha \leq x_k, \quad \forall a_k \in A^* & (7)\\ & \alpha \leq y_k, \quad \forall a_k \in A^* & (8) \end{aligned}$$

4.3 Constraints of the problem

The constraints of the optimization problem are the following:

$\sigma_\pi(a_k, b_{h_k-1}) - x_k = \lambda$, $\forall a_k \in A^*$	definition of the slack variables x_k (n)
$\sigma_\pi(a_k, b_{h_k}) + y_k = \lambda$, $\forall a_k \in A^*$	definition of the slack variables y_k (n)
$\alpha \leq x_k, \alpha \leq y_k$, $\forall a_k \in A^*$	definition of α (2n)
$\lambda \in [0.5, 1]$	interval of variation for λ (2)
$g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_{h+1})$, $\forall j \in F$, $\forall h \in B$	consistency of categories (m(p-1))
$p_j(b_h) \geq q_j(b_h)$, $\forall j \in F$, $\forall h \in B$	thresholds consistency (mp)
$k_j \geq 0$, $q_j(b_h) \geq 0$, $\forall j \in F$, $\forall h \in B$	non-negativity constraints (m+mp)

According to the general scheme given in Figure 4, some additional constraints can be added in the course of the interactive procedure in order to take into account an intuitive view of the DM on the value of some parameters. For instance, if the DM does not consider any criterion as a dictator, an appropriate constraint is: $k_j \leq \frac{1}{2}\sum_{i=1}^{m} k_i$, $\forall j \in F$

4.4 Optimization problem to be solved

The basic form of the optimization problem to be solved is the following:

$$\begin{aligned} (\alpha \quad & + \epsilon \sum_{a_k \in A^*}(x_k + y_k)) \quad \rightarrow \; max & (9)\\ s.t. \quad & \alpha \leq x_k, \quad \forall a_k \in A^* & (10)\\ & \alpha \leq y_k, \quad \forall a_k \in A^* & (11) \end{aligned}$$

$$\frac{\sum_{j=1}^{m} k_j c_j(a_k, b_{h_k-1})}{\sum_{j=1}^{m} k_j} - x_k = \lambda, \quad \forall a_k \in A^* \tag{12}$$

$$\frac{\sum_{j=1}^{m} k_j c_j(a_k, b_{h_k})}{\sum_{j=1}^{m} k_j} + y_k = \lambda, \quad \forall a_k \in A^* \tag{13}$$

$$\lambda \in [0.5, 1] \tag{14}$$

$$g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_{h+1}), \forall j \in F, \forall h \in B \tag{15}$$

$$p_j \geq q_j, \quad \forall j \in F \tag{16}$$

$$k_j \geq 0, q_j \geq 0, \quad \forall j \in F \tag{17}$$

Because of constraints (12) and (13), the above problem is a non-linear programming problem. It contains 2n+3mp+m+2 variables and 4n+3mp+2 constraints. Let us remark that the slack variables x_k and y_k can be eliminated from the problem formulation since they are defined by the constraints (12) and (13). This elimination reduces the number of variables to 3mp+m+2.

4.5 Approximation of partial concordance indices $c_j(a_k, b_h)$

The partial concordance indices $c_j(a_k, b_h)$ are piecewise linear functions (see (1)), and are hence non differentiable. This prevents from using gradient optimization techniques which would be the most suitable ones for solving the above problem. In order to circumvent this difficulty, we will approximate $c_j(a_k, b_h)$ by a differentiable sigmoidal function $f(x)$ of the following form:

$$f(x) = \frac{1}{1 + \exp\left[-\beta(x - x_0)\right]} \tag{18}$$

The sigmoidal function has the following properties:

$$\left[\begin{array}{l} \lim_{x \to \infty} f(x) = 1 \\ \lim_{x \to -\infty} f(x) = 0 \\ f(x_0) = 0.5 \\ \frac{df(x)}{dx} = \beta f(x)\left(1 - f(x)\right) \end{array}\right. \tag{19}$$

These properties make possible a "fair" representation of $c_j(a_k, b_h)$ by $f(x)$. The shape of this function is shown in Figure 6.

$c_j(a_k, b_h)$ is a function of the difference $g_j(a_k) - g_j(b_h)$ and of the thresholds $q_j(b_h)$ and $p_j(b_h)$[3]. In order to represent $c_j(a_k, b_h)$ by $f(x)$, we substitute x for $g_j(a_k) - g_j(b_h)$ and include both thresholds in the parameters x_0 and β.
As $c_j(a_k, b_h) = 0.5$ for $g_j(a_k) - g_j(b_h) = \frac{p_j(b_h)+q_j(b_h)}{2}$, we pose $x_0 = \frac{p_j(b_h)+q_j(b_h)}{2}$. The value of β minimizing the approximation error is: $\beta = \frac{5.55}{p_j(b_h)-q_j(b_h)}$ (see appendix). The resulting approximation of $c_j(a_k, b_h)$ is given below and shown in Figure 7.

[3] We assume here that the preference increases with the value of g_j.

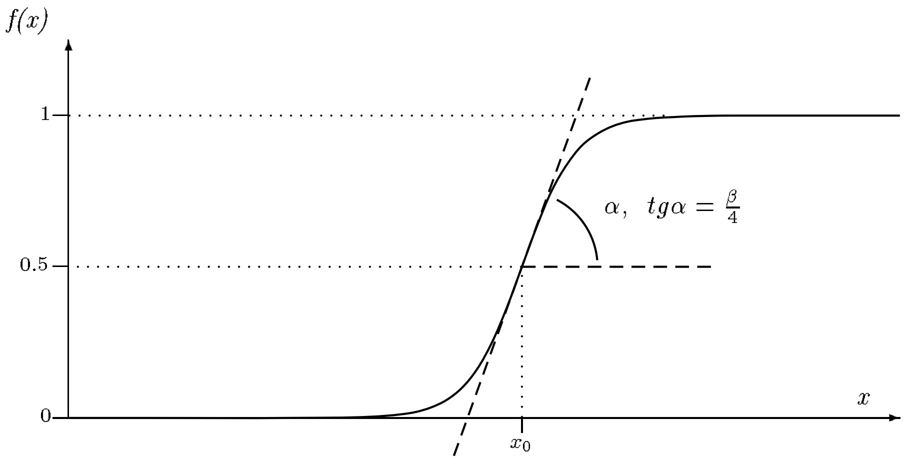

Figure 6: Sigmoidal function $f(x)$

$$\hat{c}_j(a_k, b_h) = \frac{1}{1 + \exp\left[\frac{-5.55}{p_j(b_h) - q_j(b_h)} \cdot \left(g_j(a_k) - g_j(b_h) + \frac{p_j(b_h) + q_j(b_h)}{2}\right)\right]} \qquad (20)$$

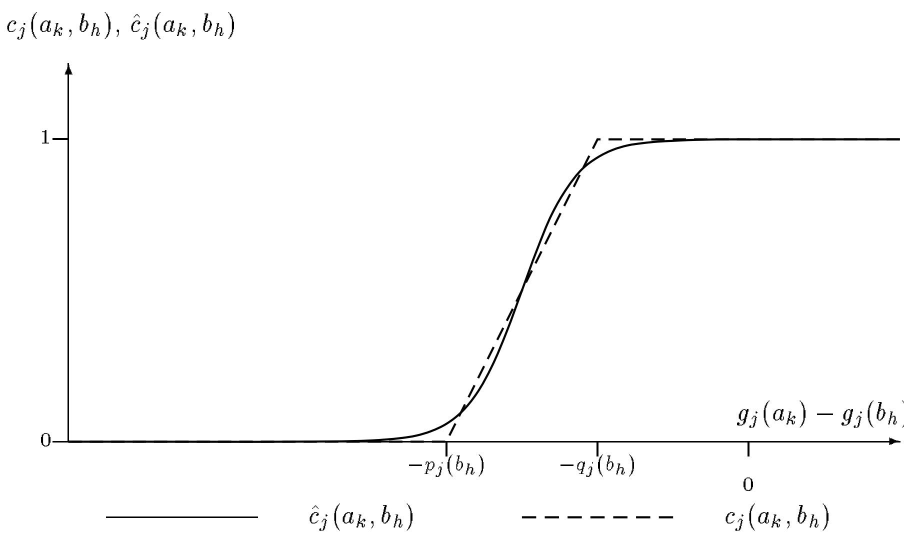

Figure 7: Approximation of $c_j(a_k, b_h)$ by $\hat{c}_j(a_k, b_h)$

5 Solving the optimization problem

5.1 Input data

In order to run the optimization phase, the model should contain "enough" information so as to infer a set of ELECTRE TRI parameters. More specifically, the set of assignment examples $a_k \rightarrow_{DM} C_{h_k}$, $\forall a_k \in A^*$, should be "sufficiently large", e.g., $n \geq m + p$). Moreover, the alternatives should be well distributed among the p+1 categories and alternatives assigned to the same category C_h should have profiles "as different as possible".
Consistently with the general scheme presented in §2.1, the DM can add information concerning the value of some parameters of the model. This information take the form of additional constraints in the optimization problem.

5.2 Output of the inference phase

The output of the inference phase consists of a set of values for ELECTRE TRI parameters. It should be checked then whether the obtained model is compatible with the assignment examples. If it is the case, the variable α takes a positive value. The inferring process stops at this point unless the DM wants to revise the value of some parameters.

In the case of a negative value of α, it is possible to find which assignment example causes this negative value, i.e., which example is the most difficult to be reproduced by the model. Let us denote by $\tilde{a}$ the corresponding alternative. This assignment example can be viewed as the most "untypical" compared to the others. In such a case, two options are possible:

- either the DM changes the "untypical" assigment of $\tilde{a}$ and then the optimization phase restarts with the modified set of assignment examples,
- or the DM confirms his/her assignment of $\tilde{a}$ and then the optimization phase restarts considering all assignment examples but $\tilde{a}$.

In the second option, the elimination of $\tilde{a}$ is temporary and aims at finding an appropriate model for the remaining examples. This elimination is not definite as $\tilde{a}$ can be re-integrated to A^* when the DM modifies another assigment example.

5.3 Choice of an optimization technique

In the case of relatively small optimization problems, the solver of Excel 5.0 gives satisfactory results in a reasonable computing time (see section 6), although we cannot be sure to attain the global optimum with this tool. When considering large problems, a more powerful solver seams necessary, mainly because of the multiplicity of local minima. As can be seen from recent works (see [8]), metaheuristics in particular based on genetic algorithms can satisfactorily deal with the local minima problems. This direction of research deserves further investigation.

6 An illustrative example

Let us consider a sorting problem in which alternatives have to be assigned to three categories, C_1, C_2 and C_3, defined by two profiles, b_1 and b_2 ($B = \{1,2\}$), taking into account their evaluations on three criteria, g_1, g_2 and g_3 ($F = \{1,2,3\}$). The evaluations on each criterion take their values in the interval [0,100]. Let us consider the 6 assignment examples given in Table 1.

	g_1	g_2	g_3	Category
a_1	70	64·75	46·25	C_3
a_2	61	62	60	C_3
a_3	40	50	37	C_2
a_4	66	40	23·125	C_2
a_5	20	20	20	C_1
a_6	15	15	30	C_1

Table 1: Set of assignment examples

The optimization problem corresponding to the search of an ELECTRE TRI model consistent with the assignment examples contains 23 variables and 44 constraints (see §4.4). Three additionnal constraints of the form $k_j \leq \frac{1}{2}\sum_{j=1}^{3} k_j, j = 1,2,3$ prevent any criterion to be a dictator.
In order to perform the optimization phase, we need to determine a starting point. In the absence of any additional information from the DM, we fix $k_j = 1, j = 1,2,3$. The initial profiles are defined by the following heuristic rule:

$$g_j(b_h) = \frac{1}{2}\left\{\frac{\sum_{a_i \to C_{h-1}} g_j(a_i)}{n_{h-1}} + \frac{\sum_{a_i \to C_h} g_j(a_i)}{n_h}\right\}$$

where n_h and n_{h-1} are the number of alternatives assigned to categories C_h and C_{h-1}, respectively.

The above heuristic rule leads to the initial profiles defined in Table 2.

	g_1	g_2	g_3
b_2	59·25	54·19	41·6
b_1	35·25	31·25	27·5

Table 2: Initial profiles defining the category limits

As to the initial values for the indifference and preference thresholds, their values are fixed arbitrarily as follows.

$$\left[\begin{array}{lcl} q_j(b_h) & = & 0.05\, g_j(b_h) \\ p_j(b_h) & = & 0.1\, g_j(b_h) \end{array}\right. \tag{21}$$

Table 3 presents the initial values of the thresholds.

	g_1	g_2	g_3
$q_i(b_2)$	2·96	2·71	2·08
$p_i(b_2)$	5·93	5·42	4·16
$q_i(b_1)$	1·77	1·56	1·38
$p_i(b_1)$	3·53	3·12	2·75

Table 3: Initial values of indifference and preference thresholds

Fixing $\lambda = 0.75$, we obtain the initial values for x_a and y_a shown in Table 4.

	x_k	y_k
a_1	0.25	0.75
a_2	0.25	0.75
a_3	0.25	0.598
a_4	-0.083	0.417
a_5	0.25	0.75
a_6	0.25	0.417

Table 4: Initial values of the slack variables x_k and y_k

The initial values of the parameters lead to a model that is not able to assign all alternatives consistently with the DM's assignments. a_4 is assigned by the model to category C_1 while the DM assigned a_4 to category C_2. As can be seen in the starting point, $\alpha = x_4 = -0.083 < 0$.

The resulting optimization problem has been solved using the Excel 5.0 solver which computes a solution within less than two minutes on a Pentium 60Mhz computer. The values of parameters in the obtained model are shown in Tables 5 and 6.

Moreover, $\alpha = 0.37$, $\lambda = 0.629$, $k_1 = 0.517, k_2 = 1, k_3 = 0.483$.

	g_1	g_2	g_3
b_2	59·25	62·75	41·6
b_1	35·25	31·25	23·65

Table 5: "Optimal" profiles defining the category limits

	g_1	g_2	g_3
$q_i(b_2)$	2·96	2·71	2·08
$p_i(b_2)$	5·925	5·419	4·16
$q_i(b_1)$	1·762	1·563	1·376
$p_i(b_1)$	3·525	3·125	2·813

Table 6: *"Optimal" values of indifference and preference thresholds*

We obtain the final values for x_k and y_k shown in Table 7. The obtained model is able to assign all alternatives from A^* consistently with the DM's assignments. The model assignments remain consistent for all $\lambda \in [0.5, 1]$ which proves a good robustness of the model.

	x_k	y_k
a_1	0.371	0.629
a_2	0.370	0.629
a_3	0.371	0.624
a_4	0.370	0.370
a_5	0.371	0.628
a_6	0.371	0.387

Table 7: Final values of the slack variables x_k and y_k

7 Conclusions and further research

We have proposed an interactive approach that infers the parameters of an ELECTRE TRI model from assignment examples. The determination of the model that best restitutes the assignement examples is formulated as an optimization problem. The interactive aspect of this approach lies in the possibility given to the DM to revise his/her assignment examples and/or to give additional information on the range of variation of some parameters before the optimization phase restarts.

The proposed approach is based on a realistic assumption that the DM prefers to give some assignment examples rather than to specify directly the values of parameters used in ELECTRE TRI. In this way, our approach transfers the interaction with the DM from the level of direct elicitation of parameters to the level of exemplary assignment decisions at which less cognitive effort is required.

As preliminary experience with this approach is encouraging, further research should be pursued in two complementary directions: improvement of computational efficiency in the optimization phase and extension of some useful features of the proposed approach.

As to the first direction, additional computational experiments should be made, both for larger problems and using different optimization techniques. The use of metaheuristics based, in particular, on genetic algorithms deserves further investigation. Another important point to be studied concerns post-optimal analysis of the obtained solution so as to estimate its stability.

As to the second direction, it should be given a possibility of considering sub-problems concerning subset of ELECTRE TRI parameters to be inferred, This may be particularily interesting from a practical point of view and can lead to linear optimization problems. Moreover, the inference procedure should be able to propose a rich interaction with the DM. The dialog with the DM should enable assignments of alternatives to multiple categories, e.g., "a_k *should be assigned to the top two categories*" or "$a_{k'}$ *will not be assigned to the two extreme categories*", etc. It should be also taken into account that the DMs are sometimes able to express a degree of confidence related to an assignment. Two assertions, "*it is possible that* a_k *meets the requirements of category* C_h" and "$a_{k'}$ *should certainly be assigned to* $C_{h'}$", should not be processed by the inference procedure in the same way. Furthermore, the approach should be extended to take into account the veto phenomenon considered in the complete version of the ELECTRE TRI method. Finally, the development of user-friendly interactive software is a necessary condition of a successful implementation of this approach in real-world decision problems.

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APPENDIX: Approximation of $c_j(a_k, b_h)$ by $\hat{c}_j(a_k, b_h)$

We approximate the partial concordance index $c_j(a_k, b_h)$ (see (1)) by the sigmoidal function $f(x) = 1/(1 + exp(-\beta(x - x_0)))$. As follows from §4.5, the value of $x_0 = (p_j(b_h) + q_j(b_h))/2$. The value of β influences the angle of inclination of the tangent to $\hat{c}_j(a_k, b_h)$ in the point $(x_0$,0.5) (see Figure 6). As a consequence, it also influences the size of the closed areas A and B created by $c_j(a_k, b_h)$ and $\hat{c}_j(a_k, b_h)$ (see Figure 8). The value of β has to be chosen such that the surface of A is equal to the surface of B. Then, the approximation error is minimal. Solving a simple integral equation, we get $\beta = \frac{5.55}{p_j(b_h) - q_j(b_h)}$.

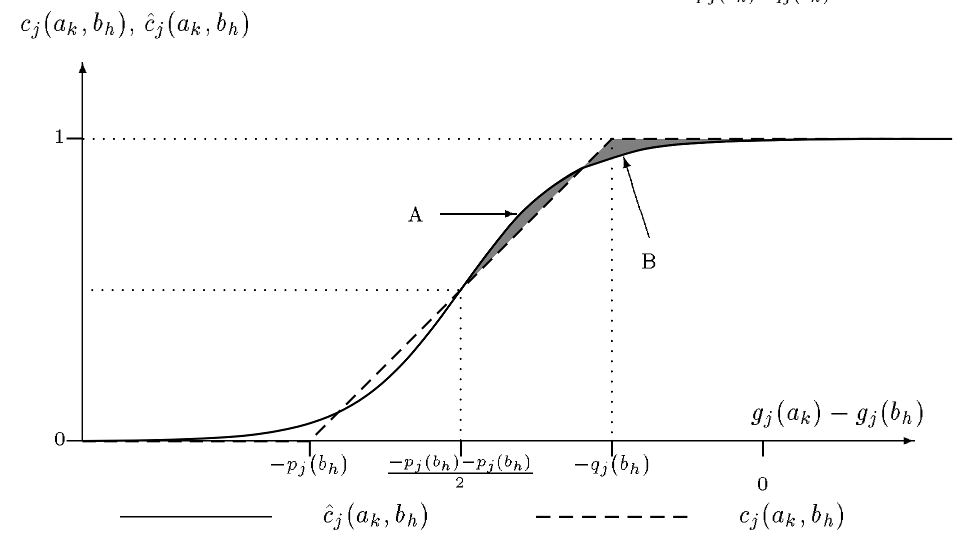

Figure 8: Determination of β minimizing the approximation error