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A user-oriented implementation of the ELECTRE-TRI method integrating preference elicitation support

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Abstract

Multiple Criteria Sorting Problem consists in assigning a set of alternatives $A = \{a_1, a_2, \dots, a_l\}$ evaluated on n criteria $g_1, g_2, \dots, g_n$ to one of the categories which are pre-defined by some norms corresponding to vectors of scores on particular criteria, called profiles, either separating the categories or playing the role of central reference objects in the categories. The assignment of an alternative a_k to a specific category results from a comparison of its evaluation on all criteria with the profiles defining the categories. This paper presents a new implementation of an existing method called ELECTRE TRI. It integrates specific functionalities supporting the decision maker (DM) in the preference elicitation process. These functionalities grouped in ELECTRE TRI Assistant aim at reducing the cognitive effort required from the DM in the phase of calibration of the preference model. The main characteristic feature of ELECTRE TRI Assistant is the inference of the ELECTRE TRI preferential parameters from assignment examples supplied by the DM. The software is presented through an illustrative example.

Scope and purpose

Decision makers (DMs) often face decision situations in which different conflicting viewpoints (goals or criteria) are to be considered. The field of multiple criteria decision aid (MCDA) offers the DMs a selection of methods and operational tools that explicitly account for the diversity of the viewpoints considered. Each method constructs first a model of DM's preferences and then exploits this model in order to workout a recommendation. A large class of methods proposed in the literature use an outranking relation as a preference model (whose semantic is "at least as good as"). In order to implement these methods in real-world applications, the values of preference parameters, like importance coefficients and discrimination thresholds, are to be given by the DM. As this is usually a difficult task, we propose to infer values of these parameters from examples of decisions supplied by the DM. Such an approach to preference modeling is

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called aggregation/disaggregation approach. The paper describes an implementation of an outranking method integrating this kind of preference elicitation support.

Keywords: Sorting problem statement; ELECTRE TRI; Software implementation; Preference elicitation support

1. Introduction: Multicriteria sorting problem

Most real-world decision problems can be represented by a model stating explicity the multiple points of view from which alternatives under consideration should be analysed, each one being formalized by a criterion function. Given a set $A = \{a_1, a_2, \dots, a_l\}$ of potential alternatives evaluated on the criteria, the analyst conducting the decision aid study on demand of the decision maker (DM), may formulate the problem in different terms. Roy [1] distinguishes among three problem statement, i.e, problem formulations (choice, sorting and ranking) that may guide the analyst in structuring the decision problem (see also [2]).

Among these problem statements, a major distinction concerns *relative vs absolute judgment* of alternatives. This distinction refers to the way alternatives are considered and to the type of result expected from the analysis.

In the first case, alternatives are directly compared one to each other and the results are expressed using the comparative notions of "*better*" and "*worse*". Choice (selecting a subset A^* of the best alternatives from A) or ranking (definition of a preference order on A) are typical examples of comparative judgments. The presence (or absence) of an alternative a_k in the set of best alternatives A^* results from the comparison of a_k to the other alternatives. Similarly, the position of an alternative in the preference order depends on its comparison to the others.

In the second case, each alternative is considered independently from the others in order to determine its intrinsic value by means of comparisons to norms or references; results are expressed using the absolute notions: "*assign*" or "*not assign*" to a category "*similar*" or "*not similar*" to a reference profile, "*adequate*" or "*not adequate*" to some norms. The sorting problem statement refers to absolute judgments. It consists in assigning each alternative to one of the categories which are pre-defined by some norms corresponding to vectors of scores on particular criteria, called profiles, either separating the categories or playing the role of central reference points in the categories. The assignment of an alternative a_k results from the intrinsic evaluation of a_k on all criteria with respect to the profiles defining the categories (the assignment of a_k to a specific category does not influence the category to which another alternative should be assigned).

The semantics of the categories can imply an ordered structure on categories or not; the former case refers to ordered Multiple Criteria Sorting Problems (MCSP), the latter to nominal MCSP. MCSP differs from clustering; the categories considered here are defined a priori and do not result from the analysis, while clusters result from a partition of A into categories unknown a priori.

Previous works on MCSP have been developped using the outranking approach and several methods have been proposed: Trichotomic Segmentation (see [3,4]), N-Tomic (see [5]) or Class (see [6,7]), ELECTRE TRI (see [8,9]). Filtering methods based on concordance and non-discordance principles have been studied in [10]. The use of rough sets theory [11–13] has also allowed significant progress in this field. The contribution of goal programming should also be mentioned (see [14,15])

Real-world case studies of MCSP have been reported in the literature in various domains:

- evaluation of applicants for loans or grants [16,17],
- business failure risk assessment [18,19],
- screening methods prior to project selection [20],
- satellite shot planning [21],
- medical diagnosis [22–24].

In this paper, we consider ordered MCSP and, more specifically, an existing method called ELECTRE TRI (see [8,9]). This method, like other methods supporting multiple criteria assignment of alternatives, exploits a preference model of the DM, characterised by a number of parameters following more or less directly from preferential information supplied by the DM. In ELECTRE TRI, the preference model is an outranking relation and parameters are weights and various thresholds on criteria. In order to support the preference elicitation process, we propose an ELECTRE TRI Assistant integrated with the method. It infers preference model parameters from assignment examples given by the DM. The paper is organized as follows: the next section gives a brief methodological presentation of the ELECTRE TRI method and Section 3 explains how the preference elicitation process can be supported. The functionalities of the software are presented through an illustrative example in Section 4.

2. ELECTRE TRI: a brief reminder

ELECTRE TRI is a multiple criteria sorting method, i.e, a method that assigns alternatives to pre-defined catagories. The assignment of an alternative a results from the comparison of a with the profiles defining the limits of the categories. Let F denote the set of the indices of the criteria $g_1, g_2, \ldots, g_m$ ($F = \{1, 2, \ldots, m\}$) and B the set of indices of the profiles defining $p+1$ catagories ($B = \{1, 2, \ldots, p\}$), b_h being the upper limit of category C_h and the lower limit of category C_{h+1}, $h = 1, 2, \ldots, p$ (see Fig. 1, where the profiles b_{p+1} and b_0 correspond to the ideal and anti-ideal alternatives, respectively). In what follows, we will assume, without any loss of generality, that preferences increase with the value on each criterion.

Schematically, ELECTRE TRI assigns alternatives to categories following two consecutive steps:

- construction of an outranking relation S that characterises how alternatives compare to the limits of categories,
- exploitation of the relation S in order to assign each alternative to a specific category.

2.1. Construction of the outranking relation

ELECTRE TRI builds an outranking relation S, i.e., validates or invalidates the assertion aSb_h (and b_hSa), whose meaning is "a is at least as good as b_h". Preferences restricted to the significance

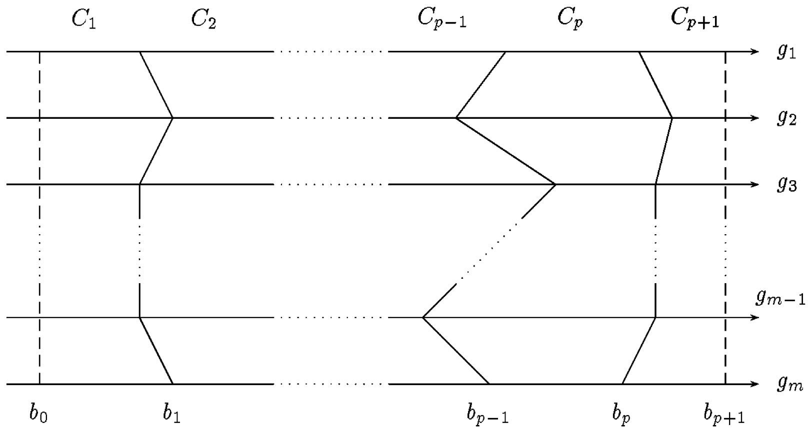

Fig. 1. Definition of categories using limit profiles.

axis of each criterion are defined through pseudo-criteria (see [25] for details on this double-threshold preference representation). The indifference and preference thresholds ($q_j(b_h)$ and $p_j(b_h)$) constitute the intra-criterion preferential information. They account for the imprecise nature of the evaluations $g_j(a)$ (see [26]). $q_j(b_h)$ specifies the largest difference $g_j(a) - g_j(b_h)$ that preserves indifference between a and b_h on criterion g_j;$p_j(b_h)$ represents the smallest difference $g_j(a) - g_j(b_h)$ compatible with a preference in favor of a on criterion g_j.

Two types of inter-criteria preference parameters intervene in the construction of S:

- the set of weight-importance coefficients $(k_1, k_2, \dots, k_m)$ is used in the concordance test when computing the relative importance of the coalitions of criteria being in favor of the assertion aSb_h,
- the set of veto thresholds $(v_1(b_h), v_2(b_h), \dots, v_m(b_h))$ is used in the discordance test; $v_j(b_h)$ represents the smallest difference $g_j(b_h) - g_j(a)$ incompatible with the assertion aSb_h.

ELECTRE TRI builds an outranking relation S using an index $\sigma(a, b_h) \in [0, 1]$ ($\sigma(b_h, a)$, resp.) that represents the degree of credibility of the assertion aSb_h (b_hSa, resp.), $\forall a \in A, \forall h \in B$.

2.2. Exploitation procedure

As the assignment of alternatives to categories does not result directly from the relation S, an exploitation phase is necessary; it requires the relation S to be "defuzzyfied" using a so called λ-cut: the assertion aSb_h (b_hSa, resp.) is considered to be valid if $\sigma(a, b_h) \geqslant \lambda(\sigma(b_h, a) \geqslant \lambda$, resp.), λ being a "cutting level" such that $\lambda \in [0.5,1]$. This λ-cut determines the preference situation between a and b_h:

- $\sigma(a, b_h) \geqslant \lambda$ and $\sigma(b_h, a) \geqslant \lambda \Rightarrow aSb_h$ and $b_hSa \Rightarrow aIb_h$, i.e., a is indifferent to b_h,
- $\sigma(a, b_h) \geqslant \lambda$ and $\sigma(b_h, a) < \lambda \Rightarrow aSb_h$ and not $b_hSa \Rightarrow a \succ b_h$, i.e., a is preferred to b_h (weakly or strongly),

- $\sigma(a, b_h) < \lambda$ and $\sigma(b_h, a) \geqslant \lambda \Rightarrow$ not aSb_h and $b_hSa \Rightarrow b_h \succ a$, i.e., b_h is preferred to a (weakly or strongly),
- $\sigma(a, b_h) < \lambda$ and $\sigma(b_h, a) < \lambda \Rightarrow$ not aSb_h and not $b_hSa \Rightarrow aRb_h$, i.e., a is incomparable to b_h.

Remark that b_0 and b_{p+1} are defined such that $b_{p+1} \succ a$ and aSb_0, $\forall a \in A$. The role of the exploitation procedure is to analyse the way in which an alternative a compares to the profiles so as to determine the category to which a should be assigned. Two assignment procedures are available:

Pessimistic (or conjunctive) procedure:

(a) compare a successively to b_i, for $i = p, p-1, \ldots, 0$,
(b) b_h being the first profile such that aSb_h,
assign a to category C_{h+1} $(a \rightarrow C_{h+1})$.

Optimistic (or disjunctive) procedure:

(a) compare a successively to b_i, $i = 1, 2, \ldots, p+1$,
(b) b_h being the first profile such that $b_h \succ a$,
assign a to category C_h $(a \rightarrow C_h)$.

If b_{h-1} and b_h denote the lower and upper profile of the category C_h, the pessimistic (or conjunctive) procedure assigns alternative a to the highest category C_h such that a outranks b_{h-1}, i.e., aSb_{h-1}. When using this procedure with $\lambda = 1$, an alternative a can be assigned to category C_h only if $g_j(a)$ equals (up to a threshold) or exceeds $g_j(b_h)$ for each criterion (conjunctive rule).

The optimistic (or disjunctive) procedure assigns a to the lowest category C_h for which the lower profile b_h is preferred to a, i.e., $b_h \succ a$. When using this procedure with $\lambda = 1$, an alternative a can be assigned to category C_h when $g_j(b_h)$ exceeds $g_j(a)$ (by some threshold) for at least one criterion (disjunctive rule). When λ decreases, the conjunctive and disjunctive characters of these rules are weakened.

3. Support for parameters elicitation: ELECTRE TRI assistant

One of the main difficulties that an analyst must face when interacting with a DM in order to build a decision aid procedure is the elicitation of various parameters of the DM's preference model. In the ELECTRE TRI method, the analyst should assign values to profiles, weights and thresholds (see Section 2). Even if these parameters can be interpreted, it is difficult to fix directly their values and to have a clear global understanding of the implications of these values in terms of the output of the model.

Mousseau and Slowinski [27] proposed a methodology that avoids this problem by substituting assignment examples for direct elicitation of the model parameters. The values of the parameters are inferred through a certain form of regression on assignment examples. ELECTRE TRI Assistant implements this methodology in a way that requires from the DM much less cognitive effort: the elicitation of parameters is done indirectly using holistic information given by the DM through assignment examples, i.e., alternatives assigned by the DM to categories according to his/her comprehensive preferences.

Assuming that a specific subset of parameters (possibly all of them) is to be assessed from assignment examples, a mathematical program infers the values for these parameters that best restitutes the assignment examples (the general form of the mathematical program to be solved is

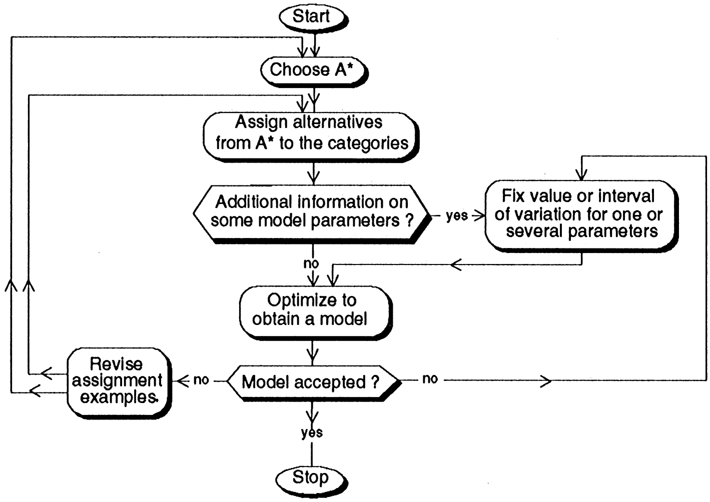

Fig. 2. General scheme of the use of ELECTRE TRI Assistant.

given in Appendix A). This is done in the course of an interactive process whose general scheme is presented in Fig. 2. Its aim is to find an ELECTRE TRI model as compatible as possible with the assignment examples given by the DM. The assignment examples concern a set A^* of, so-called, reference alternatives for which the DM has clear opinion, i.e., alternatives that the DM can easily assign to a category, taking into account their evaluation on all criteria. The reference alternatives can correspond to past decisions of the DM or to ficticious alternatives designed to make hypothetical assignments. The compatibility between the ELECTRE TRI model and the assignment examples is understood as an ability of the ELECTRE TRI method using this model to reassign the alternatives from A^* in the same way as the DM did.

In order to minimize the difference between the assignments made by ELECTRE TRI and the assignments made by the DM, an optimization procedure is used. The DM can tune up the model in the course of an interactive procedure. He/she may either (1) revise the assignment examples or (2) change the set of paramters to be optimized or (3) fix values (or intervals of variation) for some model parameters. In the first case, the DM may:

- remove and/or add some alternatives from/to A^*,
- change the assignment of some alternatives from A^*.

In the second case, he/she may remove and/or add some parameters from the set of those that are to be optimized.

In the last case, the DM can give additional information on the range of variation of some model parameters based on his/her own intuition. For example, he/she may specify:

- ordinal information on the importance of criteria,
- noticeable differences on the scales of criteria,
- incomplete definition of some profiles defining the limits between categories.

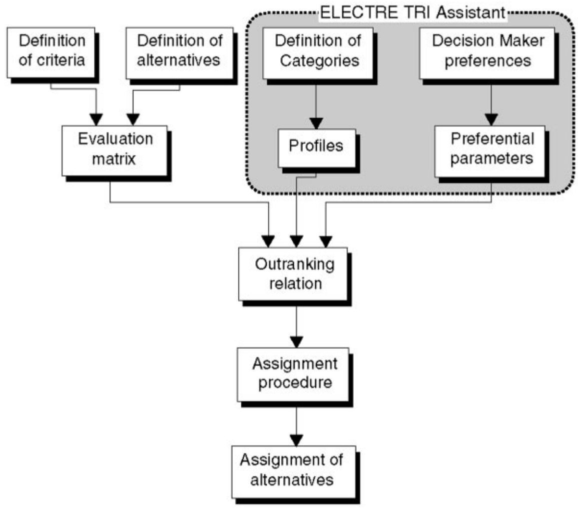

Fig. 3. General scheme of the use of ELECTRE TRI.

When the model is not perfectly compatible with the assignment examples, the procedure is able to detect all "hard cases", i.e., the alternatives for which the assignment computed by the model strongly differs from the DM's assignment. The DM is then asked to reconsider his/her judgement.

To get a representative model, the subset A^* must be defined such that the numbers of alternatives assigned to the categories are almost equal and sufficiently large to "contain enough information". The empirical behavior of the inference procedure has been studied in [28]. These experiments show that $2m$ (m being the number of criteria) is a sufficient number of assignment examples to infer the weights k_j and the cutting level λ (The other parameters being fixed).

The approach used in ELECTRE TRI Assistant is concordant with the principle of posterior rationality (see [29]) and with the aggregation–disaggregation paradigm used for the construction of a preference model in UTA-like procedures (see [30–33]). It has been also applied for the elicitation of weights used for the construction of an outranking relation in the DIVAPIME method (see [34]).

4. Implementation in the decision aid process

4.1. Structure of the decision aid process

Decision aid processes are never sequential; the different phases for the definition of an assignment model interact (for example, the assignment of some alternatives may reveal the necessity of an additional criterion). However, the general scheme of use of ELECTRE TRI method can be schematically represented in Fig. 3.

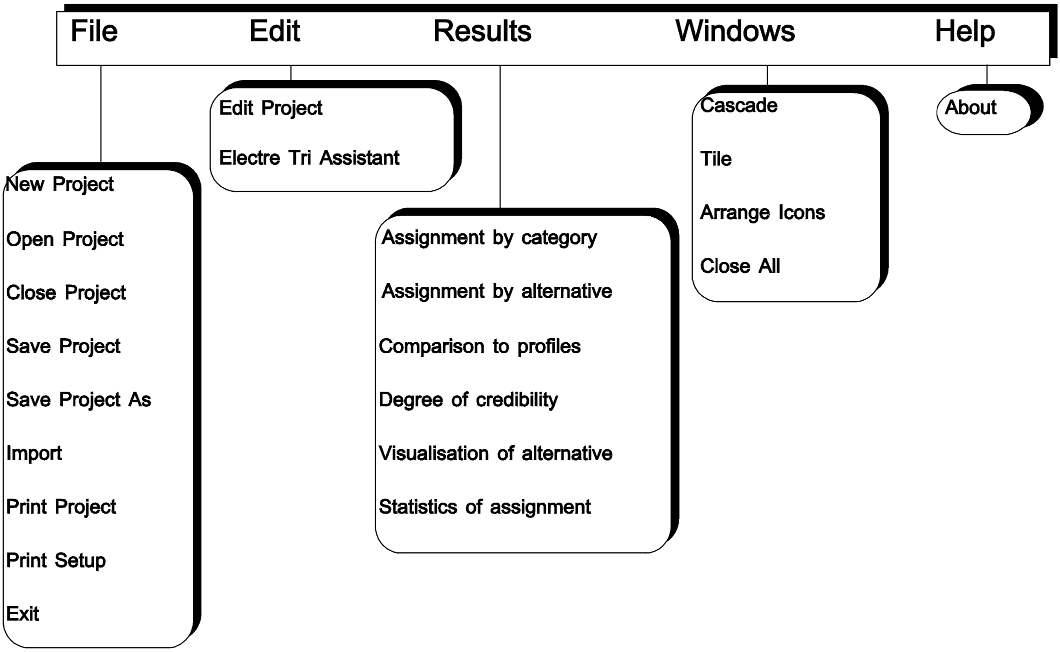

Fig. 4 . Description of the main options of the software.

4.2. Main functionalities

The ELECTRE TRI Software 2.0 has been written in the C++ programming language using the Microsoft Windows interface. The minimal hardware and software requirements are the following:

- IBM-PC compatible computer (Pentium processor with 16MB RAM),
- Microsoft Windows (3.1 or higher).

The structure of the options available in the software is described hereafter in Fig. 4. The contents of the different options is the following:

- **File**: this options allows the user to create a new project, load an existing project and save the current project. Additional print and import options are provided. Generation of project reports is also available.
- **Edit**: enables the user to enter the data required by ELECTRE TRI (criteria, alternatives, weights, profiles and thresholds) and/or to use the ELECTRE TRI assistant functionalities.
- **Results**: allows the user to visualise the results (including intermediary results such as degree of credibility of the outranking relation, comparison of alternatives to profiles, …); also gives graphical representation of alternatives and profiles.
- **Windows**: gives the possibility to organize the appearance of the windows on the screen.
- **Help**: provides the user an online help.

Table 1
Values of preference parameters [8]

k_j	g_1	g_2	g_3	g_4	g_5	g_6	g_7
	3.0	1.0	1.0	1.0	1.0	1.0	2.0
$g_j(b_2)$	14.0	29.0	29.0	29.0	29.0	29.0	27.0
$q_j(b_2)$	0.64	1.56	1.56	1.56	1.56	1.56	2.04
$p_j(b_2)$	1.28	3.17	3.17	3.17	3.17	3.17	4.15
$g_j(b_1)$	17.0	43.0	43.0	43.0	43.0	43.0	40.0
$q_j(b_1)$	0.67	1.84	1.84	1.84	1.84	1.84	2.56
$p_j(b_1)$	1.34	3.73	3.73	3.73	3.73	3.73	5.19

To illustrate the main functionalities of the new ELECTRE TRI Software, we consider a real-world application in the banking sector. Using this data we will simulate a posteriori how the modelling process could have taken place using our new ELECTRE TRI Software. More specifically, we will show how the ELECTRE TRI Assistant functionalities can support the preference elicitation process in a way that is methodologically valid (the values assigned to the parameters correspond to what the DM want them to be) and that reduces the cognitive effort required from the DM (the interaction between the analyst and the DM does not require the DM to understand the precise meaning of each parameter).

We consider a model that aims at assigning alternatives to three categories using ELECTRE TRI on the basis of their evaluations on seven criteria. All criteria are evaluated on a [0–100] scale and have a decreasing direction of preference (the lower the evaluation, the better the alternative). Forty-five alternatives are considered (their evaluations on criteria are presented in Appendix B). The parameters of the model are fixed by the DM as shown in Table 1. The cutting level λ is set equal to 0.86.

When beginning a session with ELECTRE TRI, the user should edit the data concerning the criteria, profiles, thresholds and weights. This can be done (when the user does not the ELECTRE TRI assistant functionalities) by choosing the Edit window (see Fig. 5) from the menu through the *Edit project* option (see Fig. 4). This Edit window is divided into two parts: the left side of the window gives a list of the elements to be edited (criteria, profiles and alternatives) and the data relative to the element selected in the list is input on the right side of the window using different folders. In Fig. 5, profile b_2 is selected and the thresholds attached to this profile are input on the *Thresholds* folder.

When the whole dataset is input, the user can obtain the results of both assignment procedures through the *Results* option:

- final results: assignment of alternatives (listed by categories or by alternatives), see Fig. 6,
- statistics of assignment, i.e., proportion of alternatives assigned to each category by the optimistic and the pessimistic procedure,
- intermediary results: degree of credibility of the outranking relation $\sigma(a, b_h)$ and $\sigma(a, b_h)$, comprehensive comparison of alternatives to profiles,
- visual representation of alternatives and profiles.

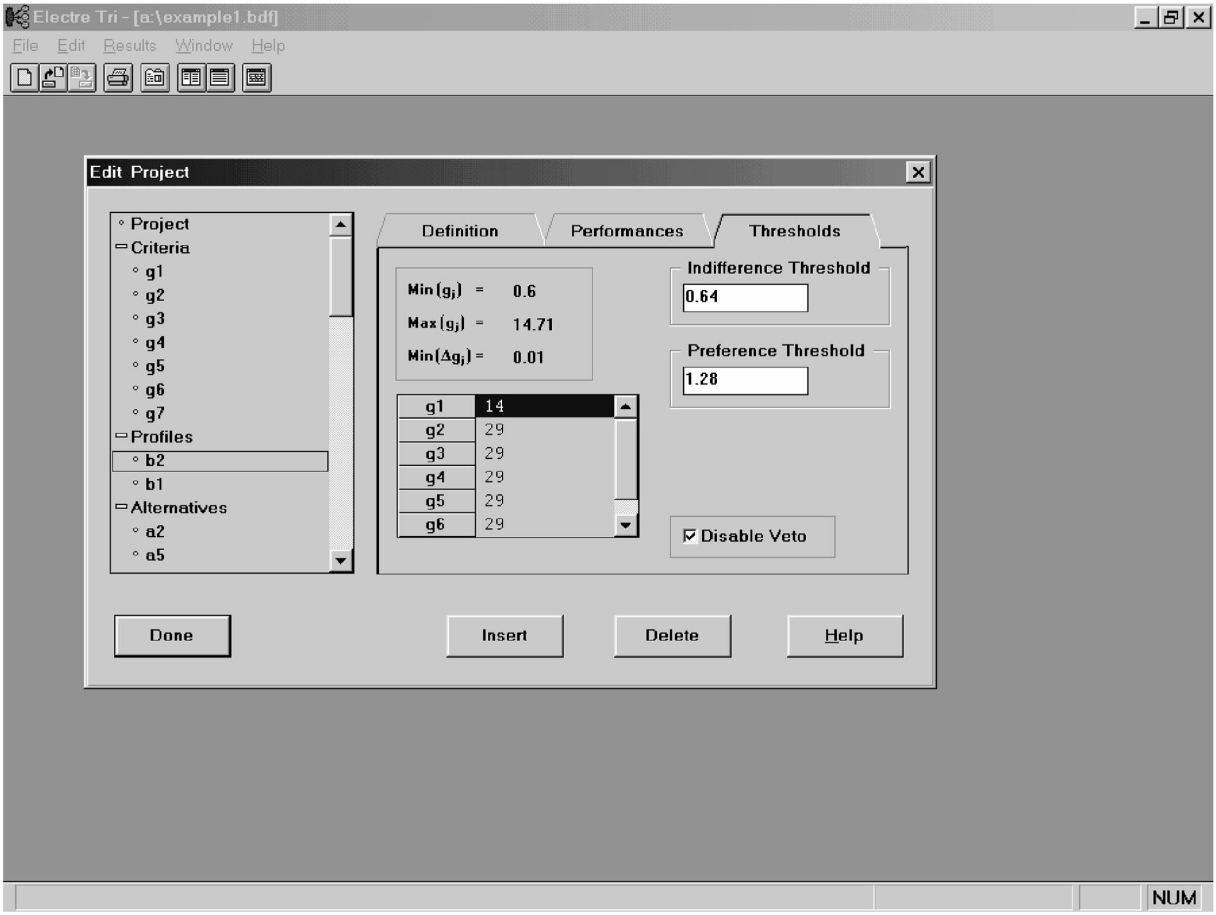

Fig. 5 . Edit window.

4.3. *Advanced functionalities: ELECTRE TRI assistant*

The ELECTRE TRI Software 2.0 includes an assistant that is able to infer preference model parameters from assignment examples provided by the user (the structure of the options available from the ELECTRE TRI Assistant submenu is described in Fig. 7). The present version supports the user in defining the weights of criteria and the cutting level λ for the pessimistic assignment procedure only (the next version will include similar functionalities for profiles and thresholds). The use of ELECTRE TRI Assistant functionalities proceeds according to the following scheme:

(a) Input the list of assignment examples composed of alternatives for which the DM gives a holistic assignment (such alternative can be an existing alternative of a fictitious one designed for this purpose); imprecise assignments are accepted, i.e., the DM can express an hesitation in the assignment of an alternative *a* by specifying a subset of consecutive categories to which *a* could be assigned.

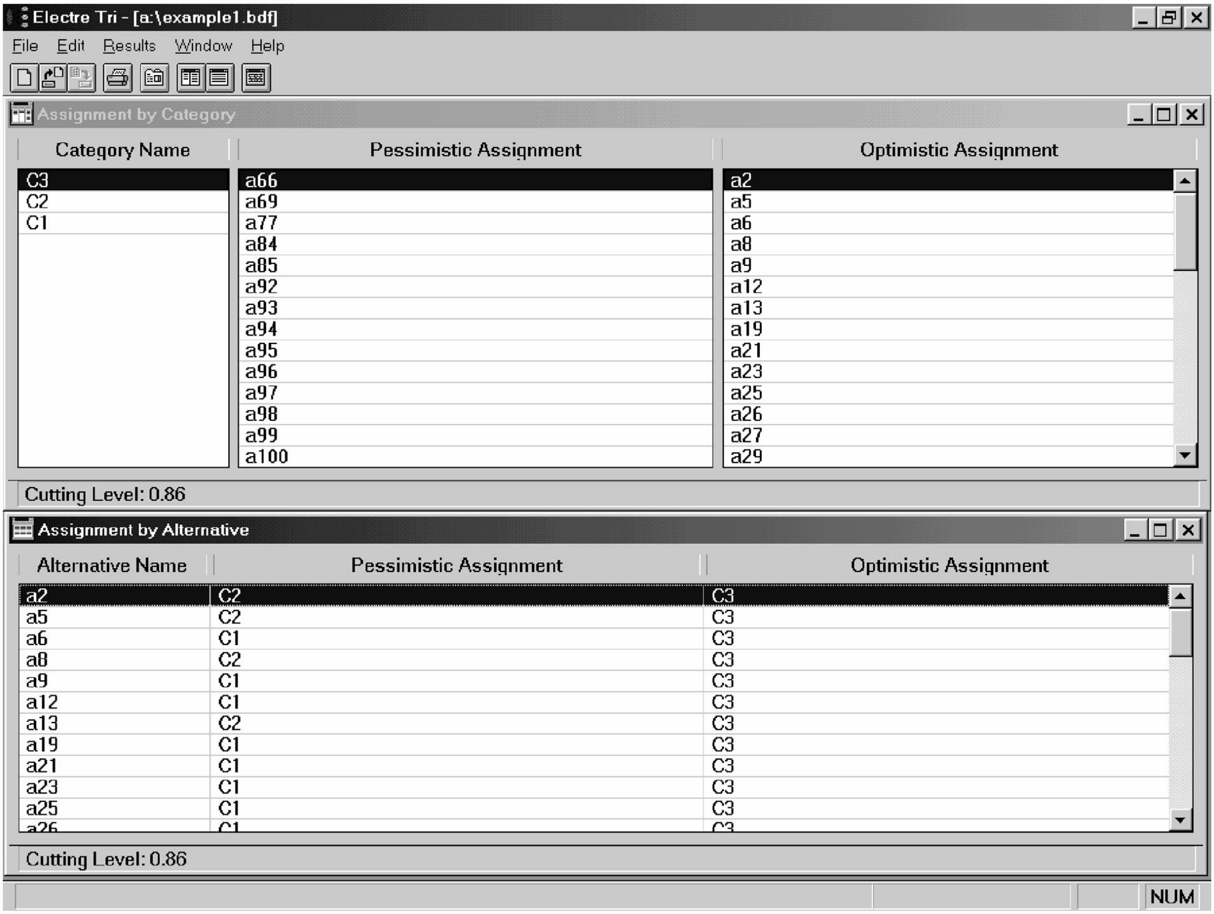

Fig. 6 . Results: Assignment of alternatives.

(b) Give preferential information on the weights (preorder, comparisons of specific coalitions, bounds on weights) — optional.
(c) Run the inference procedure to find the most adequate values of the weights.
(d) Check for the acceptability of the obtained weight vector and, either:

- accept the proposed weights so as to use it by the assignment procedure,
- or reject it and revise the information provided in the step (a) and/or (b), then perform (c) again.

The invitation window of ELECTRE TRI Assistant is presented in Fig. 8. Every ELECTRE TRI Assistant option is available from this screen; moreover, it is possible to interrupt an ELECTRE TRI Assistant session by saving the assistant data (*.eta file) and loading it back when continuing the session.

So as to illustrate this general scheme, let us consider a hypothetic preference elicitation process. Let us suppose that our fictitious DM is able to elicit directly the profiles and thresholds as

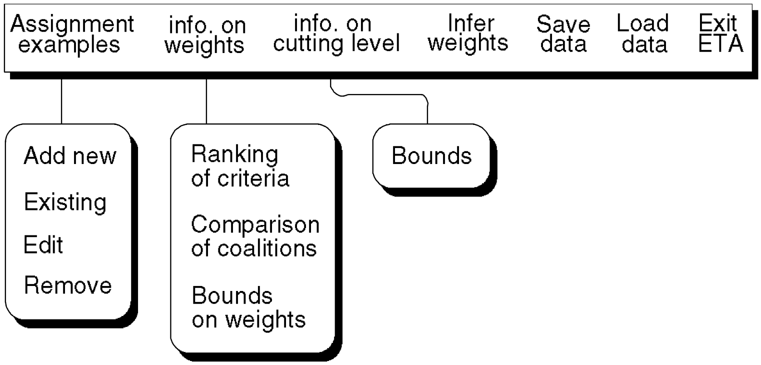

Fig. 7. Options available from the ELECTRE TRI Assistant sub-menu.

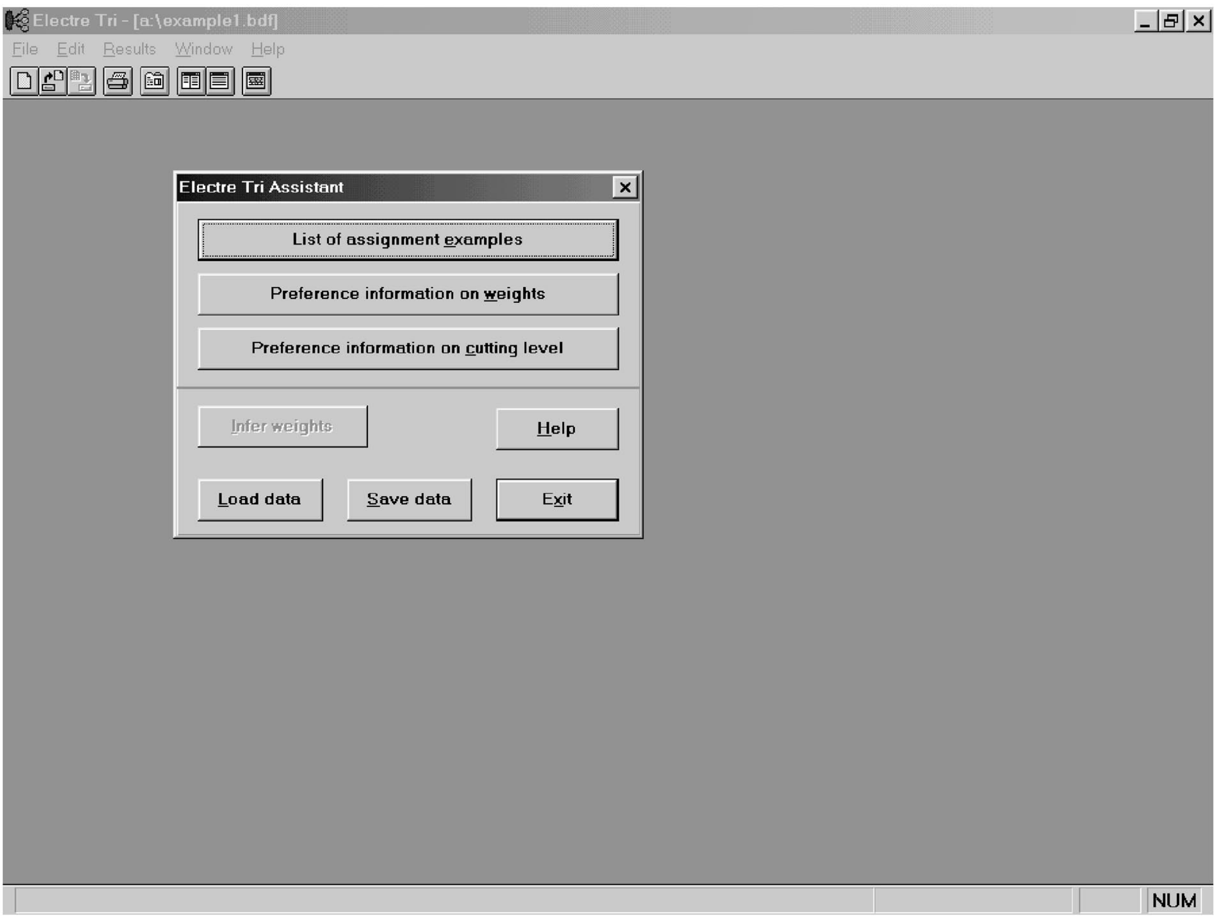

Fig. 8. ELECTRE TRI Assistant menu.

Table 2
Set of assignment examples

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	Desired category
a_{19}	13.02	15.74	18.02	7.24	79.21	42.63	79.32	C_1
a_{23}	13.66	11.01	14.11	70.55	69.01	18.77	42.39	C_1
a_{41}	13.04	7.99	22.44	7.24	31.40	14.83	58.65	C_1
a_{49}	13.48	1.05	18.02	6.45	31.40	18.77	100.00	C_1
a_{68}	9.91	7.99	14.11	7.24	12.92	3.02	58.65	C_1
a_5	14.43	11.01	18.02	29.25	22.16	31.73	39.44	C_1
a_{27}	13.51	14.02	18.02	29.25	22.16	8.91	39.44	C_1 or C_2
a_{29}	13.39	11.01	18.02	17.36	22.16	8.91	39.44	C_2
a_{55}	12.14	7.99	18.02	5.46	22.16	3.02	39.44	C_2
a_{62}	11.07	7.99	18.02	5.46	12.92	3.02	39.44	C_2
a_{66}	10.25	7.99	14.11	15.58	12.92	8.91	20.24	C_3
a_{69}	10.65	11.01	10.47	3.69	3.76	8.91	20.24	C_3
a_{84}	8.26	7.99	10.47	3.69	3.76	3.02	20.24	C_3
a_{93}	3.68	1.96	6.74	15.58	3.76	3.02	20.24	C_2 or C_3
a_{94}	3.86	1.96	3.02	3.69	3.76	3.02	20.24	C_3

presented in Table 1 but has difficulties with expressing directly the importance coefficients. Instead, our DM is able to express some assignment examples. These examples are reported in Table 2 and correspond to a subset $A^* \subset A$ of 15 alternatives intuitively assigned by the DM to a specific category. Let us remark that these examples correspond to the assignments of these alternatives done by ELECTRE TRI pessimistic procedure for the complete preference information given in Table 1 with the only exceptions of alternatives a_5, a_{27} and a_{93} for which the desired categories are C_1, C_1 or C_2, and C_2 or C_3 instead of C_2, C_2 and C_3, respectively.

ELECTRE TRI Assistant gives the possibility to edit these assignment examples through the screen shown in Fig. 9. Let us suppose that the DM wants to get a first proposal for the weights without giving any additional information.

The output of the computations (see Fig. 10) shows that the inferred model is not able to assign all alternatives to their respective "desired" category; a_5 is assigned to C_2 instead of C_1 and appears highlighted on the screen.

Considering this first result, let us suppose that the DM revises his/her judgement concerning the assignment of alternative a_5 (by stating that a_5 should be assigned to C_1 or C_2 in the Edit Assignment Examples window, see Fig. 9) and reruns the optimization phase. After this second optimization phase, all alternatives are assigned using the inferred weights consistently with the DM. The inferred weight vector (0.048, 0.048, 0.048, 0.048, 0.349, 0.048, 0.413) and $\lambda = 0.793$ are displayed on the screen (see Fig. 11).

Our DM considers that the obtained weights do not express adequately his/her opinion concerning the importance of criteria. He/she can add constraints by stating:

- a pre-order on criteria according to their relative importance,
- comparisons of coalitions of criteria,
- bounds for importance coefficients k_j.

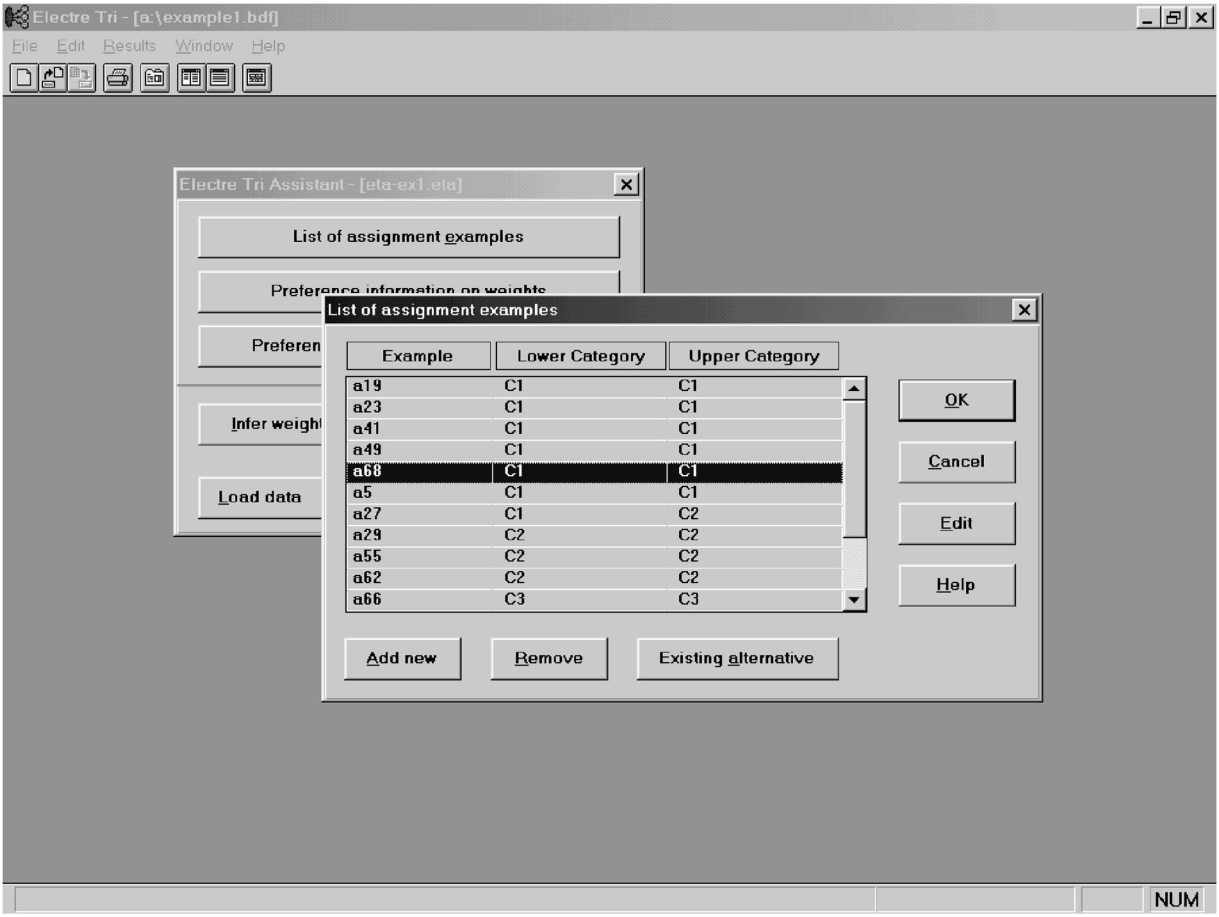

Fig. 9. Edit assignment examples window.

As our DM is surprised that the inferred weight of g_1 is low; he/she imposes g_1 to be the most important criterion, moreover he/she would like to add intuitive information concerning the ranking of criteria in terms of importance. In consequence, he/she specifies the following importance ranking on criteria:

$$g_1 \geqslant g_7 \geqslant g_2 \approx g_3 \approx g_4 \approx g_5 \approx g_6, \quad \text{i.e., } k_1 > k_7 > k_2 = k_3 = k_4 = k_5 = k_6.$$

Further, the DM considers that a preference on criterion g_1 is at least as important as g_3 and g_7 together. This information can be expressed through a comparison of coalitions of criteria in terms of importance:

$$g_1 \succcurlyeq \{g_3, g_7\}, \quad \text{i.e., } k_1 \geqslant k_3 + k_7.$$

Such information can be stated in the screen shown in Fig. 12. The optimization phase taking into account additional preference information is then performed again.

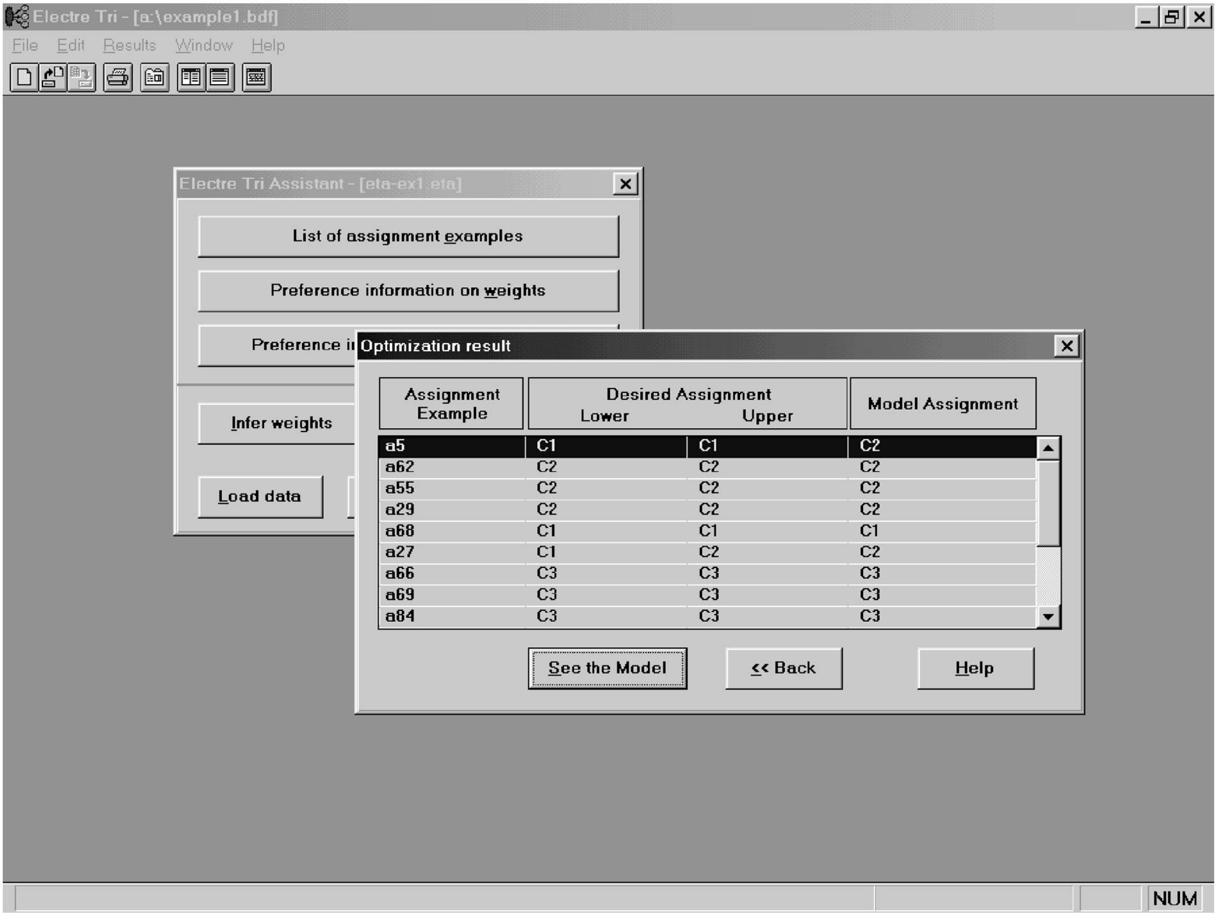

Fig. 10. Results of the inference phase.

As a result of optimization, the inferred weights still assign all alternatives consistently with the DM. The inferred weight vector (0.301, 0.107, 0.097, 0.097, 0.097, 0.097, 0.203) and $\lambda = 0.897$ and displayed on a screen similar to the one in Fig. 11. As the DM considers these weights to reflect adequately his/her opinion concerning the importance of criteria, he/she may take this vector into account (by clicking on the Accept button) so as to assign all other alternatives using these weights and the cutting level λ.

Let us remark that, in general, the weights k_j and the cutting level λ corresponding to an optimal value of the objective function (see (1) in Appendix A) are not unique. In order to learn about possible combinations of values of these parameters at the optimum, we could generate all alternative optimal solutions of the optimization problem ((A.1)–(A.7), Appendix A). Unfortunately, this is a computationally hard combinatorial problem. Moreover, the DM would have difficulties with using such a set of all possible combinations in the interactive process. For these reasons, such post-optimal analysis of the optimisation problem seems less useful than the

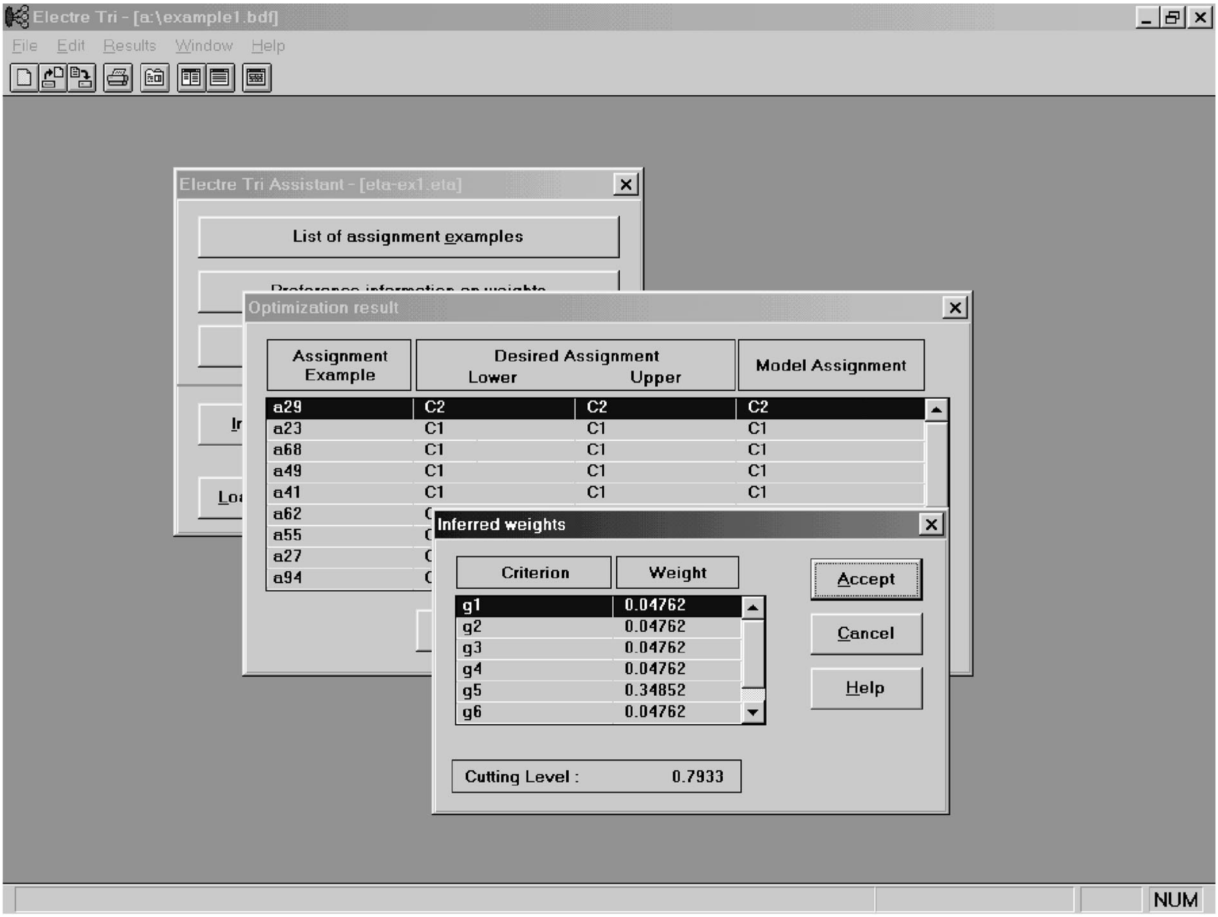

Fig. 11. Inferred weight vector.

possibility given in ELECTRE TRI Assistant to base the interaction on holistic information (assignment examples, ranking of criteria, comparisons of coalitions of criteria, …).

5. Conclusions

A new implementation of the ELECTRE TRI method is presented. Our attention has been focused on preference elicitation support. ELECTRE TRI Assistant is a specific tool that aims at reducing the cognitive effort required from the DM in the phase of calibration of the model (determination of the model parameters).

ELECTRE TRI Assistant proceeds using assignment examples which are alternatives that the DM assigns intuitively according to his/her expertise and preferences. Such information is more easily provided by DMs than values of preferential parameters; DMs are more comfortable exercising their expertise rather than analysing it. An inference procedure is provided in order to

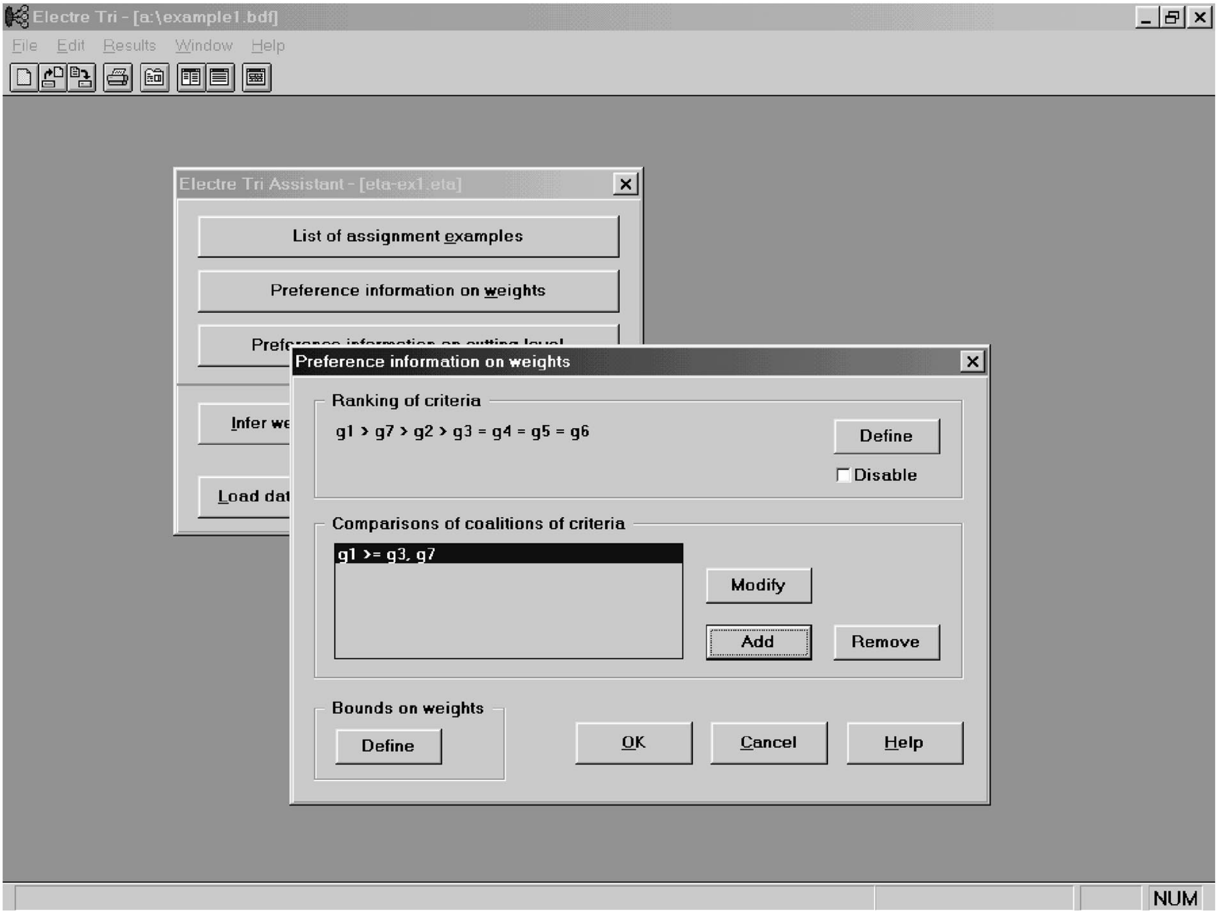

Fig. 12. Additional preferential information supplied by the DM.

determine the values of the ELECTRE TRI model parameters that best "match" these assignment examples. This procedure is able to identify the examples which create troubles during the inference process, i.e., those that correspond to rather untypical examples. The inference procedure is integrated in a trial-and-error interactive process in which the DM can check what is the impact of modifications of the input on the result of the inference procedure. ELECTRE TRI Assistant is enhancing the learning spirit of the preference elicitation. The software has been presented through an illustrative example.

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Appendix A. Mathematical programming formulation of the inference problem (see [27])

In order to infer the parameters of Electre Tri pessimistic assignment procedure (without veto) form assignment examples, optimization problem to be solved is as follows.

$$\text{Max} \quad \alpha + \varepsilon \sum_{a_k \in A^*} (x_k + y_k) \tag{A.1}$$

$$\text{s.t.} \quad \frac{\sum_{j=1}^m k_j c_j(a_k, b_{h_k - 1})}{\sum_{j=1}^m k_j} - \alpha - \lambda \geqslant 0, \quad \forall a_k \in A^*, \tag{A.2}$$

$$\lambda - \frac{\sum_{j=1}^m k_j c_j(a_k, b_{h_k - 1})}{\sum_{j=1}^m k_j} - \alpha \geqslant 0, \quad \forall a_k \in A^*, \tag{A.3}$$

$$\lambda \in [0.5, 1], \tag{A.4}$$

$$g_j(b_{h+1}) \geqslant g_j(b_h) + p_j(b_h) + p_j(b_{h+1}), \quad \forall j \in F, \ \forall h \in B, \tag{A.5}$$

$$p_j(b_h) \geqslant q_j(b_h), \quad \forall j \in F, \ \forall h \in B, \tag{A.6}$$

$$k_j > 0, q_j(b_h) \geqslant 0, \quad \forall j \in F, \ \forall h \in B, \tag{A.7}$$

where x_k and y_k represent slack variables whose meaning is such that all alternatives from the reference set A^* are "correctly" assigned for all $\lambda' \in [\lambda - \min_{a_k \in A^*}(y_k), \ \lambda + \min_{a_k \in A^*}(x_k)]$. When $\varepsilon = 0$, any non-negative value for the objective function guarantees existence of model parameters that permits "correct" assignment of all alternatives from A^*.

Because of constraints (A.2) and (A.3), the above problem is a non-linear programming problem. For n, m and p denoting the number of assignment examples, of criteria and of profiles, respectively, this problem contains $3mp + m + 2$ variables and $4n + 3mp + 2$ constraints.

The optimization problem becomes linear when optimization is limited to the inference of weights and of the cutting level, while $\sum_{j=1}^m k_j = 1$. The size of this LP being small, it can be solved by any commercial solver in a reasonable computing time.

When additional preference information on dependencies among the weights or on the range of their variation is given, constraints of the following form should be considered in the above formulation:

- $\sum_{j \in J_1} k_j \left\{ \begin{matrix} > \\ \geqslant \\ = \end{matrix} \right\} \sum_{j \in J_2} k_j$, where $J_1, J_2 \subset F$, $J_1 \cap J_2 = \emptyset$,
- $b_j \leqslant k_j \leqslant B_j$, $j \in F$, where $b_j, B_j \in (0, 1]$,

Moreover, it is always possible to limit the range of variation of the cutting level using the condition $\lambda_{\min} \leqslant \lambda \leqslant \lambda_{\max}$.

Appendix B

Assignment of alternatives using ELECTRE TRI pessimistic procedure with the initial weights is given in Table 3.

Table 3

Name	g_1	g_2	g_3	g_4	g_5	g_6	g_7	Categ.
a_6	14.15	11.01	18.02	19.13	49.87	14.83	58.65	C_1
a_9	14.54	14.02	18.02	4.65	12.92	3.02	58.65	C_1
a_{12}	14.42	11.01	18.02	4.65	12.92	8.91	58.64	C_1
a_{19}	13.02	15.74	18.02	7.24	79.21	42.63	79.32	C_1
a_{21}	13.60	4.02	22.44	4.65	31.40	31.73	58.65	C_1
a_{23}	13.66	11.01	14.11	70.55	69.01	18.77	42.39	C_1
a_{25}	13.04	7.99	22.44	40.19	40.64	6.96	100.0	C_1
a_{26}	12.97	7.99	18.02	29.66	31.40	18.77	100.0	C_1
a_{41}	13.04	7.99	22.44	7.24	31.40	14.83	58.65	C_1
a_{49}	13.48	1.05	18.02	6.45	31.40	18.77	100.0	C_1
a_{58}	10.41	1.96	18.02	19.13	22.16	18.77	100.0	C_1
a_{68}	9.91	7.99	14.11	7.24	12.92	3.02	58.65	C_1
a_{73}	8.65	1.96	18.02	71.48	59.11	7.88	60.12	C_1
a_{81}	7.58	1.96	18.02	71.48	49.87	7.88	60.12	C_1
a_{87}	6.46	4.98	18.02	50.78	12.92	6.96	100.0	C_1
a_2	14.64	14.02	18.02	5.46	22.16	3.02	39.44	C_2
a_5	14.43	11.01	18.02	29.25	22.16	31.73	39.44	C_2
a_8	14.71	14.02	18.02	6.43	12.92	3.02	39.44	C_2
a_{13}	14.65	14.02	18.02	6.43	22.16	14.83	39.44	C_2
a_{27}	13.51	14.02	18.02	29.25	22.16	8.91	39.44	C_2
a_{29}	13.39	11.01	18.02	17.36	22.16	8.91	39.44	C_2
a_{37}	13.52	11.01	14.11	6.43	22.16	3.02	39.44	C_2
a_{38}	13.39	11.01	18.02	5.46	22.16	3.02	39.44	C_2
a_{40}	13.40	7.99	14.11	6.43	22.16	8.91	39.44	C_2
a_{48}	13.24	4.98	14.11	6.43	22.16	20.32	39.44	C_2
a_{55}	12.14	7.99	18.02	5.46	22.16	3.02	39.44	C_2
a_{59}	11.01	7.99	18.02	17.36	22.16	3.02	39.44	C_2
a_{76}	8.76	4.98	14.11	17.36	22.16	3.02	39.44	C_2
a_{88}	6.65	4.98	14.11	5.46	3.76	14.83	39.44	C_2
a_{90}	4.06	1.05	18.02	39.78	12.92	3.02	39.44	C_2
a_{91}	4.64	1.96	6.74	5.46	3.76	3.02	39.44	C_2
a_{66}	10.25	7.99	14.11	15.58	12.92	8.91	20.24	C_3
a_{69}	10.65	11.01	10.47	3.69	3.76	8.91	20.24	C_3
a_{77}	9.40	7.99	10.47	3.69	3.76	8.91	20.24	C_3
a_{84}	8.26	7.99	10.47	3.69	3.76	3.02	20.24	C_3
a_{85}	8.45	7.99	6.74	3.69	3.76	8.91	20.24	C_3
a_{92}	5.11	4.98	3.02	3.69	3.76	3.02	20.24	C_3
a_{93}	3.68	1.96	6.74	15.58	3.76	3.02	20.24	C_3
a_{94}	3.86	1.96	3.02	3.69	3.76	3.02	20.24	C_3
a_{95}	2.56	1.96	6.74	15.58	3.76	14.83	20.24	C_3
a_{96}	2.81	1.96	3.02	3.69	5.41	3.02	20.24	C_3
a_{97}	1.04	1.05	14.11	49.90	3.76	3.02	20.24	C_3
a_{98}	1.48	1.05	3.02	15.58	3.76	3.02	20.24	C_3
a_{99}	1.68	1.96	3.02	15.58	5.41	3.02	20.24	C_3
a_{100}	0.60	1.05	0.71	3.69	5.41	3.02	20.24	C_3

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