EMPIRICAL VIOLATION OF THE THIRD ALTERNATIVE INDEPENDANCE PRINCIPLE: IMPLICATIONS FOR MULTICRITERIA DECISION AID

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Abstract: Multiple criteria decision aiding methodologies rely on basic principles concerning the consistency of decision maker's (DM's) preferences. Among these principles, the Third Alternative Invariance (TAI) principle states that elicited preferences on two alternatives should not be affected by the presence or availability of a third alternative. Empirical statements on cognitive limitations of decision makers tends to contradict this principle. Our work aims at analysing the behavior of DMs when answering binary and ternary choice questions. Results show a violation of the TAI principle. Such results may be explained by a shift in the reference points considered by decision makers in the binary and ternary choice questions. Implications for preference elicitation techniques in multiple criteria decision aid are discussed.

Introduction

Decision processes can be approached as involving several objectives, viewpoints or criteria, which may often be contradictory. In order to cope with these conflicts, researchers in the field of Multiple Criteria Decision Aid (MCDA) have proposed several methodologies, and corresponding elicitation techniques (see [7], [2]). These methods are grounded on hypothesis concerning the consistency of preferences. One of them is the *procedure invariance principle* which states that normatively equivalent preference elicitation procedures should lead to the same elicited preferences. The elicitation techniques are then no more and no less than tools used

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to measure an "objective reality". Many of these methods consist in proposing alternatives between which decision makers (DMs) should express their preferences. According to procedure invariance, the number of alternatives proposed to DMs in a choice should have no influence upon elicited preference. More precisely, the Third Alternative Independence (T.A.I.) principle assumes that the addition of a supplementary alternative to a choice set containing two alternatives does not provoke any change in the elicited preferences concerning the initial set.

In the early seventies, many works in experimental psychology have questioned procedure invariance. [3] and [4] first outlined preference reversals in a risky decision context. As far as T.A.I. principle is concerned, such phenomena did not inspire much literature, particularly in the domain of decision aid. In a marketing oriented approach, [1] indirectly approached the question in order to test similarity (the "market share" of a product which is added to choice set is taken from the one similarly evaluated products) and regularity (the "market share" of an option cannot be increased by enlarging the choice set, from [9]). [1] called the added alternative "decoy". Their method consisted in two stages. First, the decision makers were asked to choose from two alternatives out of a scale of products. Two or three weeks later, the same alternatives plus a decoy were proposed to the same DMs. Their experiment was realized for only one couple of alternatives, on a sample of 153 persons. [6] take account of this whole literature in a more general context of behavioral research. [9] gave a context based interpretation of phenomena such as T.A.I. violation.

Our objective is to confirm the results of these experiments and to analyse their consequences in terms of decision-aid. The notations used are presented in section 1. Section 2 specifies our hypotheses, and the method for testing them. The concrete framing and the interview-pattern are detailed in section 3. Results are given in section 4 and the last section is devoted to the discussion of these results.

1 Notations

Let us denote by:

- F, a family of n criteria $g_1, g_2, ..., g_n$ $(F = \{1, 2, ..., n\})$,
- X_j, the ordered set of possible evaluations on criterion g_j, $\forall j \in F$,
- P_j and I_j, the preference and indifference relations restricted to the criterion, $\forall j \in F$,
- $X = \prod_{j \in F} X_j$, the set of possible evaluation vectors,
- $x = (x_1, x_2, ..., x_n)$, an evaluation vector corresponding to an alternative a such that $x_j = g_j(a), \forall j \in F$,

- P, the preference relation on X (irreflexive and asymmetric),
- I, the indifference relation (reflexive and symmetric),
- Ψ=(I, P) defines a comprehensive preference system,
- The dominance relation Δ is defined by: $\forall(x,y) \in X^2, x\Delta y \iff x_j \geq y_j, \forall j \in F$,
- The p-dominance relation Δ_p is also defined by: $\forall(x,y) \in X^2, x\Delta_p y \iff x\Delta y \; and \; \exists j \in F \; / \; x_j P_j y_j$. Let us remark that $\Delta_p \subset \Delta$,
- An alternative $x \in F$ is said to be efficient in $Y \subset X$ iff $\forall y \in Y \setminus X, not[y\Delta x]$,
- In a choice set $Y = \{x, y, z\}$, z is asymetrically p-dominated iff $x\Delta_p z$ and not$[y\Delta z]$; in this case, x will be called the p-dominant alternative. The structure of such a set is illustrated in Figure 1 for a bi-criteria problem.

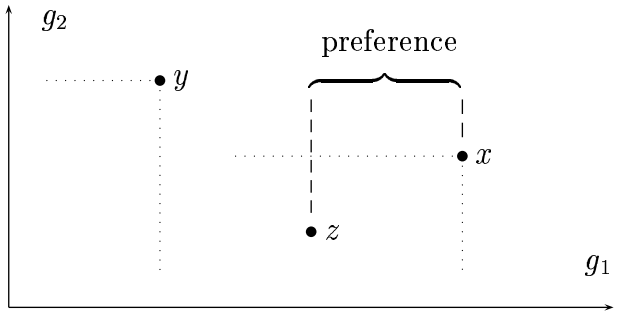


Figure 1: Set $\{x, y, z\}$ in which z is asymetrically p-dominated by x

2 Experimental scheme

2.1 Hypothesis

In our experiment, we aim at testing the third alternative independence (TAI) principle through the following hypothesis: let us consider the two choice sets $\{x, y\}$ and $\{x, y, z\}$ in which x and y are efficient, z being asymmetrically p-dominated and x the p-dominant alternative in $\{x, y, z\}$. In elicitation techniques using the choice questioning mode, if we denote $\Psi'' = (I'', P'')$ ($\Psi''' = (I''', P''')$, respectively) the comprehensive preference system elicited using the binary choice procedure (the ternary choice procedure, respectively), does Ψ''' significantly differs from Ψ'' or not? If the experiment points out differences, the TAI principle will be said to be violated.

If Ψ'' significantly differs from Ψ''', we should further analyse these differences. In such a case, the Uniformity hypothesis states that the differences between Ψ'' and Ψ''' indifferently favor x (the p-dominant alternative) or y when introducing the alternative z in the choice set. In case of violation of the uniformity hypothesis, we will determine the alternative favoured by the changes in elicited preferences. The objective is to check wether the failure of TAI is due to an asymetric dominance effector more generally results from the introduction of a third alternative in the choice set.

Hence the structure of our experimental scheme will be the following:

- H_0: T.A.I. verified,
- H_1: T.A.I. violated,
  - H'_0: uniformity hypothesis verified,
  - H'_1: uniformity hypothesis violated.

2.2 Method

The experiment consists of a questionaire in which two types of questioning appear:

- Binary choice (between x and y) consists of a choice out of a set of two alternatives. This type of questioning is denoted BC,
- Ternary choice (between x, y and z) consists of a choice out of a set of three alternatives. This type of questioning is denoted TC.

In our experiment, the choice sets are defined in order to make possible comparisons of the answers to both questionings. More precisely, each ternary choice question involving a choice set $\{x, y, z\}$ corresponds to a binary question involving a choice set $\{x, y\}$. In these choice sets, x and y are efficient in $\{x, y, z\}$ and z is asymmetrically p-dominated in $\{x, y, z\}$ ($x\Delta_p z$ and not$[y\Delta z]$).

As a consequence, the possible answers to the two questionings are the following:

1. Binary Choice :
   - choice of x in the set $\{x, y\}$, which is interpreted xPy. This case corresponds to the situation where the subject can find clear and positive reasons why he/she should significatively prefer x to y.
   - choice of y in $\{x, y\}$, which is interpreted yPx, corresponds to the opposite of the preceding situation.
   - choice of x and y in $\{x, y\}$, which is interpreted xIy, corresponds to the situation where the subject can find clear and positive reasons why he should see the two alternatives x and y as equivalent.

2. Ternary Choice :

   - choice of x in $\{x, y, z\}$, which is interpreted xPy and xPz. choice of y in $\{x, y, z\}$, which is interpreted yPx and yPz.
   - choice of x and y in $\{x, y, z\}$, which is interpreted yPz, xPz, and xIy.
   - choice of z in $\{x, y, z\}$, which is interpreted zPx and zPy.
   - choice of y and z in $\{x, y, z\}$, which is interpreted yPx, zPx and zIy.
   - choice of x and z in $\{x, y, z\}$, which is interpreted xPy, zPy, and xIz.
   - choice of x, y, and z in $\{x, y, z\}$, which is interpreted xIy, yIz, and zIx.

In our experiment, we assume that: $\forall(x, y) \in X^2,\ x\Delta_p y \implies xPy$ (*i.e.* $P \subset \Delta_p$). Hence, the last four answers are inconsistent, for:

- zPx is assumed to be incompatible with the situation in which $x\Delta_p z$;
- xIz is assumed to be incompatible with the situation in which $x\Delta_p z$.

As a consequence, only three answers are possible for both questionings (choice of x, of x and y, and of y).

Before going on with the method, let us denote:

- $Q_B \subseteq X^2$, the set of BCs proposed to the subjects,
- $Q_T \subseteq \{(x, y, z) \in X^3/(x, y) \in Q_B\}$, the set of TCs proposed to the subjects. Q_T is constructed so that each question from Q_T corresponds to a question in Q_B (see Appendix B).
- $Q_{B\times T} = \{((x, y), (x, y, z)) \in Q_B \times Q_T\}$ the set of corresponding question-couples.

In Table 1, $p_{bc=\beta}^{tc=\alpha}$ is the proportion of elements from $Q_{B\times T}$ for which the answer of a subject is β in the binary choice procedure, and α in ternary choice procedure.

		Binary choice: $\{x, y\}$			
		choice of y	choice of x and y	choice of x	Total
Ternary choice: $\{x, y, z\}$	choice of y	$p_{bc=y}^{tc=y}$	$p_{bc=xy}^{tc=y}$	$p_{bc=x}^{tc=y}$	$p^{tc=y}$
	choice of x and y	$p_{bc=y}^{tc=xy}$	$p_{bc=xy}^{tc=xy}$	$p_{bc=x}^{tc=xy}$	$p^{tc=xy}$
	choice of x	$p_{bc=y}^{tc=x}$	$p_{bc=xy}^{tc=x}$	$p_{bc=x}^{tc=x}$	$p^{tc=x}$
	Total	$p_{bc=y}$	$p_{bc=xy}$	$p_{bc=x}$	

Table 1: Definition of computed proportions

Using these notations, the hypotheses presented in §2.1 can be expressed as follows:

a) H_0 is verified if: $E\left[\frac{p_{bc=y}^{tc=y}}{p_{bc=y}}\right] = E\left[\frac{p_{bc=xy}^{tc=xy}}{p_{bc=xy}}\right] = E\left[\frac{p_{bc=x}^{tc=x}}{p_{bc=x}}\right] = 100\%$,

b) H_1 if verified if: $E\left[\frac{p_{bc=y}^{tc=y}}{p_{bc=y}}\right] \leq 100\%$, $E\left[\frac{p_{bc=xy}^{tc=xy}}{p_{bc=xy}}\right] \leq 100\%$, $E\left[\frac{p_{bc=x}^{tc=x}}{p_{bc=x}}\right] \leq 100\%$,

b$_1$) H_0' is verified if:

$$E\left[p_{bc=y}^{tc=xy} + p_{bc=y}^{tc=x}\right] = E\left[p_{bc=x}^{tc=xy} + p_{bc=x}^{tc=y}\right],$$
$$\overline{p}_{bc=xy}^{tc=x} = \overline{p}_{bc=xy}^{tc=y}$$

b$_2$) H_1' is verified in the other cases, that is to say if:

$$E\left[p_{bc=y}^{tc=xy} + p_{bc=y}^{tc=x}\right] \neq E\left[p_{bc=x}^{tc=xy} + p_{bc=x}^{tc=y}\right],$$
$$\overline{p}_{bc=xy}^{tc=x} \neq \overline{p}_{bc=xy}^{tc=y}$$

In this particular case, the nature of the inequations will allow us to validate our conjecture according which, in TC procedure, changes in preference give advantage to the p-dominant alternative x. In fact, if $E\left[p_{bc=y}^{tc=xy} + p_{bc=y}^{tc=x}\right] > E\left[p_{bc=x}^{tc=xy} + p_{bc=x}^{tc=y}\right]$ and $\overline{p}_{bc=xy}^{tc=x} > \overline{p}_{bc=xy}^{tc=y}$ then the conjecture is verified, i.e., there is a systematic asymetric dominance effect which "favors" x in the ternary choice procedure (compared to the binary choice procedure).

3 Experimental Context

3.1 Concrete framing

In this experiment, the alternatives x, y, and z are firms described on two criteria, salary and job interest (the other aspects, such as job stability, are not considered in this study; the proposed alternatives are assumed to be equivalent on these aspects). Subjects are asked to consider the different firms in order to select the ones in which they would like to postulate for a job. The two criteria are constructed as follows:

Criterion 1 : The annual salary is evaluated in thousands French Francs. Only four different salaries are considered, which amounts at discretizing the continuous scale into four levels (A, B, C, D). Every difference between consecutive levels expresses a strict preference (AP_1BP_1CP_1D). Every level is constructed regarding the subject's expectations in terms of salary (see Appendix A).

Criterion 2 : The job interest is measured on a qualitative discrete scale (four levels A, B, C and D). Each level is qualitatively defined by the subject, so that the difference between two consecutive levels reflects a strict preference. The instructions given to the subjects are consistent with the fact that the four levels have to be constructed so that:

- they can describe a large spectrum of job interests,
- the differences between two consecutive levels on the scale should be perceived by the subject as equivalent in terms of preference.

The following example shows an hypothetic output.

A : Very interesting job, formative, no repetitive aspect, evolutive and large independence at work.

B : Interesting job, slightly repetitive but formative, rather evolutive, good independence at work.

C : Job of little interest, quite repetitive, but still somehow formative, evolutive in the long run, fairly little independence.

D : Boring job, very little formative, numerous repetitive aspects, annoying hierarchy.

The two criteria are constructed in order to prevent the subject from considering one of them as preponderant. More precisely, two conditions are checked, before going on with the interview, using to the following test: check the two preference situations (A,D) P (D,C) and (D,A) P (C,D)).

3.2 Interview-pattern

After having constructed both criteria (with the analyst's support, see §3.1), each individual is asked to answer 36 questions (18 questions from Q_B, and 18 from Q_T, corresponding to 18 elements of $Q_{B\times T}$). Firms are proposed to DMs on computer screens, as shown in Figures 2 and 3.

In what follows, the p-dominant firm x offers the best salary, y offers the best job-interest, and z is the asymmetrically p-dominated firm. The questions are constructed so that z has the same evaluation as x on either salary (z_1 on Figure 4) or job interest (z_2 on Figure 4). The sets of binary and ternary questions are provided in appendix B).

The order of the questions is randomized. This precaution is to prevent the decision maker from pointing out common alternatives between binary and ternary choices. Also the position on the screen for alternatives x, y, and z is randomized in order to prevent the subject from recognizing z as the alternative to be eliminated

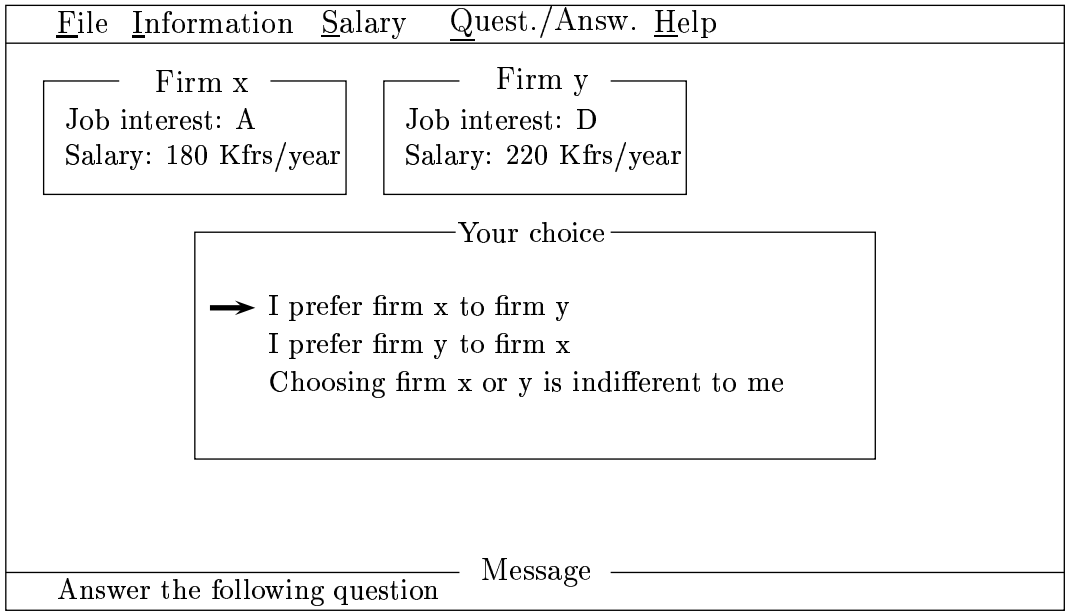


Figure 2: Binary choice

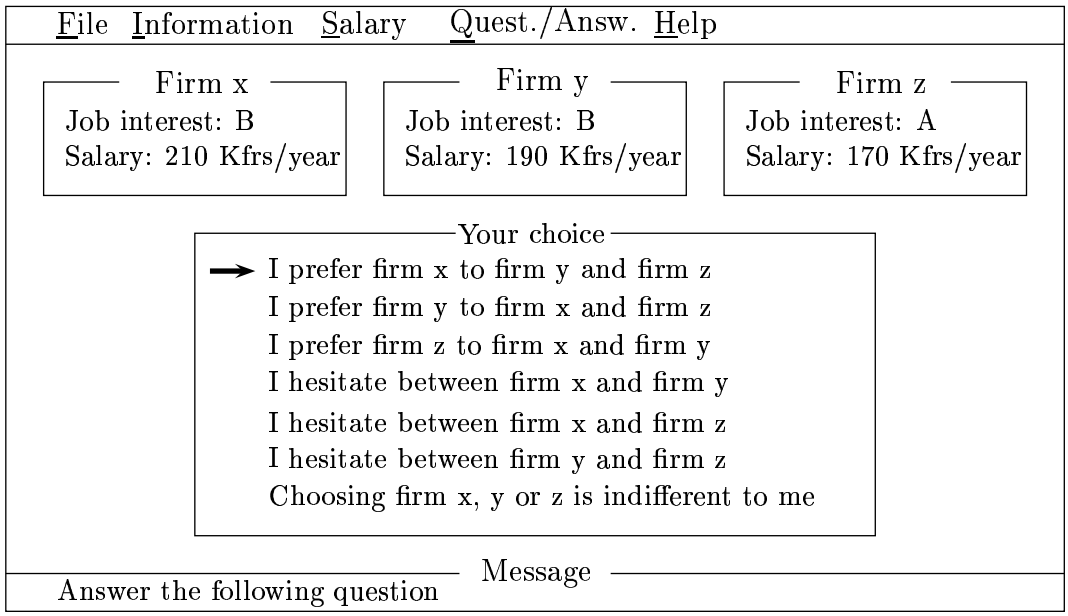


Figure 3: Ternary choice

(without further analysis). Each answer is recorded, and rates (see Table 1) are calculated at the end of the questioning period. A mean time for answering each type of question is also recorded.

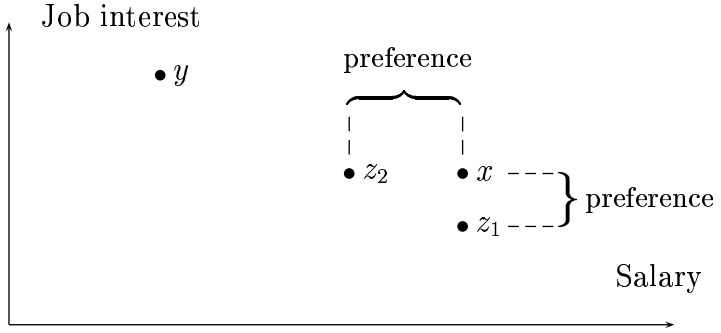


Figure 4: Alternatives in the choice sets

In every ternary question, when the p-dominated alternative is selected in the choice, the answer is considered as inconsistent. In such cases, the computer program reasks the question, and the analyst has to explain the reason for inconsistency (this explanation is expressed in terms of enterprises, job interest and salaries).

4 Results

The sample population is composed of 30 undergraduate students from University or business school. Each of them was to look for a job no later than a one year delay, which would ensure their implication to the proposed decision problem. The average subject was 23,5 years old, and thought he had to find a job in 3,2 month-delay. At the end of interviews, many of them did assert that it had been a major opportunity to give rise to many personal questions or stakes.

The answering time for ternary questions ($m = 16,2$ sec., $\sigma = 4,8$) is significantly longer than the binary one ($m = 11,9$ sec., $\sigma = 4,2$), which expresses DM made a supplementary effort to consider the ternary situations (t-student = 7,6). However, the increase in answering time from binary to ternary choice questions shows that the subjects did not proceed by pairwise comparisons for answering the ternary choice questions, but rather used simpler strategies. Such statement suggest that the subjects did not always explicitely noticed the dominance situations among the choice sets.

For each subject, the following values are computed ($\alpha = x, xy, y$ and $\beta = x, xy, y$):

- $p^{tc=\alpha}_{bc=\beta}$,the proportion of elements from $Q_{B\times T}$ for which a subject has chosen α in TC procedure, and β in BC procedure,
- $p_{bc=\beta}$,the proportion of elements from Q_B for which a subject has chosen β in BC procedure,
- $p^{tc=\alpha}$,the proportion of elements from Q_T for which a subject has chosen α in TC procedure,

4.1 Third Alternative Independance hypothesis

Table 2 presents the statistics that allow us to test the T.A.I. principle. Each $\frac{p^{tc=\alpha}_{bc=\alpha}}{p_{bc=\beta}}$ represents the number of question couples from $Q_{B\times T}$ (see §2.2) for which the choice β remains stable from binary to ternary questioning, divided by the total number of elements from $Q_{B\times T}$ for which β is chosen in the binary questions. The average for $\beta = \{x\}, \{y\}$, and $\{x, y\}$ are significatively lower than 1 (see the confidence intervals). Thus the T.A.I. principle is violated.

	Mean	Standard deviation	Confidence Interval (0.95)
$\frac{p^{tc=xy}_{bc=xy}}{p_{bc=xy}}$	0.387	0.325	[0.266, 0.508]
$\frac{p^{tc=y}_{bc=y}}{p_{bc=y}}$	0.524	0.337	[0.398, 0.650]
$\frac{p^{tc=x}_{bc=x}}{p_{bc=x}}$	0.870	0.265	[0.771, 0.969]

Table 2: Test of the Third Alternative Independance Hypothesis

4.2 Uniformity hypothesis

In Table 3, $p^{tc=xy}_{bc=y} + p^{tc=x}_{bc=y}$ represents the number of question couples from $Q_{B\times T}$ for which the presence of the asymetrically p-dominated alternative z in the TC procedure induced changes (compared to the corresponding BC) in favor of x, when y was chosen in BC. $p^{tc=xy}_{bc=x} + p^{tc=y}_{bc=x}$ represents the number of question couples from $Q_{B\times T}$ for which the presence of the asymetrically p-dominated alternative z in the TC procedure induced changes in favor of y occurred, when x was chosen in BC. $p^{tc=x}_{bc=xy}$ represents the number of question couples from $Q_{B\times T}$ for which changes in favor of x occurred, when x and y were chosen in BC. $p^{tc=y}_{bc=xy}$ represents the number of question couples from $Q_{B\times T}$ for which changes in favor of y occurred, when x and y were chosen in BC. Let us recall that $card(Q_{B\times T}) = 18$.

	Mean	Standard deviation
$p^{tc=xy}_{bc=y} + p^{tc=x}_{bc=y}$	3.70 (elements of $Q_{B\times T}$)	2.78
$p^{tc=xy}_{bc=x} + p^{tc=y}_{bc=x}$	0.50 (elements of $Q_{B\times T}$)	0.97
$p^{tc=x}_{bc=xy}$	2.60 (elements of $Q_{B\times T}$)	2.44
$p^{tc=y}_{bc=xy}$	0.67 (elements of $Q_{B\times T}$)	1.52

Table 3: Test of the Uniformity Hypothesis

A t-student test (with ρ=95%) asserts that:

- $p_{bc=y}^{tc=xy} + p_{bc=y}^{tc=x} > p_{bc=x}^{tc=xy} + p_{bc=x}^{tc=y}$ (t-student = 5.4),
- $p_{bc=xy}^{tc=x} > p_{bc=xy}^{tc=y}$ (t-student = 3,6).

As a conclusion, the uniformity hypothesis is violatedand the conjecture is confirmed: preference changes give advantage to the p-dominant alternative x. Moreover, $\overline{p}_{bc=y}^{tc=xy}$ is not significantly different from $\overline{p}_{bc=y}^{tc=x}$, while $\overline{p}_{bc=x}^{tc=xy}$ is significantly superior to $\overline{p}_{bc=x}^{tc=y}$. This means that changes in favour of the p-dominant alternative x are much more radical than the ones in favour of y. This last observation reinforces our conjecture.

5 Discussion

5.1 Analysis of DMs behavior

The results of this empirical study show a strong failure of the third alternative independance principle: in our sample, we observed a significant divergence among choices expressed on choice sets containing two and three alternatives. Moreover, when an asymetrically p-dominated alternative z is added to a binary choice set $\{x, y\}$, the changes tend to "favor" the alternative (denoted x) p-dominating z. Subjects were strongly influenced by the evaluations of the asymetrically p-dominated alternative; they answered the binary and ternary choice questions in such a way that the introduction of z "increases" the value of x compared to y. Such phenomenon is consistent with previous experiments reported in the literature (see [1], [9] and [10]).

The Reference-Dependent Model (RDM) proposed by [8] provides an attractive framework for explaining such phenomenon. In the RDM, preferences of a DM are formalized through a familly of preference relations on the set of alternatives; P_r and I_r denote the preference and indifference relations relatively to a reference point r. Preferences on each criterion are represented by S-shaped value function defined relatively to a reference point. Loss aversion induce a value function which is steeper in the losses than in the gains (relatively to the reference point). The value functions are suposed to be concave above the reference point and convex below it, due to a decrease in the perceived evaluation differences as gain and losses increase (see figure 5). P_r and I_r result from an additive aggregation of the value functions on each criterion.

Let us illustrate, on a simple example, how a shift in the reference point can affect the preferences between two alternatives x and y see Figure 6. Let us suppose that the reference point is r_1 and that the preferences of the DM are such that $xI_{r_1}y$, what will be the impact on preferences if the reference point is changed to r_2? to r_3? Let us remark that, in this example, the reference points vary only on criterion g_1 ($g_2(r_1) = g_2(r_2) = g_2(r_3)$); we will thus focus our analysis on g_1 only.

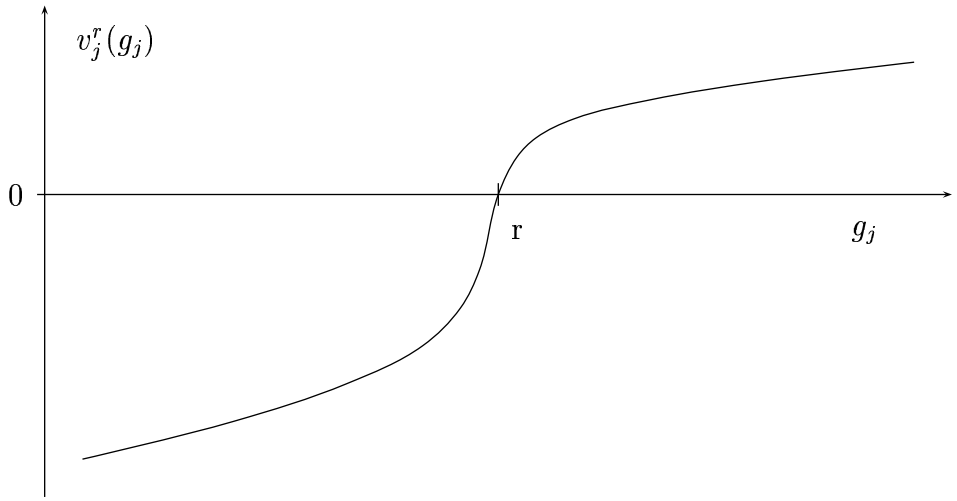


Figure 5: S-shaped value function $v_j^r(g_j)$ for criterion g_j with reference point r

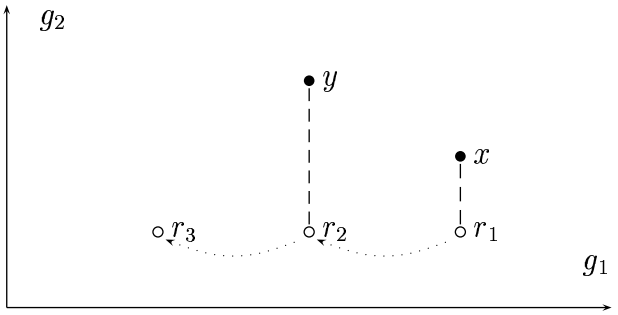


Figure 6: Impact of the reference point on preferences

When the reference point is r_1:

- as $g_1(x) = g_1(r_1)$, $g_1(x)$ is perceived as neutral (neither as a gain nor as a loss); it holds $v_1^{r_1}(g_1(x)) = 0$,
- as $g_1(y) < g_1(r_1)$, $g_1(y)$ is perceived as a loss; it holds $v_1^{r_1}(g_1(y)) < 0$,

When switching the reference point from r_1 to r_2:

- as $g_1(x) > g_1(r_2)$, $g_1(x)$ is perceived as a gain; it holds $v_1^{r_2}(g_1(x)) > 0$,
- as $g_1(y) = g_1(r_2)$, $g_1(y)$ is perceived as neutral; it holds $v_1^{r_2}(g_1(y)) = 0$,

The loss aversion principle states that losses loom larger than the corresponding gains, i.e., value functions are steeper in losses than in gains. Hence, in our example, the incresase in value for x resulting from a shift of the reference point from r_1 ro r_2 ($v_1^{r_2}(g_1(x))$) is lower than the corresponding increase in value for y ($-v_1^{r_1}(g_1(y))$). In other words, the change of the reference point favors y stronger than x; consequently, it holds $xI_{r_1}y$ and $yP_{r_2}x$.

When swiching the reference point from r_2 to r_3, the evaluation of x and y are both perceived as gains. However, let us recall that, in the RDM, the marginal value decreases with the distance from the reference point. In our case, $v_1^{r_3}(g_1(y)) - v_1^{r_2}(g_1(y)) > v_1^{r_3}(g_1(x)) - v_1^{r_2}(g_1(x))$. Consequently, changing the reference point from r_2 to r_3 increases the relative advantages of y over x (as $yP_{r_2}x$, it reinforces $yP_{r_3}x$).

In our example, it appears that decreasing $g_1(r)$ favors y in comparison to x; similarily, it can be shown that decreasing $g_2(r)$ favors x in comparison to y.

It seams natural to consider the reference point to be influenced by aspirations and expectations of the decision maker but also by the availability of alternatives. In our experiment, the presence or absence of the p-dominated alternative z might have had an implicit influence on the reference point considered by the subjects. Let us denote $r_{\{x,y\}}$ ($r_{\{x,y,z\}}$, repectively) the reference point considered by subjects in the binary choice procedure (in the ternary choice procedure, respectively).

We hypothetize that the reference point implicitly considered by subjects refer to a "medium value" of available alternatives on each criterion. Considering the evaluations on x, y and z, it appears that $g_1(r_{\{x,y,z\}}) \geq g_1(r_{\{x,y\}})$ and $g_2(r_{\{x,y,z\}}) \geq g_2(r_{\{x,y\}})$ (one of the two inequalities being strict, see Figure 4). Such a change of the reference point increases the probability of choosing x in in the ternary choice procedure compared to the binary choice procedure.

5.2 Implications for prescriptive Multiple Criteria Decision Aiding

The preference elicitation is a crucial phase in a decision aid process which is usually supported by Preference Elicitation Techniques (PET). Schematically, two different components of a PET are generally distinguished: the questioning procedure specifies how information is collected from the DM, i.e., the questioning mode and the sequence of questions. The deduction method uses information obtained by the questioning procedure, verifying if this information is compatible with the chosen aggregation procedure so as to infer values for preference parameters. PET should account for the formal definition of preferences (in the chosen aggregation procedure) and the empirical analysis of decision behavior

Empirical investigation of decision behavior provides fruitfull insights in order to conceive relevant elicitation techniques and use them properly. The way a PET interacts with the DM should account for his/her intuitive perception of his/her prefrences and for its limitations in perceiving and processing information (see [5]).

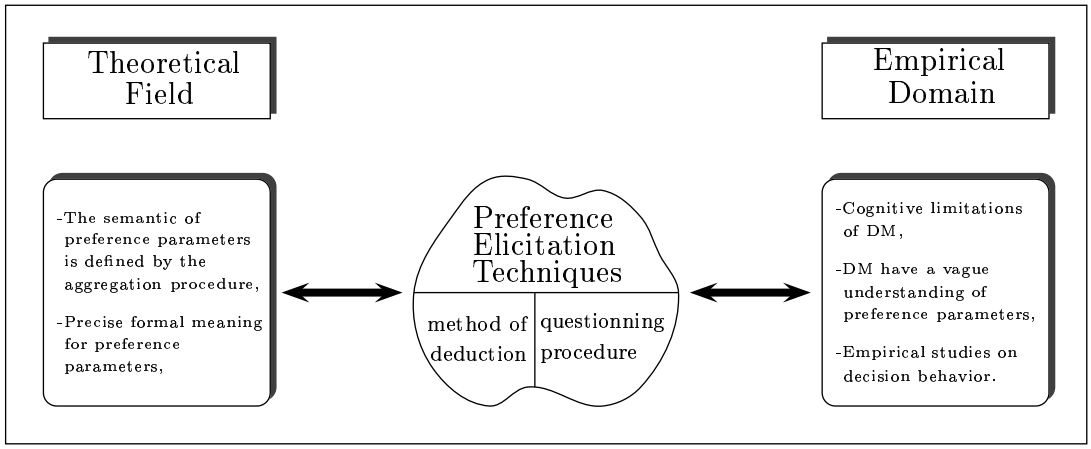


Figure 7: Preference Elicitation Techniques

However, the role devoted to such empirical studies differs according to the way we seek to give meaning to the notion of preference information, refering either to a *descriptivist* or a *constructivist* approach.

In the descriptivist approach, preferences are assumed to be well-defined in the DM's mind before the modelling process begins and the modelling process does not modify them. The preference model chosen is intended to give an account of such pre-existing preferences as objectively as possible. Within this approach, empirical observations concerning DM's behavior are the expression of the pre-existing preferences, possibly distorted by the elicitation mode. Experiments aim at identifying biases, and eventually conceive correcting procedure in order to reduce the "noise" introduced during the elicitation phase.

The constructivist approach, on the contrary, assumes that preferences are not entirely pre-formed in the DM's mind and that the very nature of the work involved in the modelling process is to specify and even to modify pre-existing elements. The decision aid process and the elicitation procedure are not intended to represent the DM preferences but rather to help him/her to structure a model that account for his/her viewpoints. Within this approach, experiments aims at analysing the way the DMs construct a preference model and the influence of elicitation procedures on such construction process. Thus, empirical results provide elements that the analyst should have in mind in order to assist the DM in the construction of a preference model.

The present study shows that the availability of alternatives can strongly affect the preferences expressed by DMs when answering to binary/ternary choice questions. Numerous PET make the use of such choice questions in order to elicit a preference model and specify indirectly values (or interval of variation) for the

parameters of the aggregation procedure. Consequently, it appears that, when using PET that infer the preference parameters values from choice questions, the obtained model is strongly contingent to the set of alternatives that the DM had in mind (either implicitly or explicitly) when answering the questions.

Such empirical evidence highlights the essential role, during the modelling process, of the construction of the set of alternatives. The explicit definition of the set of available alternatives should be completed before eliciting the preferences in order for the DM to express preferences having clearly in mind what are the available alternatives. Moreover, we would not recommend to use irrealistic fictitious alternatives in questions as it might "influence" the DM preferences in a way that does not correspond to the real set of alternatives. During the elicitation process, the use of alternatives whose evaluations could not correspond to a realistic alternative would orient the DM's reasonning in such a way that he/she might account for very specific and even marginal aspects of the decision problem.

6 Conclusion

We have presented an empirical study aiming at testing the impact of the introduction of an third alternative z in a choice set $\{x, y\}$. More specifically, the introduced alternative z was asymetrically dominated (dominated by x but not by y). We observed strong violation of the third alternative independence principle. Preferences change wether they are elicited through binary or ternary choice question. Moreover, the observed preference changes were in favour of asymetrically dominant alternatives x. These results can be explained through the reference dependent model proposed by [8]. Implications for preference elicitation techniques in prescriptive multiple criteria decision aid are studied.

References

[1] J. Huber, J. W. Payne D., and C. Puto. Adding asymetrically dominated alternatives: Violation of regularity and the similarity hypothesis. *Journal of Consumer Research*, 9:90–98, 1982.

[2] R. Keeney and H. Raiffa. *Decision with multiple objectives: preferences and value tradeoffs*. John Wiley & Sons, 1976.

[3] S. Lichtenstein and P. Slovic. Reversals of preference between bids and choices in gambling decisions. *Journal of Experimental Psychology*, 89:46–55, 1971.

[4] S. Lichtenstein and P. Slovic. Response-induced reversals of preference in gambling: An extended replication in las vegas. *Journal of Experimental Psychology*, 101:16–20, 1973.

[5] V. Mousseau. *Problèmes liés à l'évaluation de l'importance relative des critères en aide multicritère à la décision : Réflexions théoriques, expérimentations et implémentations informatiques.* PhD thesis, Université Paris-Dauphine, 1993.

[6] J.W. Paynes, J.R. Bettman, and J.E. Johnson. *The adaptative decision making.* Cambridge University Press, 1993.

[7] B. Roy. *Multicriteria Methodology for Decision Aiding.* Nonconvex Optimization and its Applications. Kluwer Academic Publishers, Dordrecht, 1996.

[8] A. Tversky and D. Kahneman. Loss aversion in riskless choice: a reference-dependent model. *Quarterly Journal of Economics*, 106:1039–1061, 1991.

[9] A. Tversky and I. Simonson. Context-dependent preferences. *Management Science*, 39(10):225–270, 1993.

[10] M. Willinger. Préférences dépendantes du contexte. *Revue économique*, 3:577–587, 1996.

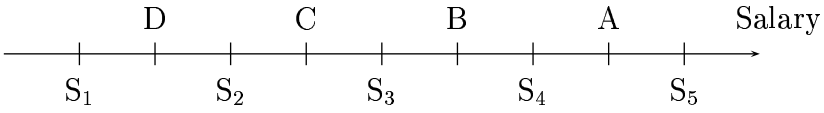
Appendix

Appendix A: Construction of criterion "salary"

Let us recall that the considered alternatives are firms, evaluated on two criteria: salary (expressed in thouthands of french francs per year) and job interest, evaluated on a qualitative scale. In order to discretize the quantitative scale of the criterion 'salary", we ask the subjects to give a monetary value to the following salary types:

- S_1: Minimum salary the subject would accept;
- S_2: Salary for which the person feels spoiled regarding his qualifications and hopes;
- S_3: "Neutral" salary, corresponding to a salary perceived as fair;
- S_4: Interesting salary corresponding to significantly advantaging salary;
- S_5: Maximum salary the subject thinks he can obtain.

These values $(S_1, S_2, S_3, S_4, S_5)$ determine four intervals, whose centers are the levels on the discrete salary scale, as shown below.



In order to consider alternatives z (asymetrically dominated) in every case of couples (x, y), we construct intermediate levels A^-, B^-, C^-, defined as follows:

- $A^- = \frac{A+B}{2}$
- $B^- = \frac{B+C}{2}$
- $C^- = \frac{C+D}{2}$

Appendix B: Construction of binary and ternary questions

Binary choices corresponds to a pair of alternatives x and y such that not$[x\Delta y]$ and not$[y\Delta x]$. In our case (2 criteria, 4 levels on each scale), there are 36 possible binary questions. We chose 18 questions couples out of 36 in order to avoid lassitude of subjects. Ternary questions were constructed on basis of binary ones, adding a third asymetrically p-dominated alternative z, which had the same evaluation as x on one criterion. We had to use the intermediate levels in order to avoid the cases where z had the same evaluation as y on criterion salary. The question couples are presented in the following table ($Q_{B\times T}$)

	Binary choice questions	Ternary choice question
1	(A,B)?(B,A)	(A,B)?(B,A)?(A^-,B)
2	(A,B)?(C,A)	(A,B)?(C,A)?(B,B)
3	(A,B)?(D,A)	(A,B)?(D,A)?(B,B)
4	(A,C)?(B,A)	(A,C)?(B,A)?(A^-,C)
5	(A,C)?(B,B)	(A,C)?(B,B)?(A^-,C)
6	(A,C)?(C,A)	(A,C)?(C,A)?(A,D)
7	(A,C)?(C,B)	(A,C)?(C,B)?(A,D)
8	(A,C)?(D,A)	(A,C)?(D,A)?(A,D)
9	(A,C)?(D,B)	(A,C)?(D,B)?(A,D)
10	(B,B)?(C,A)	(B,B)?(C,A)?(B,C)
11	(B,B)?(D,A)	(B,B)?(D,A)?(C,B)
12	(B,C)?(C,A)	(B,C)?(C,A)?(B^-,C)
13	(B,C)?(C,B)	(B,C)?(C,B)?(B^-,C)
14	(B,C)?(D,A)	(B,C)?(D,A)?(B,D)
15	(B,C)?(D,B)	(B,C)?(D,B)?(B,D)
16	(C,B)?(D,A)	(C,B)?(D,A)?(C,C)
17	(C,C)?(D,A)	(C,C)?(D,A)?(C^-,C)
18	(C,C)?(D,B)	(C,C)?(D,B)?(C,D)

Table 4: List of binary and ternary choice questions