Title: Strong equilibria under special coalition structures

Supervision: Laurent Gourvès and Julien Lesca (laurent.gourves, julien.lesca}@dauphine.fr)

Place: Université Paris Dauphine (LAMSADE), France

Description: Many practical situations can be modeled as a strategic game. Such a game consists of a player set $N$, a set of strategies and an individual cost function for each player. A state of the game is the choice of one strategy per player. For example, we can think of an election where the players are the voters and their strategy set is the set of candidates.

What is a plausible outcome of a strategic game? The Nash equilibrium is a famous solution concept in which no player can unilaterally deviate and strictly decrease his individual cost. Other relevant solution concepts can be defined. We propose to investigate equilibria that are immune to group deviations. For example, in a strong equilibrium [1], there is no coalition of players who can deviate and induce a strict reduction of the cost of every member of the coalition. In a close concept called super strong equilibrium [2], a coalition of players may form if the individual cost of at least one member strictly decreases whereas the individual costs of the other members don’t increase.

In a (super) strong equilibrium, every non-empty subset of $N$ is a possible coalition. However, not all these coalitions can form in practice if, for example, the players cannot communicate and coordinate their actions. Therefore, it is relevant to define which are the possible coalitions. An important part of the PhD thesis consists in listing the existing ways to identify the possible coalitions and the PhD candidate shall also propose novel concepts. For example, we can use an hypergraph in which each vertex is associated with a player and the possible coalitions correspond to the hyperedges. We can also use a collection of subsets of $N$ and exploit the properties of the collection.

Once a solution concept is specified, it is important to find which known game is guaranteed to admit it in any instance. Some games are known to always possess a pure strategy Nash equilibrium (e.g. congestion games [3]), a strong equilibrium (e.g. singleton congestion games) but sometimes a game fails to admit a given solution concept. The PhD candidate will have to answer to this question for a short list of games (e.g. graph games, scheduling games, congestion games, etc.).

Beyond the existence of an equilibrium, the scientific community (especially in computer science) is interested in its construction. Can we design an algorithm which efficiently builds
an equilibrium? A task of the PhD thesis is to propose algorithms and, possibly, explore the computational difficulty [5] of building an equilibrium.

The last point that may be investigated by the PhD candidate is the quality of equilibria with respect to a social optimum. This question, related to the price of anarchy [4], has received a lot of attention and it can be posed each time a new solution concept is proposed.

References


