Approximate solutions of \textsc{HappyNet} on cubic graphs

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Abstract

The \textsc{HappyNet} problem is defined as follows: Given a undirected simple graph \(G\) with integer weights \(w_{vu}\) on its edges \(vu \in E(G)\), find a function \(s : V(G) \rightarrow \{-1, 1\}\) such that \(\forall v \in V(G)\), \(v\) is \textit{happy} in \(G\), i.e. such that \(\sum_{u \in \Gamma(v)} s(v)s(u)w_{uv} \geq 0\). It is easy to see that \textsc{HappyNet} has always a solution, no matter what the input is. However, no polynomial algorithm is known for this problem, which is complete for the class \textbf{PLS}. Parberry \textit{et al.} have shown that in the case of cubic graphs (i.e. of maximum degree 3) \textsc{HappyNet} is as difficult as for arbitrary graphs. A \(\rho\)-approximate solution to a \textsc{HappyNet} instance of size \(n\) can be defined for \(0 \leq \rho \leq 1\) as a natural extension of the solution function, with at least \(\rho n\) happy vertices. In this paper, we present a polynomial-time algorithm that finds a \(\rho\)-approximate solution for the \textsc{HappyNet} problem on cubic graphs, with \(\rho \geq \frac{3}{4}\).