Fast reoptimization for the minimum spanning tree problem

Abstract

Minimum spanning tree is a classical polynomial problem very well known in operational research and in theoretical computer science. In this paper, we settle the reoptimization versions of this problem, which can be formulated as follows: given an instance of the problem for which we already know some optimal solution, and given some “small” perturbations on this initial instance, is it possible to compute a new (optimal or at least near-optimal) solution for the modified instance without ex nihilo computation? We focus on two kinds of modifications: node-insertions and node-deletions. For the former type of modifications, where \( k \) new nodes are inserted together with their incident edges, we first propose a fast strategy with complexity \( O(kn) \) which provides a \( \max\{2, 3 - (2/(k - 1))\} \)-approximation ratio, in complete metric graphs. We then devise a more elaborated strategy that computes optimal solutions in any graph with complexity \( O(kn \log n) \). When \( k \) nodes are deleted, we devise a strategy which in \( O(n) \) achieves approximation ratio bounded above by \( 2|L_{\text{max}}|/2 \) in complete metric graphs, where \( L_{\text{max}} \) is the longest deleted path and \( |L_{\text{max}}| \) is the number of its edges. For any of the approximation strategies, we also provide lower bounds on their approximation ratios.