Multicriteria decision-aid: Two applications in education management

N° 11-1977

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novembre 1977

RÉSUMÉ

Dans la première section les conditions et la formalisation du choix trichotomique sont établies sur la base des concepts de relation de surclassement et d'ensembles de références. Ils permettent la modélisation des préférences dans le contexte décisionnel étudié.

Deux applications suivent. La première appliquée à la décision du jury de sélection des candidats à une institution d'enseignement commercial, permet, grâce à un exemple numérique de préciser l'utilisation du modèle.

La seconde décrit une application faite à la répartition de subventions aux écoles d'une académie. Nous montrons que le type d'aide à la décision choisi répond au contexte organisationnel en ce qui concerne la gestion du système d'information et de relations.
SUMMARY

In the first section the conditions for and formalism of trichotomic choice problem formulation are stated on the basis of the fundamental concepts of outranking relations and reference sets, used to the modelling of global preferences within the given context. Two applications follow. The first one, applied to the decision of the admission committee of a business school, makes the explanation of the procedure more concrete. The second one is the description of an implementation for resource allocations to schools of an academic region. It is shown that the approach to decision aid we propose fits to the organisational background regarding the management of information systems as well as the human relations and organisational development.

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INTRODUCTION

The procedures resulting from the operational research are
How can we conceive such a procedure of trichotomic choice? That is the problem we are studying there (section one). Can such a procedure be implemented in the context of administrative organisation? That is what we are trying to show by describing two applications in the field of education management (section two and three).

The first application will enable us to make the explanation of the procedure and of the model used more concrete by studying the well known case of the decision in an admission committee in a business school.

The second application regarding a decision procedure for resource allocations to schools takes place in a large and rather bureaucratic administration. By describing how the procedure has been implemented we shall try to show that such an approach to decision aid particularly fits to the organisational background regarding the management of information system as well as of the human relations and organisational development.
I. THE METHODOLOGY OF MULTICRITERIA TRICHOTOMIC SEGMENTATION

1. Type of aid intended

Given, actions, objects or candidates (we are now going to speak of objects), which are not fundamentally in competition, we want to help an identified decision maker by saying whether such or such object is compatible with his requirement.

We have thus to define and model a referential system which will allow us to establish by comparison (on the basis of criteria) the intrinsic value of each object. Thanks to this particular modelling effort we don’t need to consider the whole objects at once to be able to come to a conclusion on each of them.

The procedure lends itself to a sequential examination of the objects. Nevertheless to insure a sufficient reliability we shall admit that in some cases the intrinsic value of the object can not be defined.

More precisely, on the base of just the initial information introduced in the model, the main point of the trichotomic segmentation is, to:

- accept all the objects declared without the intervention of the decision maker as sufficiently good (category $A_1$);
- refuse all the objects declared without the intervention of the decision maker as too bad (category $A_3$);
- demand a further examination by the decision maker for the others (category $A_2$).

By assigning each object to one of these categories we discriminate between

- those for which initial information is sufficient to found a solid assumption about the intrinsic value of the object (category $A_1$ and $A_3$);
- those for which initial information does not allow us to come to a conclusion.

For these objects we must have recourse to the decision maker; a search for new information and a deeper reflexion are necessary.
When the initial information allows us to build a discriminating trichotomy (population of $A_1$ and $A_2$ not too small), the implementation of these principles leads to a better organisation of the decision maker's reflections and effort by directing, in a selective way, his reflection, the mobilisation of his experience and of all diffuse information he can have, toward the examination of the most difficult cases.

To implement such a trichotomic segmentation we have to model in three stages [1]:

1- In the first stage we have to represent and consider each object $a$, the set $A$ ($a \in A$), and the consequences or main attributes of each $a$, evaluated on $n$ scales $E_1$ and represented by the vectors $\gamma_i(a) = \gamma_1(a) \ldots \gamma_n(a)
\gamma_i(a) \in E_i$

2- In a second stage we have to model the decision maker's preference: choice of a consistent family of criterion [2] $g_1 \ldots g_n$ adapted to the discriminating power on each scale $E_i$, formal definition of the global preference to be able to establish on the basis of the $n$ criteria the decision maker's preference: "$a$" is best or worst than $a'$, or "$a$" can't be compared to $a'"

3- In the last stage we have to build the rules to decide whether an object $a$ must be assigned to $A_1$, $A_2$ or $A_3$. We shall see that this assignment is chosen on the basis of "$a$" comparison to particular objects of reference.

2. Fuzzy outranking relation

Let us assume that for the objects we consider the study of the relevant consequences and attributes has been made and formalised so that we can sum them through a consistent family of $n$ criteria [2]. So let us assume that to each object $a$ is associated a vector:

$g(a) = g_1(a), \ldots g_n(a)$

Let us agree that "$a" is better as $g_1(a)$ is greater.
This vector represents the initial information. Each component $g_i$ (a) grasps more or less precisely an aspect of the intrinsic value of $a$. To compare two objects on the base of that initial information we use the concept of fuzzy outranking.

It can be introduced in a quite natural way if we accept that the modelling of preferences is not complete and only depends on the part of preferences which we are able to know with sufficient objectivity and confidence (see table 1).

The term *outranking* refers to those of the preferences thus modelled; given two potential actions $a$ and $a'$:
### Table 1: Situations to Which May Lead

The Comparison of Two Potential Actions $a, a'$

<table>
<thead>
<tr>
<th>Situations</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indifference</td>
<td>The two actions are indifferent in the sense that there exist clear and positive reasons to choose equivalence. Example: $g(a) = g(a') \forall_1$ some of the equalities not being rigorous but only $a$ approximate.</td>
</tr>
<tr>
<td>Four fundamental</td>
<td><strong>strict preference</strong> One of the two actions (which one being known) is strictly preferred to the other. Example: $g_i(a) &gt; g_i(a') \forall_1 \neq k, g_k(a)$ a significant difference.</td>
</tr>
<tr>
<td>excludive large preference</td>
<td>One of the two actions (which one being known) is not strictly preferred to the other but it is impossible to say if the other is strictly preferred to or indifferent from the first one because neither of the two former situations dominates. Example: $g_i(a) = g_i(a') \forall_1 \neq k, g_k(a') = g_k(a)$ neither sufficiently small to justify indifference nor sufficiently large to justify strict preference.</td>
</tr>
<tr>
<td>incomparability</td>
<td>The two actions are not comparable in the sense that neither of the three former situations dominates. Example: $g_i(a) &gt; g_i(a')$ for $i = 1, \ldots, p$, $g_i(a') &gt; g_i(a)$ for $i = p + 1, \ldots, n$ the majority of the differences being significant.</td>
</tr>
</tbody>
</table>
3°) \( d(a',a) = 1 \) implies a certain outranking of "a" by \( a' \)
whereas \( d(a', a) = 0 \) implies either a certain non-outranking of \( a \) by \( a' \),
or the total absence of arguments in favour of such an outranking; it
follows that:

\[ 0 \leq d(a', a) \leq 1. \]

Given a fuzzy outranking relation \( S \). It is interesting to introduce the outranking relation (non fuzzy) defined by:

\[ a \mathrel{S} a' \iff d(a,a') \geq \lambda \]

\( \lambda \) can be interpreted as a threshold generally near 1 and which is in all cases at least equal to 1/2. The outranking through \( S^\lambda \) can be stated as soon as the credibility degree has reached this threshold. To take into consideration a decreasing sequence of values \( \lambda \) comes down to introducing a nested family of outranking relations (non fuzzy) more and more richer but more and more risky. The scientist can in this way test the behavior of the model, in function of requirements of variable security and severity.

In the Annexe we describe a way to build such an outranking relation.

3. The references sets

Given two particular objects (real or fictitious) \( b \in B \) and \( c \in C \) called reference objects and characterized by the vectors \( g(b) \) and \( g(c) \).

These vectors indicate on the set of criteria the "combined limits" of what we consider sufficiently good and sufficiently bad.

\( B \) and \( C \) must lead to the following properties:

- If an object is such that for \( a \in B \), the credibility degree \( d(a, b) \) is sufficient to guarantee (without too much risk) the preference of "a" compared to \( b \), then "a" is presumed to be "sufficiently" good and merits acceptance on the basis of initial information.

- If an object "a" is such that for \( c \in C \) the credibility degree \( d(c, a) \) is sufficient to guarantee (without too much risk) the preference of \( c \) on "a", then "a" is presumed to be "for too" bad and merits rejection on the basis of initial information.
The elements of the set \( R = B \cup C \) may be real objects, if not they are introduced as combinations of limits on the \( n \) criterions.

4. Segmentation procedure

\( \lambda \) is the minimum threshold which indicates the limit above which we have a right to garantee through the model the preference of \( a'' \) compared to \( a' \) \( (a \not\succ_{\lambda} a') \). This threshold is fixed by the decision maker.

Thanks to natural graphic convention we may represent \( S_{\lambda} \) with an outranking graph (1). The analysis of such a graph (fig. 2) built on the objects of \( a' \cup R \) is useful for the scientist to locate in the context of the decision maker's preferences, the object which is being studied.

![Figure 2](image)

Thus on the example of figure 2

- \( a \) is at least as good as \( b^1 \) and \( c^1 \)
- \( a \) is less good as \( b^2 \)
- \( a \) can't be compared to \( c^2 \)
- \( a \) is equivalent to \( b^3 \)

To analyse this outranking graph let us define the subsets

- \( B^+, B^-, B^0 \) & \( C^+, C^-, C^0 \):

- \( b \in b^+ \) if \( d (a, b) \succ_{\lambda} \) thus \( a \succ_{\lambda} b \)
- \( b \in b^- \) if \( d (b, a) \not\succ_{\lambda} \) thus \( b \succ_{\lambda} a \)
- \( b \in b^0 \) if \( d (a, b) \not\succ \lambda \) and \( d (b, a) \not\succ \lambda \) thus non \( a \succ_{\lambda} b \) and non \( b \succ_{\lambda} a \)

\[ c \in C^+ \text{ if } d(c, a) \geq \lambda \text{ thus } cS^A a \]
\[ c \in C^- \text{ if } d(a, c) \geq \lambda \text{ thus } aS^A c \]
\[ c \in C^0 \text{ if } d(a, c) \leq \lambda \text{ and } d(c, a) \leq \lambda \text{ thus } \text{non } aS^A c \text{ and } \text{non } cS^A a \]

Figure 3 is an illustration of these definitions.

When the relation \( S^\lambda \) is antisymmetric these subsets form a partition of \( B \) and a partition of \( C \):
\[ B^+ \cap B^- = \emptyset, \quad C^+ \cap C^- = \emptyset \]
This partition (to which it is always possible to refer (1)) characterizes the situation of \( a \) and the scientist uses it to work out the rules which are necessary to assign \( a \) to \( A_1, A_2 \) or \( A_3 \).

Given a partition of \( B \) and \( C \), it can be characterized by
\[ x^+ = |B^+|, \quad x^- = |B^-| \]
\[ y^+ = |C^+|, \quad y^- = |C^-| \]

To each $a \in A$ we can therefore associate the vector $E_{A}^{a} = (x^+, x^-, y^-, y^+).$ Following the configuration of this vector we shall assign "a" to $A_1$, $A_2$ or $A_3$. The tree in figure 4 shows an example of decision rules.

The combination of the four basic exclusive situations (Table 1) on the set of the pairs $(a, c)$ and $(a, b)$ and the non necessary transitivity of $S^A$, leads to quite complex configurations, some of which may appear as surprising.

So on figure 5 a appears both as less good and better than two
Figure 4: Assignment rules built out of the configuration analysis of 
\[ E \Lambda (a) = (x^+, x^-, y^+, y^-) \]

\[ x^+ > 0 \Rightarrow \exists b \in B : aS^\lambda b \]
\[ x^- > 0 \Rightarrow \exists b' \in B : b'S^\lambda a \]
\[ y^+ > 0 \Rightarrow \exists c \in C : cS^\lambda a \]
\[ y^- > 0 \Rightarrow \exists c' \in C : aS^\lambda c' \]

This 0 can be replaced by \( x^+ \)

This 0 can be replaced by \( y^- \)
II. THE SELECTION OF CANDIDATES FOR A BUSINESS SCHOOL ADMISSION

1. The problem of selection

The admission to the school is pronounced by an admission committee of about 6 members (professors and professionals). The number of candidates is superior than the admissions offered. The selection rate is about 1/3.

For each candidate the admission committee has got a dossier including information about his curriculum, his past school results and his psychological profile.

The committee's task is made quite difficult for the following reasons: the dossiers are quite numerous the quality of information regarding each candidate is not very good and the decision criteria to take into account are various. As a consequence we can observe that
- there is an implicit tendency to privilege some criteria such as school results, although every body doubts the pertinence of this information
- the behavior of the committee is not always the same: some criteria only play an incidental part
- sometimes exceptional information not included in the dossier are brought in by a member.

This distortions within the procedure mainly appear during the examination of the candidates who are just at the limit of admission conditions.

The following solution can be considered
1/ selection from dossier: we try to get a partition of the candidates set $A$ in three groups
   \[ A_1 \text{ official admission from dossier} \]
   \[ A_2 \text{ convocation to a selection interview} \]
   \[ A_3 \text{ officially refusal from dossier} \]

2/ Final selection during an interview of the candidates affected to the group $A_2$, and eventually ranking on a complementary list.
2. The modelling

To affect a candidate to one of the three groups $A_1$, $A_2$ or $A_3$ we use the model which has been described before.

2.1. The consistent family of criteria

The admission committee is looking for candidates who will be able to succeed in their studies as well as in their future professional integration. The candidate should therefore be able to adapt himself both to quantitative and relatively literary matters, but his own human qualities and motivations are considered as important to his future professional integration. This initial and quite confused perception can be precised by the definition of dimensions considered as useful to value as well as possible the consequences of the admission. By defining a scale $E_1$ upon each dimension $i$ we can build a state indicator $Y_i(a)$ which is the formal information taken into account. According to the variation of the decision maker's preference along $E_1$ we can build $g_i$ out of $Y_i$.

The table 6 gives an example of that modelling work.

2.2. The references

The admission committee chooses two references for the "good" candidates $b_1$ and $b_2$. The vertices $a(b_1)$ and $a(b_2)$ mark the combined limits from which the candidate can be considered as sufficiently "good" to be officially admitted on dossier.

These two references are useful to express in a formal way the fact that the committee considers that good pupil without any particular personality ($b_1$), should be admitted as well as a hard working candidate whose personality seems to be particularly suitable, but whose school results are rather mediocre ($b_2$).
FROM THE CLOUD OF ATTRIBUTES TO THE CRITERIAS

<table>
<thead>
<tr>
<th>E&lt;sub&gt;i&lt;/sub&gt;</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 5</td>
<td>( g_1 (a) = y_1 (a) )</td>
</tr>
<tr>
<td>0 to 20</td>
<td>( g_2 (a) = y_2 (a) )</td>
</tr>
<tr>
<td>0 to 20</td>
<td>( g_3 (a) = y_3 (a) )</td>
</tr>
<tr>
<td>0 to 20</td>
<td>( g_4 (a) = y_4 (a) )</td>
</tr>
<tr>
<td>1 to 9</td>
<td>( g_5 (a) = y_5 (a) )</td>
</tr>
<tr>
<td>1 to 5</td>
<td>( g_6 (a) = y_6 (a) )</td>
</tr>
<tr>
<td>1 to 3</td>
<td>( g_7 (a) = y_7 (a) )</td>
</tr>
<tr>
<td>1 to 4</td>
<td>( g_8 (a) = y_8 (a) )</td>
</tr>
<tr>
<td>1 to 3</td>
<td>( g_9 (a) = y_9 (a) )</td>
</tr>
</tbody>
</table>
This choice enable us to give a chance to candidates whose personality is considered as very positive for a good professional integration.

One reference only is defined for the "bad" candidate. Not to
TABLE 7

| Criteria   | A.LEV | Meth | French | English | Improvement | Appraisal | Experience | Motivation | Profession int. |
|------------|-------|------|--------|---------|-------------|-----------|------------|------------|----------------|-----------------|
| Scale E₁   | 1 to 5| 1 to 20 | 1 to 20 | 1 to 20 | 1 to 9      | 1 to 5    | 1 to 3     | 1 to 4     | 1 to 3         |
| Reference : g₁ (b₁) | 4   | 12   | 12     | 14   | 5   | 3   | 1   | 2   | 1   |
| g₂ (b₂)    | 2   | 9    | 9      | 10   | 8   | 4   | 3   | 4   | 3   |
| g₃ (c)     | 1   | 8    | 8      | 10   | 3   | 2   | 1   | 1   | 1   |
| Weight p₁  | 1   | 2    | 2      | 2    | 1   | 1   | 1   | 1   | 1   |
| Discordance interval D₁ | 3   | 5    | 6      | 5    | 3   | 2   | 2   | 3   | 2   |

TABLE 8

<table>
<thead>
<tr>
<th>Concordance Table</th>
<th>b₁</th>
<th>b₂</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td>b₂</td>
</tr>
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<td></td>
<td></td>
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<td>b₂</td>
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<tr>
<td>c</td>
<td>0.16</td>
<td>0.16</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discordance Table</th>
<th>b₁</th>
<th>b₂</th>
<th>c</th>
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<tr>
<td>b₁</td>
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<td></td>
<td>b₂</td>
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<tr>
<td>b₂</td>
<td>0.8</td>
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<td>0</td>
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<tr>
<td>c</td>
<td>0.6</td>
<td>1.6</td>
<td></td>
</tr>
</tbody>
</table>
3. Numerical application

Applying the rules defined in annexes and referring ourselves to concordance and discordance tables (Table 8) we get:

\[ d(b_1, b_2) = 0.58, \quad d(b_2, b_1) = 0, \quad d(c, b_1) = 0, \quad d(c, b_2) = 0, \quad d(b_1, c) = 1, \quad d(b_2, c) = 1 \]

Then the outranking graph associated to \( S_A \) is the following:

\[ \lambda_1 = 0.9, \lambda_2 = 0.7, \lambda_3 = 0.6 \]

\[ \lambda_4 = 0.55 \]

We can see that the good pupil \( b_1 \) is preferred with a quite a low degree of credibility to the candidate with a high individual
### Table 9

Table of criteria values

<table>
<thead>
<tr>
<th></th>
<th>A, LFV</th>
<th>Math</th>
<th>French</th>
<th>English</th>
<th>IMP</th>
<th>APP</th>
<th>EXP</th>
<th>MDT</th>
<th>INP</th>
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</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>5</td>
<td>15</td>
<td>12</td>
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<td>7</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
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<tr>
<td>$b_1$</td>
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<td>12</td>
<td>12</td>
<td>14</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$b_2$</td>
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<td>9</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$c$</td>
<td>1</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Concordance vs. Discordance**

<table>
<thead>
<tr>
<th>Concordance</th>
<th>Discordance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1, b_1$</td>
<td>0.83</td>
</tr>
<tr>
<td>$a_1, b_2$</td>
<td>0.66</td>
</tr>
<tr>
<td>$a_1, c$</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Outranking graphs

- $\lambda = 0.9$
  - $b_1$
  - $a$
  - $b_2$
  - $c$
- $\lambda = 0.7$
  - $b_1$
  - $a$
  - $b_2$
  - $c$
- $\lambda = 0.6$
  - $b_1$
  - $a$
  - $b_2$
  - $c$

- $E_{0.9}(a_1) = (0, 0, 0, 0) \rightarrow A_2$
- $E_{0.7}(a_1) = (0, 0, 1, 0) \rightarrow A_2$
- $E_{0.6}(a_1) = (4, 0, 4, 0) \rightarrow A_4$

**Notice:** This candidate is too bad in English to be considered as a good pupil (discordance effect). With a quite low credibility degree he could be considered as better than the reference $b_2$ which might allow to admit him.

* In table 9, 10, 11 the discordance indicate is notice only when it is superior to 1
### Table 10

Table of criteria values

<table>
<thead>
<tr>
<th></th>
<th>A. LEV</th>
<th>Math</th>
<th>French</th>
<th>English</th>
<th>IMP</th>
<th>APP</th>
<th>EXP</th>
<th>MOT</th>
<th>INP</th>
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<tbody>
<tr>
<td>$a_2$</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>2</td>
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</table>

: **Concordance** : ** Discordance 

<table>
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<tr>
<th>Pair</th>
<th>Concordance</th>
<th>Discordance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2$, $b_1$</td>
<td>0.25</td>
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</tr>
<tr>
<td>$a_2$, $b_2$</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>$a_2$, $c$</td>
<td>0.33</td>
<td></td>
</tr>
</tbody>
</table>

Concordance: Discordance

Outranking graphs:

\[ d = 0.9 \]

\[ E_{0.9}(a_2) = (0, d, 0, 0) \rightarrow a_2 \]

\[ E_{0.7}(a_3) = (0, 0, 0, 1) \rightarrow a_3 \]

Notice: This candidate can be rejected. The outranking of $a_2$ by $c$ is obtained with a high credibility degree.
TABLE 11

<table>
<thead>
<tr>
<th></th>
<th>A.LEV</th>
<th>MAT</th>
<th>FREN</th>
<th>ENG</th>
<th>IMP.</th>
<th>APP.</th>
<th>EXP.</th>
<th>MOT.</th>
<th>INP</th>
</tr>
</thead>
<tbody>
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<td>a_3</td>
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<td>10</td>
<td>10</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>3</td>
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</tr>
<tr>
<td>b_1</td>
<td>2</td>
<td>12</td>
<td>12</td>
<td>14</td>
<td>5</td>
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<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>b_2</td>
<td>4</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>8</td>
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<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

: concordance : discordance

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>ab_1</td>
<td>0.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>ab_2</td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>ac</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Outranking graphs

\[ d = 0.9 \]
\[ a_3 \rightarrow b_2 \]
\[ d = 0.55 \]
\[ b_2 \rightarrow b_3 \]

Notice: As for candidate a, this example reveals the advantage of the use of two non-comparable references. Here although he obtains middle school results, the candidate has a personality which justifies his admission. With a lower credibility degree this candidate appears as less good than the good pupils, which explains that, if we apply the decision tree of figure 4, (based on a majority rule) he will be affected to \( A_2 \) unless with a higher credibility degree he would be admitted. This may suggest to apply an other decision tree based upon the most significant elements of B on C (1)

(1) J. MOSCAROLA - Aide à la décision en présence de critères multiples fondée sur une trichotomie - Thèse Paris IX Dauphine 1977
4. The use of the procedure by the committe

The choice of critieria as well as other elements of the model (the vectors of references \( g (b_1) \), \( g (b_2) \), \( g (c) \), the weight \( p_1 \), the threshold ...) explicitly expresses the policy of the committe (during a first meeting), which is certainly one of the most important contributions of the procedure. With respect to this formal elicitation let us point out that the use of the model may bring out some underlying dissensions among the members of the committe regarding the admission policy. Nevertheless thanks to the modelling effort the consensus to which the committe debate must necessarily lead becomes clearer and more transmissible.

In this way the model is also a report of the decisions taken by the committe.

During the second meeting the committe members have a look at the results of the procedure and decide weather the resulting partition \( A_1, A_2, A_3 \) can in all cases be applied.

Let us notice that at this stage the committe remains entirely free of taking the final decision.

Thus if during the first meeting the members of committe have not come to any agreement regarding the admission policy, the scientist may indicate what are the consequences of each policy in view.

In this way the final arbitration no more appeals to principles but rather to the consequences of these principles applied to the case of some candidates.

It is also possible to rank (using Electre Method: [1]) for example the candidates assigned to \( A_2 \) which enables us in case of desistances of candidates assigned to \( A_1 \), the admission of the first candidates of \( A_2 \).

[1] P. BERTHIER - B. ROY La méthode Electre II Une application au média planning VII Conférence internationale de recherche opérationnelle Dublin 1972
III. RESOURSE ALLOCATION TO THE SCHOOLS OF AN ACADEMIC REGION

1. The distribution procedure

1.1. The financing of the schools of an academic region: the initial situation

The French administration of the national educational system is centralized and mainly financed by public subsidies.

It is centralized at the top, the central administration controls about 20 academic regions, each of which administers about 300 colleges. The colleges receive their resources (teachers and money) from the academic regional authority, which itself receives its resources from the central authority.

We are interested here in the allocation of funds from the academic authority to the school. These subsidies are necessary to the schools to balance its budget (1) (figure 2) and result from a division of the total subsidy allocated by the central authority to the academic region. This division is made by the financial department of the regional authority and should take into account the situation and needs of each school.

An initial study on the financial situation of the schools and on the division procedure showed that:

- the most important criterion upon which the financial director officially bases his decision is the number of pupils attending the school; however, that criterion does not reflect very well the operating costs (the direct charges - increasing with the number of pupils - do not represent the fourth of the subvention);

- the financial director adjusts the theoretical amount of money computed from the number of pupils. Therefore he bases his judgement on diffuse information which is not systematically taken into account and so he tends to favour some schools, more than others often due to subjective factors such as personal relations (manager of the school, financial service)

(1) the payment of teachers is not part of that budget
there are very important disparities of financial exigencies between schools. These disparities can most frequently be explained by the history of the school and the principle of status quo.

The financial department was really conscious of the undue importance given to the number of pupils and of the fact that basing the adjustment of the figure calculated on diffuse informations was a very partial, too subjective, and biased method. It tried to take more systematically into consideration some elements such as the building structure, and the financial abilities of the school. However, the available information was too extensive and varied to be systematically taken into account and so to balance the very preponderant weight given to pupils number.

1.2. Objectives

The aim of the study is to build a model for decision aid. This model must be used to solve management difficulties resulting from the increasing number of schools (more than 300) and the complexity of information. The solution is based on an enrichment of the formal information system, a diagnosis aid and a selective orientation of the search for further information (regarding the school which can't be evaluated through the formal information system and the diagnosis aid).

More precisely the model must allow us:

- a) to carry out a multidimensionnal analysis of the school and to multiply the information elements systematically taken into consideration for the division and so to lower the importance given to the number of pupils and the appeal to badly controlled diffuse information

- b) to start from the preferences, the objectives and the policy of the decision maker, as he puts it into words; to combine this with an automatic data processing system used to compute an aggregate evaluation of the school. This diagnosis has to reveal the disparities between schools and to call into question the established situations.

- c) to implement a procedure which does not fix the division policy; and so to forbid any rigid standards. Therefore the procedure must let the decision maker entirely free of his actions, the reciprocal arrangement of which is the setting up of dialogue between the schools and the financial service; possibility for each school to defend its own case on the base of particular information exceptionally taken into account by the financial service.
1.3. Work method

As there have been already at disposal in the financial service a lot of information which have not really been used, a study constraint has been fixed not to create new basic information from the schools, but to try to make the existing information useful. This constraint led to some difficulties in defining some particular criteria.

The decision maker of the financial service as well as the school managers have been tightly associated within a working group in the definition of the model.

To make the explicitation of objectives, preferences and criteria
Table 12: The budgetary structure of a school

<table>
<thead>
<tr>
<th>Charges</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insecting and</td>
<td>Subsidies from state and local public organizations</td>
</tr>
<tr>
<td>Sources</td>
<td></td>
</tr>
<tr>
<td>45 %</td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td>Development costs from firms</td>
</tr>
<tr>
<td>maintenance</td>
<td></td>
</tr>
<tr>
<td>20 %</td>
<td></td>
</tr>
<tr>
<td>Administration</td>
<td>Diagnosis of financial basis or difficulties</td>
</tr>
<tr>
<td>20 %</td>
<td></td>
</tr>
<tr>
<td>Teaching</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Facilities contribution (1)</td>
</tr>
</tbody>
</table>

(1) Coming from the board of trustees

Table 13: The operators of the procedure

- Building infrastructure of the school
- Financial data from surveys
- Modifications in activities
- Infrastructural environment
- Diagnosis of financial basis or difficulties

Allocation decision of subsidy for t+1

Table 14: INFORMATION SYSTEM

School

- Information Basis
  - Dialogue chart

Authority of the academic region

- Diagnosis
  - Function calculation of theoretical subsidies
  - Diagnosis indicating the correction to introduce

Specific information by exception

Innocuous amount in francs

Decision
- the financial situation and urgency of needs justify a
decrease of the theoretical amount of subsidies (the school is "rich")
- the available information (introduced in the model) do not
enable us to conclude whether the theoretical amount should be corrected.

To the proposition regarding each school is associated a number
which indicates the credibility degree of the presumption of ease or
difficulty of the school.

1.4.2. The information system (figure 14)

The theoretical amount of subsidies and the diagnosis are
elements of the decision maker's "dashboard". The table 15 shows which
information is edited. In the second column one can read the diagnosis. The
sign - (+) indicates that the theoretical amount (column 4) should be
lowered (higher). The figures indicate the credibility of the proposition(1)

The decision power of the financial director remains entire. He
may add to this date his own knowledge of the schools, or a further informa-
tion when necessary.

The information basis. The dates used in the operators are
reported by the financial service to the school's managers in a document
called the "dialogue chart". To each indicator regarding each school is
associated the average level calculated from all the schools of the region
(or an objective given by the financial service).

The choice of the elements of the information basis, parameters,
the division function and the diagnosis method is made in a conference where
the school's managers may advise and give their opinions to the financial
director.

2. Modelling of the diagnosis method

We apply the methodology described in I. Given a consistent
family of n criteria we are able to associate the vector $g(a) = g_1(a) \ldots$
$g_n(a)$ to each school. This vector results from the indicators of the infor-
mation basis and dialogue chart.

Following the method described in I we affect each school to
one of the categories $A_1$, $A_2$, $A_3$. More precisely the school $a$ will be
assigned to :

(1) For further explanation see J. MOSCAROLA Thèse Paris IX Dauphine.
<table>
<thead>
<tr>
<th>NUMERO</th>
<th>IDENTIFICATION</th>
<th>ANÉCDOTE</th>
<th>SUB.74</th>
<th>SUB.75</th>
<th>SÉJEL 73</th>
<th>SÉJEL 75</th>
<th>SÉJEL 73</th>
<th>SÉJEL 75</th>
<th>OBEJ.73</th>
<th>OBEJ.75</th>
<th>VÉRIF.</th>
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</thead>
<tbody>
<tr>
<td>0070007</td>
<td>LYCÉE PLYSI D'ANGLES BURHAY</td>
<td>13 -13</td>
<td>3100</td>
<td>41090</td>
<td>143</td>
<td>77</td>
<td>14549</td>
<td>15</td>
<td>54</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>0070002</td>
<td>LYCÉE SANTO-PLATZER ANHOLAY</td>
<td>31 -33</td>
<td>10330</td>
<td>77823</td>
<td>286</td>
<td>8</td>
<td>2167</td>
<td>33</td>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>0070003</td>
<td>LYCÉE POLY MARCEL GIMOND ARBRESSE</td>
<td>11 -12</td>
<td>465000</td>
<td>286071</td>
<td>999</td>
<td>10</td>
<td>955</td>
<td>22</td>
<td>3</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>0070004</td>
<td>LYCÉE TECH ARBRESSE</td>
<td>46 -15</td>
<td>250290</td>
<td>257071</td>
<td>976</td>
<td>9</td>
<td>15</td>
<td>37</td>
<td>2</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>0070005</td>
<td>LYCÉE BOURG-SAINT-ARNOY</td>
<td>35 -35</td>
<td>302410</td>
<td>174</td>
<td>9</td>
<td>1737</td>
<td>13</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0070006</td>
<td>LYCÉE GOMMAR</td>
<td>31 -17</td>
<td>33500</td>
<td>93641</td>
<td>156</td>
<td>16</td>
<td>727</td>
<td>213</td>
<td>3</td>
<td>60</td>
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<tr>
<td>0070007</td>
<td>LYCÉE BOURG-SAINT-ARNOY</td>
<td>24 -24</td>
<td>120640</td>
<td>163141</td>
<td>133</td>
<td>11</td>
<td>1532</td>
<td>133</td>
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<td>30</td>
<td></td>
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<tr>
<td>0070008</td>
<td>LYCÉE MONTCEQUIE</td>
<td>3 -13</td>
<td>4725</td>
<td>57031</td>
<td>135</td>
<td>18</td>
<td>968</td>
<td>260</td>
<td>16</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>0070009</td>
<td>LYCÉE FALLE</td>
<td>31 -21</td>
<td>10300</td>
<td>28264</td>
<td>523</td>
<td>32</td>
<td>735</td>
<td>52</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0070010</td>
<td>LYCÉE VINCENT D'INYE PETRAS</td>
<td>22 -22</td>
<td>316232</td>
<td>337672</td>
<td>159</td>
<td>5</td>
<td>990</td>
<td>70</td>
<td>3</td>
<td>55</td>
<td></td>
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<tr>
<td>0070011</td>
<td>LE TEL</td>
<td>27 -33</td>
<td>52160</td>
<td>214</td>
<td>24</td>
<td>2224</td>
<td>253</td>
<td>3</td>
<td>51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0070012</td>
<td>LYCÉE CL. GABRIEL FAURE FOURS</td>
<td>82 -6</td>
<td>174352</td>
<td>177422</td>
<td>203</td>
<td>13</td>
<td>553</td>
<td>84</td>
<td>3</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>0070013</td>
<td>LEYSY FOURS</td>
<td>62 -9</td>
<td>2727</td>
<td>1351298</td>
<td>229</td>
<td>6</td>
<td>621</td>
<td>31</td>
<td>3</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>0070014</td>
<td>LEYSY LES VILLES LES VILLES</td>
<td>41 -13</td>
<td>19510</td>
<td>94966</td>
<td>128</td>
<td>9</td>
<td>1036</td>
<td>33</td>
<td>3</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>0070015</td>
<td>LEYSY LE TEL</td>
<td>27 -33</td>
<td>177730</td>
<td>125382</td>
<td>163</td>
<td>2</td>
<td>1761</td>
<td>185</td>
<td>3</td>
<td>45</td>
<td></td>
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<tr>
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<td>LEYSY TOURON</td>
<td>27 -18</td>
<td>214500</td>
<td>247921</td>
<td>149</td>
<td>0</td>
<td>1913</td>
<td>289</td>
<td>3</td>
<td>60</td>
<td></td>
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<tr>
<td>0070017</td>
<td>LE CHEYLARD</td>
<td>35 -15</td>
<td>1545</td>
<td>83459</td>
<td>158</td>
<td>8</td>
<td>259</td>
<td>3</td>
<td>60</td>
<td></td>
<td></td>
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<tr>
<td>0260001</td>
<td>LE BOURG-DE-PAGE</td>
<td>35 -35</td>
<td>45550</td>
<td>157958</td>
<td>145</td>
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<td>1748</td>
<td>240</td>
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<td>60</td>
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<tr>
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<td>7162</td>
<td>57074</td>
<td>232</td>
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<td>1310</td>
<td>942</td>
<td>3</td>
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<tr>
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<td>LA CAMPAGNE</td>
<td>30 -59</td>
<td>17557</td>
<td>82860</td>
<td>296</td>
<td>96</td>
<td>667</td>
<td>121</td>
<td>3</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

Example - The "lycée polyvalent Marcel Gimond" has got the mark - 2 (column 2) which indicates that the theoretical amount of subsidies (286071 Fr Col. 4) should be lowered. This proposition is established with an average degree of credibility. When in column 2 we don't get any mark that means that the school was assigned to A₂ and that further information is necessary.
A. If the initial information introduced in the model is sufficient to
<table>
<thead>
<tr>
<th>Main aspects of the problem</th>
<th>Dimensions</th>
<th>Scales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incompressible charges</td>
<td>- importance of heating and energy charges</td>
<td>- rate: total amount of charges</td>
</tr>
<tr>
<td></td>
<td>- evolution of it</td>
<td>- percent of evolution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_1 (a) = - \gamma_1 (a) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_2 (a) = - \gamma_2 (a) )</td>
</tr>
<tr>
<td>Financial autonomy</td>
<td>- transfer of income from boarding</td>
<td>- rate of shifting</td>
</tr>
<tr>
<td></td>
<td>- anticipation capacity</td>
<td>- reserve ratio</td>
</tr>
<tr>
<td></td>
<td>- evolution of anticipation capacity</td>
<td>- percent of evolution of reserve ratio</td>
</tr>
<tr>
<td></td>
<td>- own resources</td>
<td>- own resources ratio</td>
</tr>
<tr>
<td></td>
<td></td>
<td>neglected (disagreement up the variation sensus)</td>
</tr>
<tr>
<td>Quality of Management</td>
<td>- cost of heating</td>
<td>- cost per m²</td>
</tr>
<tr>
<td></td>
<td>- evolution</td>
<td>- percent of evolution of it</td>
</tr>
<tr>
<td></td>
<td>- quality of budget estimates</td>
<td>- budget estimates margin rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_6 (a) = - \gamma_8 (a) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_7 (a) = - \gamma_7 (a) )</td>
</tr>
<tr>
<td>Quality of service</td>
<td>- building degradation</td>
<td>- subjective evaluation</td>
</tr>
<tr>
<td></td>
<td>- quality of living</td>
<td>- density: pupils/m²</td>
</tr>
<tr>
<td></td>
<td>- quality of maintenance</td>
<td>- cost per m²</td>
</tr>
<tr>
<td></td>
<td>- quality of administrative service</td>
<td>- cost per pupil (x)</td>
</tr>
<tr>
<td></td>
<td>- quality of teaching service</td>
<td>- cost per pupil (x)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_8 (a) = - \gamma_8 (a) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>non monotonic criteria</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_9 (a) = \gamma_9 (a) )</td>
</tr>
<tr>
<td>Building infrastructure</td>
<td>- sport installation</td>
<td>ranking following the importance</td>
</tr>
<tr>
<td></td>
<td>- lawns, parks and grounds commodities</td>
<td>ranking</td>
</tr>
<tr>
<td></td>
<td>- heat insulation</td>
<td>subjective evaluation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_{12} (a) = - \gamma_{12} (a) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_{13} (a) = - \gamma_{13} (a) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( g_{14} (a) = - \gamma_{14} (a) )</td>
</tr>
</tbody>
</table>

(x) lacking of other information we assess that the quality of service is proportional to the cost.
In this application the choice of these thresholds must be based upon the examination of the different situations of the school, for the following reasons.

- the concept of intrinsic value, or here of "poverty" or "wealth" must take into account the situation of the schools as a whole and the extend of their disparities

- the whole procedure is based upon the restoration of balance of the theoretical subsidies within the constraints of the regional subsidy.

To make the diagnosis operational the partition $A_1$ (school to higher) $A_2$ (school to lower) $A_2$ (indetermination) have to be balanced. This objective can be introduced as a constraint in the model. Therefore we can change the values of references so long the reliability of the final affectation allows it. To make it a shorter trial and error procedure, the working group was given a graphic presentation of the distribution of school on each dimension.

2.3. The modelling of global preferences and the outranking relation.

The fuzzy outranking relation used to compare a school $a$ to the references $b$ and $c$ is described in the annex.

Let us notice that:

- for the first implementation and to make the understanding of the method easier the scientist did not introduce the discordance concept which was naturally introduced later at the decision maker's request.

- the choice of the weights $p_i$ was made after a discussion based upon several tests where different sets of weights were applied to well known schools.

The choice of the weights as well as those of other elements of preferences modelling (criteria, outranking threshold) is the occasion of an interactive research process which characterise the scientist-decision maker relations.[1]

[1] J. MOSCAROLA - B. ROY, Procédure automatique d'examen de dossiers fondée sur une segmentation trichotomique en présence de
2.4. The affectation procedure

Several decision trees may be built to decide, on the base of the fuzzy outranking graph on \((a, b, c)\), which proposition should be affected to a \(1\). Through the variations of the credibility degree we can get several diagnosis of decreasing confidence, each of them is characterised by the absolute value of the number in column 2 (table 15).

3. The effects of the model upon the administrative environment

The procedure and the diagnosis model we described has been used for now 3 years to allocate subsidies to more than 300 schools evaluated on the basis of about 15 criteria. There was some evolutions.
CONCLUSION

This decision aid approach may be situated in the background of large administrative organisation which can be characterized by:
- the great number of files which is often a "goulot d'étranglement" for the decisional procedures
- the complexity of the problems and the multidimensionality of their consequences
- the weight of too often purely qualitative externalities
- the increasing demand of having decision open and above board

The implementation of the model and the resulting procedure, may undoubtly in quiet numerous situations, contribute to solve some necessities of administrative work without loosing the control of the management.

This may be mainly achieved thanks to
- the increase of cognitive abilities of the decision maker involved by the implementation of the preferences modelling with multiple criteria
- the better management of information system based upon the selective ability introduced by the trichotomy

Finally these improvements involve a better investigation of the solving problems due to any modelling effort.
OUTRANKING RELATION [1]

Given $g_1, g_1, \ldots, g_n$ a consistent family of criteria

$$P_i \in F = 1 \ldots n$$

the weights assigned to each criteria $g_i$

$$\sum_{i \in F} P_i = 1$$

$D_i$ the maximal admitted discordant margin for each criteria $g_i, i \in F$

For each pair $(a, a')$ we can build the following partition of $F$:

$$F = F^+ \cup F^- \cup U$$

$$F^+ = \{ i/ g_i (a) > g_i (a') \}$$

$$F^- = \{ i/ g_i (a) < g_i (a') \}$$

$$F^0 = \{ i/ g_i (a) = g_i (a') \}$$

Concordance coefficient

We call concordance coefficient the number calculated as follows

$$\text{Con} (a, a') = \sum_{i \in F^+} P_i + \sum_{i \in F^-} P_i$$

Calculating for each pair $(a, a')$ of $A$ the value of the coefficients $\text{Con} (a, a')$ we build a concordance table.

Discordance coefficient

Given $d_i$ the discordant margin observed on the criteria $i$:

$$g_i (a') - g_i (a) = d_i$$

We call discordance coefficient the number calculated as follows

$$\text{Dis} (a, a') = \max_{i \in F^-} \frac{d_i}{D_i}$$

Calculating for each pair $(a, a')$ of $A$ the value of the coefficients $\text{Dis} (a, a')$ we build a discordance table.

(1) One can find similar definitions in B. ROY - E. JACQUET LAGREZE and G. HIRSCH - see Bibliography.
Making up an outranking relation

Given the thresholds of concordance and discordance C and D.
For example C = 0.6 means that we are at least exacting for the sum of the
concurring criteria ($i \in F^a \cup F^a'$) to be superior to 0.6 to make up the
outranking of $a'$ by $a$ ($a'$ is preferred to $a'$)
In the application regarding the selection of candidates we use 4 nested outranking relations

\[ S_4 \subset S_3 \subset S_2 \subset S_1 \] 

obtained by making vary the concordance threshold \( C \) for a fixed threshold of discordance \( D = 1 \):

- \( C_1 = 0.9 \) for \( S_4 \)
- \( C_2 = 0.7 \) for \( S_3 \)
- \( C_3 = 0.6 \) for \( S_2 \)
- \( C_4 = 0.5 \) for \( S_1 \)

Then we are able to characterize the fuzzy outranking thus obtained taking for the degree of credibility \( d (a, a') \) the value of the coefficient of concordance \( C (a, a') \):

When \( \text{Dis} (a, a') \leq 1 \)
\[ d (a, a') = C (a, a') \]

When \( \text{Dis} (a, a') > 1 \)
\[ d (a, a') = 0 \]
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