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An approach to solving multiple criteria
macroeconomic policy problems and an application

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RESUME

Dans ce cahier, les auteurs présentent une approche multicritère et interactive pour résoudre des problèmes de politique économique utilisant des modèles économétriques.
SUMMARY

In this paper we propose an interactive multiple criteria optimization approach to solve problems associated with macroeconomic policy. In contrast with classical optimization methods no explicit knowledge of the decision-maker's preference function in terms of the objectives is required. The first part of the study consists of an experiment to test the applicability of the approach to economic policy. Three high-level knowledgable decision-makers participated in it and used the GEOFFRION multiple criteria method to solve the problem. The reactions of the decision-makers were favorable and one of them expressed his willingness to continue the implementation work at the Confederation of Finnish Industries. Because the decision-makers experienced some difficulty in using the GEOFFRION procedure, the ZIONTS and WALLENIUS method was implemented for the UNIVAC 1108 time-sharing system as well. The system has now been used for one year for optimizing economic policy and
PLAN

1 - INTRODUCTION

2 - AN INTERACTIVE MULTICRITERION OPTIMIZATION APPROACH TO MACROECONOMIC POLICY FORMULATION
AN APPROACH TO SOLVING MULTIPLE CRITERIA MACROECONOMIC
POLICY PROBLEMS AND AN APPLICATION

In this paper we propose an approach to solving multiple
criteria macroeconomic policy problems and report on an
application using an existing econometric model.

1. Introduction

During the past few decades efforts have been made to
develop econometric models for analyzing macroeconomic
policy problems in several countries. Typical policy
problems are, for instance, whether and by how much
different categories of taxes and other income transfers
should be increased or decreased, what kind of exchange
rate policies should be followed, etc. The value of a
simultaneous equation econometric model lies in the fact
that it helps to account for the complex interrelation-
ships existing among different variables and sectors of
the economy. Once such a model has been constructed,
a computer simulation approach can be used for generating
and comparing different policies and for choosing a policy
considered to be the best among the alternatives. This
approach does not require an explicit preference function
of the decision-maker, but his preferences are implicitly
present in the selection of the alternative policies on
the basis of the values of the target variables. Computer
simulation is, however, an inefficient method of finding
the best policy decisions and optimization models have been proposed as an alternative for solving these problems. In the optimization approach a preference function is optimized subject to a set of constraints which represent economic interrelationships among the variables and describe the possible values of the instrument variables (=decision variables, parameter variables, parameters). The purpose of the optimization process is thus to select the best solution (in terms of an overall preference function) from the set of feasible solutions in an efficient way.

Recent advances in multiple criteria optimization have made it possible not to require explicit knowledge of the decision-maker's preference function in terms of the objectives prior to solving the problem. Instead, as we shall demonstrate in this paper, the decision-maker's preferences can be identified by a procedure which simultaneously leads to choosing an "optimal" solution. The practical value of an optimization procedure depends, of course, on the ability of the underlying model to describe the phenomena under study, i.e., the word optimum refers to the mathematical "play process" and not necessarily to the "real process." In order to have a realistic
into a linear optimization model involving multiple objectives by allowing certain instrument variables to vary within a feasible range each. The optimization model was used for solving an actual decision problem of the Finnish economy and subsequently presented at the Confederation of Finnish Industries.
\[ y = (I - \hat{A})^{-1}\hat{B}x = \hat{x}. \]

Usually the exogenous variables also include a number of instrument variables which may take values from given intervals \([x^a, x^b], x^a < x^b\). For the other predetermined
variations by Eckaus and Parik (3), Kornai (12), MacEwan (15), Roskamp (17), Spivey and Tamura (18), Van Eijk and Sandee (23), Zeleny and Cochrane (30), among others. An interview approach to identifying a decision-maker's preference function has been developed by Frisch (7,8). For some related research see Johansen (10,11). The use of optimal control theory has been proposed by Chow (1), Pindyck (16) and Livesey (13,14). An extensive survey of the theory of quantitative economic policy is provided by Fox, Sengupta and Thorbecke (6). Until recently, however, relatively little has been done in trying to use such methods in practice.

The approaches discussed above can all be criticized on the score of the assumption that the decision-maker is able to construct an overall preference function carried over the time period for which he is planning. Recent research in multiple criteria decision-making suggests that this assumption can be relaxed by using iterative procedures if the decision-maker is able to provide certain local information about his preferences at each cycle.

Such approaches have been developed, among others, by Geoffrion, Dyer and Feinberg (9) and Zionts and Wallenius (31). Using either of the approaches a sequence of target vectors \( \{y^1, y^2, \ldots, y^N\} \), is generated, which the decision-maker can influence in accordance with his preferences, such that for all \( k \in \{1, \ldots, N\} \): \( y^k = \hat{y}^k \cdot x^k, x^k \cdot x = [x_1^a, x_1^b] \times \ldots \times [x_m^a, x_m^b] \). The process is terminated for some \( N \), when the decision-maker
does not want to change \( y \) any more. Such \( y^N \) is called the optimal target vector, \( y^N = y \), and it satisfies \( \forall k: \sum_{k=1}^{N} y^k \geq y^k \); that is, \( y \) is preferred to alternative solutions. If certain assumptions concerning the stability and form of the decision-maker's preference function \( U \) are made, it can further be shown that this procedure leads to an optimal solution of the maximization problem (A). For this to be true the preference function need not be a linear function of the instrument variables. It suffices that the relationship is concave. As such formulation (A) is, of course, more general than the linear programming formulation, which assumes that we can estimate (e.g. by employing fictitious questions) the decision-maker's preference function and that it is linear. (A) is also more general than the approach proposed by Spivey and Tamura (18) or Theil (19).

3. Problem Representation

In this section we provide a brief description of the problem. A summary of the model currently employed is given in an appendix to this paper and more detailed information may be found in Vartia (24). The model is constructed using the Dutch short-term annual model as a starting point and adapted to the circumstances prevailing in Finland (see e.g. Verdoorn, Post and Goslinga (26)). As usual with short-term models, the emphasis is on the demand side and no explicit production function is included in the model. The model is based on
annual percentage changes and consists of 13 behavioral
equations for the volumes and prices of the main expenditure
categories, for imports, labor input, unemployment and the
wage rate. In addition, the model has a number of equations
defining other endogenous variables. The exogenous variables
of the model include usual policy variables, such as incidence
of indirect taxes, income transfers, public expenditure and
changes in the exchange rate, which were taken as instrument
variables for the problem. Neither monetary policy instruments
nor an endogenous block for the public sector have been
incorporated in the model. The model has been estimated using
data for the years 1951-1970.

The econometric simulation model was expanded to an annual
optimization model involving multiple objectives by taking
some of the endogenous variables as target variables and by
allowing certain instrument variables to vary within feasible
bounds. For target variables we selected four traditional
aggregate variables relating to the internal and external
equilibrium of the economy: the percentage change in gross
domestic product, unemployment, the rate of inflation
(measured by consumer prices) and the balance of trade\(^1\).
Values for the lagged endogenous and fixed exogenous
variables were obtained from the latest "Economic Prospects
in Finland", ETLA (4), and they were kept up-to-date to
reflect the situation in the Finnish economy each time
the model was used. In the earlier experiments bounds for
the instrument variables were determined by us. When the
optimization model was used on a more permanent basis, it
was natural to let the decision-maker himself determine the bounds defining the set of feasible solutions.

A linear version of the model presented in (24) was used in the application. Nonlinear models have often to be linearized, because linear systems allow of much easier estimation and solution techniques. However, in our case little was lost by linearizing the model, since the difference between the two versions concerns only a number of definitional equations which in the original version incorporate certain cross products. We also emphasize that, although our model is linear in percentage changes of variables, it is, obviously, nonlinear in terms of the absolute values of the variables. Different versions of the model have been used at the Research Institute of the Finnish Economy for a period of three years for forecasting and analyzing economic fluctuations, and the results have been found useful. For additional tests of the accuracy of the model we refer to (24).

4. An Application

We performed a first test of our multiple criteria approach to solving an actual decision problem of the Finnish economy in February 1976. The experiment was successful and one of the decision-makers participating in it expressed his willingness to continue the implementation work at the Confederation of
Finnish Industries to install the optimization model as a more permanent decision-making tool. For that purpose a one-year project was initiated in the fall of 1976. In this section we describe the results of the experiment as well as our experience of using the optimization model at the Confederation of Finnish Industries.

4.1. Description of the Economic Situation

We initially provide a brief description of the economic situation in Finland at the time of our study. Finland was one of the few market economies where production did not decline during the recent deep international recession (the volume of GDP at market prices rose by 4.7 % in 1974 and by 0.1 % in 1975) and where the employment situation remained fairly good until the end of 1975 (the unemployment rate averaged 1.7 % in 1974 and 2.2 % in 1975). This was due to the strong investment activity which was to a large extent financed with foreign capital. As a consequence the country's foreign indebtedness grew quickly and rose to about a fifth of the annual gross domestic product by the end of 1975. Simultaneously the sharp rise in commodity prices had amounted to an inflationary impulse, which continued as a strong internal wage-price spiral. The rate of inflation as measured by consumer prices was 18 % in 1974 and 17.4 % in 1975, year-on-year.
At the beginning of 1976 the continuing growth of foreign indebtedness was generally regarded as Finland's most difficult problem. To achieve the central goal of external equilibrium, it was considered advisable to strongly reduce the rate of inflation and thus ensure competitiveness in foreign trade— even at the cost of lower production and higher unemployment. Since the controlling of inflation was also necessary in itself, not many were in favor of improving price competitiveness by altering the international value of the Finnish currency in 1976, as this would have affected the rate of inflation unfavorably.

The measures to control inflation and decrease the trade deficit were moderately successful during 1976. However, the inflation rate was still above ten per cent and the trade deficit more than four billion Fmk. Simultaneously,
4.2. An Experiment

A linearized version of the model presented in Vartia (24) was modified in the manner described above, so as to obtain an optimization model, and this together with the Geoffrion method assuming a monotonic preference function and a linear constraint set was implemented for the UNIVAC 1108 time-sharing system at the Helsinki School of Economics. Some parameter values of the model were also changed so as to better reflect the current economic situation.

Three high-level knowledgeable decision-makers participated in the experiment. They were 1) the Chief of the Bank Inspectorate, an ex-Cabinet Member, 2) the Deputy Managing Director of the Confederation of Finnish Industries and 3) a Director of the Bank of Finland. The purpose was to evaluate the applicability of our approach to macroeconomic policy formulation. Each of the decision-makers was familiar with the general characteristics and scope of the econometric model upon which the study is based. We discussed various aspects and explained the major features of our approach and the use of the method. After the starting solution the decision-maker was expected to provide two kinds of information at each cycle concerning his preferences: 1) An estimate of his marginal rates of substitution between the objectives determining the "best" direction of search. 2) Resolution of a step-size problem determining how much of a change to make. For a more complete treatment of the method the reader is referred to Geoffrion.
In an earlier work one of us had discovered that the Geoffrion, Dyer and Feinberg (9) method was relatively difficult to use (see Wallenius 27). This was why we decided to assist the decision-makers in several ways. A variant of the original method designed to help the decision-makers to estimate their marginal rates of substitution described in Dyer (2) was implemented. We also presented the decision-makers with a number of examples computed using different responses generated by us.

A summary of the results of the test is given in Table 1. For some additional details see (29). We note that two of the decision-makers wanted the gross domestic product to grow from the preceding year, but the third was content with a decrease in its value, since this made a considerable reduction in trade deficit possible. The resulting unemployment rates represent in each case increases on the previous year's figure. The rates of inflation chosen by the three persons were close to each other and not far from the absolute minimum inflation achievable. The "optimal" balance of trade deficits varied considerably among the three persons but promised in each case an improvement on the preceding year's Fmk 7.75 billion.
Table 1: Summary of the Results of the Iterative Decision-Making Procedure for 1976

Criteria

<table>
<thead>
<tr>
<th>Cycle</th>
<th>GDP Increase, %</th>
<th>Inflation, %</th>
<th>Unemployment Rate, %</th>
<th>Trade Deficit Billions Fmk.</th>
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<tr>
<td></td>
<td>(7.18)</td>
<td>(8.16)</td>
<td>(1.88)</td>
<td>(1.21)</td>
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</table>

First Decision-Maker

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<td>2.24</td>
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<tr>
<td>2</td>
<td>0.57</td>
<td>9.00</td>
<td>2.81</td>
<td>5.27</td>
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<tr>
<td>3</td>
<td>1.81</td>
<td>8.88</td>
<td>2.64</td>
<td>6.54</td>
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Second Decision-Maker

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<td>3.28</td>
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<tr>
<td>2</td>
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<td>2.95</td>
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<td>3</td>
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Third Decision-Maker

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<td>-1.39</td>
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<td>-1.39</td>
<td>8.69</td>
<td>3.06</td>
<td>3.46</td>
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</table>

1) "Utopian" solutions obtained with one-at-a-time optimization are given in parentheses under each criterion.

Some Reactions of the Decision-Makers

The decision-makers considered the results yielded by the model and the relationships among the objectives realistic.
region of the decision variables may not have corresponded exactly to the one thus considered relevant in the current economic situation. The decision-makers seemed to appreciate the interactive processing mode providing rapid feedback and were satisfied with the way the Geoffrion method worked, with the exception that the estimation of the marginal rates of substitution among the objectives (despite the assistance provided) was not considered simple enough.

The advantages of the optimization framework were considered to be the following:

1. The model would provide information about the relationships between the objectives and between the objectives and policy instruments.
2. The decision-makers would better learn their preferences in terms of the objectives.
3. The optimization framework would indicate which are feasible and which are infeasible targets for economic policy.

Also, the possibility of using this kind of "policy formulation game" as a pedagogic instrument for different groups of decision-makers was considered a fruitful application.
4.3. Application at the Confederation of Finnish Industries

Our intention was to implement the optimization model as a more permanent decision-making tool at the Confederation of Finnish Industries. The Confederation of Finnish Industries does not have the authority to implement the results but as an interest group of various industries it can influence the economic policy pursued. Our involvement in the project terminated by writing a detailed user's manual (28) and by teaching some persons at the Confederation of Finnish Industries to use and update the system, so that its future use is no longer necessarily dependent on us. Prior to the application some of the equations had been reestimated and the model extended with an additional behavioral equation. In addition, net income transfers were disaggregated into a number of components, public expenditure was decomposed into public consumption and investment and total investment into private and public sector investment. The values of the predetermined variables were updated in the fall of 1976 and in the spring of 1977 in accordance with the latest "Economic Prospects in Finland", ETLA (4). In its present form the periodic updating of the model involves some labor, but it could easily be automatized so that the computer would do most of the routine work.

In the initial stages of the project we used the Geoffrion, Dyer and Feinberg method (9), but because the decision-makers experienced some difficulty in using it we subsequently
switched to the method developed by Zions and Wallenius (31). Both methods are fully interactive and the system allows the user to execute programs in a conversational way. In the Zions and Wallenius method, the decision.
the decision-maker corresponds to the relevant region in
the current economic situation. In practice a reasonable
approach seems to be to start with loose bounds and, as
more information becomes available, modify the feasible
Representing a first application of the general version of the Zionts and Wallenius optimization procedure, the tests provide us with some useful information about its performance. The procedure seemed generally to function well, its ease of use was appreciated and it was not sensitive to inconsistent responses by the decision-maker. On average, approximately four question sessions and ten responses were required for finding the neighborhood containing the optimal solution. The cost of a run was of the order of US $25. As the decision-maker's preferences were nonlinear in general, the method found a solution within some neighborhood of an optimum. In the tests it sometimes happened that some of the solutions neighboring to the optimal solution were relatively far from each other and it was thus necessary to use further optimization to find the optimal solution. It seemed sufficient to perform this further optimization by visualizing the neighborhood in which the true optimum was located and letting the decision-maker select the most preferred solution from this neighborhood. No sophisticated search methods were used but, if desired, could relatively easily be incorporated. An example of the use of the Zionts and Wallenius optimization procedure, performed in early 1977, will now be presented.

An initial solution was determined by using arbitrary weights for the objectives. The result, provided in terms of the objectives and the instrument variables, is presented in Table 2. In addition, the user was provided with a complete
<table>
<thead>
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<th>Value</th>
<th>Weight</th>
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<td>57</td>
<td>4.56</td>
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<td>48</td>
<td>2.58</td>
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<td>2.5</td>
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list of the values of the other variables and, if desired, the associated aggregate balance of resources and expenditure at each iteration. The next step requires the decision-maker to respond to certain efficient tradeoff questions concerning feasible changes in the values of the objectives. More specifically, the decision-maker was asked whether he was willing to accept an increase in GDP of .2005 %, an increase in inflation of .1925 %, a decrease in unemployment of .0333 %, and an increase in trade deficit of Fmk .1013 billion corresponding to a unit devaluation of the currency.
question and the decision-maker was advised to respond "I don't know." Obviously, the solution consistent with the three most recent responses remained unaltered. In such a case we generate neighboring solutions to the previous solution corresponding to yes and I don't know responses. This time there was only one such solution
currency was devalued in April 1977 by 5.7%, the impact was simulated using the model for the year 1977. Simultaneously it was investigated which effects a larger devaluation would have had. In the following we report in more detail on the results of a simulation run performed after the Government had announced in June 1977 a program for the stimulation of the growth of the economy. These results were used as one source of information in evaluating the effectiveness of the proposed measures.

The program consisted of the following measures, among others, to be implemented in the fall:

1) The turnover tax and some other indirect taxes will be reduced by Fmk 240 million.
2) Exports will be subsidized by Fmk 20 million.
3) Income transfers from the public sector to households will be increased by Fmk 28 million.
4) Employers' contributions to social security will be reduced by Fmk 138 million.
5) Funding of industrial investments will be increased by Fmk 350 million.
6) Governmental share of the capital stock of certain public companies will be increased by Fmk 30 million.

From Table 3 we see that the main effects were those on investment, imports and private consumption. GDP was estimated to grow by about 0.5%, but the trade deficit
would at the same time increase by Fmk 500 million. The program does not seem to have any major impact on the employment situation, nor does it increase exports much.

Table 3: Changes in the Aggregate Balance of Resources and Expenditure Due to the Governmental Program to Stimulate the Economy

<table>
<thead>
<tr>
<th></th>
<th>Annual Change (percentage)</th>
<th>Absolute Changes</th>
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</thead>
<tbody>
<tr>
<td>GDP (mp)</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>IMPORTS</td>
<td>0.7</td>
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<tr>
<td>TOTAL RESOURCES</td>
<td>0.9</td>
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</tr>
<tr>
<td>EXPORTS</td>
<td>0.3</td>
<td>4.1</td>
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<td>0.8</td>
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<tr>
<td>INVENTORY CHANGES</td>
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</tr>
<tr>
<td>TOTAL DEMAND</td>
<td>0.9</td>
<td>1.6</td>
</tr>
<tr>
<td>TRADE DEFICIT</td>
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<tr>
<td>UNEMPLOYMENT (%)</td>
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<td>UNEMPLOYMENT (%)</td>
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5. Conclusion

In this paper we have described work in process to implement an optimization model for formulating macroeconomic policy decisions in Finland. One of three experienced decision-makers who experimentally used the model to solve the problem for 1976 felt that the approach would be valuable in providing decision-makers with relevant quantitative information and
Appendix: A Model of the Finnish Economy

In the following list of variables all exogenous variables are underlined. Unless otherwise stated, capital letters stand for value and small letters for volume. Absolute variables are denoted by the symbol \( \cdot \); all other variables denote percentage changes from the previous year. Because no confusion is expected to arise, the subscript \( t \) denoting time has been dropped. Thus, for instance, \( C_t \) is abbreviated \( C \) and \( C_{t-1} \) (the lagged value) \( C_{-1} \).

**CONSUMPTION**

\[
\begin{align*}
&c, C \quad \text{total consumption} \\
cpr, Cpr \quad \text{private consumption} \\
cg, Cg \quad \text{public consumption} \\
&pc \quad \text{price of total consumption} \\
p_cpr \quad \text{price of private consumption} \\
p_cg \quad \text{price of public consumption}
\end{align*}
\]

**INVESTMENTS**

\[
\begin{align*}
&i, I \quad \text{total investments} \\
i_pr, Ipr \quad \text{private investments} \\
i_as \quad \text{private investments in housing} \\
i_asc \quad \text{other private investments} \\
i_g, Ig \quad \text{public investments} \\
&N \quad \text{inventory changes} \\
&pi \quad \text{price of total investments} \\
pi_pr \quad \text{price of private investments} \\
pi_g \quad \text{price of public investments}
\end{align*}
\]
FOREIGN TRADE

Imports
m, M total imports
mg, Mg commodity imports
ms, Ms imports of services
pm price of total imports
pmg price of commodity imports

Exports
mw weighted growth of industrial production in ten OECD countries (export demand variable)
x,X total exports
xE, XE bilateral commodity exports
xg, Xg commodity exports
xgw, Xgw multilateral commodity exports
xs, Xs exports of services
px price of total exports
pxcom price of competing exports
pxg price of commodity exports

Others
E trade deficit
S factor income from abroad (net)

SALARIES

H wage increases exceeding profitability
k1 effects of employers' contribution to social security
W total wage bill (inc. social security)
Ws total wage bill (exc. social security)
wI wage rate

LABOR INPUT

all total labor input
U unemployment rate
INCOME VARIABLES

K    profit margin
NI   national income
O₁   public sector income from property and enterprises
WZD  disposable income of households
Z₁₀  nonlabor income

SAVING

Q₄   corporate saving

TAXATION Q₃   direct corporate taxes
Q₅   direct taxes of households
T₁   indirect taxes minus subsidies
T₁DN incidence of indirect taxes minus subsidies

INCOME TRANSFERS

Q₆   other income transfers from households to the public sector
Q₇   income transfers from the public sector to households
Q₈   income transfers of households abroad
Q₉   income transfers from abroad to households
Q₁₀  net income transfers of households

OTHERS

d, D total demand
py \quad \text{price index of gross domestic product}

Ynfc \quad \text{net domestic product at factor prices}

Desiderata:

1. Change in the volume of gross domestic product (y) as big as possible.
2. Inflation (pc) as small as possible.
3. Unemployment rate (\bar{U}) as small as possible.
4. Trade deficit (E) as small as possible.

Behavioral equations:

Domestic expenditure:

1. \[ cpr = .365(WD - pcpr) + .435(WD - Cpr)_{-1} - 2.407\bar{U} + 2.389 \]
2. \[ iasc = 3.111\Delta y_{-3/4} + .527Z_{10} + .487Z_{10}_{-1} - .573pipr - .916 \]
3. \[ N = .321\Delta d_{-1/2} + .037\Delta pmg - .369N_{-1} + .695 \]

Foreign sector:

4. \[ xgw = 2.338mw - .520(pxg - pxcom) - .628(pxg - pxcom)_{-1} \]
\hspace{2cm} - .308(pxg - pxcom)_{-2} - 3.172
\hspace{2cm} + .374d_{dn} - 3.888 \]
9. \( \text{pcpr} = .362H + .164pmg + .207TiDN + 2.084 \)
10. \( \text{pcg} = .757H + .060pmg + .090\text{pcg}_{-1} + 2.486 \)
11. \( \text{pir} = .363H + .255pmg + .155\text{pir}_{-1} + 1.086 \)
12. \( \text{pig} = .812H + .208pmg + .136\text{pig}_{-1} - .254 \)
13. \( \text{pxg} = .189H + .780\text{pxcom} + .069pmg + .3(\text{pxg} - .189 - .069pmg)_{-1} - .297 \)

Definitional equations: 

14. \( \text{Cpr} = \text{cpr} + \text{pcpr} \)
15. \( \text{Ipr} = \text{ipr} + \text{pir} \)
16. \( \text{ipr} = (\text{tiasc/tipr})_{-1}\text{iasc} + (\text{tias/tipr})_{-1}\text{ias} \)
17. \( G = (\text{tCg/tG})_{-1}\text{Cg} + (\text{tIg/tG})_{-1}\text{Ig} \)
18. \( Xgw = xgw + pxg \)
19. \( \text{Mg} = \text{mg} + \text{pmg} \)
20. \( D = (\text{tcpr/tD})_{-1}\text{Cpr} + (\text{tipr/tD})_{-1}\text{Ipr} + (\text{tDN/tD})_{-1}\text{N} + (\text{tG/tD})_{-1}\text{G} + (\text{tX/tD})_{-1}\text{X} \)
21. \( \text{DN} = (\text{tcpr/tDN})_{-1}\text{Cpr} + (\text{tipr/tDN})_{-1}\text{Ipr} + (\text{tG/tDN})_{-1}\text{G} + (\text{tX/tDN})_{-1}\text{X} \)
22. \( \text{dn} = (\text{tcpr/tdn})_{-1}\text{cpr} + (\text{tipr/tdn})_{-1}\text{ipr} + (\text{tg/tdn})_{-1}\text{g} + (\text{tx/tdn})_{-1}\text{x} \)
23. \( \text{DN} = \text{pdn} + \text{dn} \)
24. \( \text{H} = \text{wI} - (y - \text{aII})_{-1/2} \)
25. \( \text{W} = \text{Ws} + \text{kl} \)
26. \( \text{K} = \text{pdn} - (\text{tWS/tDN})_{-1}\text{wI} - (\text{tTi/tDN})_{-1}\text{TiDN} - (\text{tM/tDN})_{-1}\text{pm} \)
27. \( \text{Ws} = \text{aII} + \text{wI} \)
28. \( \text{WZO} = (\text{tw/tWZO})_{-1}\text{w} + (\text{tZ10/tWZO})_{-1}\text{Z10} + (\text{tW + tZ10}/\text{tWZO})_{-1}\text{D10} \)

+) The symbol \( t \) in the definitional equations is used to indicate the absolute value of the variable.
29. \( Ti = DN + TiDN \)
30. \( xE = XE - px \)
31. \( xg = (txgw/txg)_{-1}xgw + (txE/txg)_{-1}xE \)
32. \( x = (txg/tx)_{-1}xg + (txs/tx)_{-1}xs \)
33. \( Xg = (tXgw/tXg)_{-1}Xgw + (tXE/tXg)_{-1}XE \)
34. \( X = (tXg/tX)_{-1}Xg + (tXS/tX)_{-1}XS \)
35. \( X = x + px \)
36. \( m = (tmg/tm)_{-1}mg + (tms/tm)_{-1}ms \)
37. \( M = (tMg/tM)_{-1}Mg + (tMs/tM)_{-1}Ms \)
38. \( yn = (tdn/tyn)_{-1}dn - (tm/tyn)_{-1}m \)
39. \( d = (tcpr/td)_{-1}cpr + (tipr/td)_{-1}ipr + (tdn/td)_{-1}N + (tg/td)_{-1}g + (tx/td)_{-1}x \)
40. \( y = (td/ty)_{-1}d - (tm/ty)_{-1}m \)
41. \( Y = (tD/ty)_{-1}D - (tM/ty)_{-1}M \)
42. \( Y = py + y \)
43. \( Ynfo = (tY/tYnfo)_{-1}Y - (tF/tYnfo)_{-1}F - (tT/tYnfo)_{-1}T \)
44. \( Nl = (tYnfo/tNl)_{-1}Ynfo + (tYnfo/tNl)_{-1}S \)
45. \( Z_{10} = (tNl/tZ_{10})_{-1}NI - (tW/tZ_{10})_{-1}W \)
46. \( c = (tcpr/tc)_{-1}cpr + (tcg/tc)_{-1}cg \)
47. \( i = (tipr/ti)_{-1}ipr + (tig/ti)_{-1}ig \)
48. \( Cg = cg + pcg \)
49. \( Ig = ig + pig \)
50. \( C = (tCpr/tC)_{-1}Cpr + (tCg/tC)_{-1}Cg \)
51. \( I = (tIpr/tI)_{-1}Ipr + (tIg/tI)_{-1}Ig \)
52. \( C = c + pc \)
53. \( I = i + pi \)
54. \( G = g + pg \)
55. \( O_{10} = -0.1 + 0.2 - 0.3 - 0.4 - 0.5 - 0.6 - 0.7 - 0.8 + 0.9 \)
56. \( \vec{X} = \vec{Mg}_{-1} + \frac{Mg}{100} \vec{Mg}_{-1} - \vec{Xg}_{-1} - \frac{Xg}{100} \vec{Xg}_{-1} \)
57. \( g = (tcg/td)_{-1}cg + (tig/tg)_{-1}ig \)
In order to take into account the effect of eventual exchange rate changes on exogenous foreign trade variables the following equations were used (variables with the bar represent forecast values of the exogenous variables excluding the effect of exchange rate changes):

58. $p_m = \bar{p}_m + D_e$
59. $pxcom = \bar{pxcom} + D_e$
60. $pmg = \bar{pmg} + D_e$
61. $Ms = \bar{Ms} + D_e$
62. $X_e = \bar{X_e} + D_e$
63. $Xs = \bar{Xs} + D_e$
References


(4) "Economic Prospects in Finland", Research Institute of the Finnish Economy, (ETLA), Bi-Annual Publication, Helsinki.


Footnotes

1 At the end of 1975 the Economic Council, chaired by the
6 A definition of an efficient neighboring solution and the procedure for identifying them is given in (31).

7 The necessary FORTRAN programs were developed by Mr. Matti Sihto, Helsinki Technical University.