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COMPENSATORINESS OF PREFERENCES
IN MATCHING AND CHOICE

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(NON-)COMPENSATION DES PRÉFÉRENCES ÉVALUÉES PAR COMPARAISONS BINAIRES ET QUESTIONS D’AJUSTEMENT

Résumé

Cet article étudie expérimentalement l’influence d’un mode de questionnement sur les préférences exprimées par un décideur. La structure de préférence entière de décideurs est évaluée à travers deux modes de questionnement différents : le choix et l’ajustement. Les résultats montrent une sérieuse mise en défaut du principe d’invariance procédurale. Les impacts en terme de modélisation des préférences et de techniques d’évaluation de paramètres d’importance sont discutés.


COMPENSATORINESS OF PREFERENCES IN MATCHING AND CHOICE

Abstract

This paper empirically studies the influence of a questioning mode on elicited preferences. The entire preference structure of decision makers is elicited using two different questioning modes: choice and matching. The results show a strong failure of procedure invariance. Impacts on preference modelling and importance parameters elicitation techniques are discussed.

Keywords: Multiple Criteria Decision Aid, Preference, (Non-)Compensatoriness, Experiment, Procedure Invariance, Importance of Criteria.
1. Introduction

Real world decision situations often involve several objectives, viewpoints or criteria. Different methodologies dealing with Multiple Criteria Decision Aid (MCDA) have been proposed in the past decades (see [Keeney & Raiffa 76], [Roy 85]) and the numerous aggregation techniques reported in the literature are useful tools to help managers facing decision tasks. Several fundamental principles concerning the consistency of preferences emerge from these works. Among these is the procedure invariance principle which states that normatively equivalent preference elicitation techniques should lead to the same elicited preferences. By analogy, when comparing the length of objects, the available tools should lead to the same lengths and thus compare the objects in the same way; no interaction appears between the tools and the objects to be measured.

Unfortunately, this principle seems to be contradicted by decision makers' (DMs) behavior. Psychological and behavioral science studies showed that DMs fail to respect the procedure invariance principle when expressing preferences. [Lichtenstein & Slovic 71,73] first highlighted the preference reversal phenomenon in the context of risky decision making. These authors found that DMs' answers to direct comparison and minimum selling price questions induce different preferences though these two questioning modes are normatively equivalent. This preference reversal phenomenon clearly contradicts the procedure invariance principle.
More recently, [Tversky et al. 88] showed that the same kind of violation may occur within the context of riskless decisions. In this paper, two different questioning modes were studied in a bi-criterion context: direct binary choice between alternatives and matching (matching questions consist in proposing two alternatives to the DM, one of the evaluations of an alternative being unfixed; the DM is to determine this evaluation in order to be indifferent between the two alternatives). The authors showed a prominence effect which states that "the more prominent attribute looms larger in choice than in matching", in other words that "information-induced prominence can be a factor of the choice behavior".

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2. (Non-)Compensatoriness of preference structures

The notion of (non-)compensatoriness of preference structures is intuitively linked to the possibility to resort to substitution rates in the construction of preferences that take all criteria into account. These substitution rates allow to compensate a disadvantage on a particular criterion by a sufficient advantage on another criterion. When an aggregation procedure uses such ideas to solve conflicts between criteria, the induced preference structure is said to be compensatory. Otherwise it is said to be non-compensatory. Similarly, preference...
More recently [Roy & Mousseau 92] proposed a definition allowing the analysis of compensation on each criterion individually (this definition is grounded on a new formalism of the notion of relative importance of criteria). However the (non-)compensatoriness of a preference structure is a notion that deserves to be studied in further details. Firstly no large consensus emerges concerning its definition. Moreover the available definitions only define extreme cases and do not allow to precise to which level a preference structure is (non-)compensatory.

2.2. Construction of a (non-)compensatoriness index

Total non-compensation is an extreme situation never reached in real world decision contexts. Let us define an index aiming at "measuring" the (non-)compensatory aspects of a preference structure $\Psi = (X,P,\Pi_p)$. $\Psi$ is even more non-compensatory since two pairs of alternatives having the same preferential profile on criteria are linked with the same preference relation on the overall level. Possibilities of compensation in a preference structure depend on the overall preference situation that link pairs $(x,y)$ and $(z,t) \in X^2$ verifying :

\[
\begin{align*}
P(x,y) &= P(z,t) \\
P(y,x) &= P(t,z) \\
x_i = y_i \text{ and } z_i = t_i \quad \forall i \in I(x,y)
\end{align*}
\]

When all pairs $(x,y)$ and $(z,t)$ verifying [1] are such that $xPy \Rightarrow zPt$, the considered preference structure is totally non-compensatory (see definition 1). Possibilities of compensation appear in a preference structure when there exist $(x,y)$ and $(z,t)$ such that

\begin{itemize}
  \item not$(xPy \Rightarrow zPt)$ i.e. $xPy$ and not$(zPt)$
  \item i.e. $(xPy \text{ and } zIt)$ or $(xPy \text{ and } tPz)$
\end{itemize}

The first case of compensation (quadruplet verifying [1] such that $xPy$ and $zIt$) corresponds to definition 2. The second situation (quadruplet verifying [1] with $xPy$ and $tPz$) corresponds to a stronger compensation (a preference reversal). However our index will account for these two situations in the same way. This index will measure proportion of quadruplets $(x,y,z,t)$ verifying [1] such that $xHy \Leftrightarrow zHt$ (H being any preference relation).

Suppose that all $X_i$ are discrete and let us define :

$T = \{(x,y,z,t) \in X^4 \text{ such that } P(x,y)=P(z,t) \text{ and } P(y,x)=P(t,z) \}$ with $P(x,y)=\{i \text{ such that } xPy\}$

$NC = \{(x,y,z,t) \in T \text{ such that } xHy \Leftrightarrow zHt \}$, H being any overall relation

Let us define the (non-)compensatoriness index of a preference structure $\Psi$ by :

\[nc(\Psi) = \frac{|NC|}{|T|} \quad nc(\Psi) \in [1/\lambda, 1]\]
In this definition, $\lambda$ is the number of possible preferential situations on overall level (when preferences are modeled using a (L,P) structure, three preferential situations are possible between two alternatives: aPb, alb and bPa). This index is built such that $\text{nc}(\Psi)=1$ means that $\Psi$ is totally non-compensatory (lexicographic order); a decreasing value for $\text{nc}(\Psi)$ means greater possibilities of compensation in $\Psi$. Thus the index $\text{nc}$ allows us to compare the (non-)compensatoriness of two preference structures; this index has only an ordinal signification (no cardinal use of this index will be made).

3. Empirical scheme

Our aim in this experiment is to highlight the violation of procedural invariance and to exhibit a link between the use of a specific questioning mode (choice and matching in our case) and the (non-)compensatoriness of the elicited preference structures. We will deal with a bi-criteria context. The scale on both criteria will be discretized on a four level scale \{A,B,C,D\} where A P_1 B P_1 C P_1 D i=1,2.

3.1. The two questioning modes

The experiment consists in the exhaustive evaluation of the preference structure using two different questioning modes:

Choice: this type of questions consists in a hollistic comparison of two alternatives defined by their evaluations on all criteria. Subjects are to choose between an indifference situation or a preference in favor of one of the alternatives. As we deal with a bi-criteria context, let us denote by \((x_1,x_2)?(y_1,y_2)\) such a question.

Matching: this type of question consists in proposing two alternatives \((x_1,x_2)\) and \((y_1,y_2)\) leaving \(y_2\) unfixed. Subjects are to determine \(y_2\) in order to obtain indifference between the two alternatives. Let us denote by \((x_1,x_2)?(y_1,?)\) such a question. As in our experiment answers are given on a continuous scale, it is necessary to analyse the answers in terms of the discretized four level scale (A→D). In order to do so we use an indifference threshold $q$ which represents the minimum discernable difference between two evaluations (see [Roy & Vincke 87]). This threshold is elicited beforehand and used to compare the value $y_2$ to the four levels of the discretized scale. The nine possible situations are represented in figure 1.
3.2. Empirical hypotheses

We use a intra-subject experimental design to test two connected hypotheses: the prominence hypothesis and the contingent compensation hypothesis.

The prominence hypothesis lies in the fact that preferences elicited by choice questions should be closer to a lexicographic order than those elicited by matching questions. In other terms, the "more important" criterion should be taken into account more often in the first case than in the second. In order to test this hypothesis, we will count in the two elicited preference structures the number of "cases" in which an advantage on the preponderant criterion is taken into account firstly (if \((x_1, x_2)P(y_1, y_2)\) with \(x_iPy_1\) and \(y_2Py_1\) then criterion \(C_i\) is taken into account in the first place). Let us denote \(C^*\) the criterion taken into account the more often. Let \(p(\Psi_m)\) (respectively \(p(\Psi_c)\)) be the proportion of cases in which an advantage on \(C^*\) is decisive on the overall level when preferences are elicited by matching questions (respectively by choice questions). The prominence hypothesis states that: \(p(\Psi_m) < p(\Psi_c)\)

The contingent compensation hypothesis states that there are more possibilities of compensation in preference structures elicited with matching questions than in the one elicited using choice questions. In order to judge the (non-)compensatoriness of elicited preferences, we will use the index \(nc(\Psi)\) proposed in section 3.2. Let us recall that this index varies between 0 and 1 and that its value increases with the non-compensatoriness of \(\Psi\). Let us denote by \(\Psi_m\) (respectively by \(\Psi_c\)) the preference structure elicited using matching questions (respectively choice questions). The contingent compensation hypothesis states that: \(nc(\Psi_m) < nc(\Psi_c)\).
3.3. Experimental framework

The real world context concerns the evaluation of firms by young executives working in computer science companies so as to postulate in these firms. The proposed firms differ in the annual salary and the job interest. The scales of both criteria (salary and job interest) are discrete and made of four levels of evaluation.

Criterion 1: The job interest is evaluated on a qualitative scale that is built with subjects. Each level is defined by linguistic terms. Instructions were given to subjects: they had to build this scale so as to perceive the "distance" between consecutive levels as equivalent. It was then checked that all levels are separated by a strict preference. For example, the scale of one of the subject was:

- A: Very interesting job, formative, no repetitive aspect, evolutive and large independence at work.
- B: Interesting job, slightly repetitive but formative, rather evolutive, good independence at work.
- C: Job of little interest, quite repetitive but still somehow formative, evolutive in the long term, fairly little independence.
- D: Boring job, very little formative, numerous repetitive aspects, annoying hierarchy.

Criterion 2: The salary is measured in thousands of francs per year. This numerical scale is discretized in a four level scale; all consecutives level are separated by a strict preference and are built so as to reflect various salaries (from very attractive to quite repulsive). The values corresponding to the four levels are determined regarding the subject's expectations in terms of salary.

3.4. Simplifying postulates
Postulate 2 (monotonicity): $\forall x,y,z,t \in X \left[ \begin{array}{l} xPy \text{ and } t\Delta_p x \Rightarrow tPy \\ xPy \text{ and } y\Delta_p z \Rightarrow xPz \end{array} \right.$

The interpretation of this postulate is the following: "when an assertion $aPb$ is established, increasing the evaluation of $x$ or decreasing the evaluations of $y$ leaves the overall preference relation between $x$ and $y$ unchanged.

Postulate 3 (a partial preference is valid on the overall level):

\[
\begin{array}{l}
x I y \text{ and } y\Delta_p z \Rightarrow xPz \\
x I y \text{ and } t\Delta_p x \Rightarrow tPx
\end{array}
\]

i.e. $I \subseteq \Delta_p \subseteq P$

As in the preceding postulate, this postulate enables us to induce assertion from previously determined assertions. In concrete terms, if $(x_1,x_2)I(y_1,y_2)$ then we have:

\[
\begin{array}{l}
\forall (z_1,z_2) \in X \quad (z_1,z_2)\Delta_p (x_1,x_2) \Rightarrow (z_1,z_2)P(y_1,y_2) \\
\forall (t_1,t_2) \in X \quad (y_1,y_2)\Delta_p (t_1,t_2) \Rightarrow (x_1,x_2)P(t_1,t_2).
\end{array}
\]
answering time could be recorded. The average answering time on the whole subject group is 24 seconds for matching questions and 30 seconds for binary comparisons. A T Student test shows that these two mean values differ significantly with \( \alpha \leq 0.01 \). This difference may be explained by the fact that only 3 answers are possible in choice questions while subjects are to answer to matching questions on a continuous scale. Moreover we observed during the interviews that subjects seem to test a few binary comparisons in order to answer to matching questions.

### 4.1. Prominence effect

Let us recall that the prominence effect states that preferences elicited by binary comparisons are closer to a lexicographic structure that those elicited by matching questions, i.e. the preponderant criterion is taken onto account more frequently in the first case than in the second one (see 4.2). Let \( \Psi_c \) (respectively \( \Psi_m \)) be the proportion in which an advantage on the criterion \( C^* \) is considered first (\( C^* \) being the criterion that is taken into account the more often) when preferences are elicited by binary comparisons (respectively by matching questions).

The results are presented in figure 4 in which each point corresponds to a subject. It is obvious that the scatterplot is located below the bisectrix. This means that an advantage on the preponderant criterion \( C^* \) is more often decisive when preferences are elicited through choice questions rather than by matching questions.

![Figure 4](image-url)
In this experiment, the mean value for $p(\Psi_c)$ (respectively $p(\Psi_m)$) is 0.800 (resp. 0.663) and its standard deviation is 0.116 (respectively 0.133). A Wilcoxon test leads to accept that the mean values of $p(\Psi_c)$ and $p(\Psi_m)$ are significantly different with $\alpha \leq 0.01$. A T Student test leads to the same conclusion with $\alpha \leq 0.01$. We can thus conclude that our experiment confirms the prominence effect.

4.2. Contingent compensation effect

The contingent compensation effect is closely related to the prominence effect and states that elicited preferences reveal more possibilities of compensation when the questioning mode is matching rather than binary comparison. So as to measure the possibilities of compensation inherent in a DM’s preference structure, we use the index $nc$ built in section 3.2. Let us recall that this index varies between $\Psi_s$ and 1 and the higher the value for $nc(\Psi)$, the less possibilities of compensation exist in $\Psi$ ($nc(\Psi)=1$ corresponding to a lexicographic order).

We compute the values of this index for both questioning modes and all subjects. Results are synthetized in figure 5 in which each point represents a subject. The scatterplot is located below the bisectrix, i.e. the values for $nc(\Psi_c)$ are higher than those of $nc(\Psi_m)$ for a large majority of subjects.

![Figure 5](image_url)
In our sample, the mean value across subjects for the computed value of the index nc is 0.535 (respectively 0.685) when preferences are elicited by matching questions (respectively choice questions) and the standard deviation is 0.145 (respectively 0.144). A Wilcoxon test leads to conclude to a significant difference between the mean values for nc(Ψₐ) and nc(Ψₑₐ) with α≤0.01. A T Student test leads to the same conclusion with α≤0.01. We can thus conclude that our experiment confirms the contingent compensation effect.

5. Discussion

The experimental results show a strong failure of the procedure invariance principle: we observed, in our sample, a significant divergence among preferences elicited through two different questioning modes (choice and matching questions). More precisely, subjects answered in such a way that matching questions elicit preference structures in which more possibilities of compensation are allowed and in which the preponderant criterion looms larger than in the one elicited with binary comparisons. Such liabilities in preferences have already been reported in the literature (see [Weber & Borcherding 93] for an overview) and seems to be a problem that is inherent to preference elicitation.

Descriptivist vs constructivist analysis of lability of preferences

Many works in the field of behavioral research have reported a great lability of elicited preferences and values (see [Fishhoff et al. 89]) such as framing effect (see [Kahneman & Tversky 84], [Slovic et al. 81], [Johnson et al. 91]), splitting effect (see [Fishoff et al. 78], [Adelman et al. 86], [Weber et al. 88]), question order effect (see [Mousseau 92]), procedure invariance violation (see [Tversky et al. 88], [Fisher & Hawkins 89]).

There are two ways to account for such lability and to explain observed phenomena. These two approaches disagree on the nature of what is being modelled: are preferences to be considered as pre-existing and thus to be discovered or are they a result of interactions between the DM and the elicitation tool (see [Roy 93])? This distinction is essential as the two approaches use experimental works in a very different way.

The first stream, the descriptivist approach (also called discoverist or realist approach) refers to stable pre-existing preferences. In this framework, there are values to be discovered reflecting DM’s true preferences. By analogy with physical measurement in which each object is assumed to have a well-defined value for a specific attribute, true preferences exist and are to be measured. Discrepancies between elicitation tools reflect the fact that these tools only provide estimates of the true preferences, the elicited preferences being biased. True
preferences are supposed to remain constant but to be distorted during the elicitation process. Elicitation techniques orient preferences in certain directions, introduce noises ... However it is assumed that, when the analyst and the DM are sufficiently careful, have enough time and use different elicitation methods, obtained preferences should converge on true preferences.

Lability of preferences may also be analysed following a constructivist approach. In this case, preferences are not assumed to be pre-existent to the modelling process. [Bell et al. 88] emphasize that "it is a planotic myth that latently probabilities and utilities really exist deep down and that the analyst merely has to cut away the fat in order to display the pre-existing structure". Observed preferences are considered as a construct of the elicitation process and analysed as a result of interactions between the DM and the elicitation tool. This does not mean that DMs are free from any opinion but the constructs refer to basic attitudes, values and opinions that cannot be observed directly and to which we have access only through the filter of an elicitation procedure. Lability of preferences results from differences in the interactions between DMs and elicitation tools.

In a descriptivist interpretation, the violation of the procedure invariance principle is then analysed as biases in the elicitation procedures (see [Hershey et al. 82]). In our case, choice questions are supposed to be biased in such a way that DMs tend to "overweight the more important criterion" while matching questions "push" DMs in the opposite direction. The "true pre-existing preferences" are supposed to be in between and the use of different questioning modes should lead to converge to these true preferences.

In the constructivist approach, DMs' preferences are not viewed as totally pre-formed; they are built (at least partially) during the modelling process. In this framework, elicited preferences are the result of an interaction between the DM and the elicitation tools; in this sense, this tool is not to be considered as neutral. Following this approach, the experimental results described in this paper may be used in two distinct ways according to the objective to be pursued. [Tversky et al. 88] obtained similar results and proposed a model (the contingent tradeoff model) that accounts for the observed divergences among the preferences elicited by different questioning modes in which "the tradeoffs among inputs depend on the nature of the output". In this case, the pursued objective is to account for the observed phenomenon through an explicative model.

We propose an alternative use of the same observed phenomenon. Our goal is to use the general tendency of the empirical observations in order to induce a rule concerning the questioning mode to be used in importance parameters elicitation techniques. We observe that binary comparisons induce a preference structure of a more non-compensatory nature than the one elicited with matching questions for the same DM. It seems to us that the choice of a questioning mode for elicitation techniques should be made in regard to the aggregation
procedure used to model preferences. It is crucial for the information obtained from the DM to be consistent with the use of this information in the aggregation procedure. Consequently if the chosen aggregation procedure is of a non-compensatory nature (lexicography, majority rule), then the use of binary comparisons seems to be more adequate than matching questions: choice questions will induce an information that is likely to be more consistent with a non-compensatory aggregation rule. Conversely, matching questions will be more adapted to elicit preferences when a compensatory aggregation is used.

6. Conclusion

The experimental study reported in this paper shows that DMs strongly violate the procedure invariance principle. Our results confirm and enlarge those of [Tversky et al. 88]: prominence effect has been observed on the entire preference structure of DMs; moreover we build a (non-)compensatoriness index and computed it for all subjects and both questioning modes (choice and matching): matching questions induces preferences in which more possibilities of compensation are allowed and in which the preponderant criterion looms larger than in the one elicited with binary comparisons. The analysis of these results differ according to basic assumptions concerning the origins of lability of preferences (descriptivist versus constructivist approaches). Following a constructivist approach, we proposed a new interpretation of these empirical results.

Further investigations should be undertaken in order to test similar hypothesis with other questioning modes. Such works provide an interesting example of how behavioral science studies may be helpfull for multiple criteria decision aid.
References


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