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COMPENSATORINESS OF PREFERENCES
IN MATCHING AND CHOICE

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(NON-)COMPENSATION DES PRÉFÉRENCES ÉVALUÉES PAR COMPARAISONS BINAIRES ET QUESTIONS D'AJUSTEMENT

Résumé

Cet article étudie expérimentalement l'influence d'un mode de questionnement sur les préférences exprimées par un décideur. La structure de préférence entière de décideurs est évaluée à travers deux modes de questionnement différents : le choix et l'ajustement. Les résultats montrent une sérieuse mise en défaut du principe d'invariance procédurale. Les impacts en terme de modélisation des préférences et de techniques d'évaluation de paramètres d'importance sont discutés.


COMPENSATORINESS OF PREFERENCES IN MATCHING AND CHOICE

Abstract

This paper empirically studies the influence of a questioning mode on elicited preferences. The entire preference structure of decision makers is elicited using two different questioning modes: choice and matching. The results show a strong failure of procedure invariance. Impacts on preference modelling and importance parameters elicitation techniques are discussed.

Keywords: Multiple Criteria Decision Aid, Preference, (Non-)Compensatoriness, Experiment, Procedure Invariance, Importance of Criteria.
1. Introduction

Real world decision situations often involve several objectives, viewpoints or criteria. Different methodologies dealing with Multiple Criteria Decision Aid (MCDA) have been proposed in the past decades (see [Keeney & Raiffa 76], [Roy 85]) and the numerous aggregation techniques reported in the literature are useful tools to help managers facing decision tasks. Several fundamental principles concerning the consistency of preferences emerge from these works. Among these is the procedure invariance principle which states that normatively equivalent preference elicitation techniques should lead to the same elicited preferences. By analogy, when comparing the length of objects, the available tools should lead to the same lengths and thus compare the objects in the same way; no interaction appears between the tools and the objects to be measured.

Unfortunately, this principle seems to be contradicted by decision makers' (DMs) behavior. Psychological and behavioral science studies showed that DMs fail to respect the procedure invariance principle when expressing preferences. [Lichtenstein & Slovic 71,73] first highlighted the preference reversal phenomenon in the context of risky decision making. These authors found that DMs' answers to direct comparison and minimum selling price questions induce different preferences though these two questioning modes are normatively equivalent. This preference reversal phenomenon clearly contradicts the procedure invariance principle.
More recently, [Tversky et al. 88] showed that the same kind of violation may occur within the context of riskless decisions. In this paper, two different questioning modes were studied in a bi-criterion context: direct binary choice between alternatives and matching (matching questions consist in proposing two alternatives to the DM, one of the evaluations of an alternative being unfixed; the DM is to determine this evaluation in order to be indifferent between the two alternatives). The authors showed a prominence effect which states that "the more prominent attribute looms larger in choice than in matching", in other words that "preferences induced from choice are likely to be closer to the lexicographic ordering than those induced by matching".

Our work takes [Tversky et al. 88] as a starting point and aims at confirming and extending the obtained results. Unlike the above mentioned study that tests the prominence effect on a small number of specific pairs of alternatives, we want to analyse this effect on the overall preference structures of DMs. The knowledge of these overall PSs allows us to evaluate the "amount of compensation" the DM accepts when answering to choice and matching questions.

Our analysis of the phenomenon initially observed by [Tversky et al. 88] differ from the one proposed by these authors. We use the general tendency of the empirical observations in order to induce a rule concerning the questioning mode to be used in importance parameters elicitation techniques. The questioning mode of an elicitation techniques is to be chosen in regard to the aggregation procedure used to model preferences. The information obtained from the DM should be consistent with the use of this information in the aggregation procedure. Consequently, if the chosen aggregation procedure is of a non-compensatory nature (lexicography, majority rule), then the use of binary comparisons seems to be more adequate than matching questions: choice questions will induce an information that is likely to be more consistent with a non-compensatory aggregation rule. Conversely, matching questions will be more adapted to elicit preferences when a compensatory aggregation is used.

In the next section we will present two different ways of analysing lability of preferences. Section 3 is devoted to the clarification of the notion of (non-)compensatoriness of preference structures; a index is built to "measure" this notion. The experimental design and hypothesis tested are exposed in section 4. Results are presented and discussed in the two last sections.
2. (Non-)Compensatoriness of preference structures

The notion of (non-)compensatoriness of preference structures is intuitively linked to the possibility to resort to substitution rates in the construction of preferences that take all criteria into account. These substitution rates allow to compensate a disadvantage on a particular criterion by a sufficient advantage on another criterion. When an aggregation procedure uses such ideas to solve conflicts between criteria, the induced preference structure is said to be compensatory, otherwise it is said to be non-compensatory. Similarly, preferences expressed by a DM denote a preference structure of more or less compensatory nature.

2.1. Definitions

A first formal definition of this notion has been proposed by [Fishburn 76] :
Let us define by
P and I the preference and indifference relations on a product set \( X = X_1 \times X_2 \times \ldots \times X_n \)
P_i a preference relation on \( X_i \)
P(x,y) = \{ i \text{ such that } x P_i y \} ; \text{ we denote } \Pi_P = \{ P_1, P_2, \ldots, P_n \}

**Definition 1 :** a preference structure \((X,P,\Pi_P)\) is totally non-compensatory if and only if :

\[
\forall x,y,z,t \in X \quad \begin{cases} P(x,y) = P(z,t) \\ P(y,x) = P(t,z) \end{cases} \iff [ xP y \Rightarrow zP t ]
\]

Roughly speaking, a preference structure is totally non-compensatory when all pairs of alternatives \((x,y)\) and \((z,t)\) that have the same "preferential profile" (i.e. whose comparisons are similar on all criteria) compare themselves in the same way on the overall level (i.e. when all criteria are taken into account). A generalization of this notion may be found in [Bouyssou & Vansnick 85]. [Bouyssou 86] proposes a definition of a minimaly compensatory preference structure.

**Definition 2 :** a preference structure \((X,P,\Pi_P)\) is minimaly compensatory if and only if :

\[
\exists x,y,z,t \in X \text{ such that } \begin{cases} P(x,y) = P(z,t) \\ P(y,x) = P(t,z) \end{cases} \quad \text{ with } [ xP y \text{ and } zP t ]
\]

\[
x_i = y_i \text{ and } z_i = t_i \quad \forall i \in I(x,y)
\]

\[I(x,y) = \{ i / i \notin P(x,y) \text{ and } i \notin P(y,x) \}\]
More recently [Roy & Mousseau 92] proposed a definition allowing the analysis of compensation on each criterion individually (this definition is grounded on a new formalism of the notion of relative importance of criteria). However the (non-)compensatoriness of a preference structure is a notion that deserves to be studied in further details. Firstly no large consensus emerges concerning its definition. Moreover the available definitions only define extreme cases and do not allow to precise to which level a preference structure is (non-)compensatory.

2.2. Construction of a (non-)compensatoriness index

Total non-compensation is an extreme situation never reached in real world decision contexts. Let us define an index aiming at "measuring" the (non-)compensatory aspects of a preference structure $\Psi = (X, P, I_p)$. $\Psi$ is even more non-compensatory since two pairs of alternatives having the same preferential profile on criteria are linked with the same preference relation on the overall level. Possibilities of compensation in a preference structure depend on the overall preference situation that link pairs $(x,y)$ and $(z,t) \in X^2$ verifying:

\[
\begin{align*}
P(x,y) &= P(z,t) \\
P(y,x) &= P(t,z) \\
x_i = y_i \text{ and } z_i = t_i \quad \forall i \in I(x,y)
\end{align*}
\]

When all pairs $(x, y)$ and $(z, t)$ verifying [1] are such that $xPy \Rightarrow zPt$, the considered preference structure is totally non-compensatory (see definition 1). Possibilities of compensation appear in a preference structure when there exist $(x, y)$ and $(z, t)$ such that $\text{not}(xPy \Rightarrow zPt)$ i.e. $xPy$ and $\text{not}(zPt)$

i.e. $(xPy$ and $zIt)$ or $(xPy$ and $tPz)$

The first case of compensation (quadruplet verifying [1] such that $xPy$ and $zIt$) corresponds to definition 2. The second situation (quadruplet verifying [1] with $xPy$ and $tPz$) corresponds to a stronger compensation (a preference reversal). However, our index will account for these two situations in the same way. This index will measure proportion of quadruplets $(x, y, z, t)$ verifying [1] such that $xHy \Leftrightarrow zHt$ (H being any preference relation).

Suppose that all $X_i$ are discrete and let us define:

$T = \{(x, y, z, t) \in X^4 \text{ such that } P(x, y) = P(z, t) \text{ and } P(y, x) = P(t, z) \text{ with } P(x, y) = \{i \text{ such that } xPy\} \}

NC = \{(x, y, z, t) \in T \text{ such that } xHy \Leftrightarrow zHt \}$, H being any overall relation

Let us define the (non-)compensatoriness index of a preference structure $\Psi$ by:

\[
nc(\Psi) = \frac{|NC|}{|T|} \quad nc(\Psi) \in [1/\lambda, 1]
\]
In this definition, \( \lambda \) is the number of possible preferential situations on overall level (when preferences are modeled using a \((I,P)\) structure, three preferential situations are possible between two alternatives: \( aPb, aLb \) and \( bPa \)). This index is built such that \( \text{nc}(\Psi) = 1 \) means that \( \Psi \) is totally non-compensatory (lexicographic order); a decreasing value for \( \text{nc}(\Psi) \) means greater possibilities of compensation in \( \Psi \). Thus the index \( \text{nc} \) allows us to compare the (non-)compensatoriness of two preference structures; this index has only an ordinal signification (no cardinal use of this index will be made).

3. Empirical scheme

Our aim in this experiment is to highlight the violation of procedural invariance and to exhibit a link between the use of a specific questioning mode (choice and matching in our case) and the (non-)compensatoriness of the elicited preference structures. We will deal with a bi-criteria context. The scale on both criteria will be discretized on a four level scale \( \{A,B,C,D\} \) where \( A \leq P_i \leq B \leq C \leq D \), \( i=1,2 \).

3.1. The two questioning modes

The experiment consists in the exhaustive evaluation of the preference structure using two different questioning modes:

*Choice*: this type of questions consists in a hollistic comparison of two alternatives defined by their evaluations on all criteria. Subjects are to choose between an indifference situation or a preference in favor of one of the alternatives. As we deal with a bi-criteria context, let us denote by \((x_i,x_j) \neq (y_i,y_j)\) such a question.

*Matching*: this type of question consists in proposing two alternatives \((x_i,x_j)\) and \((y_i,y_j)\) leaving \(y_2\) unfixed. Subjects are to determine \(y_2\) in order to obtain indifference between the two alternatives. Let us denote by \((x_i,x_j)\neq (y_i,y_j)\) such a question. As in our experiment answers are given on a continuous scale, it is necessary to analyse the answers in terms of
3.2. Empirical hypotheses

We use a intra-subject experimental design to test two connected hypothesis: the prominence hypothesis and the contingent compensation hypothesis.

The prominence hypothesis lies in the fact that preferences elicited by choice questions should be closer to a lexicographic order than those elicited by matching questions. In other terms, the "more important" criterion should be taken into account more often in the first case than in the second. In order to test this hypothesis, we will count in the two elicited preference structures the number of "cases" in which an advantage on the preponderant criterion is taken into account first (if \( x_1, x_2 \) \( P(y_1, y_2) \) with \( x_i Py_1 \) and \( y_2 Py_1 \) then criterion \( C_1 \) is taken into account in the first place). Let us denote \( C^* \) the criterion taken into account the more often. Let \( p(\Psi_m) \) (respectively \( p(\Psi_c) \)) be the proportion of cases in which an advantage on \( C^* \) is decisive on the overall level when preferences are elicited by matching questions (respectively by choice questions). The prominence hypothesis states that: \( p(\Psi_m) < p(\Psi_c) \)

The contingent compensation hypothesis states that there are more possibilities of compensation in preference structures elicited with matching questions than in the one elicited using choice questions. In order to judge the (non-)compensatoriness of elicited preferences, we will use the index \( nc(\Psi) \) proposed in section 3.2. Let us recall that this index varies between \( \Psi \) and 1 and that its value increases with the non-compensatoriness of \( \Psi \). Let us denote by \( \Psi_m \) (respectively by \( \Psi_c \)) the preference structure elicited using matching questions (respectively choice questions). The contingent compensation hypothesis states that: \( nc(\Psi_m) < nc(\Psi_c) \).
3.3. Experimental framework

The real world context concerns the evaluation of firms by young executives working in computer science companies so as to postulate in these firms. The proposed firms differ in the annual salary and the job interest. The scales of both criteria (salary and job interest) are discrete and made of four levels of evaluation.

Criterion 1: The job interest is evaluated on a qualitative scale that is built with subjects. Each level is defined by linguistic terms. Instructions were given to subjects: they had to build this scale so as to perceive the "distance" between consecutive levels as equivalent. It was then checked that all levels are separated by a strict preference. For example, the scale of one of the subject was:

A: Very interesting job, formative, no repetitive aspect, evolutive and large independence at work.
B: Interesting job, slightly repetitive but formative, rather evolutive, good independence at work.
C: Job of little interest, quite repetitive but still somehow formative, evolutive in the long term, fairly little independence.
D: Boring job, very little formative, numerous repetitive aspects, annoying hierarchy.

Criterion 2: The salary is measured in thousands of francs per year. This numerical scale is discretized in a four level scale; all consecutives level are separated by a strict preference and are built so as to reflect various salaries (from very attractive to quite repulsive). The values corresponding to the four levels are determined regarding the subject’s expectations in terms of salary.

3.4. Simplifying postulates

Eliciting the subject's overall preference structure amounts at determining the cartesian product of $X$ (the number of couples $(x_i,x_j),(y_i,y_j)\in (X\times X)^2$ is $4^2=256$). In order to minimize the number of questions to be posed to subjects, we pose three postulates that impose relatively weak conditions on preference structures and do not interfere with the questioning modes. Let us consider the $P$-dominance relation defined by:

$$\forall x,y\in X \quad x \Delta_p y \iff \exists i, \not y_i P_i x_i \wedge \exists j, x_j P_j y_j$$

Postulate 1 (preference contains $P$-dominance): $\forall x,y\in X \ x \Delta_p y \Rightarrow xPy$.

This first postulate amounts at reducing the number of couples of alternatives between which the preferential situation is initially undetermined. The number of undetermined comparisons is reduced down to 36.
Postulate 2 (monotonicity): \( \forall x, y, z, t \in X \begin{eqnarray*} xPy \text{ and } t \Delta_p x & \Rightarrow & tPy \\ xPy \text{ and } y \Delta_p z & \Rightarrow & xPz \end{eqnarray*} \)

The interpretation of this postulate is the following: "when an assertion \( aPb \) is established, increasing the evaluation of \( x \) or decreasing the evaluations of \( y \) leaves the overall preference relation between \( x \) and \( y \) unchanged.

Postulate 3 (a partial preference is valid on the overall level):
\[
\begin{align*}
xIy \text{ and } y \Delta_p z & \Rightarrow xPz \\
xIy \text{ and } t \Delta_p x & \Rightarrow tPx
\end{align*}
\text{i.e.}
\[
\begin{align*}
I \Delta_p & \subset P \\
\Delta_p & \subset I \subset P
\end{align*}
\]

As in the preceding postulate, this postulate enables us to induce assertion from previously determined assertions. In concrete terms, if \((x_1, x_2)I(y_1, y_2)\) then we have:
\[
\begin{align*}
\forall (z_1, z_2) \in X & \ (z_1, z_2) \Delta_p (x_1, x_2) \Rightarrow (z_1, z_2)P(y_1, y_2) \\
\forall (t_1, t_2) \in X & \ (y_1, y_2) \Delta_p (t_1, t_2) \Rightarrow (x_1, x_2)P(t_1, t_2)
\end{align*}
\]

3.5. Data acquisition software

According to the preceding postulates, we build an algorithm aiming at reducing the number of questions to be posed to the DM in order to elicit his overall preference structure. Let us ground our algorithm on the oriented graph \( G=(C, U) \) where \( C \) is composed of every pair \((x, y)\) with \( x \neq y \) whose comparison is not determined by postulate 1 and \( U \) represents possible deductions (according to postulates 2 and 3). Thus, \( U \) is defined by:
\[
\forall ((x, y), (z, t)) \in C^2, \ (x, y), (z, t) \in U \ \Rightarrow \ z \Delta x \text{ or } y \Delta t
\]

Begin

While a comparison remains unmarked

Do

Determine by a question an unmarked comparison.

If the first alternative is preferred to the second one,

then mark P the current comparison and all its successors

If the second alternative is preferred to the first one,

then mark -P the current comparison and all its predecessors

If the two alternatives are indifferent,

then mark I the current comparison, P all its successors and -P all its predecessors

End

During the experiment, the two questioning modes were used simultaneously and randomly mixed. So as to implement this experiment, we have built a data acquisition support software that allows to make instantaneously the deductions (corresponding to the postulates) during the questioning procedure. In the software, the two questioning modes were phrased as shown in the screen dumps (see figures 2 and 3). More details concerning the software supporting this experiment may be found in [Mousseau 93].
4. Results

The 33 subjects are young executives having a high level of education, most of them working in the computer science sector. The mean age is 28 (σage=3.2) and their professional situation is such that they can consider several job types with different salaries. In this sense, comparing jobs offered by companies is for them a pertinent and experimented decision problem.

The mean number of questions is 31 and the total duration of interview is 40 minutes in average. As the experiment has been supported by a data acquisition support software, the
answering time could be recorded. The average answering time on the whole subject group is 24 seconds for matching questions and 30 seconds for binary comparisons. A T Student test shows that these two mean values differ significantly with $\alpha \leq 0.01$. This difference may be explained by the fact that only 3 answers are possible in choice questions while subjects are to answer to matching questions on a continuous scale. Moreover we observed during the interviews that subjects seem to test a few binary comparisons in order to answer to matching questions.

4.1. Prominence effect

Let us recall that the prominence effect states that preferences elicited by binary comparisons are closer to a lexicographic structure that those elicited by matching questions, i.e. the preponderant criterion is taken onto account more frequently in the first case than in the second one (see 4.2). Let $\text{p}(\Psi_c)$ (respectively $\text{p}(\Psi_m)$) be the proportion in which an advantage on the criterion $\text{C}^*$ is considered first ($\text{C}^*$ being the criterion that is taken into account the more often) when preferences are elicited by binary comparisons (respectively by matching questions).

The results are presented in figure 4 in which each point corresponds to a subject. It is obvious that the scatterplot is located below the bissectrix. This means that an advantage on the preponderant criterion $\text{C}^*$ is more often decisive when preferences are elicited through choice questions rather than by matching questions.

![Figure 4](image-url)
In this experiment, the mean value for \( p(\Psi_c) \) (respectively \( p(\Psi_m) \)) is 0.800 (resp. 0.663) and its standard deviation is 0.116 (respectively 0.133). A Wilcoxon test leads to accept that the mean values of \( p(\Psi_c) \) and \( p(\Psi_m) \) are significantly different with \( \alpha \leq 0.01 \). A T Student test leads to the same conclusion with \( \alpha \leq 0.01 \). We can thus conclude that our experiment confirms the prominence effect.

4.2. Contingent compensation effect

The contingent compensation effect is closely related to the prominence effect and states that elicited preferences reveal more possibilities of compensation when the questioning mode is matching rather than binary comparison. So as to measure the possibilities of compensation inherent in a DM’s preference structure, we use the index \( nc \) built in section 3.2. Let us recall that this index varies between \( \Psi_s \) and 1 and the higher the value for \( nc(\Psi) \), the less possibilities of compensation exist in \( \Psi \) (\( nc(\Psi) = 1 \) corresponding to a lexicographic order).

We compute the values of this index for both questioning modes and all subjects. Results are synthetized in figure 5 in which each point represents a subject. The scatterplot is located below the bisectrix, i.e. the values for \( nc(\Psi_c) \) are higher than those of \( nc(\Psi_m) \) for a large majority of subjects.

![Figure 5](image-url)
In our sample, the mean value across subjects for the computed value of the index \( nc \) is 0.535 (respectively 0.685) when preferences are elicited by matching questions (respectively choice questions) and the standard deviation is 0.145 (respectively 0.144). A Wilcoxon test leads to conclude to a significant difference between the mean values for \( nc(\Psi_c) \) and \( nc(\Psi_m) \) with \( \alpha \leq 0.01 \). A T Student test leads to the same conclusion with \( \alpha \leq 0.01 \). We can thus conclude that our experiment confirms the contingent compensation effect.

5. Discussion

The experimental results show a strong failure of the procedure invariance principle: we observed, in our sample, a significant divergence among preferences elicited through two different questioning modes (choice and matching questions). More precisely, subjects answered in such a way that matching questions elicit preference structures in which more possibilities of compensation are allowed and in which the preponderant criterion looms larger than in the one elicited with binary comparisons. Such labilities in preferences have already been reported in the literature (see [Weber & Borcherding 93] for an overview) and seems to be a problem that is inherent to preference elicitation.

Descriptivist vs Constructivist analysis of lability of preferences

Many works in the field of behavioral research have reported a great lability of elicited preferences and values (see [Fishhoff et al. 89]) such as framing effect (see [Kahneman & Tversky 84], [Slovic et al. 81], [Johnson et al. 91]), splitting effect (see [Fishhoff et al. 78], [Adelman et al. 86], [Weber et al. 88]), question order effect (see [Mousseau 92]), procedure invariance violation (see [Tversky et al. 88], [Fisher & Hawkins 89]).

There are two ways to account for such lability and to explain observed phenomena. These two approaches disagree on the nature of what is being modelled: are preferences to be considered as pre-existing and thus to be discovered or are they a result of interactions between the DM and the elicitation tool (see [Roy 93])? This distinction is essential as the two approaches use experimental works in a very different way.

The first stream, the descriptivist approach (also called discoverist or realist approach) refers to stable pre-existing preferences. In this framework, there are values to be discovered reflecting DM's true preferences. By analogy with physical measurement in which each object is assumed to have a well-defined value for a specific attribute, true preferences exist and are to be measured. Discrepancies between elicitation tools reflect the fact that these tools only provide estimates of the true preferences, the elicited preferences being biased. True
preferences are supposed to remain constant but to be distorted during the elicitation process. Elicitation techniques orient preferences in certain directions, introduce noises ... However it is assumed that, when the analyst and the DM are sufficiently careful, have enough time and use different elicitation methods, obtained preferences should converge on true preferences.
procedure used to model preferences. It is crucial for the information obtained from the DM to be consistent with the use of this information in the aggregation procedure. Consequently if the chosen aggregation procedure is of a non-compensatory nature (lexicography, majority rule), then the use of binary comparisons seems to be more adequate than matching questions: choice questions will induce an information that is likely to be more consistent with a non-compensatory aggregation rule. Conversely, matching questions will be more adapted to elicit preferences when a compensatory aggregation is used.

6. Conclusion

The experimental study reported in this paper shows that DMs strongly violate the procedure invariance principle. Our results confirm and enlarge those of [Tversky et al. 88]: prominence effect has been observed on the entire preference structure of DMs; moreover we build a (non-)compensatoriness index and computed it for all subjects and both questioning modes (choice and matching): matching questions induces preferences in which more possibilities of compensation are allowed and in which the preponderant criterion looms larger than in the one elicited with binary comparisons. The analysis of these results differ according to basic assumptions concerning the origins of lability of preferences (descriptivist versus constructivist approaches). Following a constructivist approach, we proposed a new interpretation of these empirical results.

Further investigations should be undertaken in order to test similar hypothesis with other questioning modes. Such works provide an interesting example of how behavioral science studies may be helpfull for multiple criteria decision aid.
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