ELICITING INFORMATION CONCERNING THE
RELATIVE IMPORTANCE OF CRITERIA

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OBTENTION D'INFORMATION SUR
L'IMPORTANCE RELATIVE DES CRITÈRES

Résumé : La notion d’importance relative des critères (IRC) est centrale dans le domaine de l’aide multicritère à la décision (AMCD). Elle a pour but de différencier le rôle de chacun des critères dans la construction des préférences globales, permettant ainsi de discriminer entre les actions pareto-optimales. Dans la plupart des procédures d’agrégation cette notion prend la forme de paramètres d’importance.

L’acquisition d’information concernant l’IRC peut se faire à travers l’utilisation de méthodes d’évaluation des paramètres d’importance (MEPI). La conception de telles méthodes doit prendre en compte à la fois la signification que chaque agrégation confère à ses paramètres, et la compréhension qu’ont les décideurs de la notion d’IRC. Plus précisément, les MEPI doivent assurer une bonne cohérence entre la manière avec laquelle l’analyste utilise les réponses du décideur dans le modèle et l’information que celui-ci souhaite exprimer à travers ses affirmations.

Dans cet article, nous présentons une MEPI adaptée aux méthodes de type ELECTRE et qui procède par comparaisons par paire d’actions fictives. Cette MEPI est implémentée dans un logiciel appelé DIVAPIME et permet de déterminer des intervalles de variation pour les paramètres préférentiels des méthodes ELECTRE.

Mots Clé : AMCD, Importance Relative des Critères, Poids, méthodes ELECTRE

ELICITING INFORMATION CONCERNING
THE RELATIVE IMPORTANCE OF CRITERIA

Abstract: The notion of Relative Importance of Criteria (RIC) is central in the domain of Multiple Criteria Decision Aid (MCDA). It aims at differentiating the role of each criterion in the construction of comprehensive preferences, thus allowing to discriminate among pareto-optimal alternatives. In most aggregation procedures, this notion takes the form of importance parameters.

The acquisition of information concerning the RIC may be supported by Elicitation Techniques for Importance Parameters (ETIP). The design of such techniques should account for both the meaning that each aggregation confers on its parameters and the decision makers’ (DMs) understanding of the notion of RIC. More precisely, ETIPs should be able to provide a good fit between the way the analyst uses the DM’s assertions in the model and the information that he/she expresses through his/her statements.

In this paper, we present an ETIP adapted to the ELECTRE methods that proceeds by means of pairwise comparisons of fictitious alternatives. Implemented in a software program called DIVAPIME, this ETIP supports the elicitation of variation intervals for the ELECTRE methods’ preferential parameters.

Keywords: MCDA, Importance of Criteria, Weights, Elicitation, ELECTRE Methods.
Introduction

When the analysis of a decision problem is grounded in the definition of a set of criteria, it is difficult to discriminate between alternatives whose evaluations on several criteria are in conflict. Multiple criteria preference modeling requires that the analyst obtains from the decision maker (DM) some preference information so as to discriminate between pareto-optimal alternatives.

A classical approach to Multiple Criteria Decision Aid (MCDA) consists of linking restricted preferences (corresponding to the n criteria) with the comprehensive preferences (taking all criteria into account) through a so called Multiple Criteria Aggregation Procedure (MCAP). In the MCAP, all criteria are not supposed to play the same role; the criteria are commonly said not to have the same importance. This is why there are parameters in the MCAP that aim at specifying the role of each criterion in the aggregation of evaluations. We will call such parameters importance parameters. They aim at introducing preferential information concerning the importance that the DM attaches to the points of view modelled by the criteria.

The nature of these parameters varies across MCAPs. The way of formalizing the
1. What does the notion of the relative importance of criteria cover?

What does a decision-maker mean by assertions such as "criterion $g_j$ is more important than criterion $g_i$," "criterion $g_j$ has a much greater importance than criterion $g_i$," etc. Let us recall, by way of comparison, that the assertion "b is preferred to a" reflects the fact that, if the decision-maker must choose between the alternatives b and a, he is supposed to decide in favor of b. So, it is possible to test whether this assertion is valid or not. There is no similar possibility for comparing the Relative Importance of Criteria (RIC). Moreover, the way this notion is taken into account within the framework of the different models mentioned above by means of importance parameters reveals significant differences in what this notion deals with.

The first statement we should make when trying to analyse the notion of RIC is the following: the information underlying this notion is much richer than that contained in the importance parameters of the various multicriteria models. In fact, these parameters are mainly scalars and constitute a simplistic way of taking RIC into account, as this notion is by nature of a functional type.

In the comparison of two alternatives a and b, when one or several criteria are in favor of a and one or several others in favor of b, the way each MCAP solves this conflict and determines a comprehensive preference (i.e., taking all criteria into account) denotes the importance attached to each criterion (and to the logic of the aggregation used). Thus, the result of such conflicts (i.e., the comprehensive preference situation between a and b) constitutes the elementary data providing information on the relative importance of the criteria in conflict.

When we analyse the RIC notion, it appears that this notion represents a certain form
where \( q \), the indifference threshold, represents the maximum difference of evaluation compatible with an indifference situation;
- \( p \), the preference threshold, represents the minimum difference of evaluation compatible with a preference situation;
- \( aPb \) is to be interpreted as a preference for \( a \) over \( b \) on criterion \( g \);
- \( aIb \) is to be interpreted as indifference between \( a \) and \( b \) on criterion \( g \);
- \( aQb \) is to be interpreted as an hesitation between the two preceding situations.

We will call \( P \) the partial preference relation on the \( j \)-th criterion (the same terminology holds for \( I \) and \( Q \)). \( (I, Q, P) \) defines a pseudo-order.

The comprehensive preferences are modelled through three preference relations: \( P \) a preference relation (asymmetric, irreflexive), \( I \) an indifference relation (symmetric, reflexive), \( R \) an incomparability relation (symmetric, reflexive). For every pair of alternatives \((a, b)\), one and only one of the four following assertions is valid:

\[
\begin{bmatrix}
aPb \\
aIb \\
bPa \\
aRb
\end{bmatrix}
\]

Let us consider the outranking relation \( S=P \cup I \), \( aSb \) being interpreted as "\( a \) is at least as good as \( b \)". As \( aPb \Rightarrow aSb \Rightarrow \text{not}[bSa] \), the four assertions of system \([1]\) correspond to the following assertions:

\[
\begin{bmatrix}
aPb \\
aSb \text{ and not}[aPb] \\
bPa \\
\text{not}[aSb] \text{ and not}[bPa]
\end{bmatrix}
\]

When the preference model is of a \((I, P, R)\) type, it does not seem restrictive to delimit the information concerning the relative importance of a criterion using only preference and outranking relations (see system \([2]\)). On such a basis, we will assume that the empirical content of the RIC notion refers to the nature and variety of cases in which a partial preference on a given criterion \( g \) leads us (i) to accept the same preference on the comprehensive level, or (ii) to accept only an outranking in the same direction, or (iii) to refuse the inverse preference. In other words, the relative importance of criterion \( g \) is characterized by:

- the set of "situations" in which \( aPb \) and \( aPb \) hold simultaneously,
- the set of "situations" in which \( aPb \) and \( aSb \) hold simultaneously,
- the set of "situations" in which \( aPb \) and \( \text{not}[bPa] \) hold simultaneously.

In this characterization, the contribution of criterion \( g \) to the preferences at the comprehensive level is considered only through the situations of strict preference at the restricted level of criterion \( g \). This does not seem restrictive when all criteria are quasi-criteria (such that \( p = q \)), but it can become restrictive when \( F \) contains pseudo-criteria.
On the basis of the preceding considerations [Roy & Mousseau 95] propose a formal
definition of the notion of RIC and a theoretical framework to analyse it. Within this
framework, it clearly appears that the importance of criteria is taken into account in very
different ways in the various aggregation procedures. In particular, this means that the values
attached to importance parameters are meaningless as long as the aggregation rule in which
they are used is not specified. Interesting theoretical proposals can also be found in
[Podinovskii 88, 94].

2. What do we aim at when evaluating the importance of criteria ?

The elicitation of information about RIC is a crucial phase in a decision aid process.
Basic assumptions concerning the nature of what is being done during this phase have an
impact on the way to proceed to elicit such information.

2.1. The descriptivist and constructivist approaches to MCDA

The way we seek to give meaning to the notion of importance will differ according
to which of these two approaches we adopt.

The descriptivist approach assumes that the way in which any two alternatives are
compared on the comprehensive level (that is, taking all criteria into account) is well-defined
in the decision-maker’s mind before the modelling process begins. Moreover, it supposes that
the modelling process does not modify such comparisons. The preference model chosen is
intended to give an account of such pre-existing preferences as objectively as possible. Under
these conditions, the role which devolves to each criterion as a function of its importance, is
apprehended by the set of values attributed to the importance parameters. Thus, it is the
capacity of a model to adjust to a well-defined, real-world situation which confers meaning
on the notion of importance and allows to assign a numerical value to these parameters.
aggregating the evaluations $g_j(a)$ and for building comprehensive preferences. Under these conditions, the numerical values assigned to importance parameters reflect a working hypothesis accepted for decision-aid. These are convenient numerical values with which it seems reasonable and instructive to work. The importance parameters, therefore, can be seen as keys which allow us to differentiate the role played by each criterion in the preference model selected for use. "True numerical values," to which we can refer to give meaning to the language of estimation, do not necessarily exist. Importance parameters and the numerical values we assign to them are, nonetheless, instruments for reasoning, investigating and communicating among the stakeholders in a decision-making process. The values (or interval
The various ETMs reported in the literature (see [Mawson 97] for a review) may...
in [Roy & Bouyssou 93]). Besides the implementation, our work consisted of restructuring and extending the questioning procedure to make it more precise, as well as improving the algorithmic aspects of the method.

### 3.1. The preferential parameters of the ELECTRE methods

ELECTRE methods build an outranking relation $S$, i.e., validating or invalidating, for any pair of alternatives $(a, b)$, an assertion $a \preceq b$, whose meaning is "$a$ is at least as good as $b$". In the ELECTRE methods, preferences restricted to the significance axis of each criterion are defined through pseudo-criteria (see section 1). The indifference and preference thresholds $(q_{ij}$ and $p_{ij}$) model the intra-criterion preferential information. They account for the imprecise nature of the evaluations $g_i(a)$. The $q_{ij}$ threshold specifies the largest difference of evaluation $g_i(a) - g_i(b)$ that preserves indifference between $a$ and $b$ ($a \sim b$); $p_{ij}$ represents the smallest difference $g_i(a) - g_i(b)$ compatible with a preference situation in favor of $a$ ($a \preceq b$). The weak preference relation $Q_{ij}$ should be interpreted as hesitating between opting for a preference or indifference situation.

At the comprehensive level of preferences, in order for the assertion $a \preceq b$ to be valid, two conditions should be verified:
- concordance: for an outranking $a \preceq b$ to be accepted, a "sufficient" majority of criteria should be in favor of this assertion,
- non-discordance: when the concordance condition holds, none of the criteria in the minority should oppose to the assertion $a \preceq b$ in "a too strong way".

Two types of importance parameters intervene in the construction of $S$:
- the set of importance coefficients $(k_1,k_2, ..., k_n)$ takes into account the relative importance of coalitions of criteria and intervenes in the construction of a concordance index $c(a \preceq b)$ which is defined by:

$$c(a \preceq b) = \frac{\sum_{i=1}^{n} k_i c_i(a \preceq b)}{\sum_{i=1}^{n} k_i}$$

with $c_i(a \preceq b) = 1$ if $aP_i b$ or $aQ_i b$ or $aI_j b$

- the set of veto thresholds $(v_1(g_1), v_2(g_2), ..., v_n(g_n))$ is used in the discordance concept.
3.2. Determination of a polyhedron of admissible values for importance parameters

In this section, we present a technique which aims at determining a non-empty polyhedron of admissible values for \( k = (k_1, k_2, \ldots, k_n) \) starting from linear inequalities on these coefficients. These inequalities come from DM's answers to binary comparisons of fictitious alternatives (see 3.3). We do not aim at determining a single vector of values \( k \), but a set of vectors consistent with assertions expressed by the DM.

3.2.1. Formulating information through a segmented description

Let us suppose that criteria can be ordered by importance and let us renumber them so that \( k_1 < k_2 < \ldots < k_n \). We want to specify an interval \( [m_i, M_i] \) containing each \( k_i \) (\( \forall i \neq 1 \)). Each of these intervals constitutes a segment in which the value of \( k_i \) may vary. We aim at building intervals \( [m_i, M_i] \) such that \( m_i \) and \( M_i \) depend only on the \( k_j \) for \( j < i \); Hence we will denote them \( m_i(k_1,k_2,\ldots,k_{i-1}) \) and \( M_i(k_1,k_2,\ldots,k_{i-1}) \). The system of inequalities is then:

\[
\begin{align*}
m_1(k_1) &< k_2 < M_1(k_1) \\
m_2(k_1,k_2) &< k_3 < M_2(k_1,k_2) \\
m_3(k_1,k_2,k_3) &< k_4 < M_3(k_1,k_2,k_3) \\
& \vdots \\
m_{n-1}(k_1,k_2,\ldots,k_{n-2}) &< k_{n-1} < M_{n-1}(k_1,k_2,\ldots,k_{n-2}) \\
m_n(k_1,k_2,\ldots,k_{n-1}) &< k_n < M_n(k_1,k_2,\ldots,k_{n-1})
\end{align*}
\]

In what follows, \( m_i(k_1,k_2,\ldots,k_{i-1}) \) will be defined by the maximum of a list of lower bounds of \( k_i \), and \( M_i(k_1,k_2,\ldots,k_{i-1}) \) by the minimum of a list of upper bounds of \( k_i \). Each of these lower or upper bounds takes the form of a linear combination of \( k_j \) for \( j < i \). For example, the value of \( m_2(k_1,k_2,k_3,k_4) \) can be \( \{k_1+k_4, k_2+k_4\} \). So as to add a new inequality to the system\(^2\), we proceed as follows: such an inequality may always be written as \( \sum_{i=1}^{n} \alpha_i k_i > 0 \), with \( \alpha_i \in \mathbb{R} \). If \( \text{imax} \) is defined as the index of the greatest non-null coefficient \( \alpha_i \) \( (\text{imax} = \max\{i/\alpha_i \neq 0 \ \forall j > i\}) \), we may rewrite the preceding inequality as \( \alpha_{\text{imax}} k_{\text{imax}} < \sum_{j \neq \text{imax}} -\alpha_j k_j \). According to the sign of \( \alpha_{\text{imax}} \), this inequality specifies a new upper or lower bound for \( k_{\text{imax}} \). It is then necessary to check that each lower bound of \( k_{\text{imax}} \) is still lower than each upper bound of \( k_{\text{imax}} \). This verification may lead to adding new bounds. This step is called the saturation of the segmented description.

\(^1\) The representation \( m_i(k_1,k_2,\ldots,k_{i-1}) < k < M_i(k_1,k_2,\ldots,k_{i-1}) \) of the domain of variation of \( k = (k_1,k_2,\ldots,k_n) \) exists whether this domain is a polyhedron or not; the functions \( m_i(k_1,k_2,\ldots,k_{i-1}) \) and \( M_i(k_1,k_2,\ldots,k_{i-1}) \) may not be linear (see [Roy 70], chap. 10).

\(^2\) This inequality comes from the answer to a pairwise comparison of alternatives (see 3.3.1).
3.2.2. Saturation of the segmented description

At a given stage, adding an inequality may produce supplementary inequalities. If the added inequality specifies a new upper or lower bound for $k_i$ so that $m_i(k_1, k_2, \ldots, k_{i-1}) < k_i < M_i(k_1, k_2, \ldots, k_{i-1})$, by transitivity, it should hold:

$$m_i(k_1, k_2, \ldots, k_{i-1}) < M_i(k_1, k_2, \ldots, k_{i-1})$$  \hspace{1cm} \text{[i]}

If [i] is verified $\forall k_1, k_2, \ldots, k_{i-1}$, then no additional inequality should be generated; in the opposite case, this inequality [i] should be integrated into the system, and will produce one (or several) upper and/or lower bound(s) from which one (or several) additional inequalities might be generated.

Hence, saturating a segmented description consists of producing all possible (upper or lower) bounds from inequalities similar to [i] that are not verified $\forall k_1, k_2, \ldots, k_{i-1}$. The generation of a new bound can itself produce an additional bound. The saturation algorithm stops when no bound can be generated.

In certain cases, the addition of an inequality to the system can lead to an empty polyhedron. Such a situation occurs when a lower bound of a $k_i$ is greater than one of the upper bounds of this $k_i$, $\forall k_1, k_2, \ldots, k_{i-1}$. This situation characterizes a contradiction between the last inequality integrated into the system and one or several others.

The algorithm can detect such inconsistencies and determine which inequalities generate the contradiction. In order for the polyhedron of admissible solutions not to become empty, it is necessary to delete one or several inequalities generating the inconsistency together with all bounds that have been generated by the bound(s) to be deleted.

The convergence of the saturation mechanism has been proved (see [Roy 70]); we are then sure to reach a saturated segmented description. Let us give, for explanatory purposes, some elements that justify the convergence of the algorithm: at the saturation step, the production of an additional bound comes from the fact that the inequality $m_i(k_1, k_2, \ldots, k_{i-1}) < M_i(k_1, k_2, \ldots, k_{i-1})$ is not verified $\forall k_1, k_2, \ldots, k_{i-1}$. In this case, the new bound will always be added on a segment $k_j$ such that $k_i < k_j$ (indeed, $m_i(k_1, k_2, \ldots, k_{i-1})$ and $M_i(k_1, k_2, \ldots, k_{i-1})$ are functions of $k_j$ such that $j \leq i$). It follows that such deductions can be carried out only a finite number of times and that the saturation should be computed linearly, starting from the segment corresponding to the most important criterion without any backtracking to saturated segments.

---

3 Each upper or lower bound comes either from an original inequality, or from two "parent bounds". It is then easy to retrieve the "ancestry chain" of a bound so as to identify its origins.
3.3. Questioning procedure

3.3.1. Questioning mode: pairwise comparisons of fictitious alternatives

As this ETIP is an indirect ETIP, the questioning mode does not directly refer to the concept of importance. We can, from the answers of the DM to the questions, infer information through the MCAP used. The questioning mode selected is a pairwise comparison of fictitious alternatives. For each question, the DM has to define the comprehensive preference situation between two evaluation vectors.

The fictitious alternatives involved in the comparisons are chosen so as to provide specific information. Moreover, they should be able to correspond to real alternatives (their evaluations should be plausible and respect possible statistical links between criteria). The questioning procedure is founded upon the following fictitious alternatives:

- \( b_0 \): A reference alternative whose evaluations on each criterion are "average".
- \( b_i \): Alternatives whose evaluations are identical to \( b_0 \) on all criteria except on criterion \( g_i \) on which its evaluation is increased by a significant amount (but not exceeding the veto threshold) relative to the scale of \( g_i \) (\( b_i \) P \( b_0 \)).
- \( b_{ij} \): Alternatives identical to \( b_0 \) on all criteria except \( g_i \) and \( g_j \) on which its evaluation is increased by a significant amount (but not exceeding the veto threshold) relative to the scales of \( g_i \) and \( g_j \) (\( b_{ij} \) P \( b_0 \) and \( b_{ij} \) P \( b_0 \)).
- \( b_J \): Alternatives identical to \( b_0 \) on all criteria except on criteria contained in the coalition \( J \) \( (J \subseteq F) \) on which its evaluation is increased by a significant amount (but not exceeding the veto threshold) relative to the scales of the considered criterion (\( b_J \) P \( b_0 \), \( \forall i \in J \)).

If the DM and the analyst agree to ground decision aid on an ELECTRE type MCAP, then the knowledge of the comprehensive preference situation between two fictitious alternatives allows us to infer information on importance parameters of this MCAP. In fact, when the DM states \( b_{11} \) P \( b_{12} \), it means that the advantages on criteria contained in \( J_1 \) loom larger than the ones on criteria contained in \( J_2 \), i.e., the coalition \( J_1 \) is more important than the coalition \( J_2 \) \( (J_1 \gg J_2) \). As each assertion of the relation \( \gg \) \( (more \ important \ than \ between \ disjoint \ coalitions \ of \ criteria) \) is formalized, in the considered MCAP, by \( \forall J_1, J_2 \subseteq F \ J_1 \gg J_2 \iff \sum_{i \in J_1} \kappa_i > \sum_{i \in J_2} \kappa_i \), we infer from each preference relation between \( b_{11} \) and \( b_{12} \) an inequality on \( \kappa_i \)s.

Similarly, an indifference between \( b_{11} \) and \( b_{12} \) leads to stating an equality on \( \kappa_i \)s:
\[
\sum_{i \in J_1} \kappa_i = \sum_{i \in J_2} \kappa_i
\]
So as to soften the highly reductive effect of such an equality on the set of admissible values for \( \kappa_i \)s, we propose to treat indifferences in the following way:
\[ b_j \top b_j \iff | \sum_{j \in I_j} k_j - \sum_{j \in I_{-j}} k_j | < k_{\text{min}} \quad \text{with} \quad k_{\text{min}} = \min_{j \in P} \{k_j\} \]

The "strength" of the indifference is then reduced to an equality on \( k_j \)'s "to within \( k_{\text{min}} \)."

Moreover, it is important to stress what underlies the technique of saturated segmented description. When the polyhedron of admissible values for \( k_j \)'s becomes empty, this means that it is not possible to find values for \( k_j \)'s compatible, in the considered model, with the assertions expressed by the DM.

### 3.3.2. Preliminary step: eliciting discrimination thresholds.

The role of indifference and preference thresholds \( q_j \) and \( p_j \) is to specify the preferences restricted to the significance axis of a criterion; they do not refer directly to the notion of RIC. However, these thresholds interact with the inter-criteria preferential parameters and may have an indirect influence on the role of each criterion in the aggregation. Moreover, the definition of \( b_j \) requires the knowledge of \( p_j \). So as to determine values for these thresholds, the analyst may refer to the ill-determined nature of some constituent elements of \( [\underline{x}, \bar{x}, \{\underline{x}, \bar{x}\}] \) and \( \{\underline{x}, \bar{x}\} \). In this situation, these thresholds may be elicited through an interaction with the DM.

In the implementation of the ELECTRE methods (see [Vallée & Zielniewicz 94]), \( p_j \) and \( q_j \) are defined as affine functions of evaluations, i.e., such that:

\[
\begin{align*}
q_j(g_j(a)) &= a_{q_j} \cdot g_j(a) + b_{q_j} \\
p_j(g_j(a)) &= a_{p_j} \cdot g_j(a) + b_{p_j}
\end{align*}
\]
For each criterion \( j \)

Do

\[
\begin{align*}
\text{min} & \leftarrow n_j, \\
\text{max} & \leftarrow a_j
\end{align*}
\]

While \( \text{max}-\text{min} > \varepsilon \)  

(\( \varepsilon \): proportion of the range of the scale)

Do

If \( \text{max}+\text{min}/2 \geq n_j \)

Then \( \text{min} \leftarrow \text{max}+\text{min}/2 \)

Else \( \text{max} \leftarrow \text{max}+\text{min}/2 \)

End

\( q_j(n_j) \leftarrow \text{max}+\text{min}/2 \)

\( \text{min} \leftarrow q_j(n_j) \)

\( \text{max} \leftarrow a_j \)

While \( \text{max}-\text{min} > \varepsilon \)

Do

If \( \text{max}+\text{min}/2 \geq P_j n_j \)

Then \( \text{max} \leftarrow \text{max}+\text{min}/2 \)

Else \( \text{min} \leftarrow \text{max}+\text{min}/2 \)

End

End

\( p_j(n_j) \leftarrow \text{max}+\text{min}/2 \)

End

Determining the discrimination thresholds in this way supposes that:

- criteria are evaluated on a continuous scale (or discrete with a large number of levels),
- criteria are not the result of a sub-aggregation,
- the DM knows precisely how evaluations are determined.

### 3.3.3. Step 1: Rank criteria by order of importance

The first step in the questioning procedure consists of searching for a pre-order on the \( k_s \). To achieve this, the alternatives \( b_1, b_2, \ldots, b_n \) are presented to the DM; he should then determine the alternative \( b_h \) he considers the best. Then, it holds \( b_h P b_i \) \( \forall i \neq h \); hence \( k_h > k_i \) \( \forall i \neq h \). When several alternatives \( b_1, b_2, \ldots, b_n \) are judged to be the best and indifferent to one another, we have: \( \forall h \in H, \forall h' \in \mathcal{H} \) \( b_h P b_{h'} \) and \( \forall h, h' \in H \) \( b_h I b_{h'} \) with \( H = \{h_1, h_2, \ldots, h_p\} \). Then \( k_{h_1} = k_{h_2} = \ldots = k_{h_p} > k_{h'}, \forall h' \in \mathcal{H} \).

The best alternative(s) is (are) then deleted from the initial list and the DM must choose the best in the list of remaining alternatives, etc. We then obtain a pre-order on the \( k_s \) and it is always possible to renumber the criteria so that: \( k_1 \leq k_2 \leq \ldots \leq k_n \). When several criteria are of equal importance, we keep only one representative of the equivalence class, during the rest of the questioning procedure, so as to obtain, after a second renumbering: \( k_1 < k_2 < \ldots < k_n \).
3.3.4. Step 2: determining groups of criteria which are close in importance

The second step aims at partitioning the set of criteria (ranked by importance beforehand) according to a specific condition that can be interpreted as defining a partition into groups of "relatively close" criteria when considering their relative importance. Each group of criteria $G_i$ is defined by the index $h(i)$ of the least important criterion of the group.
The second step of the questioning procedure is summarized in the following algorithm:

\[
\begin{align*}
  i &\leftarrow 1 \\
  h(i) &\leftarrow 1 \\
  \text{While } h(i) + 2 \leq \text{number-of-crit} \\
  \text{Do} \\
  &\begin{align*}
    j &\leftarrow \text{number-of-crit} \\
    \text{Repeat} \\
    &\begin{align*}
      \text{Compare } b_j \text{ to } b_{\bar{x}(i), \bar{y}(i)+1} \\
      j &\leftarrow j - 1
    \end{align*} \\
    \text{Until not}(b_j \text{ P } b_{\bar{x}(i), \bar{y}(i)+1} \text{ or } h(i)+2 > j) \\
    \text{If } b_{\bar{x}(i), \bar{y}(i)+1} \text{ P } b_j \\
    &\begin{align*}
      h(i+) &\leftarrow j + 2 \\
      \text{Else } h(i+) &\leftarrow h(i-1)+2
    \end{align*} \\
  \text{Endif} \\
  i &\leftarrow i+1 \\
\text{End} \\
\end{align*}
\]

number-of-group \leftarrow i-1

So as to obtain the required information, this algorithm contains linear sequences of questions. It is possible to reduce significantly the number of questions by applying a dichotomic segmentation rule: instead of comparing \( b_{\bar{x}(i), \bar{y}(i)+1} \) to \( b_y, b_{y-1}, \text{ etc.} \) successively, it is more efficient to determine \( h(i+1) \) using a dichotomic search in the interval \([h(i)+2, n]\).

### 3.3.3.5. Step 3: Evaluating of the "distance" between groups of criteria

At the end of the first two steps, we obtain a partition of the set of criteria (subsequently ordered) in groups of "relatively close" criteria, from the point of view of their relative importance. The third step aims at evaluating the "distance" between these groups. The procedure seeks, for each group, the coalition composed of the two least important criteria which is more important than the least important criterion of the group just above it. More precisely, for each group \( G_i \) we search for \( m_1(i) \) and \( m_2(i) \) such that:

\[
\begin{align*}
  \forall a, b \in F & \quad a \leq m_1(i) \text{ et } b \leq m_2(i) \\
  \text{(one inequality at least being strict)} & \Rightarrow g_{h(i+1)} > \{a, b\}
\end{align*}
\]
- Do you prefer $b_{h(i)+1,k(i)+4}$ or $b_{h(i)+1}$?
  - If the answer is $b_{h(i)+1,k(i)+4}$ P $b_{h(i)+1}$, then $k_{h(i)+2}+k_{h(i)+4}>k_{h(i)+1}$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1,k(i)+4}$ I $b_{h(i)+1}$, then $|k_{h(i)+2}+k_{h(i)+4}-k_{h(i)+1}|<k_1$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1}$ P $b_{h(i)+1,k(i)+4}$, then we go on until we find $\lambda$ such that $b_{h(i)+1,k(i)+\lambda}$ P $b_{h(i)+1}$. If it is not possible to find such a $\lambda$, then we ask the following question:

- Do you prefer $b_{h(i)+2,k(i)+3}$ or $b_{h(i)+1}$?
  - If the answer is $b_{h(i)+2,k(i)+3}$ P $b_{h(i)+1}$, then $k_{h(i)+2}+k_{h(i)+3}>k_{h(i)+1}$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1}$ I $b_{h(i)+2,k(i)+3}$, then $|k_{h(i)+2}+k_{h(i)+3}-k_{h(i)+1}|<k_1$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1}$ P $b_{h(i)+2,k(i)+3}$, then we ask the following question:

- Do you prefer $b_{h(i)+2,k(i)+4}$ or $b_{h(i)+1}$?
  - If the answer is $b_{h(i)+2,k(i)+4}$ P $b_{h(i)+1}$, then $k_{h(i)+2}+k_{h(i)+4}>k_{h(i)+1}$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+2,k(i)+4}$ I $b_{h(i)+1}$, then $|k_{h(i)+2}+k_{h(i)+4}-k_{h(i)+1}|<k_1$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1}$ P $b_{h(i)+2,k(i)+4}$, then we continue until we find $\lambda_1$ and $\lambda_2$ such that not[$b_{h(i)+1}$ P $b_{h(i)+\lambda_1,h(i)+\lambda_2}$].

In the third step, the sequence of questions follows the algorithm given below:

For each group $G_i$:

Do

1. $m_1(i) \leftarrow h(i)$, $m_2(i) \leftarrow h(i)+1$
2. stop $\leftarrow$ false
3. Repeat
4. If $m_2(i)+1 < h(i+1)$
5. Then $m_2(i) \leftarrow m_2(i)+1$
6. Else
7. If $m_1(i)+2 < h(i+1)$
8. Then $m_1(i) \leftarrow m_1(i)+1$
9. $m_2(i) \leftarrow m_1(i)+1$
10. Else stop $\leftarrow$ true
11. Endif
12. If not stop
13. Then Compare $b_{h(i)+1}$ to $b_{m_1(i),m_2(i)}$
14. Endif
15. Until stop or not[$b_{h(i)+1}$ P $b_{m_1(i),m_2(i)}$]

End
3.3.6. Step 4: obtaining information so that each \( k_j \) has an upper bound

At the end of the third step, each \( k_j \) has a lower bound (at least \( k_{j-1} \)), but it does not necessarily have an upper bound. This means that the domain of admissible values for \( k=(k_1, k_2, \ldots, k_n) \) is open. This step aims at providing an upper bound for each \( k_j \). A sufficient condition for the polyhedron to be closed is that \( k_n \) has an upper bound and so has \( k_j/k_1 \). Indeed we may deduce from \( k_j/k_1 < \alpha \) that \( k_j < \alpha \cdot k_1 \) (\( \forall j \)). So as to bound this ratio, the analyst may ask the following question: suppose that we can imagine \( \alpha \) criteria having the same importance as \( g_j \); let us consider the fictitious alternative \( b_j^\alpha \) whose evaluations are average (i.e., identical to \( b_j \)) except on these \( \alpha \) criteria on which its evaluation is improved by a significant amount. Formally, the comparison of \( b_j^\alpha \) and \( b_n \) (\( \alpha \) being variable) allows us to obtain an upper bound for the ratio \( k_j/k_1 \). This technique puts only few questions to the DM; however, the complexity of the required interaction can lead to an uncertain and imprecise answer.

Another technique may be used. We consider \( J \subseteq F \) the set of criteria \( g_j \) for which \( k_j \)
3.3.7. Step 5: adding supplementary inequalities so as to reduce the polyhedron of admissible values

The fifth step aims at reducing the polyhedron of admissible values for k. This step is necessary when the obtained polyhedron leads to considering a very large number of sets of importance coefficients to be valid.

The questions deal with two alternatives \( b_{ij} \) et \( b_{kj} \) whose evaluations vary on 4 criteria. In order for the answers to provide supplementary information, it is necessary that \( k_i < k_k, k_i < k_j \). The choice of these 4 criteria (verifying \( k_i < k_k, k_i < k_j \)) must be made with regard to the polyhedron obtained. It is not necessary, therefore, to automate this step (the choice of these questions is left to the analyst). It should be noted that inconsistencies usually appear at this stage (see 3.2.2 how to reduce such inconsistencies).

3.3.8. Determining several admissible weight vectors

When the polyhedron of admissible values for k is closed, an interval of variation for each \( k_i \) can easily be inferred from the saturated segmented description: each \( k_i \) lies in a segment whose bounds are functions of the \( k_i \) verifying \( k_i < k_j \). These intervals are the following:

\[
\begin{align*}
m_2(k_i) &< k_2 < M_2(k_i) \\
m_3(k_i, m_1(k_i)) &< k_3 < M_3(k_i, M_1(k_i)) \\
m_4(k_i, m_2(k_i), m_3(k_i, m_2(k_i))) &< k_4 < M_4(k_i, M_2(k_i), M_3(k_i, M_2(k_i)))
\end{align*}
\]

etc.

The value of the lower and upper bound of each coefficient \( k_i \) is obtained by assigning to the \( k_i \) (such that \( i < j \)) in the function \( m_i(k_i, k_2, \ldots, k_j) \) and \( M_i(k_i, k_2, \ldots, k_j) \) their minimum or maximum value, respectively, according to the sign of the coefficient in the linear forms
3.3.9. Elicitation of the veto thresholds

Like the coefficients $k_j$, the veto thresholds $v_j(g)$ are inter-criteria preferential parameters. They account for an aspect of the notion of RIC which is distinct from that modelled by the coefficients $k_j^6$. We should recall that the veto thresholds aim at impoverishing the outranking relation through the invalidation of assertions $a \preceq b$ that satisfy the concordance condition.

Before trying to elicit thresholds $v_j(g)$, it is important to determine whether the DM wants to attach some veto power to the criteria. Moreover, the analyst must verify whether this veto power may become effective in comparing alternatives, i.e., if $v_j(g) < L_j$ with

$$L_j = \max_{a \in A} (g_j(a)) - \min_{a \in A} (g_j(a)).$$

When the veto has a significant effect on the result, further interaction with the DM is necessary. The software implementation of the ELECTRE methods considers the thresholds $v_j(g)$ as affine functions of evaluations. Several interaction modes have been proposed so as to determine values for $v_j(g)$ with the DM.

It is possible to take advantage of the meaning that the DM gives to the criteria and the role he wants them to play. [Roy & Bouyssou 93] (chap. 8 et 9) propose using the ratio $v_j/p_j$. We will propose below a method for determining an interval of variation for $v_j$. We will determine values for $v_j(n_j)$ and $v_j(a_j)$ ($n_j$ and $a_j$ being neutral and attractive evaluations respectively).

So as to determine these values, we proceed indirectly by asking questions concerning fictitious alternatives. We denote $b^0=(n_1,n_2,...,n_n)$ and $b^a=(a_1,a_2,...,a_n)$; let $b^j$ and $b^j_+$ be the alternatives having the same evaluations as $b^0$ and $b^a$ respectively except on the criterion $j \in J$ ($J \subseteq P$) on which its evaluation is increased by $p_j$. We denote $b^j_{+x}$ and $b^j_{-x}$ the alternatives having the same evaluations as $b^0$ and $b^a$ respectively except on criterion $j$ on which their evaluations are increased by $x$. So as to bound $v_j(n_j)$ and $v_j(a_j)$, the DM should compare $b^j_{+x}$ to $b^j$ and $b^j_{-x}$ to $b^j$, $x$ being variable and $J=F\{j\}$ (in what follows, we will explain only how to determine $v_j(n_j)$; we will proceed similarly with $v_j(a_j)$).

It should be noted that the assertion $b^j_+ \succ b^j_{-x}$ satisfies the concordance condition$^7$. In the comparison of $b^0$ and $b^j_{+x}$, the DM may either prefer $b^j$ or hesitate because of difficulties when comparing these alternatives. In the first case, $x$ constitutes a lower bound for $v_j(n_j)$ while in the second case, it specifies an upper bound for $v_j(n_j)$. So as to bound $v_j(n_j)$ we proceed through a dichotomic segmentation of the interval $[p_j,L_j]$ ($L_j$ being the range of the scale); this leads to the following algorithm:

---

$^6$ These two facets of the importance of criteria are usually linked; however, it is formally possible for a criterion to have simultaneously a low weight and a large veto power.

$^7$ except in very particular cases in which $d_k$ is very large ($k_j \gg \sum_{i=1}^{n} k_i$).

18
\[
\text{lower} \leftarrow p_i \\
\text{upper} \leftarrow L_j \\
\text{While upper-lower} > \epsilon \quad (\epsilon \text{ being a proportion of } L_j-p_i) \\
\text{Do} \\
\quad x \leftarrow \text{upper+lower}/2 \\
\quad \text{Compare } b_i^n \text{ and } b_{i,x}^n \\
\quad \text{If } b_i^n \not\equiv b_{i,x}^n \\
\quad \quad \text{Then lower} \leftarrow \text{upper+lower}/2 \\
\quad \quad \text{Else upper} \leftarrow \text{upper+lower}/2 \\
\text{Endif} \\
\text{End}
\]

Determining intervals for \(v_i(n_i)\) and \(v_i(a_i)\) allows us to define the coefficients of two lower and upper bound functions \(v_i^M(g_i)\) and \(v_i^M(g_i)\) (we should verify that these functions are such that \(v_i^M(g_i(a)) < v_i^M(g_i(a))\) \(\forall a \in A\)). The remarks concerning the conditions for using the algorithm for determining discrimination thresholds (see 3.3.2) also apply for the present algorithm.

### 3.4. Implementation of the method

#### 3.4.1 Role of such a tool in the decision aid process
present form, intended for the DM. Its role is not necessarily to transfer some of the activities of the analyst to the DM. It was not designed as a substitute for the analyst, whose role remains essential: the latter must explain to the DM the nature and the meaning of each of the preferential parameters as well as the way in which the DM's answers will be interpreted.

However, an independent use of this software by a DM with good background knowledge of the ELECTRE methods is possible. The proposed tool may then be viewed as a DSS that helps the DM in formalizing his preferences and eliciting the preferential parameters required by the ELECTRE methods. On the other hand, it may be dangerous to put such a tool in the hands of a novice in MCDA, since it would be difficult to know what the basis of the output obtained would be.

3.4.2. Presentation of the DIVAPIME software: general description

DIVAPIME software (Détermination d'Intervalles de VAriation des Paramètres d'Importance des Méthodes Electre) works with MS-DOS 3.2 or higher on a IBM PC with a minimum of 640 Kb memory and a VGA color monitor. It is implemented with Borland Turbo Pascal. The different options proposed are described in figure 3.

The content of the options in the main menu is as follows:

- **Files:** provide the list of criteria files, alternatives files and parameter set files.
- **Info:** provides general information on the ETIP (in particular the way answers are used) and explanations concerning each option. This option may be activated from any option as an on-line help feature.
- **Criteria:** input, loading and/or modification of a set of criteria.
- **Alternatives:** input, loading and/or modification of a set of alternatives.
- **Preferences:** Evaluation of preferential parameters through a questioning procedure.
- **Results:** display or print obtained intervals of variation for preferential parameters.

---

9 However, DMs frequently lead real world decision aid processes without the help of an analyst. The use of DIVAPIME in such a context will in any case be less controversial than the most frequently used method, i.e., direct numerical evaluation of the parameters.
The standard scheme of a DIVAPIME session is the following:

- Input (or loading/modification) of a set of criteria.
- Input (or loading/modification) of the set of alternatives\(^{10}\).
- Determination of the discrimination thresholds \( p_j \) and \( q_j \).
- Determination of intervals of variation for the importance coefficients \( k_j \) and possibly for the veto thresholds \( v_j \)^{11}.
- Generation of one or several sets of preferential parameters.

The interested reader will find in [Mousseau 93] an illustrative example of how DIVAPIME may be used in a decision aid process. It should also be mentioned that a new Windows version of this software will be implemented and will be adapted to a multi-actor situation.

3.5. Extensions

3.5.1. Adapting the method to the problem formulation

We believe that it is erroneous to conceive an ETIP without taking into account the problem formulation adopted in the modelling of the decision problem: the questioning procedure should correspond to the way the DM analyses the problem (P\(\alpha\): choice, P\(\beta\): assignment or P\(\gamma\): ranking, see [Roy 85] and [Bana e Costa 93]). In our presentation, the proposed method is mainly adapted to a problem formulated in ranking terms. The questions put to the DM should be modified so as to become appropriate for the choice and assignment problem formulation (P\(\alpha\) and P\(\beta\)).

---

\(^{10}\) This option is included in the software because the DM must have a precise perception of the set of alternatives so as to express judgments on the importance of criteria.

\(^{11}\) This option is not explicitly mentioned, but it is implied in the context of the software's functionality.
In the case of a choice problem, the answer to a pairwise comparison of alternatives could be "I do not want to choose either of them". Such an answer is hardly interpretable and appears when both alternatives are judged to be insufficiently attractive in order to be selected in the final prescription. It is then possible to "force" the DM's answer by putting him/her in a situation in which one alternative is the only available option. Another way to avoid
before answering each question. Hence, no adaptation of the method is required\textsuperscript{14}. This approach is particularly suitable when criteria represent viewpoints of specific actors. In this case, it is difficult for some of these actors to answer all questions as they may have a precise opinion on the importance of criteria only with regard to a subset of criteria. During a collective questioning procedure, it may occur that several DMs disagree on the answer to one (or more) question(s). It is then advisable to put aside temporarily the inequalities corresponding to these questions and to reintegrate them at the end in order to generate the corresponding polyhedrons.

A second approach consists of determining intervals of variation for importance parameters with each DM individually and trying to group DMs whose opinions are "closer"
with some assertions stated by the DM.

Section 3 is devoted to the presentation of an ETIP adapted for the ELECTRE methods. The interaction with the DM proceeds by means of pairwise comparisons of fictitious alternatives. This technique tests the consistency of the DM’s answers with the aggregation rule used and provides, as output, an interval of variation for each parameter. This output constitutes an interesting starting point for robustness analysis.
References

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