ELICITING INFORMATION CONCERNING THE RELATIVE IMPORTANCE OF CRITERIA

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OBTENTION D'INFORMATION SUR
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Introduction

When the analysis of a decision problem is grounded in the definition of a set of criteria, it is difficult to discriminate between alternatives whose evaluations on several criteria are in conflict. Multiple criteria preference modeling requires that the analyst obtains from the decision maker (DM) some preference information so as to discriminate between pareto-optimal alternatives.

A classical approach to Multiple Criteria Decision Aid (MCDA) consists of linking restricted preferences (corresponding to the n criteria) with the comprehensive preferences (taking all criteria into account) through a so called Multiple Criteria Aggregation Procedure (MCAP). In the MCAP, all criteria are not supposed to play the same role; the criteria are commonly said not to have the same importance. This is why there are parameters in the MCAP that aim at specifying the role of each criterion in the aggregation of evaluations. We will call such parameters importance parameters. They aim at introducing preferential information concerning the importance that the DM attaches to the points of view modelled by the criteria.

The nature of these parameters varies across MCAPs. The way of formalizing the relative importance of each criterion differs from one aggregation model to another. All this is done, for instance, by means of:

- scaling constants in Multiattribute Utility Theory (see [Keeney & Raiffa 76,93]),
- a weak-order on F in lexicographic techniques or a complete pre-order on F in the ORESTE method (see [Roubens 82]),
- intrinsic weights in PROMETHEE methods (see [Brans et al. 84]),
- intrinsic weights combined with veto thresholds in ELECTRE methods (see [Roy 91]),
- eigen vectors of a pairwise comparison matrix in AHP method (see [Saaty 80]).

Many authors have studied the problem of the elicitation of the relative importance of criteria (a critical overview may be found in [Mousseau 92]), but few of them have tried to give a precise definition of this notion. A careful analysis of this notion is still needed to build theoretically valid elicitation techniques. This paper aims at highlighting some of the difficulties that may be encountered when eliciting information concerning the relative importance of criteria and presents a way of eliciting such information when preferences are modeled through an outranking relation based on a concordance principle (see [Roy 91]).

In the first section, we will state precisely what the information underlying the notion of importance of criteria is. The second section will be devoted to the analysis of the meaning of an elicitation process. This will lead us to specify some basic requirements for importance parameters elicitation techniques. In the last section, we will present a technique, called DIVAPIME, in order to define a polyhedron of acceptable values for importance parameters in an ELECTRE type method.
1. What does the notion of the relative importance of criteria cover?

What does a decision-maker mean by assertions such as "criterion $g_j$ is more important than criterion $g_i$," "criterion $g_j$ has a much greater importance than criterion $g_i$," etc. Let us recall, by way of comparison, that the assertion "b is preferred to a" reflects the fact that, if the decision-maker must choose between the alternatives b and a, he is supposed to decide in favor of b. So, it is possible to test whether this assertion is valid or not. There is no similar possibility for comparing the Relative Importance of Criteria (RIC). Moreover, the way this notion is taken into account within the framework of the different models mentioned above by means of importance parameters reveals significant differences in what this notion deals with.

The first statement we should make when trying to analyse the notion of RIC is the following: the information underlying this notion is much richer than that contained in the importance parameters of the various multicriteria models. In fact, these parameters are mainly scalars and constitute a simplistic way of taking RIC into account, as this notion is by nature of a functional type.

In the comparison of two alternatives a and b, when one or several criteria are in favor of a and one or several others in favor of b, the way each MCAP solves this conflict and determines a comprehensive preference (i.e., taking all criteria into account) denotes the importance attached to each criterion (and to the logic of the aggregation used). Thus, the result of such conflicts (i.e., the comprehensive preference situation between a and b) constitutes the elementary data providing information on the relative importance of the criteria in conflict.

When we analyse the RIC notion, it appears that this notion represents a certain form of regularity in the link between restricted and overall preferences. In order to delimit exhaustively the importance of a criterion, we should analyse the contribution of any
where the indifference threshold represents the maximum difference of evaluation.
On the basis of the preceding considerations [Roy & Mousseau 95] propose a formal definition of the notion of RIC and a theoretical framework to analyse it. Within this framework, it clearly appears that the importance of criteria is taken into account in very different ways in the various aggregation procedures. In particular, this means that the values attached to importance parameters are meaningless as long as the aggregation rule in which they are used is not specified. Interesting theoretical proposals can also be found in Podinovskii 88, 94.

2. What do we aim at when evaluating the importance of criteria?

The elicitation of information about RIC is a crucial phase in a decision aid process. Basic assumptions concerning the nature of what is being done during this phase have an impact on the way to proceed to elicit such information.

2.1. The descriptivist and constructivist approaches to MCDA

The way we seek to give meaning to the notion of importance will differ according to which of these two approaches we adopt.

The descriptivist approach assumes that the way in which any two alternatives are compared on the comprehensive level (that is, taking all criteria into account) is well-defined in the decision-maker's mind before the modelling process begins. Moreover, it supposes that the modelling process does not modify such comparisons. The preference model chosen is intended to give an account of such pre-existing preferences as objectively as possible. Under these conditions, the role which devolves to each criterion as a function of its importance, is apprehended by the set of values attributed to the importance parameters. Thus, it is the capacity of a model to adjust to a well-defined, real-world situation which confers meaning on the notion of importance and allows to assign a numerical value to these parameters. Several authors would even talk in terms of estimating the numerical value of certain parameters, such as weight \( w_j \). Such language is meaningless unless a true numerical value for \( w_j \) exists, in which case the goal would be to estimate this true value as precisely as possible. In such an approach, observed lability in elicited preferences (see [Fishhoff et al. 89] and [Weber & Borcherdinger 93]) are explained by the existence of several biases in the elicitation techniques. [Beattie & Baron 91] argues that there is a "distinction between true and estimated weights and it is possible that subjects' true weights remain constant at all times, but become distorted in the elicitation process."

The constructivist approach, on the contrary, assumes that preferences are not entirely pre-formed in the decision-maker's mind and that the very nature of the work involved in the modelling process (and, a fortiori, in decision-aid) is to specify and even to modify pre-existing elements. The multiple criteria aggregation procedure (MCAP), which underlies the preference model chosen, is thus nothing other than a set of rules deemed appropriate for
aggregating the evaluations $g_i(a)$ and for building comprehensive preferences. Under these conditions, the numerical values assigned to importance parameters reflect a working hypothesis accepted for decision-aid. These are convenient numerical values with which it seems reasonable and instructive to work. The importance parameters, therefore, can be seen as keys which allow us to differentiate the role played by each criterion in the preference model selected for use. "True numerical values," to which we can refer to give meaning to the language of estimation, do not necessarily exist. Importance parameters and the numerical values we assign to them are, nonetheless, instruments for reasoning, investigating and communicating among the stakeholders in a decision-making process. The values (or interval of variation) for these parameters reflect, in the MCAP selected, a certain number of assertions expressed by the DM during the elicitation process (see [Roy 93], [Mousseau 93] and [Paynes et al. 92]).

2.2. Importance parameters elicitation methods

Developing Elicitation Techniques for Importance Parameters (ETIP) is an area of research that lies between the theoretical analysis of the notion of RIC and the empirical investigation of decision behavior (see figure 1). In fact, any ETIP should account both for

![Figure 1]

the precise meaning of the importance parameters in the MCAP used and for the DMs related empirical studies. For this interaction to be pertinent, it is crucial that the
The various ETIPs proposed in the literature (see [Mousseau 92] for a review) may be classified into two categories. In the first category, direct ETIPs, require an information on the concept of importance from the DM (direct evaluation of parameters, comparison of criteria in term of importance, etc.). The way the values for importance parameters are derived is defined independently of the aggregation rule in which these values will be used. Proceeding in this way, these ETIPs are not able to ensure that the information expressed in the DM’s answers matches the use of this information in the MCAP.

On the contrary, indirect ETIPs explicitly intergrate the MCAP selected for use. The interaction with the DM is not based directly on the concept of importance but on indirect information (binary comparison or ranking of alternatives, for example) from which information concerning the RIC is inferred through the aggregation rule.

It follows from the preceding considerations that any ETIP should satisfy two types of requirements:

i) Any ETIP should explicitly refer to the MCAP that is used to model the DM’s preferences: The logic of the various MCAP implies different meanings for their importance parameters. This means that the result of conflicts between criteria is determined both by the values assigned to importance parameters and by the logic of the MCAP. The knowledge of the value of these parameters is not sufficient to discriminate between pareto optimal alternatives; thus the meaning of importance parameters is only defined in relation to the MCAP in which they are used (see section 1).

ii) The way an ETIP interacts with the DM should account for his perception of the notion of RIC and for its limitations in perceiving and processing information: Any ETIP proceeds through a phase of interaction with the DM. Many studies in behavioral science and experimental psychology are useful for defining questioning procedures, pertinent from the DMs’ point of view (for an overview, see [Von Winterfeld & Edwards 86], [Paynes et al. 93]).

3. DIVAPIME: a way to elicit the importance of criteria in an MCAP
in [Roy & Bouysson 93]). Besides the implementation, our work consisted of restructuring and extending the questioning procedure to make it more precise, as well as improving the algorithmic aspects of the method.

3.1. The preferential parameters of the ELECTRE methods

ELECTRE methods build an outranking relation $S$, i.e., validating or invalidating, for any pair of alternatives $(a, b)$, an assertion $aSb$, whose meaning is "$a$ is at least as good as $b$". In the ELECTRE methods, preferences restricted to the significance axis of each criterion are defined through pseudo-criteria (see section 1). The indifference and preference thresholds ($q_i$ and $p_i$) model the intra-criterion preferential information. They account for the imprecise nature of the evaluations $g_i(a)$. The $q_i$ threshold specifies the largest difference of evaluation $g_i(a)-g_i(b)$ that preserves indiscernibility between $a$ and $b$ ($aI_b$); $p_i$ represents the smallest difference $g_i(a)-g_i(b)$ compatible with a preference situation in favor of $a$ ($aP_b$). The weak preference relation $Q_i$ should be interpreted as hesitating between opting for a preference or indifference situation.

At the comprehensive level of preferences, in order for the assertion $aSb$ to be valid, two conditions should be verified:
- concordance: for an outranking $aSb$ to be accepted, a "sufficient" majority of criteria should be in favor of this assertion,
- non-discordance: when the concordance condition holds, none of the criteria in the minority should oppose to the assertion $aSb$ in "a too strong way".

Two types of importance parameters intervene in the construction of $S$:
- the set of importance coefficients $(k_1, k_2, \ldots, k_n)$ takes into account the relative importance of coalitions of criteria and intervenes in the construction of a concordance index $c(aSb)$ which is defined by:
  $$c(aSb) = \frac{\sum_{i=1}^{n} k_i c_i(aSb)}{\sum_{i=1}^{n} k_i}$$
  with $c_i(aSb) = \begin{cases} 1 & \text{if } aP_b \text{ or } aQ_b \text{ or } aI_b \\ 0 & \text{if } bP_a \\ \in [0,1] & \text{if } bQ_a \end{cases}$
- the set of veto thresholds $(v_1(g_1), v_2(g_2), \ldots, v_n(g_n))$ is used in the discordance concept. $v_i(g_i)$ represents the greatest difference of evaluation $g_i(b)-g_i(a)$ compatible with the assertion $aSb$. 
3.2. Determination of a polyhedron of admissible values for importance parameters

In this section, we present a technique which aims at determining a non-empty polyhedron of admissible values for \( k=(k_1,k_2,\ldots,k_n) \) starting from linear inequalities on these coefficients. These inequalities come from DM's answers to binary comparisons of fictitious alternatives (see 3.3). We do not aim at determining a single vector of values \( k \), but a set of vectors consistent with assertions expressed by the DM.

3.2.1. Formulating information through a segmented description

Let us suppose that criteria can be ordered by importance and let us renumber them so that \( k_1 < k_2 < \ldots < k_n \). We want to specify an interval \( [m_i,M_i] \) containing each \( k_i \) (\( \forall i \neq 1 \)). Each of these intervals constitutes a segment in which the value of \( k_i \) may vary. We aim at building intervals \( [m_i,M_i] \) such that \( m_i \) and \( M_i \) depend only on the \( k_j \) for \( j<i \); Hence we will denote them \( m_i(k_1,k_2,\ldots,k_{i-1}) \) and \( M_i(k_1,k_2,\ldots,k_i) \). The system of inequalities is then:

\[
\begin{align*}
m_2(k_1) &< k_2 < M_2(k_1) \\
m_3(k_1,k_2) &< k_3 < M_3(k_1,k_2) \\
m_4(k_1,k_2,k_3) &< k_4 < M_4(k_1,k_2,k_3) \\
& \vdots \\
m_{i-1}(k_1,k_2,\ldots,k_{i-2}) &< k_{i-1} < M_{i-1}(k_1,k_2,\ldots,k_{i-2}) \\
m_i(k_1,k_2,\ldots,k_{i-1}) &< k_i < M_i(k_1,k_2,\ldots,k_{i-1}) 
\end{align*}
\]

In what follows, \( m_i(k_1,k_2,\ldots,k_{i-1}) \) will be defined by the maximum of a list of lower bounds of \( k_i \), and \( M_i(k_1,k_2,\ldots,k_{i-1}) \) by the minimum of a list of upper bounds of \( k_i \). Each of these lower or upper bounds takes the form of a linear combination of \( k_j \) for \( j<i \). For example, the value of \( m_3(k_1,k_2,k_3) \) can be \( (k_1+k_2+k_3) \). So as to add a new inequality to the system, we proceed as follows: such an inequality may always be written as \( \sum_{i=1}^n \alpha_i k_i > 0 \), with \( \alpha_i \in \mathbb{R} \). If \( \text{imax} \) is defined as the index of the greatest non-null coefficient \( \alpha_i \) (\( \text{imax} = \max\{i/\alpha_i \neq 0 \land j>i\} \)), we may rewrite the preceding inequality as \( \alpha_{\text{imax}} k_{\text{imax}} < \sum_{j<i} -\alpha_j k_j \). According to the sign of \( \alpha_{\text{imax}} \), this inequality specifies a new upper or lower bound for \( k_{\text{imax}} \). It is then necessary to check that each lower bound of \( k_{\text{imax}} \) is still lower than each upper bound of \( k_{\text{imax}} \). This verification may lead to adding new bounds. This step is called the saturation of the segmented description.

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1 The representation \( m_i(k_1,k_2,\ldots,k_i) > k_i < M_i(k_1,k_2,\ldots,k_i-1) \) of the domain of variation of \( k=(k_1,k_2,\ldots,k_i) \) exists whether this domain is a polyhedron or not; the functions \( m_i(k_1,k_2,\ldots,k_i) \) and \( M_i(k_1,k_2,\ldots,k_i) \) may not be linear (see [Roy 70], chap. 10).

2 This inequality comes from the answer to a pairwise comparison of alternatives (see 3.3.1).
3.2.2. Saturation of the segmented description

At a given stage, adding an inequality may produce supplementary inequalities. If the added inequality specifies a new upper or lower bound for \( k_i \) so that \( m_i(k_1, k_2, \ldots, k_{i-1}) < k_i < M_i(k_1, k_2, \ldots, k_{i-1}) \), by transitivity, it should hold:

\[
m_i(k_1, k_2, \ldots, k_{i-1}) < M_i(k_1, k_2, \ldots, k_{i-1}) \quad \text{[i]}
\]

If \([i]\) is verified \( \forall k_1, k_2, \ldots, k_{i-1} \), then no additional inequality should be generated; in the opposite case, this inequality \([i]\) should be integrated into the system, and will produce one (or several) upper and/or lower bound(s) from which one (or several) additional inequalities
3.3. Questioning procedure
\[ b_i^T \Delta b_j^T \Leftrightarrow \left| \sum_{j \in I_i} k_j - \sum_{j \in I_j} k_j \right| < k_{\min} \text{ with } k_{\min} = \min_{j \in F} k_j \]

The "strength" of the indifference is then reduced to an equality on \( k_j \)s "to within \( k_{\min} \)."

Moreover, it is important to stress what underlies the technique of saturated segmented description. When the polyhedron of admissible values for \( k_j \)s becomes empty, this means that it is not possible to find values for \( k_j \)s compatible, in the considered model, with the assertions expressed by the DM.

### 3.3.2. Preliminary step: eliciting discrimination thresholds.

The role of indifference and preference thresholds \( q_j \) and \( p_j \) is to specify the preferences restricted to the significance axis of a criterion; they do not refer directly to the notion of RIC. However, these thresholds interact with the inter-criteria preferential parameters and may have an indirect influence on the role of each criterion in the aggregation. Moreover, the definition of \( b_j \) requires the knowledge of \( p_j \). So as to determine values for these thresholds, the analyst may refer to the ill-determined nature of some constituent elements of criteria (see [Roy 85], chap. 9 and [Roy et al. 86]). In certain situations, these thresholds may be elicited through an interaction with the DM.

In the implementation of the ELECTRE methods (see [Vallée & Zielniewicz 94]), \( p_j \) and \( q_j \) are defined as affine functions of evaluations, i.e., such that:

\[
\begin{align*}
q_j(g_j(a)) &= a_q \cdot g_j(a) + b_q \\
p_j(g_j(a)) &= a_p \cdot g_j(a) + b_p
\end{align*}
\]

So as to obtain such an information, we propose to proceed in the following way:

- Determine a neutral evaluation \( n_j \) (neither attractive, nor repulsive, neither representing an advantage, nor a drawback on the considered criterion).
- Determine an attractive evaluation \( a_j \) representing a significant advantage with regard to the neutral level.

Given \( a_j \) and \( n_j \), we elicit \( q_j(n_j) \), \( p_j(n_j) \), \( q_j(a_j) \) and \( p_j(a_j) \). So as to minimize the duration of the interaction, questions are asked following a dichotomic search. The following algorithm determines \( q_j(n_j) \) and \( p_j(n_j) \) (\( q_j(a_j) \) and \( p_j(a_j) \) are obtained similarly).

---

4 The analyst should check that \( q \) and \( n \) are such that \( q \cdot p \cdot n \).

5 It is possible to determine more points so as to make a linear regression, but it requires more questions.
For each criterion $j$
\begin{align*}
\text{Do} & \quad \min \leftarrow n_j \\
& \quad \max \leftarrow a_j \\
\text{While } \max - \min > \varepsilon & \quad (\varepsilon: \text{proportion of the range of the scale}) \\
\text{Do} & \quad \text{If } \max + \min / 2 \leq n_j \\
& \quad \quad \text{Then } \min \leftarrow \max + \min / 2 \\
& \quad \quad \text{Else } \max \leftarrow \max + \min / 2 \\
\text{End} \\
& \quad q_j(n_j) \leftarrow \max + \min / 2 \\
& \quad \min \leftarrow q_j(n_j) \\
& \quad \max \leftarrow a_j \\
\text{While } \max - \min > \varepsilon & \quad (\varepsilon: \text{proportion of the range of the scale}) \\
\text{Do} & \quad \text{If } \max + \min / 2 \leq P_j n_j \\
& \quad \quad \text{Then } \max \leftarrow \max + \min / 2 \\
& \quad \quad \text{Else } \min \leftarrow \max + \min / 2 \\
\text{End} \\
& \quad p_j(n_j) \leftarrow \max + \min / 2 \\
\text{End}
\end{align*}

Determining the discrimination thresholds in this way supposes that:
- criteria are evaluated on a continuous scale (or discrete with a large number of levels),
- criteria are not the result of a sub-aggregation,
- the DM knows precisely how evaluations are determined.

3.3.3. Step 1: Rank criteria by order of importance

The first step in the questioning procedure consists of searching for a pre-order on the $k$'s. To achieve this, the alternatives $b_1, b_2, \ldots, b_n$ are presented to the DM; he should then determine the alternative $b_h$ he considers the best. Then, it holds $b_h P_i b_i \forall i \neq h$; hence $k_h > k_i \forall i \neq h$. When several alternatives $b_h, b_{h_2}, \ldots, b_{h_p}$ are judged to be the best and indifferent to one another, we have: $\forall h \in H, \forall h' \notin H \ b_h P_h b_i$ and $\forall h, h' \in H \ b_h P b_{h'}$ with $H = \{h, h_2, \ldots, h_p\}$. Then $k_h = k_{h_2} = \ldots = k_{h_p} = \varepsilon_h, \forall h' \notin H$.

The best alternative(s) is (are) then deleted from the initial list and the DM must choose the best in the list of remaining alternatives, etc. We then obtain a pre-order on the $k$'s and it is always possible to renumber the criteria so that: $k_1 \leq k_2 \leq \ldots \leq k_n$. When several criteria are of equal importance, we keep only one representative of the equivalence class, during the rest of the questioning procedure, so as to obtain, after a second renumbering: $k_1 < k_2 < \ldots < k_n$. 

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3.3.4. Step 2: determining groups of criteria which are close in importance

The second step aims at partitioning the set of criteria (ranked by importance beforehand) according to a specific condition that can be interpreted as defining a partition into groups of "relatively close" criteria when considering their relative importance. Each group of criteria \( G_i \) is defined by the index \( h(i) \) of the least important criterion of the group. The \( p \) groups are such that:

\[
\begin{align*}
\{ g_1, g_2, \ldots, g_{h(1)-1}, g_{h(2)-1} \} \\
\{ g_{h(2)}, g_{h(2)+1}, \ldots, g_{h(3)-1} \} \\
\vdots \\
\{ g_{h(p)}, g_{h(p)+1}, \ldots, g_{n} \}
\end{align*}
\]

The \( h(i) \) are defined by: \( h(i+1) = \min \{ j \mid g_j \succ [g_{h(0)} g_{h(0)+1}] \} \) with \( h(1) = 1 \). The must be interpreted as: \( g_{h(i)+1} \) is the least important criterion that remain more important than the coalition \( \{ g_{h(0)} g_{h(0)+1} \} \). The information necessary for determining the partition is the following:

\[
\begin{align*}
\{ g_{h(0)} \succ [g_{h(0)} g_{h(0)+1}] \\
\text{non}\{ g_{h(i)+1} \succ [g_{h(0)} g_{h(0)+1}] \}
\end{align*}
\] \( \forall i = \{1, 2, \ldots, p\} \)

In order to obtain this information, the DM must respond to pairwise comparisons of alternatives whose evaluations vary on 3 criteria. The protocol is the following:

- Do you prefer \( b_{i,2} \) or \( b_n \)?
  - if the answer is \( b_{i,2} \ P \ b_n \) then \( k_1 + k_2 > k_n \), we stop here for this step.
  - if the answer is \( b_{i,2} \ I \ b_n \) then \( |k_n - k_1 - k_2| < k_1 \), we stop here for this step.
  - if the answer is \( b_n \ P \ b_{i,2} \) then we ask the following question.

- Do you prefer \( b_{i,2} \) or \( b_{n,1} \)?
  - if the answer is \( b_{i,2} \ P \ b_{n,1} \) then \( k_1 + k_2 > k_{n,1} \), we stop here for this step.
  - if the answer is \( b_{i,2} \ I \ b_{n,1} \) then \( |k_{n,1} - k_1 - k_2| < k_1 \), we stop here for this step.
  - if the answer is \( b_{n,1} \ P \ b_{i,2} \) then we go on until we find the smallest \( h(2) \) such that \( \text{not}[b_{h(2)} \ P \ b_{i,2}] \).

We then continue by asking the question:

- Do you prefer \( b_{h(2),h(2)+1} \) or \( b_n \)
  - if the answer is \( b_{h(2),h(2)+1} \ P \ b_n \) then \( k_{h(2)} + k_{h(2)+1} > k_n \), we stop here for this step.
  - if the answer is \( b_{h(2),h(2)+1} \ I \ b_n \) then \( |k_n - k_{h(2)} - k_{h(2)+1}| < k_1 \), we stop here for this step.
  - if the answer is \( b_n \ P \ b_{h(2),h(2)+1} \) then we go on until we find the smallest \( h(3) \) such that \( \text{not}[b_{h(3)} \ P \ b_{h(2),h(2)+1}] \).

- and so on until the whole family of criteria is partitioned.
The second step of the questioning procedure is summarized in the following algorithm:

\[
\begin{align*}
i & \leftarrow 1 \\
h(i) & \leftarrow 1 \\
\text{While } h(i) + 2 & \leq \text{number-of-crit} \quad \text{Do} \\
& \quad j \leftarrow \text{number-of-crit} \\
& \quad \text{Repeat} \\
& \quad \quad \text{Compare } b_j \text{ to } b_{h(i), h(i)+1} \\
& \quad \quad j \leftarrow j - 1 \\
& \quad \quad \text{Until } \text{not } [b_j \text{ P } b_{h(i), h(i)+1} \text{ or } h(i)+2 > j] \\
& \quad \quad \text{If } b_{h(i), h(i)+1} \text{ P } b_j \\
& \quad \quad \quad h(i+1) \leftarrow j + 2 \\
& \quad \quad \quad \text{Else } h(i+1) \leftarrow h(i) + 1 + 2 \\
& \quad \quad \text{Endif} \\
& \quad i \leftarrow i + 1 \\
\text{End} \quad \text{number-of-group} \leftarrow i - 1
\end{align*}
\]

So as to obtain the required information, this algorithm contains linear sequences of questions. It is possible to reduce significantly the number of questions by applying a dichotomic segmentation rule: instead of comparing \( b_{h(i), h(i)+1} \) to \( b_w \), \( b_{w-1} \), etc. successively, it is more efficient to determine \( h(i+1) \) using a dichotomic search in the interval \([h(i)+2, n]\).

### 3.3.5. Step 3: Evaluating of the "distance" between groups of criteria

At the end of the first two steps, we obtain a partition of the set of criteria (subsequently ordered) in groups of "relatively close" criteria, from the point of view of their relative importance. The third step aims at evaluating the "distance" between these groups. The procedure seeks, for each group, the coalition composed of the two least important criteria which is more important than the least important criterion of the group just above it.

More precisely, for each group \( G_i \), we search for \( m_1(i) \) and \( m_2(i) \) such that:

\[
\begin{align*}
& \{ g_{m_1(i)}, g_{m_2(i)} \} \triangleright g_{h(i)+1} \\
& \forall a, b \in F \\
& \quad a \leq m_1(i) \text{ et } b \leq m_2(i) \\
& \quad \text{(one inequality at least being strict)} \\
& \Rightarrow g_{h(i)+1} \triangleright \{ g_a, g_b \}
\end{align*}
\]

In order to find \( m_1(i) \) and \( m_2(i) \), the sequence of questions and answers is the following:

- Do you prefer \( b_{h(i)+1, h(i)+3} \) or \( b_{h(i)+1} \)?
  - If the answer is \( b_{h(i)+1, h(i)+3} \text{ P } b_{h(i)+1} \), then \( k_{h(i)+1} + k_{h(i)+3} > k_{h(i)+1} \), we stop for this group and we go on to the next group.
  - If the answer is \( b_{h(i)+1, h(i)+3} \text{ I } b_{h(i)+1} \), then \( k_{h(i)+1} + k_{h(i)+3} \cdot k_{h(i)+1} < k_1 \), we stop for this group and we go on to the next group.
  - If the answer is \( b_{h(i)+1} \text{ P } b_{h(i)+1, h(i)+3} \), then we ask the following question:
- Do you prefer $b_{h(i)+1,h(i)+4}$ or $b_{h(i)+1}$?
  - If the answer is $b_{h(i)+1,h(i)+4}$ P $b_{h(i)+1}$, then $k_{h(i)+1}+k_{h(i)+4} > k_{h(i)+1}$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1,h(i)+4}$ I $b_{h(i)+1}$, then $|k_{h(i)+1}+k_{h(i)+4} - k_{h(i)+1}| < k_1$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1}$ P $b_{h(i)+1,h(i)+4}$, then we go on until we find $\lambda$ such that $b_{h(i)+1,h(i)+4} \lambda$ P $b_{h(i)+1}$. If it is not possible to find such a $\lambda$, then we ask the following question:

- Do you prefer $b_{h(i)+2,h(i)+3}$ or $b_{h(i)+1}$?
  - If the answer is $b_{h(i)+2,h(i)+3}$ P $b_{h(i)+1}$, then $k_{h(i)+2}+k_{h(i)+3} > k_{h(i)+1}$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+2,h(i)+3}$ I $b_{h(i)+1}$, then $|k_{h(i)+2}+k_{h(i)+3} - k_{h(i)+1}| < k_1$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1}$ P $b_{h(i)+2,h(i)+3}$, then we ask the following question:

- Do you prefer $b_{h(i)+2,h(i)+4}$ or $b_{h(i)+1}$?
  - If the answer is $b_{h(i)+2,h(i)+4}$ P $b_{h(i)+1}$, then $k_{h(i)+2}+k_{h(i)+4} > k_{h(i)+1}$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+2,h(i)+4}$ I $b_{h(i)+1}$, then $|k_{h(i)+2}+k_{h(i)+4} - k_{h(i)+1}| < k_1$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1}$ P $b_{h(i)+2,h(i)+4}$, then we continue until we find $\lambda_1$ and $\lambda_2$ such that $\text{not}[b_{h(i)+1} \lambda_1, b_{h(i)+4} \lambda_2]$. 

In the third step, the sequence of questions follows the algorithm given below:

For each group $G_i$

Do

\[
\begin{align*}
m_1(i) &\leftarrow h(i), \quad m_2(i) \leftarrow h(i) + 1 \\
\text{stop} &\leftarrow \text{false} \\
\text{Repeat} \\
\text{If } m_2(i) + 1 < h(i + 1) \\
\text{Then } m_2(i) &\leftarrow m_2(i) + 1 \\
\text{Else If } m_1(i) + 2 < h(i + 1) \\
\text{Then } m_1(i) &\leftarrow m_1(i) + 1 \\
\quad m_2(i) &\leftarrow m_1(i) + 1 \\
\text{Else stop } &\leftarrow \text{true} \\
\text{Endif} \\
\text{If not } &\text{stop} \\
\text{Then Compare } b_{h(i)+1} &\text{ to } b_{m_1(i),m_2(i)} \\
\text{Endif} \\
\text{Until stop or not}[b_{h(i)+1} \lambda_1, b_{h(i)+4} \lambda_2] \\
\text{End}
\end{align*}
\]
3.3.6. Step 4: obtaining information so that each $k_j$ has an upper bound

At the end of the third step, each $k_j$ has a lower bound (at least $k_{j-1}$), but it does not necessarily have an upper bound. This means that the domain of admissible values for $k=(k_1,k_2,\ldots,k_n)$ is open. This step aims at providing an upper bound for each $k_j$. A sufficient condition for the polyhedron to be closed is that $k_n$ has an upper bound and so has $k_j/k_1$. Indeed we may deduce from $k_j/k_1 \leq \alpha$ that $k_j \leq \alpha k_1$ $\forall j$. So as to bound this ratio, the analyst may ask the following question: suppose that we can imagine $\alpha$ criteria having the same importance as $g_1$; let us consider the fictitious alternative $b_i^\alpha$ whose evaluations are average (i.e., identical to $b_i$) except on these $\alpha$ criteria on which its evaluation is improved by a significant amount. Formally, the comparison of $b_i^\alpha$ and $b_i$ ($\alpha$ being variable) allows us to obtain an upper bound for the ratio $k_j/k_1$. This technique puts only few questions to the DM; however, the complexity of the required interaction needed to ensure mutual and increasing agreement is high.
3.3.7. Step 5: adding supplementary inequalities so as to reduce the polyhedron of admissible values

The fifth step aims at reducing the polyhedron of admissible values for k. This step is necessary when the obtained polyhedron leads to considering a very large number of sets of importance coefficients to be valid.

The questions deal with two alternatives (b_ij et b_kj) whose evaluations vary on 4 criteria. In order for the answers to provide supplementary information, it is necessary that k_1<k_2<k_3<k_4. The choice of these 4 criteria (verifying k_1<k_2<k_3<k_4) must be made with regard to the polyhedron obtained. It is not necessary, therefore, to automate this step (the choice of these questions is left to the analyst). It should be noted that inconsistencies usually appear at this stage (see 3.2.2 how to reduce such inconsistencies).

3.3.8. Determining several admissible weight vectors.
3.3.9. Elicitation of the veto thresholds

Like the coefficients $k_j$, the veto thresholds $\nu_j(g_j)$ are inter-criteria preferential parameters. They account for an aspect of the notion of RIC which is distinct from that modelled by the coefficients $k_j^6$. We should recall that the veto thresholds aim at impoverishing the outranking relation through the invalidation of assertions aSb that satisfy the concordance condition.

Before trying to elicit thresholds $\nu_j(g_j)$, it is important to determine whether the DM wants to attach some veto power to the criteria. Moreover, the analyst must verify whether this veto power may become effective in comparing alternatives, i.e., if $\nu_j(g_j) < L_j$ with $L_j = \max_{a \in A}(g_j(a)) - \min_{a \in A}(g_j(a))$. When the veto has a significant effect on the result, further interaction with the DM is necessary. The software implementation of the ELECTRE methods considers the thresholds $\nu_j(g_j)$ as affine functions of evaluations. Several interaction modes have been proposed so as to determine values for $\nu_j(g_j)$ with the DM.

It is possible to take advantage of the meaning that the DM gives to the criteria and the role he wants them to play. [Roy & Bouyssou 93] (chap. 8 et 9) propose using the ratio $\nu_j/p_j$. We will propose below a method for determining an interval of variation for $\nu_j$. We will determine values for $\nu_j(n_j)$ and $\nu_j(a_j)$ ($n_j$ and $a_j$ being neutral and attractive evaluations respectively).

So as to determine these values, we proceed indirectly by asking questions concerning fictitious alternatives. We denote $b^\emptyset= (n_1, n_2, ..., n_n)$ and $b^a= (a_1, a_2, ..., a_n)$; let $b^\emptyset_j$ and $b^a_j$ be the alternatives having the same evaluations as $b^\emptyset$ and $b^a$ respectively except on the criterion $j \in J$ ($j \in F$) on which its evaluation is increased by $p_j$. We denote $b^\emptyset_{j^x}$ and $b^a_{j^x}$ the alternatives having the same evaluations as $b^\emptyset$ and $b^a$ respectively except on criterion $j$ on which their evaluations are increased by $x$. So as to bound $\nu_j(n_j)$ and $\nu_j(a_j)$, the DM should compare $b^\emptyset_{j^x}$ to $b^\emptyset_j$ and $b^a_{j^x}$ to $b^a_j$, $x$ being variable and $J= F\{j\}$ (in what follows, we will explain only how to determine $\nu_j(n_j)$; we will proceed similarly with $\nu_j(a_j)$).

It should be noted that the assertion $b^\emptyset_j \leq S b^a_{j^x}$ satisfies the concordance condition$^7$. In the comparison of $b^\emptyset$ and $b^a_{j^x}$, the DM may either prefer $b^\emptyset_j$ or hesitate because of difficulties when comparing these alternatives. In the first case, $x$ constitutes a lower bound for $\nu_j(n_j)$ while in the second case, it specifies an upper bound for $\nu_j(n_j)$. So as to bound $\nu_j(n_j)$ we proceed through a dichotomous segmentation of the interval $[p_j, L_j]$ ($L_j$ being the range of the scale); this leads to the following algorithm:

---

$^6$ These two facets of the importance of criteria are usually linked; however, it is formally possible for a criterion to have simultaneously a low weight and a large veto power.

$^7$ except in very particular cases in which $k_j$ is very large ($k_j > \sum_{i} k_i$).
lower ← \( p_i \)
upper ← \( L^2 \)
While upper-lower > \( \epsilon \) (\( \epsilon \) being a proportion of \( L^2 - p_i \))
Do
\[
\begin{align*}
x &\leftarrow \text{upper+lower}/2 \\
\text{Compare } b^n_{ij} \text{ and } b^n_{ix} \\
\text{If } b^n_{ij} &\, \text{ P } b^n_{ix} \\
&\text{Then lower }\leftarrow \text{upper+lower}/2 \\
&\text{Else upper }\leftarrow \text{upper+lower}/2 \\
\end{align*}
\]
End

Determining intervals for \( v_j(n_i) \) and \( v_j(a_i) \) allows us to define the coefficients of two lower and upper bound functions \( v_j^m(g_i) \) and \( v_j^M(g_i) \) (we should verify that these functions are such that \( v_j^n(g_i(a)) < v_j^M(g_i(a)) \forall a \in A \)). The remarks concerning the conditions for using the algorithm for determining discrimination thresholds (see 3.3.2) also apply for the present algorithm.

3.4. Implementation of the method

3.4.1. Role of such a tool in a decision aid process

DIVAPIME is to be inserted in a decision aid process in which a multiple criteria model is used. The elicitation of preferential information is a crucial phase of the modelisation that is often problematical. The proposed software aims at supporting this elicitation step (when an ELECTRE type MCAP is chosen to model the decision process). Figure 2 places DIVAPIME in a classical multicriteria decision aid process.

The way this tool should be inserted in a decision process must be analysed. Several studies concerning the elicitation of the RIC have shown that there is a stumbling stock for any ETIP, namely the existence of a gap between the information underlying the answers given to the analyst by the DM, and the way these answers are interpreted in the model. We believe that the presence of an analyst may partially avoid this problem as he/she can verify whether the DM’s understanding and interpretation of the questions is correct.

Hence, this software is essentially intended to the analysts and aims at facilitating their work while enhancing the user-friendliness of the interaction with the DM. Although this tool has some of the characteristics of a decision support system (DSS)\(^8\), it is not, in its

\(^8\) Assisting the DM in ill-structured tasks, helping rather than replace the DM’s judgment, using the interactive possibilities of the tools (see [Sprague & Carlson 82], [Levine & Pomerol 89]).
present form, intended for the DM. Its role is not necessarily to transfer some of the activities of the analyst to the DM. It was not designed as a substitute for the analyst, whose role remains essential: the latter must explain to the DM the nature and the meaning of each of the preferential parameters as well as the way in which the DM's answers will be interpreted.

However, an independant use of this software by a DM with good background knowledge of the ELECTRE methods is possible. The proposed tool may then be viewed as a DSS that helps the DM in formalizing his preferences and eliciting the preferential parameters required by the ELECTRE methods. On the other hand, it may be dangerous to put such a tool in the hands of a novice in MCDA, since it would be difficult to know what the basis of the output obtained would be.

![Figure 2]

### 3.4.2. Presentation of the DIVAPIME software: general description

DIVAPIME software (Détermination d'Intervalles de VAriation des Paramètres d'ImpOrtance des Méthodes Electre) works with MS-DOS 3.2 or higher on a IBM PC with a minimum of 640 Kb memory and a VGA color monitor. It is implemented with Borland Turbo Pascal. The different options proposed are described in figure 3.

The content of the options in the main menu is as follows:

- **Files**: provide the list of criteria files, alternatives files and parameter set files.
- **Info**: provides general information on the Etip (in particular the way answers are used) and explanations concerning each option. This option may be activated from any option as an on-line help feature.
- **Criteria**: input, loading and/or modification of a set of criteria.
- **Alternatives**: input, loading and/or modification of a set of alternatives.
- **Preferences**: Evaluation of preferential parameters through a questioning procedure.
- **Results**: display or print obtained intervals of variation for preferential parameters.

---

9 However, DMs frequently lead real world decision aid processes without the help of an analyst. The use of DIVAPIME in such a context will in any case be less controversial than the most frequently used method, i.e., direct numerical evaluation of the parameters.
The standard scheme of a DIVAPIME session is the following:
- Input (or loading/modification) of a set of criteria.
- Input (or loading/modification) of the set of alternatives.
- Determination of the discrimination thresholds $p_j$ and $q_j$.
- Determination of intervals of variation for the importance coefficients $k_j$ and possibly for the veto thresholds $v_j$.
- Generation of one or several sets of preferential parameters.

The interested reader will find in [Mousseau 93] an illustrative example of how DIVAPIME may be used in a decision aid process. It should also be mentioned that a new Windows version of this software will be implemented and will be adapted to a multi-actor situation.

3.5. Extensions

3.5.1. Adapting the method to the problem formulation

We believe that it is erroneous to conceive an ETIP without taking into account the problem formulation adopted in the modelling of the decision problem: the questioning procedure should correspond to the way the DM analyses the problem (Pα: choice, Pβ: assignment or Pγ: ranking, see [Roy 85] and [Bana e Costa 93]). In our presentation, the
In the case of a choice problem, the answer to a pairwise comparison of alternatives could be "I do not want to choose either of them". Such an answer is hardly interpretable and appears when both alternatives are judged to be insufficiently attractive in order to be selected in the final prescription. It is then possible to "force" the DM's answer by putting him/her in a situation in which two alternatives are the only available options. Another way to avoid such a problem consists in proposing comparisons to the DM, in which both alternatives are "attractive". In the method proposed here, this may be done by changing the definition of the reference alternative $b_0$ (see 3.3.1) whose evaluations on all criteria should be attractive (rather than neutral).

When the decision situation is modelled through an assignment problem formulation, questioning the DM on the basis of pairwise comparisons of alternatives poses a more fundamental problem: the assignment of alternatives to a category is not founded on the comparisons of alternatives but on an absolute evaluation. In some cases, pairwise comparison of alternatives may be used. However, this mode of interaction is usually unsuitable and should be modified for the dialogue between the analyst and the DM to conform to the logic of Pβ.

One way to proceed is to ask the questions in terms of an assignment as follows. We denote $\{K_1, K_2, \ldots, K_p\}$ the ordered set of predefined categories ($i>j \Rightarrow$ categorie $K_i$ is better than categorie $K_j$). $b_0$ is a fictitious alternative conceived of in such a way that $b_0$ is assigned to $K_i$ (we note $b_0 \rightarrow K_i$). $b_1$ and $b_2$ ($J_1 \subseteq F_i$, $J_2 \subseteq F$ and $J_1 \cap J_2 = \emptyset$) are two fictitious alternatives defined as in 3.3.1. If the DM assigns $b_1$ and $b_2$ to the categories $K_{x_1}$ and $K_{x_2}$ respectively, then it holds: $\lambda_1 > \lambda_2 \Rightarrow J_1 \gg J_2$. From this information, we can infer values (or intervals of variation) for importance parameters through the MCAP used\(^{12}\).

3.5.2. Extension to a multiple DM framework

DIVAPIME is presently intended to be used in a decision situation in which a single DM is involved. Several ways can be considered to extend this ETIP (and its implementation) to decision situations in which several DMs interact\(^{13}\).

The first of these is to force the DMs to answer collectively the questions asked during the procedure. From this perspective, the multi-actor aspect of the problem is not directly managed by the method but is taken over by the DMs who must discuss their arguments

\(^{12}\) This approach, although more suitable for the assignment problem formulation, poses problems when the number of categories is low.

\(^{13}\) However such an extension is conceivable only in consensual multi-actor decision situations. In fact, in a decision problem in which opposition between actors is acute, it will be difficult for DMs to reach an agreement concerning the values of preferential parameters; seeking such values will only underline points of conflict but will not be able to orient the decision process towards a compromise solution.
before answering each question. Hence, no adaptation of the method is required\textsuperscript{14}. This approach is particularly suitable when criteria represent viewpoints of specific actors. In this case, it is difficult for some of these actors to answer all questions as they may have a precise opinion on the importance of criteria only with regard to a subset of criteria. During a collective questioning procedure, it may occur that several DMs disagree on the answer to one (or more) question(s). It is then advisable to put aside temporarily the inequalities corresponding to these questions and to reintegrate them at the end in order to generate the corresponding polyhedrons.

A second approach consists of determining intervals of variation for importance parameters with each DM individually and trying to group DMs whose opinions are "close". The simplest situation occurs when the intersection of all polyhedrons of admissible values for \( k_j \) is non-empty; a vector \( k=(k_1,k_2,\ldots,k_n) \) can then be chosen in the polyhedron defined by this intersection. Nevertheless, this polyhedron is frequently empty and such a vector cannot be found. In this case, another way to proceed would be to take advantage of automatic classification techniques so as to constitute a partition of the set of DMs. It is then simple to
with some assertions stated by the DM.

Section 3 is devoted to the presentation of an ETIP adapted for the ELECTRE methods. The interaction with the DM proceeds by means of pairwise comparisons of fictitious alternatives. This technique tests the consistency of the DM's answers with the aggregation rule used and provides, as output, an interval of variation for each parameter. This output constitutes an interesting starting point for robustness analysis.
References


Mousseau V. (1992) "Analyse et classification de la littérature traitant de l’importance relative des


Roy B. (1993) "Decision science or decision-aid science", EJOR, 66 (2), 184-203.


Vansnick J-Cl. (1986) "On the problem of weight in multiple criteria decision making (the non compensatory approach)", EJOR, 24, 288-294.

Vincke Ph. (1990) "Basic concepts of preference modelling", in Bana e Costa (ed.) "Readings in multiple criteria decision aid", Springer Verlag, 101-118.