MODELLING INACCURATE DETERMINATION,
UNCERTAINTY, IMPRECISION
USING MULTIPLE CRITERIA (*)

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LA MODÉLISATION DE L’INCERTAIN, DE L’IMPRECIS ET DE L’INDETERMINATION
DANS UNE APPROCHE MULTICRITÈRE

Résumé

Le but de ce cahier est de montrer que la prise en compte de critères multiples dans une étude d’aide à la décision offre à l’homme d’étude des moyens puissants et originaux de gestion de l’incertain, l’imprécis et l’indétermination. Après avoir rappelé brièvement quelles sont les principales sources d’incertitude, d’imprécision et d’indétermination dans une étude d’aide à la décision, on montre que la prise en compte de critères multiples permet d’établir des relations de préférence partielles, de discuter du pouvoir discriminant de chaque critère et de séparer nettement la phase d’agrégation de ces critères du reste de l’étude. Ces spécificités de l’approche multicritère présentent de nombreux avantages du point de vue de la gestion de l’incertitude, de l’imprécision et de l’indétermination.

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Abstract

The purpose of this paper is to study how the consideration of several criteria, as opposed to a more traditional mono-criterion approach, helps the modelling of imprecision, uncertainty and inaccurate determination (I.U.I.D.) in a decision-aid study. After a brief review of the main sources of inaccurate determination, uncertainty and imprecision that arises in a decision-aid situation, we show that the use of multiple criteria allows to build partial preference structures, to discuss in a powerful way the precision of the evaluation on each criteria and to create a clear language between the actors via the use of the evaluation tableau. We argue that this proves useful in dealing with I.U.I.D.
1- Introduction.

The use of multiple criteria in decision-aid models is often justified (see e.g. Zeleny (1982) or Schärlig (1985)) by the fact that the world is governed by multiple objectives and that any decision implies to balance "pros" and "cons". This widely-shared point of view can however be criticized (see Bouyssou (1987) and Roy (1988b)). Using a mono-criterion approach to decision-aid does not imply that one considers that "reality" is governed by a single criterion. It is well-known that, in this kind of models, multiple objectives are often taken into account e.g. via the use of constraints, sensitivity analysis and "prices" allowing to convert heterogeneous consequences into a single unit. As emphasized by Roy (1988b), the use of multiple criteria does not simply appear as a generalization of traditionnal approaches but constitutes a new paradigm for analysing and helping decisions.

In this paper we wish to outline what we consider as an important justification for entering this new paradigm: the management of imprecision, uncertainty and inaccurate determination (I.U.I.D) that is part of most decision situations. Our analysis follows that of Roy (1988a). He distinguished four main sources of I.U.I.D. that the analyst has to deal with. We briefly present them in section 2. In section 3, we recall the main originalities of models explicitly using several criteria. In section 4 we try to show how the consideration of several
parison is made possible through the use of "maps" of these complex "territories". For an alternative, a map consists of a model of the consequences of its implementation (in order to describe an alternative it is possible to use several maps of different "scale" using e.g. a hierarchical model). These maps create a tractable language that allows an effective communication between the various actors of the decision process and provides an adequate basis for the comparison of the alternatives. However the establishment of the maps inevitably involves many simplifications, omissions and distortions which introduce in the model an important source of arbitrariness. Indeed, there are often several and equally valid ways of building these maps. While forced to use maps in order to compare territories, the analyst has to make a tradeoff between the richness and the readability of the maps: the "richer" is one map the closer it is to the territory, but the more difficult it may be to compare it to other maps.

b) The "future" is not a "present" to come.

The alternatives that are to be compared will only be implemented in a more or less distant future. Thus, at the time of the study, the consequences of the implementation of an alternative are very often unpredictable for they depend on environmental factors and/or the strategy of other actors that are still unknown and may well be influenced by the implementation of that alternative. This is the most classical source of I.U.I.D. that is mentioned in every textbook on decision models. Many efforts have been devoted to cope with this unpredictability using, e.g., probability distributions, plausibility measures, scenarios, etc.

As Roy (1988a) mentioned, the unpredictability of the consequences of implementing an alternative also stems from the fact that the alternatives are not completely specified at the time of the study. When a firm tries to compare several sites for locating a new plant, the precise characteristics of each site may not have been completely investigated yet. Furthermore, the precise draft of the plant to be built may not be available and may well depend on the site chosen. Thus, even if one could predict with a very high precision the consequences of an alternative, an element of inaccurate determination would remain since the alternatives are still "projects".
c) The data are not the result of exact measurement.

The establishment of a map usually involves the consideration of two types of data. Data of type I are closely linked to the territory that the analyst wishes to describe. The modelling of uncertainty mentioned in the previous section will rely on this first type of data. For
d) The model is not the description of a real entity independent of the model.

Data of the second type are connected with certain aspects of the preference system(s) of the actor(s) involved in the decision process. It is well known that the questioning process used by the analyst in order to obtain these data may significantly influence the answers (see Bouyssou (1984)). This is all the more true since the preference system of an actor may not be completely structured at the time of the study: areas of firm conviction may well coexist with areas of hesitation and ambiguity in which the influence of the model on what is to be "captured" is overwhelming. Furthermore, the various actors may well disagree and, as a result of a discussion, some actors may change their mind on some point thus creating some "inconsistencies" with previously stated judgements. In such cases the management of these hesitations, contradictions and conflicts seems a prerequisite to any convincing decision-aid model. This is linked to what Roy and Bouyssou (1986) called a constructive attitude towards decision-aid, as opposed to a descriptive one, in which the role of the analyst is not to describe as accurately as possible supposedly pre-existing preferences but to provide information and tools that are useful for justifying, building and arguing preferences.

3- The multiple criteria approach to decision-aid.

From the point of view of the management of I.U.I.D., the main feature of an approach using multiple criteria is to break down the modelling process into two different phases: the construction of the criteria (which gives rise to the evaluation tableau) and the aggregation of these criteria (see Fig. 1). As advocated by Roy (1985), the analyst should use the smallest possible amount of type II data in the construction of the various criteria. We noted in section 2, that some data of type II such as utility functions have to be taken into account in order to build the criteria. However, sensitive information such as the tradeoffs between the various criteria are only introduced in the aggregation phase, contrary to what is usually done in a mono-criterion approach in which the construction of the unique criterion involves at the same time data of both types.
This approach is based on what could be called an "act of faith", i.e., the belief that the explicit construction of several criteria will have a "positive role" in the modelling process. It rests on an underlying assumption stating that in most decision-aid studies it is possible to identify a small number of "points of view" (usually between three and no more than ten, at least at the upper level if a hierarchical model is used) around which it is possible to build a family of criteria that is exhaustive and simple enough to be accepted as a basis of discussion by all the actors of the decision process.

Figure 1: Mono-criterion and multiple criteria approaches to decision-aid.

4. The management of imprecision, uncertainty and inaccurate determination.

We will center our discussion in this section around what we consider to be the three main originalities of the multiple criteria approach in the management of I.U.I.D.
a) The establishment of partial preference structures.

In a multiple criteria approach, a criterion is used to "sum up" evaluations on consequences related to a same point of view (see Roy (1985)), e.g. cost, safety, environment, etc. Formally a criterion can be defined as a function associating a real number $g_i(a)$ to each alternative "a", such that every actor in the decision process admits that if $g_i(a) \geq g_i(b)$ then alternative "a" is at least as good as alternative "b" on the point of view that underlies the definition of criterion $g_i$.

The interest of such partial preference relations for the management of I.U.I.D. is tied to the fourth source we mentioned in section 2. These partial preference relations can be seen as the stable part of the preference structure of the actors and thus as a possible basis for discussion between the actors. The criteria are designed in order to aggregate consequences that are very "close" to one another for they relate to the same point of view and are as free as possible from potentially highly conflictual type II data such as tradeoffs or
than g(b) then "a" is considered as being strictly preferred to "b", even if the difference between the evaluations is very small. Given the first three sources of I.U.I.D. we mentioned, this mode of comparison may lead to "unconvincing" preference situations. Small differences in somewhat arbitrary maps do not imply that the territories really differ. Furthermore, the way the analyst has dealt with the imprecision and/or uncertainty affecting most data in order to obtain the evaluations on the unique criterion is certainly not the only sensible one. Other reasonable ways of doing could have lead to different evaluations and, possibly, to a reversal of the comparison of "a" and "b". In this type of models, the only way to test the significance of a conclusion is to perform a thorough sensitivity analysis. A sensitivity analysis should ideally combine all plausible values for the parameters. However, given the number of these parameters and the complexity of the calculations leading to the map g(a), such a thorough sensitivity analysis can rarely be performed. It very often appears as a "one-dimensional" sensitivity analysis testing the robustness of the conclusions by varying only one parameter at a time.

These difficulties are still present in a multiple criteria approach¹. Yet, in this framework, the analyst may try to deal with these difficulties separately on each criterion. Since the model leading to the definition of each criterion is usually far less complex and uses much fewer parameters than the one that would lead to unique criterion this may be seen as an advantage. Let us note however that the analyst has often to introduce mostly "volontarist" hypotheses (such as probabilistic independence of some probability distribution for instance) in order to be able to cope with uncertainty and imprecision separately on each criterion.

Dealing with uncertainty and imprecision separately on each criterion, may prove useful from the point of view of sensitivity analyses. Furthermore, as argued by Roy (1985), the analyst may also try to define on each criterion a preference structure taking into account the fact

¹ For instance, when constructing a criterion "cost", using the expected utility of a probability distribution defined on actualized cash flows, it is clear that a small difference in the value of that criterion should not be considered as representative of a strict preference: the probability distribution used is rarely the only reasonable one, the utility function may have been greatly influenced by the type of assessment method used, etc.
that small differences may not be significant. This can be done using thresholds, a difference between two evaluations being significant only if it is "sufficiently" large. The evaluation of these thresholds is not an easy task. As long as the model leading to definition of the crite-
seems than in most real-world studies, it is possible to reach a consensus concerning the various points of view to be taken into account.

The map created by the evaluation tableau is operational in many respects. The establishment of a prescription implies the assessment of many data of type II. This assessment is usually based on "imaginary" alternatives (see Keeney and Raiffa (1976)). Using a mono-criterion approach one is bound in order to speak of such alternatives either to describe each of their consequences and characteristics, which is often very difficult, or to give their evaluations on the unique criterion in which case the alternatives have very little intuitive appeal. For instance in large linear programs, an alternative can either be described using a vector of hundreds of decision variables or by giving the value of the objective function\(^1\). The use of multiple criteria often allows to reach a convenient compromise for speaking of imaginary alternatives\(^2\). This gives the analyst a sound framework for assessing data of type II and discussing their "precision".

The evaluation tableau allows the actors to implement simple reasonings such as dominance, the use of aspiration levels or a simple lexicographic method in order to justify and elaborate preferences. In face of an evaluation tableau, most actors will recognize that a decision will inevitably be the result of a compromise between several conflictual objectives. It is then difficult for the analyst working into this framework to convince people to accept his recommendations just because of the sophistication of the methods he uses. This may allow the analyst to avoid some difficulties that are frequently encountered when using a unique criterion (see Roy (1981) and also Ackoff (1979)) and, thus, gives him tools for the management of hesitation and conflicts. In this approach, the analyst often proposes an "optimal" solution. Confronted to that optimal solution, an actor is likely either to accept it without restriction because of its "scientificity" or to reject it because the many simplifications, omissions and distortions contained in the model are incompatible with his value system (on these aspects, see GRETU

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1. This type of difficulty has lead some authors to make a great use, when possible, of visualizations of alternatives in linear programs (see Jacquet-Lagrèze and Meziani (1988)).

2. Let us also mention that the use of multiple criteria may be useful in order to generate new alternatives as noted by Starr and Greenwood (1977).
(1980)). The use of multiple criteria allows and, sometimes, forces the analyst to regard its model as support to reflexion, negotiation and creativity tolerating hesitations, ambiguities and iterations.

If the analysis goes further than the construction of an evaluation tableau then it is clear that highly sensitive data of type II will be needed either through the assessment of weights and/or tradeoffs or through a dialogue driven by an interactive procedure. By separating as much as possible this information from the rest of the data, an approach using multiple criteria allows to clearly locate conflicts between actors and perform thorough sensitivity analysis. This is true even if the aggregation method aims at building a unique criterion on the basis of the family of criteria contained in the evaluation tableau (which is done for instance in Multiattribute Utility Theory, see Keeney and Raiffa (1976)). This particular form of aggregation should not be confounded with an approach directly aiming at building a unique criterion. In the latter case, data linked to the description of the alternatives are often inextricably mixed with data linked to a particular preference system.

Using multiple criteria can be seen as a "diplomacy of small steps", trying to model what can be modelled in spite of the presence of hesitations, conflicts and ambiguities. Many other intermediate steps could be envisaged apart from the evaluation tableau. It seems however that the use of multiple criteria gives to the analyst clear and sound tools to deal with I.U.I.D. Central to the the management of I.U.I.D. into this framework is what could be called a "volontarist analytical" approach which is only very partially covered by the classical "divide and conquer" approach. Division appears here less as a cartesian device than as a pragmatic way of building a convincing prescription for decision-aid.
REFERENCES

Ackoff R.L. (1979), The future of operational research is past, JORS, Vol. 30, pp. 93-104.


Grassin N. (1986), Constructing criteria "population" for the comparison of different options of high voltage line routes, EJOR, Vol. 26, pp. 42-47.

GRETU (1980), Une étude économique a montré... mythes et réalités des études de transport, Cujas, Paris.


