EXPERIMENTAL EVALUATION
OF A MULTIOBJECTIVE
LINEAR PROGRAMMING SOFTWARE ¹

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EVALUATION D'UN LOGICIEL INTERACTIF DE PROGRAMMATION LINEAIRE MULTICRITERE PAR EXPERIMENTATION EN LABORATOIRE

RESUME

Le niveau de la recherche empirque en matière de logiciels d’aide à la décision multicritère est relativement faible comparativement au taux de développement de nouvelles méthodes d’analyse multicritère.

Ce cahier présente la méthodologie et les résultats d’une étude empirique ayant pour but l’évaluation de la capacité d’un certain logiciel interactif (il s’agit du logiciel ADELAIS pour microordinateur) de fonctionner comme un instrument efficace d’aide à la décision pour des problèmes de programmation linéaire multicritère.

Le cadre méthodologique de l’étude comprenait une expérimentation systématique du logiciel sur un problème tricritère de gestion agricole. Le rôle des décideurs a été joué par une population d’étudiants qui ont utilisé le système ADELAIS de façon individuelle pour soutenir leur propre processus de décision.

Les résultats de l’étude ont été obtenus par: (1) évaluation de quelques indices de performance de ADELAIS, comme la capacité de converger, le volume d’information, la possibilité de modéliser les préférences du décideur et le temps de calcul; (2) recensement des avantages et des faiblesses des diverses composantes du système au cours du processus de décision pour chaque utilisateur; (3) dépouillement des questionnaires auxquels ont répondu les utilisateurs concernant quelques caractéristiques spécifiques du logiciel.

Mots-clés: Programmation Linéaire Multicritère; Recherche Empirique; Logiciel Interactif.

EXPERIMENTAL EVALUATION OF A MULTIOBJECTIVE LINEAR PROGRAMMING SOFTWARE

ABSTRACT

The level of empirical research activity in multicriteria decision-aiding software is relatively low with respect to the rate of development of new multicriteria decision making methods.

This paper presents the methodology and the results of an empirical study which was conducted for the investigation of the capability of a certain software package (the ADELAIS microcomputer software) to operate as an effective decision supportive tool for multiobjective linear programming systems.

The methodological framework of the study included systematic experimentation on a three-criteria agricultural management problem. The role of the decision makers was undertaken by a population of undergraduate students who used individually the ADELAIS system in order to support their decision process.

The results of the study were obtained: (1) by measuring some performance indices of ADELAIS, such as its convergence capability, the information load, its capability in assessing the DM’s preferences and the computer effort; (2) by monitoring and recording the extent to which the users took advantage of the various components of the system during the decision process; (3) by recording some specific characteristics of the system as they were evaluated by the users with the help of a questionnaire.

Key-words: Multiobjective Linear Programming; Empirical Research; Microcomputer Software.
1. INTRODUCTION

Empirical research in decision support systems (DSSs) and in decision-aiding software in general mainly concerns the investigation of the degree to which the use of such decision tools improves the effectiveness of the decision making processes. One major class of empirical DSS studies is based on the experimental approach, according to which the performance of a DSS is tested in laboratories over simulated decision environments and controlled populations of decision makers (DMs). Representative works on this field have been recently reviewed by Sharda, Barr and McDonnell [15].

Empirical research in multicriteria decision support systems and multicriteria decision-aiding software in general is relatively sparse with respect to the rate of de-
The work presented in this paper can be listed in the latter category of empirical studies. Its purpose is twofold. First, to evaluate the performance of the ADELAIS multiobjective linear programming (MOLP) software as a decision-aid tool. Second, to outline a general framework for relative experimental tests. A comparative study of ADELAIS with other relative MOLP softwares was avoided for two main reasons: First, a software permitting the use of different MOLP methods, including the ADELAIS underlying methodology, in a homogenous computer environment was not available. Existing MOLP softwares differ in their design philosophy and show many particularities in operation. These factors were expected to influence undesirably the results of a comparative study. Second, many interesting, from the methodological aspect, MOLP algorithms have not been as yet integrated into interactive computer programs.

The paper is organized as follows. In section 2 the operational principles of ADELAIS are outlined. Section 3 presents the criteria which were used for the evaluation of the performance of the system. The experimental procedure is presented in section 4. In section 5 the results of the study are presented and discussed in some detail. Finally, some suggestions for improvement, which was deduced from the study, are given in the conclusion.

2. OPERATIONAL PRINCIPLES OF ADELAIS

ADELAIS is a fully interactive and menu driven computer program which is designed to support decisions in MOLP problems of the general form:

\[
\begin{align*}
& \text{max} \quad g_1 (x) = c_{1T} x \\
& \quad \vdots \\
& \text{max} \quad g_n (x) = c_{nT} x \\
\end{align*}
\]

subject to

\[x \in A = \{x \in \mathbb{R}^m : A x \leq b, x \geq 0\}\]

where \(x = (x_1, \ldots, x_m)\) is the vector of the decision variables, \(A\) is the matrix of the technological coefficients, \(b\) is the right-hand side of the constraints and \(c_j = (c_{j1}, \ldots, c_{jm})\) are the coefficients of the objective \(g_j\).

ADELAIS consists of twelve independent modules, which as a whole support extensive data management and realize a coherent MOLP methodology. Detailed information about the underlying methodology of ADELAIS, which is also presented briefly in the rest of this section, as well as its software structure and the user interface are given in two papers, respectively Siskos and Despotis [16] and Despotis and Siskos [3].
The MOLP method incorporated in ADELAIS operates in four stages. (An interesting three-stage method has been proposed by Jacquet-Lagrèze, Meziani and Slowinski [7]).

**Preliminary stage**

In this stage upper and lower bounds for the objectives (say $g_i^*$ and $g_i^*$ respectively) are obtained by maximizing and minimizing respectively each objective on the feasible set A. Particularly, if all or some of the minimization problems are unbounded, and this may happen even though the original MOLP problem has been well formulated in order to have a finite maximum, the lower bounds are computed with a heuristic (cf. [16]). Afterwards, an initial efficient solution (i.e., a solution which is not inferior to any other feasible solution) is estimated in a way similar to that in Step Method (STEM) of Benayoun et al. [1]. This technique guarantees that the objective values which correspond to the estimated solution will be as close as possible to the upper bounds with respect to the weighted Tchebycheff norm.

The iterative part of the method can be resolved in three successive stages.

**Stage I**

At each iteration the system provides the DM with a new efficient solution and the corresponding objective values. These solutions, except the initial one which comes from the preliminary stage, are calculated in stage III (see below). In stage I the system screens the attained objective values, the achievement percentages with respect to the upper bounds and the satisfaction levels (i.e., the revised lower bounds) established in previous iterations. The DM compares the attained objective values with the upper bounds and then he/she is asked to indicate which objectives he/she insists on increasing and if he/she intends to decrease some of the others in compensation. The DM's answers are combined with relative answers of previous iterations and then are used by the system for the establishment of new satisfaction levels. These new satisfaction levels limit the decision space but the DM can relax them, whenever he/she wants, by analysing the local trade-offs among the objectives. This possibility allows the DM to remove the consequences of previous answers which eventually contradict his/her current desires. That is to say the DM can dilate the decision space in order to reexamine solutions that had been rejected in previous iterations. The iterative process terminates within stage I when a best compromise is achieved, i.e. when the DM is not willing to decrease any objective.
Stage II

Stage II implies a learning process of the DM's preferences. At first, a simple technique is set up to build a reference set of decision profiles (i.e., a set of n vectors that might be assumed by the n objective functions). These reference alternatives are presented in pairs to the DM, who is asked to rank order them according to his/her preferences. Then a concave additive utility function, which is as consistent as possible with the DM's ranking, is assessed by a modified version of the UTA ordinal regression algorithm (cf. Jacquet-Lagrèze and Siskos [3] and Deprima and Vannucchi [4]). The system plots the curves of the...
Convergence

An iterative procedure is said to have good convergence properties if it is able to approach some final solution in a finite number of iterations. This means that the investigation of convergence is directly related to the definition of the final solution. However, in decision problems involving multiple objectives, there is no solution which could be objectively judged as the final one. Particularly, in MOLP problems the final solution (i.e., the "most satisfactory" solution) is exclusively defined by the DM's individual preference system and not by mathematical conditions. Thus, mathematical convergence is not easy to investigate in MOLP methods. Moreover, requiring "absolute" convergence from interactive methods seems to be against the principle of the "learning mechanism" on which these methods are based.

In multiple objective interactive methods it is more convenient to investigate "requisite" convergence (Phillips [12]). This property reflects the capability of an interactive method to model progressively the preference system of an individual
Information load

The information load is a considerable factor that influences the general performance of an interactive method and more precisely its applicability. As the interactive MCDM methods differ in the way they assess the DM’s preferences, the information processing operations performed by the DM vary from one method to another.

Larichev and Nikiforov [11] identified eleven information processing operations, which are widely employed in the interactive MCDM methods, and assigned to each of them a general estimate reflecting their complexity. Some of these operations are elementary (i.e., they can not be broken into other operations) while others can be analyzed in a sequence of elementary operations.

The requirements of ADELAIS in information processing operations are limited. In fact, in stage I the DM must discriminate between satisfactory and not satisfactory objective values, with respect to the solution obtained at each iteration. This operation involves comparisons of the obtained objective values against the respective upper bounds and is sufficiently reliable as it can be performed by the DM without many contradictions. Following Larichev and Nikiforov’s terminology the operation employed in stage I can be judged as "admissible" and easily performed. Thus, it is assumed that this operation does not increase the information load. On the contrary, the operations which must be performed within stage II are more complex and need to be investigated. Actually, in stage II the DM is invited to define a preference ranking on some reference alternatives. For this purpose the DM compares two alternatives at a time and chooses the one preferred. If \( k \) is the number of the reference alternatives then the number of pairwise comparisons that the DM should perform is bounded by \( k(k+1)/2 \). But the comparisons that the DM actually makes are in general less, as the system disregards the pairs of alternatives for which the choice is suggested by the transitivity of preferences. Obviously, the number of comparisons performed by each DM is not constant but depends on the structure of his/her preference system. Therefore, the mean number of pairwise comparisons needed for the elicitation of the DMs' preference rankings on a standard number of reference alternatives (8 in the case of the experiment) was taken as an index of the information load.

Man-model consistency

One of the major operations of ADELAIS is the assessment of an analytical utility model capable of representing the DM's preferences. The input of the utility assessment process is a preference ranking on the reference alternatives. Thus, the ability of the utility model and consequently of ADELAIS to represent the DM's preferences can be expressed by the degree to which the model
can reproduce the DM's subjective ranking. This latter is easily obtained by Kendall's $\tau$, whose value results from the number of violations caused by the model on the input ranking ($\tau=1$ for full consistency and $\tau=-1$ for complete inconsistency).

*Computational time*

The time spent by an interactive system in computations is a considerable factor that influences its applicability. In fact, this time determines how long the user should wait until the system responds to his inquiries. In the case of interactive MOLP systems, in which the information-retrieval operations are limited, the computational time is the most considerable factor that influences the response time of the system. The computational time is a function of the computational load of the system but depends also on other factors such as the efficiency of the algorithms and the computational speed of the computer on which the system is implemented.

In MOLP methods the computational load is a function of the number and the dimensions of the linear programs solved at each iteration.

The computational load of ADELAIS is accumulated in stages II and III (see [16] for a detailed analysis of the dimensions of the linear programs solved).

*Additional indices*

Some other features of ADELAIS, such as its factionality, the ease of use and its applicability were evaluated by using the subjective judgments of the participants after the experience they had with the system. All relative data were recorded with the help of a questionnaire.

4. THE EXPERIMENT

*Subjects*

Participants in the study were 20 students of the Piraeus Graduate School of Industrial Studies enrolled in a game theory and business policy course. In the framework of this course students had the opportunity to become familiar with decision making in simulated business environment by working on business simulation games via microcomputer. Furthermore, all participants showed some homogeneity with respect to their academic background relative to the study, as all had been taught subjects on multiple objective mathematical programming.
The decision problem

The decision problem which was used in the study concerned the planning of an annual cultivation program for a Spanish agricultural cooperative. The case study was initially presented in *Romero, Amador and Barco* [13] as an application of compromise programming.

The linear programming model formulated for this study had 25 decision variables and 21 constraints. Three objectives were under consideration in this problem as they were determined by the Agrarian Reform Law for Andalusia:

- minimize seasonal labor (measured as the mean absolute deviation from the average labor utilization for the four quarters of the year);
- maximize employment;
- maximize gross margin.

Table 1: Pay-off table for the three objectives (source: *Romero et al.* [13])

<table>
<thead>
<tr>
<th>optimizing</th>
<th>Seasonal labor (hours/ha)</th>
<th>Employment (hours/ha)</th>
<th>Gross margin (pesetas/ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seasonal labor</td>
<td>15.97</td>
<td>156.18</td>
<td>82,321</td>
</tr>
<tr>
<td>Employment</td>
<td>235.28</td>
<td>451.90</td>
<td>172,107</td>
</tr>
<tr>
<td>Gross margin</td>
<td>229.90</td>
<td>421.73</td>
<td>174,116</td>
</tr>
</tbody>
</table>

Table 1 presents the upper bounds of the objectives (underlined entries of the diagonal of the table). Each row of this table corresponds to the values achieved by the objectives when one of them takes its upper bound. The usefulness of this table lies in the fact that it provides the DM with important information about the conflict among the objectives.

Preparation and conduction of the experiment

Participants in the study attended two seminars. During the first one, the decision problem was presented and discussed. During the second one, the operational principles of ADELAIS were explained and the students, in two-persons groups, practised on the implemented version of the system in a computer environment. After the students having been experienced with the system, an efficient solution
was assigned at random to each one. These solutions had been calculated prior to the experiment by applying the first step of the algorithm by Choo and Atkins [2] to the data of the MOLP problem under consideration. For the conduction of the experiment, each participant was invited individually to assume the role of the DM within the farm planning problem and to reach the efficient solution assigned to him/her by modeling his/her preferences to this
Convergence

All participants reached the final solution within acceptable accuracy after a small number of iterations (mean number equal to 3.4). Participants carried out at least two iterations until to reach the final solution but 95%, 40% and 5% of them proceeded to a third, a fourth and a fifth iteration, respectively.

Information load

The number of pairwise comparisons performed by the participants among the reference alternatives for the assessment of their preference ranking varied from 11 to 19 with a mean of 13.9. As the number of the reference alternatives was kept constant (8 alternatives) during the experiment, the maximum number of pairwise comparisons that each participant should have to perform was 28.

In the light of the above results it is clear that the technique of successive partitions (cf. Siskos and Despotis [16]) which is employed in ADELAIS to support the elicitation of the preference ranking, exploits in the greatest extent the transitivity of preferences and reduces the number of comparisons.
Figure 1: Mean duration of each iteration.

The total time spent by the participants until to reach the final solution varied from 59 to 121 minutes (mean time 81.8 min).

It is worthy to be mentioned that the mean duration per iteration (cf. fig. 1) decreased from the first to the last iteration. This fact may well be rendered to the progressive familiarization of the participants with the system on the one hand, on the other hand to the acceleration of the information processing operations resulted from the comprehension of the decision task.

Extent to which the system was used

Table 3 presents the percentages of the participants who used the revisionary operations (feedbacks) authorized by the system at each iteration.

Table 3: Frequency of use of the revisionary operations

<table>
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<th>Operation</th>
<th>Iteration</th>
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<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Modification of the satisfaction levels</td>
<td>-</td>
</tr>
<tr>
<td>Modification of the preference ranking</td>
<td>65</td>
</tr>
<tr>
<td>Use of trade-off analysis</td>
<td>15</td>
</tr>
</tbody>
</table>

(*) i.e., % of the participants who proceeded to the 3rd iteration and modified their satisfaction level.
The revisionary operations referring to the formulation of the MOLP problem were excluded from the experiment and consequently are not included in table 3 as no modification of the MOLP model was assumed during the decision process. Among the revisionary operations provided by the system the one most used was the operation related to the revision of the preference ranking during the assessment of the DM's judgment policy. This fact may be rendered to the ability of the system of assigning the rank of higher judgment among
- **Information processing requirements are limited**

Participants did not meet difficulty in discriminating between satisfactory and not satisfactory objective values when they evaluated a new compromise solution. Indeed, such an operation does not show in general any innate difficulty but it is simplified more when facilitated, as in ADELAIS, by auxiliary elements concerning the attained solution, such as the satisfaction levels, the upper bounds of the objectives and the rates of achievement with respect to the upper bounds. Contrarily, the definition of a preference ranking on the reference alternatives is not an easy task to go through as it includes pairwise comparisons among the reference alternatives. However, 35% of the participants did not meet difficulty in performing pairwise comparisons. This fact may be rendered to the relatively small number of objectives considered in the decision problem, as well as to the way the system brings together the reference alternatives. Indeed the system, in order to facilitate the DM to exteriorize his judgment policy, does not simply puts side by side the alternatives to be compared but underlines the pros and cons of preferring one than another.

- **The system permits the DM to revise and to readjust his/her preferences**

The free readjustment of the preferences, according to their consequences in the course of the decision process, is an innate property of ADELAIS which is promoted in two levels: Globally, by means of the revisionary operations which permit the re-examination of solutions excluded in previous iterations and locally, during the assessment of the utility function and the analysis of inconsistencies where the DM can revise his judgment policy and thus to alter the search direction.

- **The system helps the DM to improve his/her knowledge about the decision problem**

In the course of the decision process participants showed progressively greater facility in expressing their preferences as the trade-off analysis provided by the system helped them perceiving more and more the relation between the objectives and what was feasible and what was not. This fact may be correlated to some extent with the declining tendency of the mean duration of the iterations (fig. 1)
6. CONCLUSION

A framework for testing interactive MOLP softwares is outlined in this paper. The empirical study conducted within this framework showed that the ADELAIS software package succeeded satisfactorily in its role as a decision-aid tool for multiobjective linear programming.

In this study, ADELAIS was applied to a MOLP problem of small size but its general performance, except the computational time for the maximization of the utility function, is not expected to change when applied to problems of medium size, i.e., up to 300 variables and 300 constraints, which are the limits of the current version.

The current version of ADELAIS, although it is user friendly, is more scientific than commercial and remains far from being considered as an end-user system. The software interface should be developed further in order to meet this requirement.

As it is deduced from the experience gained by the empirical study, some further development should include extension of graphics and incorporation of new functions, such as routines supporting a more direct adjustment of the satisfaction levels and the decision space.
REFERENCES


