A new bound for solving the recourse problem of the 2-stage robust location transportation problem

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Abstract

In this paper, we are interested in the recourse problem of the 2-stage robust location transportation problem. We propose a solution process using a mixed-integer formulation with an appropriate tight bound.

Keywords: Location transportation problem, robust optimization, mixed-integer linear programming.

1 Introduction

Robust optimization is a recent methodology for handling problems affected by uncertain data, and where no probability distribution is available. In robust optimization two decisional contexts are considered for taking decision under uncertainty. The first one is the *single-stage* context where the decision-maker has to select a solution before knowing the realization (values) of the uncertain parameters. Generally, the single-stage approaches provide the worst case

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solutions (Soyster [18]) that are very conservative and far from optimality in real-world applications. The maximum regret criterion can also be applied as a single-stage approach to problems affected by uncertain costs (see [10], [13], [14], [15] to name a few). The second approach concerns the *multi-stage* context (or dynamic decision-making) where the information is revealed in stages, and some recourse decision can be made. The multi-stage approach was firstly introduced by Ben-Tal et. al. [2], and initial focus was on two-stage decision making on linear programs with uncertain feasible set. Note that the formulations obtained following this approach are generally untractable.

In this paper, we are interested in a robust version of the location transportation problem with an uncertain demand using a 2-stage formulation. Recently, Atamturk and Zhang [1] used a two-stage robust optimization in network flow and design problem to obtain a good approximation of the robust solutions. Furthermore, Thiele et. al. [19] describe a two-stage robust approach to address general linear programs affected by uncertain right hand side. The robust formulation they obtained is a convex (not linear) program, and they propose a cutting plane algorithm to exactly solve the problem. Indeed, at each iteration, they have to solve an NP-hard recourse problem on an exact way, which is time-expensive. Here, we go further in the analysis of the recourse problem of the location transportation problem, in particular we define a tight bound for the mixed-integer reformulation.

The paper is organized as follows: in Section 2, the nominal location transportation problem is introduced and its corresponding 2-stage robust formulation. A mixed integer program is then proposed in Section 3 to solve the quadratic recourse problem with a tight bound. Finally, in Section 4, the results of numerical experiments are discussed.

2 Robust location transportation problem

We consider the following location transportation problem: a commodity is to be transported from each of m potential sources, to each of n destinations. The sources capacities are $C_i,\ i = 1, \ldots, m$ and the demands at the destinations are $\beta_j,\ j = 1, \ldots, n$. To guarantee feasibility, we assume that the total sum of the capacities at the sources is greater than or equal to the sum of the demands at the destinations. The fixed and variable costs of supplying from source $i = 1, \ldots, m$ are f_i and d_i, respectively. The cost of transporting one unit of the commodity from source i to destination j is μ_{ij}. The goal is to determine which sources to open (r_i), the supply level y_i and the amounts t_{ij} to be transported such that the total cost is minimized. The mathematical formulation of the location transportation problem is the following linear program, (T):

$$
(T)\begin{cases}
\min \sum_{i=1}^{m} d_i y_i + \sum_{i=1}^{m} f_i r_i + \sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{ij} t_{ij} & \\
\text{s.t. } \sum_{j=1}^{n} t_{ij} \le y_i & i = 1 \dots m \\
\sum_{i=1}^{m} t_{ij} \ge \beta_j & j = 1 \dots n \\
y_i \le C_i r_i & i = 1 \dots m \\
r_i \in \{0,1\},\ y_i,\ t_{ij} \ge 0 & i = 1 \dots m,\ j = 1 \dots n
\end{cases}
$$

In case of uncertainty on the demands, we model each demand by intervals, such that every β_j varies in $[\overline{\beta}_j - \hat{\beta}_j, \overline{\beta}_j + \hat{\beta}_j]$ where $\overline{\beta}_j$ represents the nominal value of β_j and $\hat{\beta}_j \ge 0$ its maximum deviation. Clearly, each demand β_i can take on any value from the corresponding interval regardless of the values taken by other coefficients. We denote (T^β) the location transportation problem for a given $\beta \in [\overline{\beta} - \hat{\beta}, \overline{\beta} + \hat{\beta}]$, with a nonempty feasible set. Finally, we denote $Z(T^\beta)$ the optimal value (bounded value) of (T^β) for a given β.

Following the approach suggested by [1], [8] and [19], which is a natural adaptation of the original Bertsimas and Sim approach (see [4],[3]), we define a parameter Γ, called *the budget of uncertainty* representing the maximum range of uncertain demands that can deviate from their nominal values. We have $\Gamma \in [0, n]$. For $\Gamma = 0$, every right hand side is equal to its nominal value, while $\Gamma = n$ leads to consider the problem with the worst demands.

We are interested in solving a robust version of the problem (T^β) with a 2-stage formulation. Indeed, the problem is to determine the minimum cost of choosing the facility $i,\ i = 1, \dots, m$ to be opened (with the r_i variables), and the supply level y_i, such that the worst demand is satisfied with a minimum cost. In this case, r_i and y_i variables are decided before the realization of the uncertainty (first stage decisions), while the t_{ij} variables represent the recourse variables to decide after the demands are revealed (second stage decisions). The robust problem is the following:

$$T_{Rob}(\Gamma)\left\{\begin{array}{l} \min\limits_{\substack{y_i \le C_i r_i,\ i=1\ldots m \\ y_i \ge 0,\ r_i \in \{0,1\}}} \{\sum_{i=1}^{m} d_i y_i + \sum_{i=1}^{m} f_i r_i + \max\limits_{\beta \in U} \min\limits_{\substack{\sum_{j=1}^{n} t_{ij} \le y_i,\ i=1\ldots m \\ \sum_{i=1}^{m} t_{ij} \ge \beta_j,\ j=1\ldots n \\ t_{ij} \ge 0,\ i=1\ldots m,\ j=1\ldots n}} \sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{ij} t_{ij}\} \end{array}\right.$$

where

$$U = \{\beta \in \mathbb{R}^n : \beta_j = \overline{\beta}_j + z_j \hat{\beta}_j,\ j = 1, \ldots, n,\ z \in Z\} \tag{1}$$

and

$$Z = \{z \in \mathbb{R}^n : \sum_{j=1}^{n} |z_j| \le \Gamma,\ \ -1 \le z_j \le 1,\ \ j = 1 \ldots n\}. \tag{2}$$

The problem $T_{Rob}(\Gamma)$ is a convex optimization problem that can be solved using *Kelley's algorithm* (see [11], [19]) that optimizes iteratively the master problem and the recourse problem by generating cuts. In this work, we focus on the recourse problem, namely

$$P(y,\Gamma)\left\{\begin{array}{l} \max\limits_{\substack{\sum_{j=1}^{n} |z_j| \le \Gamma \\ -1 \le z_j \le 1,\ j=1,\ldots,n}} \min\limits_{\substack{\sum_{j=1}^{n} t_{ij} \le y_i,\ i=1\ldots m \\ \sum_{i=1}^{m} t_{ij} \ge \overline{\beta}_j + \hat{\beta}_j z_j,\ j=1\ldots n \\ t_{ij} \ge 0,\ i=1\ldots m,\ j=1\ldots n}} \sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{ij} t_{ij} \end{array}\right.$$

At optimality $Z(P(y,\Gamma))$ represents the transportation cost value for a fixed capacity level y, and Γ worst deviations. Furthermore, we assume that $P(y,\Gamma)$ has a nonempty feasible set.

Because of the sense of the constraints of $P(y,\Gamma)$, the optimal values of the z_j variables will never be negative, and necessarily belong to $[0,1]$. Moreover, by strong duality theorem, one can replace the minimization problem by its dual (since the problem is always feasible):

$$
Q(y,\Gamma)\left\{\begin{array}{ll}
\max -\sum_{i=1}^{m} y_i u_i + \sum_{j=1}^{n} \overline{\beta}_j v_j + \sum_{j=1}^{n} \hat{\beta}_j v_j z_j & \\
\text{s.t. } v_j - u_i \leq \mu_{ij} & i = 1 \dots m, \quad j = 1 \dots n \\
\qquad \sum_{j=1}^{n} z_j \leq \Gamma & \\
\qquad 0 \leq z_j \leq 1 & j = 1 \dots n \\
\qquad u_i, \quad v_j \geq 0 & i = 1 \dots m, \quad j = 1 \dots n
\end{array}\right.
$$

where $u_i, \ v_j$ are the dual variables.

The obtained program has a quadratic shape with $(m + 2n)$ variables and $(nm + n + 1)$ constraints. More precisely, it is a bilinear program subject to linear constraints, which is a class of convex maximization problems proven NP-hard (see [7] and [20]). Several authors have been interested in solving bilinear problems. Initial work are those of Falk [6] and Konno [12] who proposed a cutting algorithm, improved by Sherali and Shetty in [17]. More recently, Bloemhof [5] gives an application to a production system.

From a complexity viewpoint, the resulting problem is not solvable in polynomial time. Instead of solving it on a direct way, we will reformulate $Q(y,\Gamma)$ as a mixed integer program. We present this formulation in next Section.

3 Mixed-integer program reformulation

In the current formulation of $Q(y,\Gamma)$, Γ is a real number varying between 0 and n. Nevertheless, one can assume Γ to be integer, representing the number of constraints for which $\beta_j \neq \overline{\beta}_j$. In this case, proposition 3.1 is required to give a MIP formulation of the problem $Q(y,\Gamma)$.

3.1 Linearization using a MIP

Proposition 3.1 *If Γ is an integer number then there exists an optimal solution (u^*, v^*, z^*) of $Q(y,\Gamma)$ such that $z_j^* \in \{0,1\}, \ j = 1,\dots,n$.*

Proof. Let us define the following polyhedra $Y = \{(u,v) \in \mathbb{R}^m \times \mathbb{R}^n : -u_i + v_j \leq \mu_{ij}, \ u,v \geq 0\}$ and $Z = \{z \in \mathbb{R}^n : \sum_{j=1}^{n} z_j \leq \Gamma, \ 0 \leq z_j \leq 1, \ \ j = 1 \dots n\}$. As $Q(y,\Gamma)$ is a bilinear problem, we know that if the problem has a finite optimal value (which is guarantee here because both polyhedra are bounded, by assumption) then an optimal solution (u^*, v^*, z^*) exists such that (u^*, v^*) is an extreme point of Y and z^* is an extreme point of Z (see [16]). This implies that when Γ is an integer number, z^* are 0-1. $\square$

From Proposition 3.1 and assuming that $\Gamma \in \mathbb{N}$ ($\Gamma \leq n$), we deduce that, at optimality either β_j is equal to its nominal value $\overline{\beta}_j$, or its worst value $\overline{\beta}_j + \hat{\beta}_j$. Furthermore, because of binary variables z_j we are able to linearize the problem $Q(y, \Gamma)$ by replacing each product $v_j z_j$ in the objective function with a new variable ω_j and adding constraints that enforce ω_j to be equal to v_j if $z_j = 1$, and zero otherwise (see [9]). The problem becomes a mixed integer program:

$$Q'(y,\Gamma)\begin{cases} \max -\sum_{i=1}^{m} y_i u_i + \sum_{j=1}^{n} \overline{\beta}_j v_j + \sum_{j=1}^{n} \hat{\beta}_j \omega_j & \\ \text{s.t. } v_j - u_i \leq \mu_{ij} & i = 1\dots m, \quad j = 1\dots n \\ \sum_{j=1}^{n} z_j \leq \Gamma & \\ \omega_j \leq v_j & j = 1\dots n \\ \omega_j \leq M z_j & j = 1\dots n \\ z_j \in \{0,1\} & j = 1\dots n \\ u_i, \quad v_j, \quad \omega_j \geq 0 & j = 1\dots n, \quad i = 1\dots m \end{cases}$$

where M is a sufficiently large constant.
For reducing the integrality gap, M needs to be as small as possible. We give the following tight bound for M:

$$M_j = v_j^*(n)$$

where $v_j^*(n)$, $j = 1, \dots, n$ is the optimal solution value of v variables in $Q'(y, n)$ (see Theorem 3.2).

Theorem 3.2 *The dual of the classical transportation problem can be written as follows*

$$(D^*)\begin{cases} \max -\sum_{i=1}^{m} y_i u_i + \sum_{j=1}^{n} \beta_j v_j & \\ s.t. -u_i + v_j \leq \mu_{ij} & i = 1\dots m, \quad j = 1\dots n \\ u_i, v_j \geq 0 & i = 1\dots m, \quad j = 1\dots n \end{cases}$$

We set (u^*, v^*) the optimal solution of the problem (D^*) and $Z^*(u^*, v^*)$ its optimal value.

Let us consider an instance of the transportation problem, such that the demand of the first customer is equal to $\beta_1 - \hat{\beta}_1$ with $\hat{\beta}_1 > 0$. The dual (D') of such a problem is the following linear program:

$$(D') \begin{cases} \max -\sum_{i=1}^{m} y_i u_i + (\beta_1 - \hat{\beta}_1)v_1 + \sum_{j=2}^{n} \beta_j v_j \\ \quad s.t. \ -u_i + v_j \leq \mu_{ij} & i = 1 \dots m, \quad j = 1 \dots n \\ \qquad u_i, v_j \geq 0 & i = 1 \dots m, \quad j = 1 \dots n \end{cases}$$

There exists an optimal solution (u', v') of (D') such that $u' \leq u^*$ and $v' \leq v^*$.

Proof. In the simple case where (u^*, v^*) is also optimal for the problem (D'), then the theorem 3.2 is verified. We are interested in the opposite case. We set (u', v') the optimal solution of (D') which does not satisfy the theorem 3.2. We define the solution (u'', v'') as follows:

$$u''_i = \min\{u^*_i, u'_i\} \ \text{ for all } i = 1 \dots m$$

$$v''_j = \min\{v^*_j, v'_j\} \ \text{ for all } j = 1 \dots n$$

Let us prove that (u'', v'') is an optimal solution for the problem (D').

First, we prove that (u'', v'') is feasible. By contradiction, suppose that there exists i_1 and j_1 such that $-u''_{i_1} + v''_{j_1} > \mu_{i_1 j_1}$

- If $u''_{i_1} = u^*_{i_1}$ then
$$u^*_{i_1} < -\mu_{i_1 j_1} + v''_{j_1} \tag{3}$$
Moreover, by definition $v''_{j_1} \leq v^*_{j_1}$, which implies that
$$-\mu_{i_1 j_1} + v''_{j_1} < -\mu_{i_1 j_1} + v^*_{j_1} \tag{4}$$
from (3) and (4) we deduce that $u^*_{i_1} < -\mu_{i_1 j_1} + v^*_{j_1}$, which contradicts the feasibility of the solution (u^*, v^*).
- If $u''_{i_1} = u'_{i_1}$ then
$$u'_{i_1} < -\mu_{i_1 j_1} + v''_{j_1} \tag{5}$$
By definition $v''_{j_1} \leq v'_{j_1}$ and thus
$$-\mu_{i_1 j_1} + v''_{j_1} < -\mu_{i_1 j_1} + v'_{j_1} \tag{6}$$
from (5) and (6) we deduce that $u'_{i_1} < -\mu_{i_1 j_1} + v'_{j_1}$ which contradicts the feasibility of the solution (u', v').

Thus, the solution (u'', v'') is feasible.

Before proving that (u'', v'') is optimal for (D'), let us prove that $v''_1 = v'_1$. By contradiction, suppose that $v''_1 = v^*_1$ (thus, $v^*_1 \leq v'_1$). We have already supposed that (u^*, v^*) is not optimal for the problem (P'), then

$$Z'(u^*, v^*) < Z'(u', v') \tag{7}$$

Furthermore, for each feasible solution (u, v) we have

$$\begin{aligned} Z'(u, v) &= -\sum_{i=1}^{m} y_i u_i + \sum_{j=1}^{n} \beta_j v_j - \hat{\beta}_1 v_1 \\ Z^*(u, v) &= -\sum_{i=1}^{m} y_i u_i + \sum_{j=1}^{n} \beta_j v_j \end{aligned}$$

which implies that

$$Z'(u, v) = Z^*(u, v) - \hat{\beta}_1 v_1 \tag{8}$$

from (7) and (8) we obtain

$$Z^*(u^*, v^*) - \hat{\beta}_1 v^*_1 < Z^*(u', v') - \hat{\beta}_1 v'_1 \tag{9}$$

$$Z^*(u^*, v^*) - Z^*(u', v') < \hat{\beta}_1 v^*_1 - \hat{\beta}_1 v'_1 \tag{10}$$

If we suppose that $v^*_1 \leq v'_1$, then $\hat{\beta}_1 v^*_1 - \hat{\beta}_1 v'_1 \leq 0$. Thus, from (10)

$$Z^*(u^*, v^*) - Z^*(u', v') < 0 \tag{11}$$

which provides a contradiction with the fact that (u^*, v^*) is an optimal solution for (D^*). Thus, necessarily $v^*_1 \geq v'_1$ and

$$v''_1 = v'_1 \tag{12}$$

Let us prove now that (u'', v'') is an optimal solution for (D'). The cost of such a solution is equal to

$$Z'(u'', v'') = -\sum_{i=1}^{m} y_i u''_i + (\beta_1 - \hat{\beta}_1) v''_1 + \sum_{j=2}^{n} \beta_j v''_j \tag{13}$$

Let $\overline{I} \subseteq I$ be the subset of indices of $I = 1 \ldots m$ such that $i \in \overline{I}$ if $u''_i = u'_i$, and thus $u'_i \leq u^*_i$. And define $\overline{J} \subseteq J$ as being the subset of indices of $J = 1 \ldots n$ such that $j \in \overline{J}$ if $v''_j = v'_j$, and thus $v'_j \leq v^*_j$. The cost of the solution (u'', v'') is

$$\begin{aligned} Z'(u'', v'') = &-\sum_{i \in \overline{I}} y_i u'_i + \sum_{j \in \overline{J} \setminus \{1\}} \beta_j v'_j + (\beta_1 - \hat{\beta}_1) v''_1 - \sum_{i \in I \setminus \overline{I}} y_i u^*_i + \\ &\sum_{j \in J \setminus \overline{J}} \beta_j v^*_j \end{aligned}$$

From (12) one can replace v_1'' by v_1' and thus

$$\begin{aligned} Z'(u'',v'') &= -\sum_{i\in I} y_i u_i' + (\beta_1 - \hat{\beta}_1)v_1' + \sum_{j\in J\setminus\{1\}} \beta_j v_j' + \sum_{i\in I\setminus\overline{I}} y_i(u_i' - u_i^*) - \\ &\quad \sum_{j\in J\setminus\overline{J}} \beta_j (v_j' - v_j^*) \\ &= Z'(u',v') + \sum_{i\in I\setminus\overline{I}} y_i(u_i' - u^*) - \sum_{j\in J\setminus\overline{J}} \beta_j(v_j' - v_j^*) \end{aligned}$$

Suppose now that (u'', v'') is not an optimal solution of (D'), then the amount

$$A = \sum_{i\in I\setminus\overline{I}} y_i(u_i' - u_i^*) - \sum_{j\in J\setminus\overline{J}} \beta_j(v_j' - v_j^*) \tag{14}$$

should be strictly negative. We define the solution $(\tilde{u}, \tilde{v})$ as :

$$\begin{aligned} \tilde{u}_i &= \max\{u_i^*, u_i'\} \quad \text{pour tout } i = 1\ldots m \\ \tilde{v}_j &= \max\{v_j^*, v_j'\} \quad \text{pour tout } j = 1\ldots n \end{aligned}$$

One can easily prove that the solution $(\tilde{u}, \tilde{v})$ is feasible (following the same reasoning as for (u'', v'')). The optimal value of $(\tilde{u}, \tilde{v})$ for the problem (D^*) is equal to

$$\begin{aligned} Z^*(\tilde{u},\tilde{v}) &= -\sum_{i=1}^{m} y_i \tilde{u}_i + \sum_{j=1}^{n} \beta_j \tilde{v}_j \\ &= -\sum_{i\in\overline{I}} y_i u_i^* - \sum_{i\in I\setminus\overline{I}} y_i u_i' + \sum_{j\in\overline{J}} \beta_j v_j^* + \sum_{j\in J\setminus\overline{J}} \beta_j v_j' \\ &= -\sum_{i\in I} y_i u_i^* + \sum_{j\in J} \beta_j v_j^* - \sum_{i\in I\setminus\overline{I}} y_i(u_i' - u_i^*) + \sum_{j\in J\setminus\overline{J}} \beta_j(v_j' - v_j^*) \\ &= Z^*(u^*,v^*) - \sum_{i\in I\setminus\overline{I}} y_i(u_i' - u_i^*) + \sum_{j\in J\setminus\overline{J}} \beta_j(v_j' - v_j^*) \\ &= Z^*(u^*,v^*) - A \end{aligned}$$

Assuming $A < 0$ contradicts the optimality of the solution (u^*, v^*) for (D^*). Thus, $A \geq 0$ and $Z'(u'', v'') \geq Z'(u', v')$. In fact, $A = 0$ and the solution $(\tilde{u}, \tilde{v})$ is optimal for (D^*). We conclude that the solution (u'', v'') is feasible and optimal for (D') and verifies Theorem 3.2. □

Following Theorem 3.2, we deduce that the values of u_i^* and v_j^* for $i = 1\ldots m,\ j = 1\ldots n$ are a kind of upper bounds for respectively u_i and v_j variables, in all instances of the transportation problem where one or many demands decrease. Indeed, one can build a sequence of one by one decreasing demands and apply successively Theorem 3.2. Going back to the problem $Q'(y,\Gamma)$, we recall that, when $\Gamma = n$ all demands $j = 1 \dots n$ are equal to their highest values $\overline{\beta}_j + \hat{\beta}_j$. When Γ decreases, some of the demands will also be decreasing. Thus, we deduce the bound $v_j^*(n)$ for the problem $Q'(y,\Gamma)$. In the next Section, we are interested in numerical experiments, performed on the transportation problem in order to compare the tight bound previously defined with an arbitrarily large M.

4 Numerical experiments

4.1 The data

Several series of tests were performed for various values of the parameters of the transportation problem, namely the number of sources, the number of demands, the amounts available at each source, the nominale and the highest demands at each destination and the transportation costs. To be closer to the reality, we choose to set the number of demands greater than the number of sources. All other numbers are randomly generated as follows: for all $j = 1,\dots,n$, the nominal demand $\overline{\beta}_j$ belongs to $[10, 50]$, and the deviation $\hat{\beta}_j = p_j\overline{\beta}_j$, such that p_j represents the percentage of maximum augmentation of each demand j. We take p_j in $[0.1, 0.5]$, which ensures $\hat{\beta}_j$ to be strictly positive. The amounts y_i at each source $i = 1,\dots,m$ are obtained by an equal distribution of the sum of the maximum demands. Finally, the costs are in the interval $[1, 50]$.

4.2 Solution process

The problem $Q'(y,\Gamma)$ was solved with CPLEX 11.2. For each (n,m), ten instances have been generated. Table 1 shows results of average running time and percentage of solved instances, for each one of the two bounds previously mentioned (see Section 3), such that the computation was stopped after 35 minutes.

The results described in Table 1 show that the computing time obtained by setting M to the bound $v^*(n)$ is significantly lower than the arbitrarily bound. Moreover, we remark that the running time increases for the value of Γ between $n/2$ and n whatever the bound is (see figure 1.a). Figure 1.b illustrates the evolution of the objective value versus Γ for a sample $m = 100$ and $n = 250$. The curve obtained is an increasing concave function, where

Table 1
Running time results

		Running time (s)		% solved instances	
$n \times m$	Γ	M	$v^*(n)$	M	$v^*(n)$
	25%	2.63	0.76	100	100
250×10	50%	1178.14	14.05	50	100
	75%	215.24	0.97	90	100
	25%	14.86	2.09	100	100
250×50	50%	434.85	4.94	90	100
	75%	645.52	6.80	70	100
	25%	8.57	1.82	100	100
250×100	50%	18.93	2.69	100	100
	75%	46.05	12.66	100	100
	25%	4.99	1.00	100	100
500×10	50%	1051.82	1050.25	50	50
	75%	843.92	2.29	60	100
	25%	10.49	1.60	100	100
500×50	50%	223.63	5.52	90	100
	75%	612.10	11.94	80	100
	25%	12.03	1.90	100	100
500×100	50%	10.98	2.26	100	100
	75%	170.57	21.34	100	100
	25%	35.59	3.26	100	100
1000×10	50%	1472.13	1260.21	30	40
	75%	1530.47	10.42	30	100

$Z(Q'(y,\Gamma))$ increases quickly for small values of Γ and slowly for high values. This is due to the model itself, since whenever Γ increases, the most influent uncertain parameters will be chosen.

Additional experiments have been performed on the uncertain transporta-

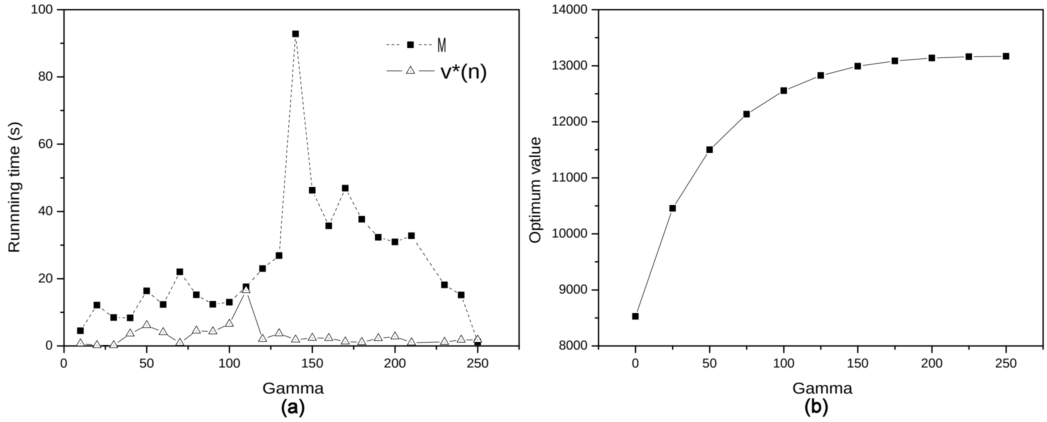

Fig. 1. A sample m=100 and n=250: **a.** Running time vs Gamma. **b.** Optimal value vs Gamma

tion problem using the bound $v^*(n)$, in order to determine the limit size of the problem that can be solved within one hour of CPU time. Figure 2.a shows the numerical results for a number of the uncertain demands set to $n = 500$. We observe that the running time grows as the number of sources m increases. Indeed, for $m = 10$ the problem takes few seconds to be solved. An average of 20 minutes is needed for instances with $m = 80$ and $\Gamma = 60\%$ of total deviation, and one hour for those with $m \in [300, 500]$ and $\Gamma = 50\%$.

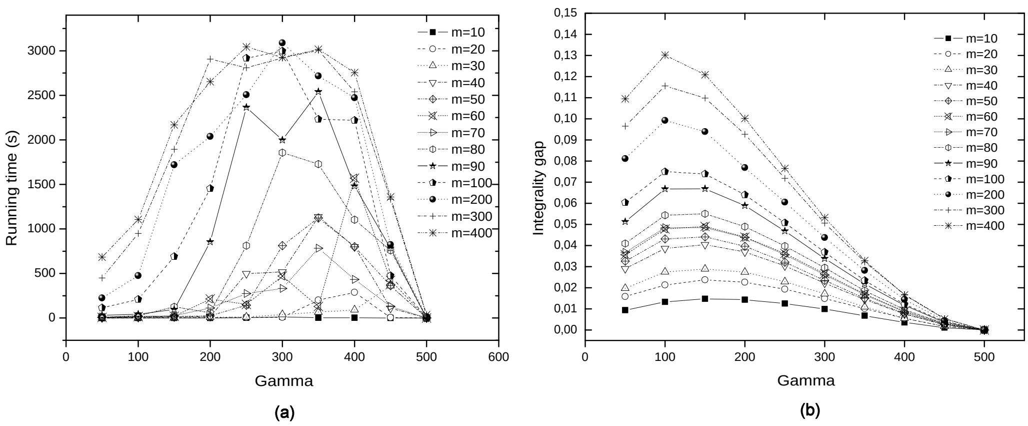

Fig. 2. Tests n=500 : **a.** Running time vs Gamma. **b.** Integrality gap vs Gamma.

Figure 3.a shows the limit running time, such that for $n = 1000$ uncertain

demands, all instances containing $m = 10$ sources are solved within one hour, whatever is Γ between 10% and 100%. For $100 \leq m \leq 500$ the solver is not able to reach the optimum within this time for $\Gamma = 50\%$, and for $m \geq 600$ there are memory issues with the solver.

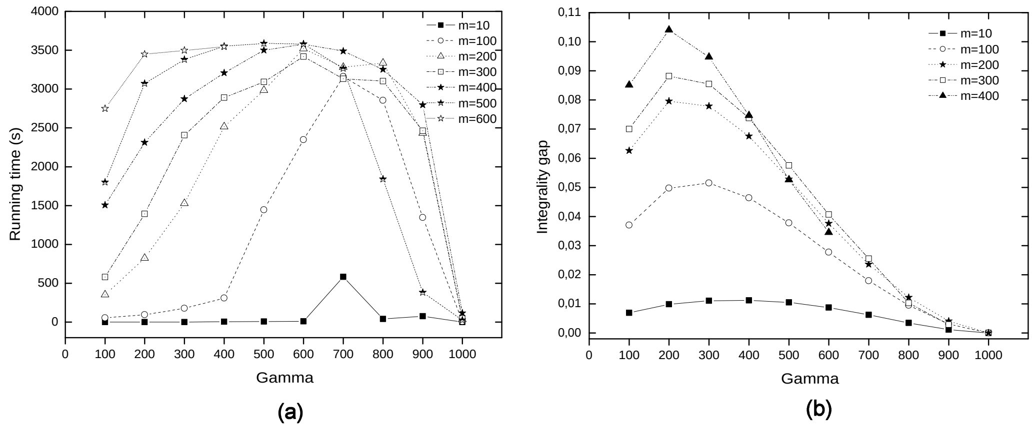

Fig. 3. Tests n=1000 : **a.** Running time vs Gamma. **b.** Integrality gap vs Gamma.

Finally, as is a very tight bound, one wants to know the behavior of the relaxation of the mixed-integer program. Figure 2.b and Figure 3.b show the integrality gap for the instances corresponding to $n = 500$ and $n = 1000$ demands respectively. We remark that this ratio is increasing as the number of sources m grows, reaching its maximum when Γ is around 20% of total deviation. Furthermore, the extra cost generated by the linear relaxation varies between 0 and 13% (comparing with the exact solution). For instance, if the decision maker is interested to know the worst optimal value for 70% of the total deviation from the nominal problem (which represents the most difficult instances), one can solve the linear relaxation in few secondes and have only 3% of extra cost at most. This represents a considerable saving of time.

5 Conclusion

The aim of this paper is to solve the recourse problem of the robust 2-stage location transportation problem. Previously, the 2-stage formulation has already been considered in [1] and [19]. Nevertheless, the limit size of solved instances with Kelley's algorithm, was performed for about 30 uncertain parameters. Here, we present the first (to our knowledge) extensive computation analysis on a particular recourse problem (namely, the location transportation problem), which is the most difficult part of the 2-stage robust optimization. Indeed, the tight bound we propose allows us to solve big size instances. Furthermore, this work seems to be promising to solve big size problems of the general 2-stage robust location transportation problem. This will be the aim of future research.

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