Cooperative games: homework 1

deadline: Monday February 27th, 3pm

Your answers must be explained and justified. The homework is due **before the start** of the lecture. It is your responsability to submit your work on time. Passed the deadline, I will accept the homework up to a week late and your grade will be reduced by 50% (the same consequence applies whether you are 15min or 7days late). You can either hand in your solution before the start of the lecture, or submit it by email. Finally, submit your *own personal* work, the homeworks are about cooperation, but not an exercise of cooperation. If you need clarification, you can contact me by email or you can stop by my office (Science Park 904, C3.125).

Excercise 1. *Convex games*

Prove that a game (N,v) is convex iff $\forall S, T \subseteq N$, $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$.

Excercise 2. *Market games*

a) *Prove that every market game is superadditive.*

b) *Give an example of a market game that is not convex.*

Excercise 3. *Example.*

Consider the following three-player game, presented in the first class: $N = \{1, 2, 3\}$, $v(\{1\}) = 0$, $v(\{2\}) = 0$, $v(\{3\}) = 0$, $v(\{1, 2\}) = 90$, $v(\{1, 3\}) = 80$, $v(\{2, 3\}) = 70$, $v(\{1, 2, 3\}) = 105$.
Determine whether the core of this game is empty using the barycentric method. In addition, provide an argument using the Bondareva Shapley theorem.

Excercise 4. *2-person superadditive game.*

a) *Provide a payoff distribution in the core of a 2-person superadditive game.*

b) *The answer to the previous question guarantees that the core of a 2-person superadditive game is non-empty, hence, a 2-person superadditive game is balanced. Prove a superadditive 2-player game is balanced from the definition of a balanced game.*