Cooperative games: homework 4

deadline: Monday May 14th, 15pm

Your answers must be explained and justified. The homework is due **before the start** of the lecture. It is your responsability to submit your work on time. Passed the deadline, I will accept the homework up to a week late and your grade will be reduced by 50% (the same consequence applies whether you are 15min or 7days late). You can either hand in your solution before the start of the lecture, or submit it by email. Finally, submit your own personal work, the homework is about cooperation, but not an exercise of cooperation. If you need clarification, you can contact me by email or you can stop by my office (Science Park 904, C3.125).

Excercise 1. *Let N=6. The preference of the game are given by the following table:*

agents	*1*	*2*	*3*	*4*	5	6
$\uparrow$	$\{12\}$	$\{23\}$	$\{34\}$	$\{14\}$	$\{156\}$	$\{256\}$
$\uparrow$	$\{156\}$	$\{256\}$	$\{23\}$	$\{34\}$	$\{256\}$	$\{156\}$
$\uparrow$	$\{14\}$	$\{12\}$	$\{3\}$	$\{4\}$	$\{5\}$	$\{6\}$
$\uparrow$	$\{1\}$	$\{2\}$				

For example, agent 1 preference is $\{1,2\} \succeq_1 \{156\} \succeq_1 \{1,4\} \succeq_1 \{1\} \succeq \mathcal{C}$ *for all* $\mathcal{C} \subseteq N$ *and* $\mathcal{C} \notin \{\{1,2\},\{1,5,6\},\{1,4\},\{1\}\}$.

1. *Show that the core of this game is empty.*
2. *Provide some examples of individually rational coalition structures (and give an argument why they are stable).*

Excercise 2. *Let us consider a set M of n men and a set W of n women. Suppose that each member i of $N = M \cup W$ has a strict preference relation i over the members of the opposite sex. A matching f is a function $f : N \to N$ that pairs men and women into couples.*

We call a matching f unstable if for some $m \in M$ and $w \in W$ we have $w \succ_m f(m)$ and $m \succ_w f(w)$. The matching is called stable if it is not unstable. We can represent this game with an NTU game. Is it always possible to find a stable matching? Remarkably, yes, using what is now called the deferred acceptance algorithm. We describe the male-proposal version of the algorithm.

Deferred Acceptance Algorithm, male-proposals

First, each man proposes to his top-ranked choice. Next, each woman who has received at least two proposals keeps (tentatively) her top-ranked proposal and rejects the rest. Then, each man who has been rejected proposes to his top-ranked choice among the women who have not rejected him. Again each woman who has at least two proposals (including ones from previous rounds) keeps her top-ranked proposal and rejects the rest. The process repeats until no man has a woman to propose to or each woman has at most one proposal. At this point the algorithm terminates and each man is assigned to a woman who has not rejected his proposal. Notice that no man is assigned to more than one woman. Since each woman is allowed to keep only one proposal at any stage, no woman is assigned to more than one man. Therefore the algorithm terminates in a matching.

Prove that the algorithm ends on a core allocation.