Most Critical Elements for Multiobjective Optimization Problems

M2 internship

For security or reliability problems, it is important to assess the capacity of a system to resist to an attack or a failure of a number of its entities (elements). This amounts to identifying critical entities - that must be the most protected to avoid strong degradations of the system - which can be determined with respect to a performance measure or a cost associated to the system. Hence, one way of identifying critical entities is to determine a subset of elements whose damage causes the largest degradations regarding several performance measures. A dual approach is to find a minimum cardinality subset of elements for which, when they become inaccessible, the optimal cost in the residual system is larger than a given threshold. In the literature these problems are referred to respectively as the "k most vital entities" problem and "min entities blocker" problem.

In many real world applications, e.g. those involving the use of communication or transportation networks, a system can be modeled as a weighted connected graph G where entities are edges or nodes and costs are weights associated to edges. Let P be a combinatorial optimization problem consisting of maximizing or minimizing a certain objective function defined on G. For instance, P could be the minimum spanning tree problem or the shortest path problem between two nodes. The k most vital edges/nodes P problem is to find a subset of k edges/nodes whose removal from G causes the largest perturbation on P (maximize the value of a shortest path or a minimum spanning tree). And the min edge/node blocker P problem consists of determining a subset of edges/nodes of minimum cardinality whose deletion from G is such that the optimal value of P is less/larger than or equal to a given threshold. Due to the immediate practical relevance, the k most vital edges/nodes and min edge/node blocker problems have been studied, in the single objective case, for several problems including Shortest path [1, 6], Minimum spanning tree [2, 3, 5].

Many real-world problems require taking into account several conflicting criteria. For example, for investment selection problems, it is natural to consider simultaneously a profitability criterion and a risk criterion. Also in the definition of distribution or routing policies, criteria such as cost and time could be conflicting. In this context, solutions of interest are the efficient (or Pareto-optimal) solutions, a solution being efficient if there is no other solution that dominates it on all objectives. In general, it is difficult to identify the set of efficient solutions for multiobjective combinatorial optimization problems, even for instances of medium size [4]. This is due to the following two main reasons. First, the number of efficient solutions can be extremely large, even exponential in the size of the input. Second, the associated decision problem is usually weakly NP-hard, even if the underlying single objective problem can be solved in polynomial time.

During this internship the goal is to study the problem of determining the most critical elements (edges, nodes, items) for multiobjective optimization problems. Since these problems are difficult, we will focus on the extension of the easiest combinatorial optimization problems: Shortest path, Minimum Spanning tree and a restricted version of the Knapsack problem. The objective is to develop relevant concepts, establish some theoretical results on the complexity of these problems, but also propose some exact or approximation algorithms.

This subject could be a starting point for a PhD thesis.

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