Claude Berge and the “Oulipo”

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Claude Berge (June 5, 1926 – June 30, 2002) is well-known in the Operational Research community for his path-breaking contributions to graph theory. He received several awards for his scientific achievements, including the EURO Gold Medal (1989) and the Euler Prize (1995, jointly with R.L. Graham).

The name of Claude Berge is likely to remain in history for several other reasons. For instance he was an expert in the Asmat art from New-Guinea. He was himself a famous sculptor.

We would like here to mention yet another facet of Claude Berge. He was a founding member of Oulipo (for “OUvroir de LIttérature POtentielle”, i.e., “workroom of potential literature”). This group was created on 24 November 1960 under the impulsion of Raymond Queneau and François Le Lionnais (the other founding members were Albert-Marie Schmidt, Jean Queval, Jean Lescure, Jacques Duchateau and Jacques Bens; they were joined later by several well-known members including Italo Calvino, Harry Mattews, Georges Perec and Jacques Roubaud). Oulipo was a group of writers and mathematicians aiming at exploring in a systematic way formal constraints on the production of literary texts. Although poets had explored for years the formal constraints of versification, the originality of the group was to develop new or rarely studied constraints and to explore them systematically. With this objective in mind, the collaboration between writers and mathematicians proved extremely useful and Oulipo is considered to be among the most influential literary groups in France in the twentieth century.

The group is still active today. According to the rules of the group, Claude Berge will be “excusé pour cause de décès” at each meeting of Oulipo.

It is impossible here to give a complete overview of the incredible variety of constraints that the members of the group have explored and are still exploring. It is nevertheless quite certain that Claude Berge has participated in the development and study of many of them besides producing his own texts.

Georges Perec was perhaps one of the most talented and prolific member of Oulipo. In his book *La Vie, mode d’emploi* [12], Perec tells a story happening in a building of ten floors with ten rooms in each floor, based on the rather recent (at that time) discovery of orthogonal Latin squares of size 10: it is a 10 by 10 array in each entry of which we have one of the numbers 1 to 10 and one letter a, b, c, ..., h, i, j; these are arranged in such a way that, in each row and in each column, all figures are different and all letters are different, and furthermore all contents of the 100 entries are different; the stories of the 100 rooms of this 10 × 10 building are told consecutively by following an order which is the one of a knight making a Hamiltonian tour in a 10 × 10 chessboard! C. Berge helped G. Perec in devising this structure (see [9] for more details about the design of *La Vie, mode d’emploi*, novel that D. E. Knuth places in [10] as one of the most important of the century). Similarly, *La Disparition* [11], still by G. Perec, is a *lipogram*, i.e. a text written without a given letter, namely without any “e”, though this is the most frequent letter in French (it has been translated in English as *A Void*, still without any “e”, although “e” is also the most frequent letter in English).

With this respect, the short story *Qui a tué le duc de Densmore ?* [5] (*Who killed the Duke of Densmore?*) is surely the most famous written contribution of C. Berge to the Oulipo. In this detective short story, written in 1994, C. Berge applies a graph theoretic theorem due to G. Hajós in order to identify the killer. Prior to this short story, C. Berge published in 1983 *La Reine aztèque* [4] (*The Aztec Queen*), in the style of the geometric illusions by Sam Loyd or DeLand (see Figure 1); these geometric illusions are based on pictures with figures such that, when cut and reorganized in an appropriate way, the number of figures is not the same before and after the reorganization, though the material forming the figures remains the same.

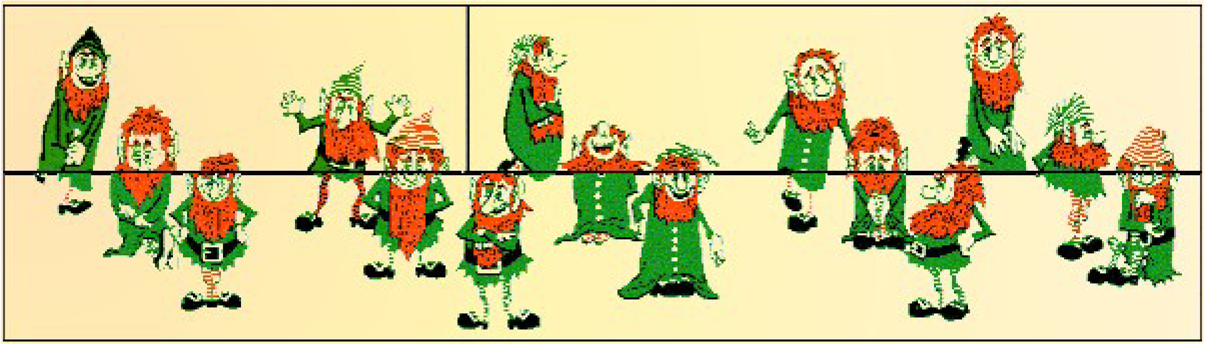

Figure 1: 15 dwarfs; if the picture is cut according to the lines and if the two upper parts are switched, then the number of dwarfs is no longer 15!

Similarly, in *La Reine aztèque*, C. Berge designs a sonnet (hence with 14 verses of 12 feet) and reorganizes these 14 verses into a poem of 15 verses still of 12 feet! How can this be possible? First, consider the 14-verse sonnet below on left. Then, split the verses at the points enhanced by vertical lines, and switch the right parts of the first lines until the empty line included with the right parts of the lines located after the empty line, while keeping the empty line between these two parts. This gives another poem, with 15 verses of 12 feet!

Tandis qu'en frissonnant elle | égrenait des vers
L'| Aztèque imperturbable à la touque imprécise
Serrait sa souveraine une blonde | aux yeux verts
D'un lien| libidineux que la froidure attise
Dans l'Ouest enfoui dit-elle à son amant | pervers,
C'est là que l'art | jaillit, que l'Inca prosaïse,
Et que la pyramide abolit l'univers | !
Nul n'entend le muet qui | tout doucement s'enlise...

Comme le perspicace inouï| conjecturait,
Jeune | ami présomptueux plus fou qu'il ne paraît,
N'offre pas de pactole à ton gardien | farouche
Si le verbe | à la fois oppresseur et charmant
D'un tel triomphateur ne trouble le diamant | ...
Même Xipe Totec | fuit et détruit sa souche

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L' | ami présomptueux plus fou qu'il ne paraît,
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C'est là que l'art | fuit et détruit sa souche
Et que la pyramide abolit l'univers !|
Nul n'entend le muet qui | égrenait des vers...
| Aztèque imperturbable à la touque imprécise
Comme le perspicace inouï | aux yeux verts,
Jeune | libidineux que la froidure attise
N'offre pas de pactole à ton gardien | pervers,
Si le verbe | jaillit, que l'Inca prosaïse,
D'un tel triomphateur ne trouble le diamant | !
Même Xipe Totec | tout doucement s'enlise...

We leave to the reader the joy of deciphering this stylistic tour de force (several "tricks" are involved).

The contributions of C. Berge to Oulipo are diverse (see for instance [2]; see also [7] for more detailed materials about C. Berge's contributions to the Oulipo). In number 4 of the *Bibliothèque oulipienne* [3], dedicated to R. Queneau, he considers Queneau's booklet *Cent mille milliards de poèmes* [13] from a graph theoretic point of view. Remember that this booklet contains ten sonnets; by picking one of the ten first verses of these ten sonnets, then one of the ten second verses of these ten sonnets, and so on until picking a fourteenth verse among the ten last verses of the ten sonnets, we may build a new poem; there are 10^{14} such possibilities (hence the title of the booklet). We may associate a vertex to each one of the 140 verses of the ten sonnets and link two vertices x and y by the arc (an arc is a directed edge) (x, y) if the rank of the verse y (in y's poem) is the rank of the verse x (in x's poem) plus 1. Thus, we get a multi-layered directed graph without circuit (a circuit is a directed cycle), and with 14 layers and 10 vertices per layer. Like C. Berge, we may then consider that the construction proposed by R. Queneau is a directed path in this graph, starting from any vertex of the first layer and ending at any vertex of the last layer. Instead of a graph without circuit, C. Berge suggests a graph without any cocircuit (remember that a cocircuit is a nonempty set A of arcs such that, if we call X the set of the heads of the arcs of A, then the tails of all the arcs of A are outside X; in other words, if Y denotes any proper subset of vertices of a graph G without any cocircuit, there will always be at least one arc going inside Y and one arc going outside Y in G; see for instance [1] for more details), the verses still being associated with the vertices. Moreover, he requires that, if the obtained poem contains all the verses exaclty once, then it is impossible to repeat the first verse while respecting the construction process (we must follow the arcs), but this must become possible if any vertex is removed. In graph theoretic terms, it means that the considered graph does not contain a Hamiltonian circuit, but the removal of any vertex leads to a subgraph which does. If such a graph exists, what is the smallest one (with respect to the number of vertices, but the question can be set also with respect to the number of arcs) ? C. Berge gave the answer in the number 4 of the *Bibliothèque oulipienne*: the graph that he called Q_6 (see Figure 2) suits (Q stands to pay tribute to his friend R. Queneau, 6 because this graph contains 6 vertices). Notice that, in spite of an ambiguous formulation by C. Berge, Q_6 is not the only graph with the property stated above (if we replace in Q_6 the two circuits defined on three vertices on left and on right by any circuit defined on an odd number of vertices while keeping the pairs of arcs between homologous vertices of these two circuits, we get a graph which does not contain a Hamiltonian circuit, but such that the removal of any vertex leads to a subgraph which does), but Q_6 is the smallest one, both with respect to the number of vertices and to the number of arcs.

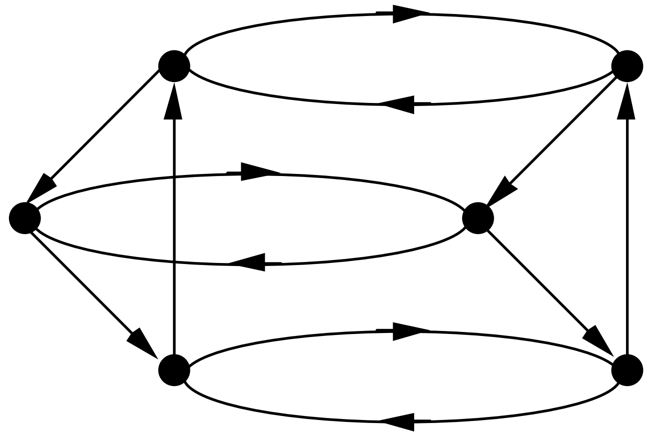

Figure 2: The graph Q_6

Claude Berge also practiced the so-called *Eulerian poem*, for which a graph is given, where a verse is assigned to every arc; by following any directed path, one gets a reasonable text. The example given by Figure 3 (drawn from [8]) is a graph of which the vertices are labelled G, R, A, P, H, E, S; every verse is an alexandrine (12 feet) and starts with the letter given to its origin vertex; furthermore most of the words (in French) relate also to graph theoretical concepts.

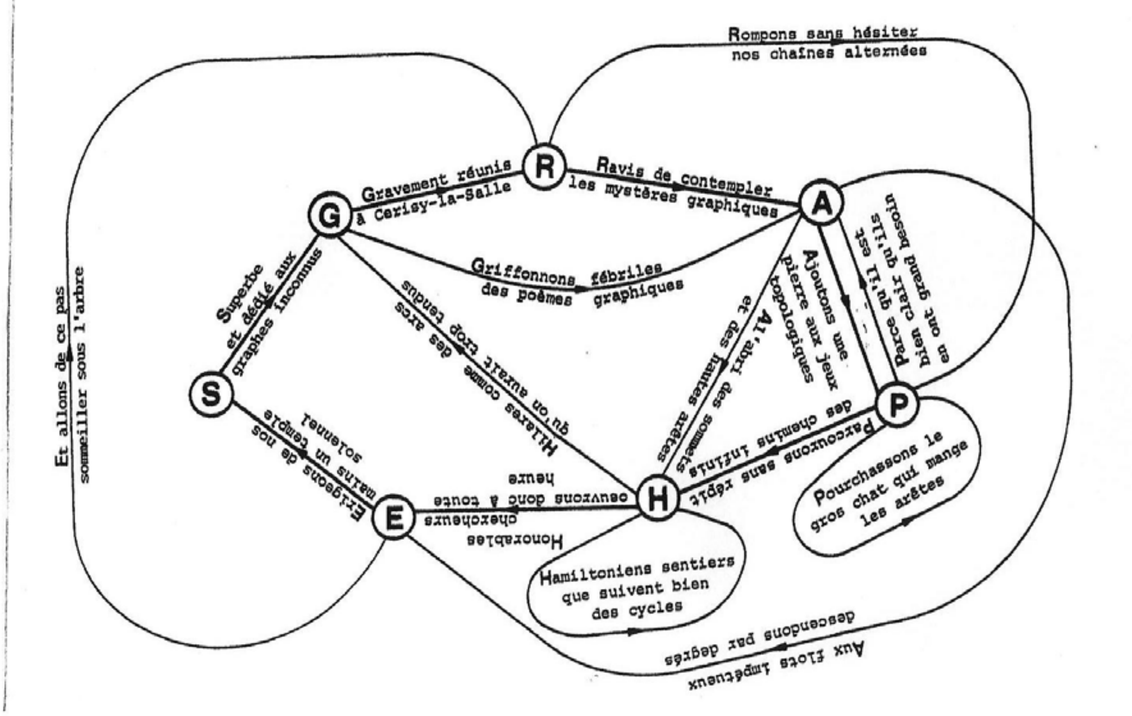

Figure 3: A Eulerian poem dedicated to C. Berge

Finally we would also like to mention an amusing application of hypergraphs to the creation of texts. Many texts involve three characters that are linked by some ternary relation. A very common relation is “*x* takes *y* for *z*” (variants “*x* prepares a plot with *y* against *z*”, “*x* is in love with *y* married to *z*”, etc.). Each of such stories could then be described by a table summarizing the ternary relation. The least interesting (but most normal) situation corresponds to the following table:

	x	*y*	*z*
x	*x*	*y*	*z*
y	*x*	*y*	*z*
z	*x*	*y*	*z*

in which everyone is right about the identity of the others. Obviously any variant of this ternary relation may give rise to interesting situations. A common one is the following:

	x	*y*	*z*
x	*x*	*z*	*y*
y	*z*	*y*	*x*
z	*y*	*x*	*z*

in which a general quiproquo happens (everyone is right about is own identity but is systematically wrong about the identity of the others. Yet another interesting one is:

	x	*y*	*z*
x	*Napoléon*	*z*	*y*
y	*z*	*Napoléon*	*x*
z	*y*	*x*	*Napoléon*

which might characterize a common psychiatric hospital.

The link between mathematics and literature has always been strong in the life of C. Berge. Quite significantly, the last conference given by C. Berge (it was in Lausanne, in 2001; see [6]) dealt with Mathematics and Literature; such a choice was not just a curious coincidence...

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