A common framework for describing most aggregation procedures in decision aiding.

Thierry Marchant*
Ghent University
H. Dunantlaan 1
B-9000 Ghent

Denis Bouyssou
ESSEC, B.P. 105
F-95021 Cergy Pontoise Cedex

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Abstract

Many axiomatic results concerning aggregation procedures in decision aiding have been obtained in the framework of conjoint measurement or social choice theory. We show that these frameworks, although they helped us to better understand some aggregation procedures, are not totally appropriate for decision aiding. We propose a new framework, very general, in which most aggregation procedures can fit and more appropriate for the axiomatization of aggregation procedures in decision aiding context. We present some axiomatic results obtained in this framework and showing its interest.

1 Introduction

When a decision-maker uses an aggregation procedure (MAUT, ELECTRE, PROMETHEE, AHP, ...), most of the time, the following elements are available to him:

- a set of alternatives,
- a set of viewpoints or criteria,
- some information about the alternatives with respect to each criterion.
- some information about the role or the relative importance of the criteria
- some a priori or initial preferences.

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These elements will be called the primitives. Of course, the set of alternatives may vary a little bit during the decision process. Some alternatives can be added or deleted. The same holds for the criteria set. Finally, the information about the alternatives can also be subject to some modifications.

1.1 Relevance of conjoint measurement

Conjoint measurement (see e.g. [Luce et al. 90] for general conjoint measurement or [Wakker 89] for additive conjoint measurement) is a part of measurement theory and, as such, is intended to study how and when (under what conditions) some binary relations can be described or represented by a numerical model. It has numerous succesful applications in many fields (e.g. physics, psychology, ...).

Conjoint measurement has often been chosen as a framework in which multicriteria decision aiding methods can be described and, eventually, axiomatised (see e.g. [Keeney and Raiffa 76]). Even though this approach has led to many interesting results and has helped us to better understand some aggregation procedures, we think that it is not totally appropriate to the problem of decision aiding.

In conjoint measurement, the main primitive is a preference relation over the alternatives with a lot of "good" properties. Most of the time, it is assumed to be a weak order.

In decision aiding, the situation is very different : the decision-maker can have some a priori or initial preferences. These are preferences that the decision-maker can express with some confidence before using an aggregation procedure. The a priori preferences can take many different forms: a (very) partial preference relation, a set of definitely "bad" alternatives, a set of promising alternatives, ... But the a priori or initial preferences are definitely not a complete and transitive binary relation[1] and it is not possible to derive from them a complete and transitive binary relation. Otherwise no aggregation procedure would be needed. Let us now distinguish two different ways of considering the axioms in conjoint measurement : the descriptive and normative approaches.

1.1.1 Descriptive approach

From a descriptive viewpoint, a model (additive or not) can be used to represent the preferences of a decision-maker only if the axioms characterising that model are satisfied by the preferences of the decision-maker. The validity of an axiom can be experimentally studied. Most models that have been characterised in the framework of conjoint measurement (including the classical additive utility) require that the preference relation be transitive and complete. But the preferences of a decision-maker resorting to a decision aiding method cannot be both complete and transitive. Otherwise he would not resort to decision aiding.

[1] A binary relation R on X is complete iff, for any x and y in X, xRy or yRx. It is transitive iff, whenever xRy and yRz, we also have xRz.

Hence, none of the models implying transitivity and completeness can be used to describe his preferences.

Let us now turn to the models that are not transitive[2] and complete. Some of them have been characterised in the framework of conjoint measurement (see e.g. [Bouyssou et al. 97, Fishburn and Nakamura 91]). These models have an advantage over the previous ones : they can be used when the preferences of the decision-maker are not transitive and complete. Nevertheless, from a decision aiding viewpoint, these models are not very useful : they allow us to represent the incomplete and intransitive preferences of a decision-maker but they do not help the decision-maker to make up his mind. They do not tell him anything about the pairs of alternatives that are not in his a priori preference relation.

In the two previous paragraphs, we discussed only transitivity and completeness but we could also discuss conditions such as the independence or cancellation conditions. The preference relation of the decision-maker is usually so incomplete that none of these conditions can be satisfied.

In conclusion, if we consider the axioms characterising an aggregation procedure in the framework of conjoint measurement from a descriptive viewpoint, we are not really helped. Let us now turn to the other approach.

1.1.2 Normative approach

Instead of checking if the axioms characterising a particular model are satisfied (descriptive approach), why not impose them ? Why not consider them as defining rationality or as a set of conditions ensuring the coherence of the decision process ? This is the normative approach.

Then, we can start from the single criterion or single attribute preferences and try to construct the global preference relation in such a way that it satisfies the axioms of a particular model, for example additive conjoint measurement. The problem is now that the theory of conjoint measurement does not tell us how to do this. Most proofs of representation theorems in measurement theory share a common characteristic : they are constructive. They tell us how to construct the utility functions, starting from the global preference relation. From a descriptive perspective, this is most useful but, since we are now in a normative approach, it doesn't work any more. We cannot mimic the proof because the starting point (the global preference relation) is missing.

Since the theory of conjoint measurement does not tell us how to construct the global preference relation, we can imagine different methods to assess the single-attribute utility functions. The global preference is then derived by means of a model (for example, the additive utility model). Such methods have been proposed by many authors (see e.g. [Keeney and Raiffa 76, Section 3.4.6, 3.4.7 and 3.7]). The problem is that, inside the theory of conjoint measurement, all methods designed to assess utility functions are equivalent. This should not be a surprise : measurement theory is descriptive in nature and does not contain any

[2]When we say that a relation is not transitive or when we speak of a non-transitive model, we mean that there can be x, y, z such that xRy, yRz and NOT xRz. We do not mean that there exists x, y, z such that xRy, yRz and zRx.

element that could help us distinguish among assessment methods. To make this point clear, let us use an example.

Suppose a decision-maker has to choose among three cars : x, y and z. The decision-maker considers two criteria as relevant to his problem : power and price (see table 1).

	price	power
x	100	4
y	100	5
z	80	4

Table 1: Performances of three cars on two criteria.

A priori, the decision-maker can tell us that he prefers y to x and z to x. The only pair about which he is undecided is (z, y). Using different methods, we could arrive at the conclusion that z should be preferred to y or the opposite. The global preference relation can thus be nay of the linear orders yzx or zyx. The reader will easily check that any of these two linear orders satisfy all the axioms of additive conjoint measurement. Therefore, inside the framework of conjoint measurement, it is not possible to make a distinction between the methods that would produce these quite different linear orders. Note that this has nothing to do with the fact that our alternatives set is discrete. Another example could be built with infinite sets.

In the next few paragraphs, we use another example to illustrate the fact that there are actually different assessing methods that could possibly lead to different conclusions, as in the above example.

Suppose that a decision-maker has to choose among a finite set of alternatives and he considers the axioms of additive conjoint measurement as particularly compelling. Therefore, what he needs to do is to assess the utility functions. Once this is done, the problem is completely solved : the best alternative is the alternative such that the sum of the utilities is maximum. In order to assess the utility functions, we are now going to use two different assessment methods: the midvalue splitting technique [Keeney and Raiffa 76, Section 3.4.7] and the PROMETHEE II method (see [Brans and Vincke 85] or section 3.1.1, p. 14).

The midvalue splitting technique. This technique is very classical in MAUT. For each attribute, the utilities 0 and 1 are given to the worst and best performances on that attribute. The analyst presents a pair of hypothetical alternatives to the decision-maker. On at least one attribute, one alternative has the worst possible performance while the other one has the best possible performance. The decision-maker must find a single value on that dimension such that substituting it for the original values produces two alternatives that are indifferent to the original ones. This allows us to determine the attribute value with a utility 0.5. Then a new pair of alternatives is presented in order to determine other points of that utility function. The same process is used for all attributes.

The Promethee II method. This method is usually presented as an outranking method, in the same family as the ELECTRE method but, in our opinion, it is much closer to MAUT than to outranking methods. We briefly discuss how this method works. For a given attribute, the decision-maker must assess his indifference threshold, i.e. the largest difference on that attribute such that he is still indifferent between two almost identical (and hypothetical) alternatives, differing only on one criterion by that value. The decision-maker must also assess his preference threshold, i.e. the smallest difference on that attribute such that he definitely prefers one of two almost identical (and hypothetical) alternatives, differing only on one criterion by that value. Then, between these two alternatives, the preference intensity is said to be equal to 1. Between the indifference and preference threshold, it is assumed that the preference intensity varies from 0 to 1 according to some non decreasing function. The same process is repeated for each attribute. Then, using the so called net flow method, a real valued function on the set of alternatives is built for each attribute :the so-called single-criterion net flow. These single-criterion net flows can then be combined using a weighted additive model, just like utility functions. And it is precisely what the PROMETHEE method does.

The two above-described method–weighted addition of utilities obtained through the midvalue splitting technique and weighted addition of single-criterion net flows–satisfy all the axioms of additive conjoint measurement. To be more precise, we should say that the global preference relations obtained by means of the two methods satisfy the axioms of additive conjoint measurement. This is more precise because conjoint measurement is concerned with relations, not with methods. In fact, all methods based on a weighted additive model will lead to a global preference relation satisfying the axioms of additive conjoint measurement. That is why it is not possible to distingish among different methods in the framework of conjoint measurement.

Nevertheless, it is clear that the two above-described methods are very different : the questioning and construction process are so different that, in most decision problems, it is very likely that the global preference relations will be different. And, actually, the global preference relation can differ greatly, as shown in the following example.

Suppose we have a problem with two dimensions (criteria, attributes). The set of levels on the first dimension is $A = \{a, b, c\}$; and on the second dimension, $Y = \{x, y, z\}$. Suppose also that we know the preferences of the decision-maker on each dimension.

$$\succeq_A : c \succ_A b \succ_A a$$

and

$$\succeq_Y : z \succ_Y y \succ_Y x.$$

The set of available or feasible alternatives is

$$X = \{ax, ay, az, bx, by, cx\}.$$

If the complete cartesian product $(A \times Y)$ would be available, then cz would clearly dominate all other alternatives and there would be no decision problem at all. But this almost never happens in real life. Suppose also that the a priori preference relation of the decision-maker, call it $\rhd$, is given by

$$\rhd = \{(az, ay), (az, ax), (ay, ax), (by, bx), (by, ay), (cx, ax), (cx, bx), (bx, ax), (by, ax)\}.$$

The situation is depicted in Fig. 1.

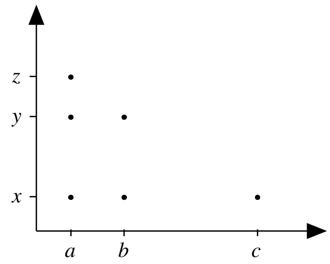

Figure 1: The six available alternatives in the A, Y space.

Define

$$\succeq_1 = \rhd \cup \{(cx, by), (cx, az), (by, az), (cx, ay), (bx, ay), (az, bx)\}.$$

$\succeq_1$ is an additive conjoint structure, with maximal element cx. To check this, use the following values for the two utility fucntions f and g : $f(a) = 0, f(b) = 3, f(c) = 6, g(x) = 0, g(y) = 2, g(z) = 4$.

Define

$$\succeq_2 = \rhd \cup \{(by, cx), (az, cx), (az, by), (ay, cx), (ay, bx), (bx, az)\}.$$

$\succeq_2$ is an additive conjoint structure, with maximal element az.

Define

$$\succeq_3 = \rhd \cup \{(by, cx), (by, az), (az, cx), (az, bx), (cx, ay), (ay, bx)\}.$$

$\succeq_3$ is an additive conjoint structure, with maximal element by. To check this, use the following values for the two utility fucntions f and g : $f(a) = 0, f(b) = 3, f(c) = 5, g(x) = 0, g(y) = 4, g(z) = 6$.

There are therefore at least three different preference relations compatible with $\rhd$ (containing $\rhd$), all satisfying the axioms of additive conjoint measurement and with three different maximal elements.

Let us now consider a more constrained version of this example : let$\rhd' = \rhd \cup \{(bx, ay), (by, az), (az, bx), (cx, ay)\}$. $\succeq_1$ is still compatible with $\rhd'$ (it contains $\rhd'$).

Define

$$\succeq_4 = \rhd' \cup \{(by, cx), (az, cx)\}.$$

$\succeq_4$ is an additive conjoint structure, with maximal element by. To check this, use the following values for the two utility fucntions f and g: $f(a) = 0, f(b) = 4, f(c) = 5, g(x) = 0, g(y) = 3, g(z) = 6$.

In the case of $\rhd'$, we have at least two different preference relations compatible with $\rhd'$ (containing $\rhd'$), all satisfying the axioms of additive conjoint measurement and with two different maximal elements.

Note that, in the case of $\rhd'$, the a priori relation is almost complete. Only two couples are missing. But if we take the axioms of additive conjoint measurement as normative, then, actually, only one pair is missing, because the second one automatically derives, by transitivity. Yet, on this highly constrained case, additive conjoint measurement doesn't tell us which alternative wins. It can be cx (in $\succeq_1$) or by (in $\succeq_4$).

Note that if the decision-maker can a priori have a preference relation richer than $\rhd'$, then $\succeq$ is uniquely and completely determined. Therefore, no decision aiding is required.

This shows that conjoint measurement is not an appropriate framework for describing decision aiding methods. It can not tell us anything about different methods that would lead to different outcomes and yet would be consistent with conjoint measurement.

Remark: our examples are based on additive conjoint measurement but other examples could be built for other conjoint measurement.

Let us answer to a possible objection: some people claim that PROMETHEE is not a MAUT method because it doesn't satisfy the condition of independance of irrelevant alternatives. We do not agree nor disagree. In fact, inside the framework of measurement theory, this claim doesn't really make sense. There are many different forms of independence of irrelevant alternatives (IIA), originating in social choice theory, but they all share a common characteristic: they consider what happens if the set of alternatives changes or if we focus our attention to some subset of the alternatives. But, in all measurement theory studies that we are aware of, the set of alternatives doesn't change. The set of alternatives is a complete cartesian product and the theory doesn't contain any element that would allow to describe a change. If we could develop a measurement theory working with a "universal" set of alternatives and all possible subsets, with a possibly different preference relation for each subset and conditions linking these preferences, then an IIA condition could be formulated in the framework of measurement theory. But, nowadays, this seems rather chimerical.

Before concluding this section, we want to make clear that we do not support any aggregation method against another one. We are just looking for a framework allowing us to meaningfully describe various aggregation procedures. We do not criticize measurement theory : it is a fascinating theory that lead to deep and enlightening results. Our aim is just to show that measurement theory leaves many questions unanswered in the field of decision aiding because it is not intended to describe decision aiding, even though it has allowed generations of researchers to deepen their understanding of some aggregation procedures.

1.2 Relevance of social choice theory

Another problem arises when axiomatic results of social choice theory are used in decision aiding (for a good introduction to social choice theory, see [Kelly 87]). In social choice theory, the primitives are much like those of decision aiding (if we replace criteria by voters). But there are some important differences.

- It is usually assumed that the assessment structures are very simple structures: total orders (e.g. [Pérez and Barba-Romero 95]) or weak orders. Even when more general structures are considered, it is assumed that the information provided by each voter is of the same kind (e.g. [Marchant 96]). But in decision aiding, most of the time, the information along each criterion is very different. For one criterion, we have a weak order, for another one, we have a real valued function on the set of alternatives and for a third one, we have linguistic assessments. The information on some criteria may be uncertain while it is certain for other criteria.

- Most of the time, all voters are treated equally. This is called anonymity. But in decision aiding, some criteria are more important or more relevant than some others. The information for some criteria is more reliable than the information for some other criteria. Therefore, we do not need such a condition.

- In some axiomatic results (e.g. [Marchant 96]), the number of voters is allowed to vary without limits (and varies dramatically in the proofs). In decison aiding, such variations in the number of criteria are not possible. If a decision maker starts a decision process with 10 criteria, he might, at some further stage, use 15 or 20 criteria but definitely not several thousands.

- There are no initial preferences. This is probably the most fundamental difference. We can think of voting theorems that would avoid the three above mentioned problems but a voting theory with a priori preferences is hardly thinkable. Yet, the existence of a priori preferences is essential in decision aiding. When we question a decision-maker in order to determine the parameters of a model (utility functions, subjective probabilities, indifference thresholds, weights, concordance thresholds, ...), his answers are mainly based upon his knowledge of the problem, his values, his experience, ... , all those things that we group under the expression *a priori preferences*. Of course, his answers are also influenced by the questioning process, by the decision-aiding process but the a priori preferences are essential. Otherwise all decision-makers using the same method would have the same global preferences. Therefore, trying to describe a decision aiding method without speaking of the a priori preferences (as it is the case in voting theory) seems not adequate.

1.3 Decision aiding in presence of risk or uncertainty

In our discussion, we focused on conjoint measurement and decision in presence of several attributes but no uncertainty nor risk. But our conclusions apply to other fields. For example, the axioms of (subjective) expected probability are not totally adequate for describing a decision aiding method aimed at decision problems in presence of risk (uncertainty). Here also the axioms are descriptive and tell us when it is possible to represent a given preference relation by utilities and probabilities. They do not tell us much about procedures that would construct a preference relation on the basis of the consequences, the states of the world and the a priori preference relation of the decision-maker.

1.4 Outline

In order to choose and meaningfully use aggregation procedures, we need to better understand the intrinsic properties of these procedures ([Bouyssou et al. 93]). Therefore, decision aiding needs its own framework, in which relevant axiomatic results can be derived. A framework aiming at the description of outranking methods has been proposed by Pirlot ([Pirlot 97]). It suffers at least two drawbacks: the information along each criterion is of the same kind and it concerns only outranking procedures.

In section 2, we present a generalization of Pirlot's framework that avoids these two drawbacks. Of course, a framework that allows to describe everything but does not allow to derive any axiomatic result is of no use. In order to prove that our framework is useful, we give, in section 3, some examples of results that we were able derive in this framework. We hope that more results are to come. We conclude in section 4 and section 5 is devoted to the proofs.

2 A general framework

Let us begin by defining the primitives of our framework. In order to be very general, we will put very few restrictions on the primitives. Of course, to be able to derive some results about very specialised aggregation procedures, we will have to impose other restrictions. These additional restrictions will be mentionned in the corresponding propositions.

2.1 Alternatives, criteria and assessment structures

The set of criteria is denoted by $C = \{1, 2, \ldots, i, \ldots, k\}$ and the set of alternatives by $X = \{x, y, \ldots\}$. For each criterion, some information about the alternatives is available: we call it an *assessment structure*. It can be a ranking of the alternatives, a real valued function on X, a linguistic assessment of the alternatives (e.g. a mapping from X into the set { "very good", "good", "average", "bad" }) and so on. Note that an assessment structure can even contain parameters, utility curves, indifference thresholds, probability distributions, ...

Nevertheless, it is more realistic to consider that such parameters are not available to the decision maker when he turns to aggregation procedures for help. These parameters, if needed for the aggregation, emerge during the use of the procedure and should not be included in the axioms. For each criterion i, the set of the possible assessment structures is denoted by E_i.

Until now, our definition of an assessment structure is extremely large: it can be anything. In order to avoid a vacuous definition, we impose four conditions on the assessment structures.

1. Let e be an element of E_i. Considering only this criterion i, there is a function δ^i that maps e on a reflexive[3] binary preference relation $\delta^i(e)$ on X. The relation $\delta^i(e)$ is interpreted as a preference relation on X, when criterion i and only that one is taken into account. Therefore, $\delta^i(e)$ will be called a *single-criterion preference relation*. If e is a weak order representing the preferences of the decision maker along criterion i, then $\delta^i(e) = e$. If e is a mapping from X to $\Re$, then $\delta^i(e)$ could be defined as follows:

$$x\delta^i(e)y \Leftrightarrow \begin{cases} e(x) \geq e(y) \text{ and } i \text{ is to be maximized} \\ \text{or} \\ e(x) \leq e(y) \text{ and } i \text{ is to be minimized.} \end{cases}$$

   A single-crtiterion preference relation does not need to be complete. For example, if e is a mapping from X to $\Re$, then $\delta^i(e)$ could be defined as follows:

$$x\delta^i(e)y \Leftrightarrow \begin{cases} e(x) \geq e(y) + \epsilon \text{ and } i \text{ is to be maximized} \\ \text{or} \\ e(x) \leq e(y) - \epsilon \text{ and } i \text{ is to be minimized,} \end{cases}$$

   where ϵ is a positive constant. In this case, $\delta^i(e)$ is the asymmetric part of a semiorder. There are cases where the single-criterion preference relation would probably be almost empty. Suppose that an assessment structure maps each alternative on a set of real numbers and a probability distribution over this set. It is very likely that the decision-maker, even when he considers only that criterion, for most pairs, cannot tell if he prefers x to y.

2. Let π be a permutation on X and e an assessment structure. Then $\pi(e)$ represents an assessment structure such that the roles of the alternatives are exchanged according to the permutation π. For example, if e is a binary relation, then $\pi(e)$ is another binary relation defined by

$$xey \Leftrightarrow \pi(x)\pi(e)\pi(y).$$

[3]A binary relation R on X is reflexive iff, for any x in X, xRx.

If e is a mapping from X to some set, then $\pi(e)$ is another mapping from X into the same set, defined by

$$e(x) = \pi(e)(\pi(x)).$$

3. Let Y be a subset of X such that Y contains at least two alternatives. An assessment structure e must be such that $e|_Y$, its restriction to Y, is defined. For example, if e is a binary relation, then $e|_Y$ is the binary relation defined by $e|_Y = \{(x, y) : x \in Y,\ y \in Y \text{ and } (x, y) \in e\}$. If e is a quaternary relation, then $e|_Y$ is the quaternary relation defined by $e|_Y = \{(x, y, z, w) : x \in Y,\ y \in Y,\ z \in Y,\ w \in Y, \text{ and } (x, y, z, w) \in e\}$. When Y contains only two elements, say x and y, we write $e|_{xy}$ instead of $e|_{\{x,y\}}$

4. Let e and f be two assessment structures belonging to E_i such that $\delta^i(e)|_Y = \delta^i(f)|_Y$. Then, there are e' and f' in E_i such that
   - $\delta^i(e') = \delta^i(e)$ and $\delta^i(f') = \delta^i(f)$
   - $e'|_Y = f'|_Y$.

An assessment structure can be anything, provided that it satisfies these four conditions. The first one tells us that the structure actually contains at least some minimal preferential information. The three other conditions have a more practical interest. They allow us to be sure that some manipulations (needed in the axioms) will be possible. We do not know any aggregation procedure using assessment structure that would violate one of these conditions. Therefore, we include them in the definition of an assessment structure.

A profile is defined as a point $p = (p_1, p_2, \ldots, p_k)$ in $E_1 \times E_2 \times \ldots \times E_k$. When all sets E_i contain only mappings from X into some set, the profile is usually called a performance tableau. Given X and $E_1, E_2, \ldots, E_k$, the set of all possible profiles is denoted by $P(X, E_1, \ldots, E_k)$.

2.2 Importance structures

An *importance structure* is a structure that tells us what is the role or the relative importance of the criteria. It can be almost anything. It can even be empty, as in ELECTREIV [Hugonnard and Roy 82]. In some cases, it is a binary relation over C, as in QUALIFLEX [Paelinck 78], ORESTE [Roubens 81] and MELCHIOR [Leclercq 84]. In other cases, it is a vector of weights, as in MAUT or PROMETHEE [Brans and Vincke 85]. It could also be a description of the power indices of each coalition of criteria.

We could impose some conditions on the importance structures so as to be sure that it is one of the above cited structures (weight vector, binary relation, ...). In order to derive some axiomatic results, in the next section, we will have to add some conditions. But we do not want to do it right now, in the definition of the importance structure for the following reason. In most cases, when a decision-maker decides that he is going to use an aggregation procedure (to be chosen), he doesn't know yet if the method that he will use will require weights or binary relations or ... And if he is going to use weights, will they be substitution rates or weights as in PROMETHEE (that we don't know yet how to interpret). If weights (or other strong structures) are to be used, they should emerge during the use of the aggregation procedure, like the indifference thresholds, utility curves, and other parameters.

Therefore, considering the importance structure as a primitive with a strong structure (weight vector, binary relation, ...) is not realistic. Obviously, imposing some restrictions on the importance structures is necessary. Otherwise our definition might be useless. But at the time present, we are not able to do it.

The set of the importance structures used in a given aggregation procedure is denoted by W and its typical element is w.

2.3 Initial preferences

Our last primitive is an *initial preference*, that reflects the knowledge of the decision-maker about his preferences before entering the aggregation process. As told earlier, it can take different forms. As we cannot make an exhaustive list of these possible forms, we are going to assume (as we did for the assessment structures) that all possible initial preferences share some common characteristics. Let I denote the set of all initial preferences on X.

1. Let π be a permutation on X and $\rhd$ an initial preference. Then $\pi(\rhd)$ represents an initial preference such that the roles of the alternatives are exchanged according to the permutation π. For example, if $\rhd$ is a binary relation, then $\pi(\rhd)$ is another binary relation defined by

$$x \rhd y \Leftrightarrow \pi(x)\pi(\rhd)\pi(y).$$

   If $\rhd$ is a mapping from X to some set, then $\pi(\rhd)$ is another mapping from X into the same set, defined by

$$\rhd(x) = \pi(\rhd)(\pi(x)).$$

2. Let Y be a subset of X and Y contains at least two alternatives. An initial preference $\rhd$ must be such that $\rhd|_Y$, its restriction to Y, is defined. For example, if $\rhd$ is a binary relation, then $\rhd|_Y$ is the binary relation defined by $\rhd|_Y = \{(x,y) : x \in Y,\ y \in Y \text{ and } (x,y) \in \rhd\}$. When Y contains only two elements, say x and y, we write $\rhd|_{xy}$ instead of $\rhd|_{\{x,y\}}$

2.4 Aggregation procedure

Let R denote the set of all relexive binary relations. An aggregation procedure is a mapping $\succeq: P(X, E_1, \ldots, E_k) \times W \times I \to R : (p, w, \rhd) \to \succeq(p, w, \rhd)$.

3 Some results

In this section, we are going to show some interesting results that we were able to derive in our framework. There are two groups of results: one is related to aggregation procedures using weights that are specified a priori; the other group is related to quite similar procedures, using weights as well, but not specified a priori. An interesting aspect of these results is that, for some of them, we do not need to put any additional restrictions on the assessment structures. A weaknesses of the first group of results is that we put a lot of additional restrictions on the importance structures. We require that they are weight vectors. These restrictions are needed in order to be able to formulate some of the axioms.

3.1 Aggregation procedures with a priori specified weights

A weight vector is a vector $w = (w_1, \ldots, w_k)$ in $(\Re^+)^C \setminus \{0\}$, where $\Re^+$ is the set of the nonnegative real numbers and $0 = (0, \ldots, 0)$. The weight vector u^j such that $u^j_j = 1, u^j_i = 0$, for all $i \neq j$ will play a special role. In the remainder of this section, we suppose that each importance structure contains at least a weight vector and we don't use any other element that the importance structure might contain. Therefore, we use the following simplified notation: the symbol w will denote the weight vector of a particular importance structure, even if that structure contains more than a weight vector.

Let us now formulate the axioms, before presenting the results.

A 1 Independence of initial preferences (IIP). $\succeq(p, w, \rhd_1) = \succeq(p, w, \rhd_2)$.

This axiom tells us that the initial preference relation will not be used in the aggregation procedure. The next one imposes that the result of the aggregation be a complete relation.

A 2 Completeness. *For all $x \neq y$, $x \succeq(p, w, \rhd) y$ or $y \succeq(p, w, \rhd) x$.*

The next four axioms are about weights.

A 3 Convexity. *$x \succeq(p, w, \rhd) y$ and $x \succeq(p, w', \rhd) y$ implies $x \succeq(p, w + w', \rhd) y$. In addition, $x \succ(p, w, \rhd) y$ and $x \succ(p, w', \rhd) y$ implies $x \succ(p, w + w', \rhd) y$.*

A 4 Monotonicity. *$x \succ(p, w, \rhd) y$ and $x \succeq(p, w', \rhd) y$ implies $x \succ(p, w + w', \rhd) y$.*

A 5 Archimedeanness. *$x \succ(p, w, \rhd) y$ implies that there is β such that, for any $\alpha > \beta$, $x \succ(p, \alpha w + w', \rhd) y$, where α and β are real numbers.*

By Archimedeanness, we know that, if we raise the weight of a criterion, we can make it a kind of dictator.

3.1.1 Antisymmetric, additive and non transitive aggregation procedures

We say that an aggregation procedure is *additive and non transitive* if and only if, for each criterion i and each pair of alternatives (x, y), there is a mapping $s^i_{xy} : P(X, E_1, \ldots, E_k) \times I \to \Re : (p, \rhd) \to s^i_{xy}(p, \rhd)$ such that

- $s^i_{xy}(p, \rhd) \geq 0$ iff $y \succeq (p, u^i, \rhd) x$ and
- $x \succeq (p, w, \rhd) y \Leftrightarrow \sum_{i=1}^{k} s^i_{yx}(p, \rhd) w_i \geq 0$.

If, in addition, $s^i_{xy}(p, \rhd) = -s^i_{yx}(p, \rhd)$, then we say that the procedure is *antisymmetric*.

In [Jacquet-Lagrèze82], Jacquet-Lagrèze describes a family of aggregation methods which is very much like ours and he shows that many popular procedures are particular cases of his family. It is very important to remark that, contrary to what happens in the family considered by Jacquet-Lagrèze, s^i_{yx} depends on p and not just on p_i; s^i_{yx} corresponds to the i-th criterion but can be influenced by other criteria as well. Before characterizing the family of all antisymmetric, additive and non transitive aggregation procedures, let us also have a look at some important procedures belonging to it.

Promethee Let each assessment structure contain a real valued function v_i on X and a *preference function*, F_i, as defined in [Brans and Vincke 85], i.e. a non decreasing function from $\Re$ to $[0, 1]$ such that $F_i(0) = 0$. Let $s^i_{xy}(p, \rhd)$ be equal to $\Phi_i(y) - \Phi_i(x)$, where $\Phi_i(y)$ is the *single criterion net flow* of alternative y as defined in [Mareschal and Brans 88], i.e.

$$\Phi_i(y) = \sum_{x \neq y} F_i[v_i(y) - v_i(x)] - \sum_{x \neq y} F_i[v_i(x) - v_i(y)].$$

Then, the aggregation procedure that we obtain is exactly PROMETHEE II.

Additive MAUT Let each assessment structure contain a mapping from X to some set and a utility function from that set to the reals. Let $s^i_{xy}(p, \rhd)$ be equal to $u_i(y) - u_i(x)$, where $u_i(y)$ is the single attribute utility of alternative y (for criterion i). This is nothing but an additive MAUT based aggregation procedure.

Weighted sum Let each assessment contain a mapping from X into the reals. To obtain a weighted sum, we just have to let $s^i_{xy}(p, \rhd)$ be equal to $v_i(y) - v_i(x)$, where $v_i(y)$ is the real number on which alternative y is mapped, for criterion i.

Simple weighted majority Let $s^i_{xy}(p, \rhd) = 1$ if $y\delta^i(p_i)x$ AND NOT $x\delta^i(p_i)y$. Let $s^i_{xy}(p, \rhd) = 0$ if $y\delta^i(p_i)x$ AND $x\delta^i(p_i)y$. We call this procedure *simple weighted majority* because $x \succeq (p, w, \rhd) y$ iff the sum of the weights of the criteria such that x is better than y is larger than or equal to the sum of the weights of the criteria such that y is better than x.

AHP Let each assessment structure contain a matrix of pairwise comparisons of the alternatives, evaluated on a ratio scale [Saaty 80]. Let the importance structure contain a matrix of pairwise comparisons of the criteria, evaluated on a ratio scale. Let $s^i_{xy}(p, \rhd)$ be the x coordinate of the eigen vector of the matrix of assessment structure i minus the y coordinate of the same eigen vector. Let w_i be the i coordinate of the eigen vector of the matrix in the importance structure. This is AHP.

Note that all above mentioned procedures share an additional characteristic: $s^i_{xy}(p, \rhd)$ depends only on p_i.

If we drop antisymmetry, we can obtain a procedure that we call *simple weighted majority with threshold*, described hereafter.

Simple weighted majority with threshold Let $s^i_{xy}(p, \rhd) = \mu > 0$ if $y\delta^i(p_i)x$ AND NOT $x\delta^i(p_i)y$. Let $s^i_{xy}(p, \rhd) = 0$ if $y\delta^i(p_i)x$ AND $x\delta^i(p_i)y$. Let $s^i_{xy}(p, \rhd) = \nu < -\mu$ if $x\delta^i(p_i)y$ AND NOT $y\delta^i(p_i)x$. We call this procedure *simple weighted majority with threshold* because $x \succeq (p, w, \rhd) y$ iff the sum of the weights of the criteria such that x is better than y is larger than or equal to the threshold multiplied by the sum of the weights of the criteria such that y is better than x. The threshold is equal to $-\nu/\mu$.

This procedure is additive and non transitive. But it is not antisymmetric. It is worth noting that simple weighted majority is a special case of simple weighted majority with threshold, where $\mu = -\nu$, i.e. the threshold is equal to 1. Let us remark as well that simple weighted majority with threshold is very close to the concordance principle of ELECTRE[Roy 68].

Proposition 1 *An aggregation procedure $\succeq$ satisfies completeness (A2), convexity (A3), monotonicity (A4) and Archimedeanness (A5) if and only if it is an antisymmetric, additive and non transitive aggregation procedure. If, in addition, independence of initial preferences (IIP, A1) is satisfied, then $s^i_{xy}(p, \rhd)$ doesn't depend on $\rhd$.*

Note that this proposition and its proof has strong links with a proposition in [Myerson 95] characterizing scoring rules in social choice.

A possible circumstance under which it could be reasonable to assume independence of initial preferences is when the decision-maker has no idea about his preferences.

Compatibility with $\rhd$. The initial preferences can take various forms. Sometimes, the decision-maker can state that an alternative is strictly better than another one. We thus have a partial binary relation. In other instances, he might be able to say that some given alternatives are definitely bad ones, and so on. The information contained in the initial preference can be used, for example, to fix the value of some parameters used by an aggregation procedure as, for example, in UTA [Jacquet-Lagrèze and Siskos 82]. In the case of antisymmetric, additive and non transitive aggregation procedures , the parameters are the mappings $s^i_{xy}(p, \rhd)$.

It seems reasonable to assume that some consistency should exist between p, w and $\rhd$. For example, if $x\delta^i(p_i)y$ and NOT $y\delta^i(p_i)x$ for all criteria, then it would be strange that the decision-maker considers y as strictly better than x or that he puts x and not y in the set of the definitely bad alternatives or ... If some degree of consistency exists, then it might be reasonable to expect that we can find values for the parameters of an aggregation procedure, such that the final preference relation $\succeq(p, w, \rhd)$ is compatible (we will give a precise definition later) with $\rhd$.

We are going to show that the family of aggregation procedures characterized by proposition 1 is so large that it contains many aggregation procedures that are probably not reasonable. It shows that there is almost always an aggregation procedure compatible with $\rhd$: the minimum consistency between p and $\rhd$ is very weak.

In order to proceed formally, we suppose that $\rhd$ is a binary relation with the following meaning.

- $x \rhd y$ and NOT $y \rhd x$: x is definitely strictly better than y,
- $x \rhd y$ and $y \rhd x$: x is definitely equivalent to y,
- NOT $x \rhd y$ and $y \rhd x$: y is definitely strictly better than x,
- NOT $x \rhd y$ and NOT $y \rhd x$: no opinion.

If $\rhd$ is a binary preference relation, we say that a preference relation $\succeq(p, w, \rhd)$ is compatible with $\rhd$ iff

- $x \rhd y$ and NOT $y \rhd x \Rightarrow x{\succ}(p, w, \rhd)y$ and
- $x \rhd y$ and $y \rhd x \Rightarrow x{\sim}(p, w, \rhd)y$.

Let us now introduce a new axiom.

A 6 Faithfulness. $\succeq(p, u^i, \rhd) = \delta^i(p_i)$.

Thanks to faithfulness, we know that, if only one criterion is considered, the aggregation procedure will be faithful to the information contained in the assessment structure for that criterion.

Proposition 2 *Let $\rhd$ be a binary preference relation. Given any $(p, w, \rhd)$, there is an aggregation procedure $\succeq$ satisfying completeness (A2), convexity (A3), monotonicity (A4) and Archimedeanness (A5) and such that $\succeq(p, w, \rhd)$ is compatible with $\rhd$.*

In addition, let us impose faithfulness (A6). Then, there is an aggregation procedure $\succeq$ such that $\succeq(p, w, \rhd)$ is compatible with $\rhd$ if and only if, whenever $x \rhd y$ and NOT $y \rhd x$, we have $x\delta^i(p_i)y$ and NOT $y\delta^i(p_i)x$, for some i. Furthermore, for all i, $\delta^i(p_i)$ is complete.

Let us rephrase the second part of proposition 2. When a decision-maker faces a given problem, i.e. a triplet $(p, w, \rhd)$, the initial preference $\rhd$ can be as "strange" as we want, with respect to p; as long as x is not dominated by y whenever the decision maker definitely prefers x to y, there is an aggregation procedure $\succeq$ such that $\succeq(p, w, \rhd)$ is compatible with $\rhd$. This (probably to) wide variety of aggregation procedures is due to the fact that the $s^i_{xy}(p, \rhd)$ in proposition 1 depends on x and y.

Proposition 2 concerns only the case where $\rhd$ is a binary preference relation. But it can easily be rephrased for other cases.

Independence of the axioms of proposition 1. In order to prove the independence of our axioms, we present four examples of aggregation procedures. In each example, three axioms are verified; one is not. As these examples can help the reader to understand more deeply what an antisymmetric, additive and non transitive aggregation procedure is, we do not defer the proof of the independence to the *Proofs* section.

Completeness Let $\succeq(p, w, \rhd) = \{(x, x) : x \in X\}$, for all p, w and $\rhd$.

Monotonicity We already met an example: simple weighted majority with threshold (when the threshold is strictly positive).

Archimedeanness Let ω be the sum of the weights. For all z and z' different of x and y, $z \sim(p, w, \rhd) z'$. For the pair (x, y),

$w_1/\omega > .5 \Rightarrow x \succ(p, w, \rhd) y$,

$w_1/\omega = .5$ and $w_2/\omega > .5 \Rightarrow x \succ(p, w, \rhd) y$,

$w_1/\omega = .5$ and $w_2/\omega = .5 \Rightarrow x \sim(p, w, \rhd) y$,

$w_1/\omega = .5$ and $w_2/\omega < .5 \Rightarrow y \succ(p, w, \rhd) x$,

$w_1/\omega < .5 \Rightarrow y \succ(p, w, \rhd) x$.

Convexity If $\omega < 1$, then $x \sim(p, w, \rhd) y$, for all x and y. If $\omega \geq 1$, then $\succeq(p, w, \rhd)$ is a given total order.

In the next section, we consider a particular case of proposition 1.

3.1.2 A uniqueness result: simple weighted majority

Let us define some new axioms. The permutation of a profile, $\pi(p)$, where π is a permutation on X, is the profile defined by $(\pi(p))_i = \pi(p_i)$, for all i.

A 7 Neutrality. $\succeq(\pi(p), w, \pi(\rhd)) = \pi(\succeq(p, w, \rhd))$.

The next two axioms are about the roles of the criteria. Weighted anonimity tells us that all pairs of criteria and weight play the same role. Let σ be a permutation on C. We denote by $\sigma(w)$ the weight vector such that $\sigma(w)_i = w_{\sigma(i)}$.

A 8 Weighted anonymity. *Let p and q be two profiles. If there is a permutation σ on C such that $\delta^i(p_i) = \delta^{\sigma(i)}(q_{\sigma(i)})$ for all i, then $\succeq(p, w, \rhd) = \succeq(q, \sigma(w), \rhd)$.*

Let $D(p, q) = \{i \in C : p_i \neq q_i\}$.

A 9 Independence of Irrelevant Criteria (IIC). *If $w_i = 0$ for all criteria in $D(p, q)$, then $\succeq(p, w, \rhd) = \succeq(q, w, \rhd)$.*

The restriction of a profile p to a subset $Y \subset X$ is denoted by $p|_Y$ and defined by $(p|_Y)_i = p_i|_Y$, for all i in C.

A 10 Independence of Irrelevant Alternatives (IIA). *If $p|_{xy} = q|_{xy}$, then $\succeq(p, w, \rhd)|_{xy} = \succeq(q, w, \rhd)|_{xy}$.*

By ordinality, our next axiom, the result of the aggregation can depend only on the ordinal information contained in the assessment structures.

A 11 Ordinality. *If $\delta^i(p_i) = \delta^i(q_i)$, for all criteria, then $\succeq(p, w, \rhd) = \succeq(q, w, \rhd)$.*

Proposition 3 *For each criterion, let the single-criterion preference relation be complete. The only aggregation method that satisfies completeness (A2), convexity (A3), monotonicity (A4), Archimedeanness (A5), neutrality (A7), weighted anonymity (A8), IIC (A9), IIA (A10), faithfulness (A6) and ordinality (A11) is the simple weighted majority.*

Note that IIP (A1) doesn't appear in this characterization though it is satisfied by the simple weighted majority. It is easy to see that the simple weighted majority with threshold violates only one of the axioms of proposition 3: monotonicity. Note also that, because of completeness and faithfulness, $\delta^i(p_i)$ is complete for all i.

3.2 Simple weighted majority without weights

A very interesting result can be found in [Fishburn 73]. It deals with the problem of binary choice in a committee. The primitives used by Fishburn are the same as those used in [May 52] to characterize simple majority; they can easily be reinterpreted in a decision-aiding context and form a subset of our primitives. Therefore, it is straightforward to adapt Fishburn's result to our framework. Here are, slightly adapted, the axioms used by Fishburn.

A 12 Unanimity. *$x\delta^i(p_i)y$ and NOT $y\delta^i(p_i)x$, for all i in C, implies $x{\succ}(p, w, \rhd)y$.*

The next axiom is a kind of monotonicity axiom. Unlike monotonicity (A4), it deals with changes in p and not in w.

A 13 Non negative responsiveness. *If, for all i in C,*
NOT $y\delta^i(p_i)x \Rightarrow$ NOT $y\delta^i(q_i)x$ and
$x\delta^i(p_i)y \Rightarrow x\delta^i(q_i)y$,
then
$x{\succ}(p, w, \rhd)y \Rightarrow x{\succ}(q, w, \rhd)y$ and
$x{\succeq}(p, w, \rhd)y \Rightarrow x{\succeq}(q, w, \rhd)y$.

A 14 Strong duality. *Let us consider m profiles* $p^1, \ldots p^j, \ldots p^m$. *If, for all criteria, the number of profiles in* $\{p^1, \ldots p^j, \ldots p^m\}$ *such that* $x\delta^i(p^j_i)y$ *and NOT* $y\delta^i(p^j_i)x$ *is the same as the number of profiles such that* $y\delta^i(p^j_i)x$ *and NOT* $x\delta^i(p^j_i)y$, *then* $x{\succ}(p^j, w, \rhd)y$ *for some j and* $y{\succ}(p^{j'}, w, \rhd)x$ *for some* j'.

Proposition 4 [Fishburn] *Let* $X = \{x, y\}$. *For each criterion, let the single-criterion preference relation be complete. If an aggregation method satisfies completeness (A2), ordinality (A11), unanimity (A12), non negative responsiveness (A13) and strong duality (A14), then, for all w in W and all* $\rhd$ *in I, there are non negative real numbers* $c_i^{w,\rhd}$ *such that*

$$x{\succeq}(p, w, \rhd)y \quad \text{iff} \sum_{i:x\delta^i(p_i)y} c_i^{w,\rhd} \geq \sum_{i:y\delta^i(p_i)x} c_i^{w,\rhd}$$

and

$$c_i^{w,\rhd} > 0 \text{ for some } i.$$

Given this proposition, the next one is trivial but, nevertheless, interesting for it deals with sets containing more than two alternatives.

Proposition 5 *Let W contain only one importance structure. For each criterion, let the single-criterion preference relation be complete. If an aggregation method satisfies independence of initial preferences (A1), completeness (A2), neutrality (A7), independence of irrelevant alternatives (A10), ordinality (A11), unanimity (A12), non negative responsiveness (A13) and strong duality (A14), then, for all* $\rhd$ *in I, there are non negative real numbers* c_i *such that*

$$x{\succeq}(p, w, \rhd)y \quad \text{iff} \sum_{i:x\delta^i(p_i)y} c_i \geq \sum_{i:y\delta^i(p_i)x} c_i$$

and

$$c_i > 0 \text{ for some } i.$$

The family of aggregation procedures characterized by this proposition looks very much like our simple weighted majority but there is a tremendous difference: in proposition 5, the weights c_i are part of the aggregation procedure while, in simple weighted majority, they are part of the primitives. In the sequel, the aggregation procedures characterized by proposition 5 will be called *simple unspecified weighted majority*, in order to make clear that the weights are not specified a priori.

In proposition 5, we assume that W contains only one element. If we consider that this unique importance structure is such that it tells nothing about the relative importance or role of the criteria, then we have a characterization of simple unspecified weighted majority when no information is available about the relative importance of the criteria. Such a case is likely to happen often (see section 2.2 about importance structures). Therefore, proposition 5 is of very practical use.

If we drop the assumption that W contains only one element but impose a kind of *independence of importance structure*, we can obtain the same result. But it is of poor practical use because, when information about the relative importance of the criteria is available, it is rather strange not to use it.

Starting again from the result of Fishburn (proposition 4), we can have a look at more general families.

Proposition 6 *Let W contain only one importance structure. For each criterion, let the single-criterion preference relation be complete. If an aggregation method satisfies completeness (A2), independence of irrelevant alternatives (A10), ordinality (A11), unanimity (A12), non negative responsiveness (A13) and strong duality (A14), then, for all $\rhd$ in I, there are non negative real numbers $c_i^{\rhd}(x,y)$ such that*

$$x \succeq(p,w,\rhd) y \quad \textit{iff} \sum_{i: x\delta^i(p_i)y} c_i^{\rhd}(x,y) \geq \sum_{i: y\delta^i(p_i)x} c_i^{\rhd}(x,y),$$

$$c_i^{\rhd}(x,y) = c_i^{\rhd}(y,x)$$

and

$$\forall x, y \in X, \forall \rhd \in I, \textit{ there is a criterion such that } c_i^{\rhd}(x,y) > 0.$$

Note that, even if IIA is satisfied, $c_i^{\rhd}(x,y)$ can depend on $z \neq x, y$ through $\rhd$.

The following result, like proposition 2, shows us that the family of aggregation procedures characterized by proposition 6 contains many aggregation procedures that are probably not reasonable. It concerns only the case where $\rhd$ is a binary preference relation. But it can easily be rephrased for other cases.

Proposition 7 *Let $\rhd$ be a binary preference relation. Let W contain only one importance structure. For each criterion, let the single-criterion preference relation be complete. Given any $(p, w, \rhd)$, there is an aggregation procedure $\succeq$ satisfying completeness (A2), independence of irrelevant alternatives (A10), ordinality (A11), unanimity (A12), non negative responsiveness (A13) and strong duality (A14) and such that $\succeq(p,w,\rhd)$ is compatible with $\rhd$ if and only if, whenever $x \rhd y$ and NOT $y \rhd x$, we have $x\delta^i(p_i)y$ and NOT $y\delta^i(p_i)x$, for some i.*

This (probably to) wide variety of aggregation procedures is due to the fact that the "weights" of proposition 6 depend on x and y. To avoid this, a possible way is to impose neutrality. But the full strength of neutrality can be used only if we impose also the following condition.

A 15 Extended Independence of Irrelevant Alternatives (EIIA). *If $p|_{xy} = q|_{xy}$ and $\rhd|_{xy} = \rhd'|_{xy}$, then $\succeq(p,w,\rhd)|_{xy} = \succeq(q,w,\rhd)|_{xy}$.*

Proposition 8 *Let W contain only one importance structure. For each criterion, let the single-criterion preference relation be complete. If an aggregation method satisfies completeness (A2), neutrality (A7), ordinality (A11), unanimity (A12), non negative responsiveness (A13), strong duality (A14) and extended independence of irrelevant alternatives (A15), then, for all $\rhd$ in I, there are non negative real numbers $c_i^{\rhd}$ such that*

$$x \succeq (p, w, \rhd) y \quad \textit{iff} \sum_{i: x\delta^i(p_i)y} c_i^{\rhd} \geq \sum_{i: y\delta^i(p_i)x} c_i^{\rhd},$$

and

$$\forall \rhd \in I, \textit{ there is a criterion such that } c_i^{\rhd} > 0.$$

It is worth mentioning a characterization of another kind of simple weighted majority. It can be found in [Nitzan and Paroush 85] and deals also with the problem of binary choice by a committee. Each member of the committee is supposed to maximize the same utility function. Thus, there is a "correct" choice but each member has a certain probability to make the wrong choice. Nitzan and Paroush show that, under some assumptions, the best way to choose is to use a simple weighted majority, where the weights are function of the probabilities of incorrect choice. Their primitives are so different from ours that we see no way to reinterpret their results in our framework.

4 Conclusions

We presented a new theoretical framework whose advantage is twofold.

- It allows us to describe many different aggregation procedures in common terms. Therefore, characterizations of these procedures (when they will be available in this new framework!) will be comparable.
- The primitives of this framework do not contain elements such as weights, utility curves, indifference thresholds, ... that, actually, cannot be supposed to exist before using an aggregation procedure.

The results that we derived in our framework show that the concept of assessment structure, although very vague, is useful. We didn't need to define it more precisely. More results like proposition 5, not involving a specialization of the importance structure, are needed. For example, a characterization of UTA [Jacquet-Lagrèze and Siskos 82], where the weights (or more exactly, the extrema of each utility function) are derived from $\rhd$ would be very interesting.

Ultimately, a still more general framework is needed: a framework in which even the set of criteria is not a primitive. Some procedures, taking into account the interaction between criteria [Grabisch 96], can probably be used with sets of criteria in which some criteria are correlated and/or dependent (see [Roy and Bouyssou 93]). Other procedures cannot. Therefore, the construction of the set of criteria must be coupled to the choice of an aggregation procedure and the set of criteria cannot be considered as a primitive.

It is most likely that we could also find some reasons to even reject the set of alternatives as a primitive. But we think that we are far from being able to derive any interesting result in such a framework.

Another possible modification to our framework is the following. We could define $\succeq$ as a mapping from $\tilde{P} \times \tilde{W} \times \tilde{I}$ into R, where $\tilde{P}$ (resp. $\tilde{W}, \tilde{I}$) is a subset of P (resp. W, I). Indeed, there is no reason for a decision maker to impose that conditions such as IIC be satisfied for all profiles. In the particular problem that the decision maker faces, only some profiles are possible. Most profiles have nothing to do with his problem. Therefore, axioms need not to be satisfied for those irrelevant profiles. The same reasonnig applies to importance structures and initial preference relations. Obviously, such a framework is extremely difficult to handle.

From a purely formal viewpoint, many results of conjoint measurement can be easily transposed in our framework. Assume the following conditions.

A 16 Independence of profile (IP). $\succeq(p, w, \rhd) = \succeq(q, w, \rhd)$.

A 17 Independence of importance structure (IIS). $\succeq(p, w, \rhd) = \succeq(p, w', \rhd)$.

Then $\succeq$ depends only on $\rhd$ and we are back to conjoint measurement. But this has almost no interest since we are in a decision aiding context.

Last remark: contrary to what happens in conjoint measurement, the emphasis is put on the aggregation procedure and not on the preference relation. We showed that this viewpoint is more appropriate in a decision aiding context.

5 Proofs

The proofs of propositions 1 and 3 are presented in the next two sections. The other proofs are easy and left as an exercise for the reader.

5.1 Antisymmetric, additive and non transitive aggregation procedures

Before proving proposition 1, we are going to introduce a new condition and prove a serie of five lemmas.

A 18 Homogeneity. *For any positive real number α, $\succeq(p, w, \rhd) = \succeq(p, \alpha w, \rhd)$.*

Note that convexity, together with a continuity condition implies homogeneity.

Lemma 1 *Let $\succeq$ be an aggregation method satisfying the following conditions: completeness (A2), convexity (A3) and Archimedeanness (A5). Then it satisfies homogeneity (A18).*

Proof. Suppose that homogeneity is not satisfied. Then, by completeness, there are $w, p, \rhd, x, y, \gamma$ such that $x \succ (p, w, \rhd) y$ and $y \succeq (p, \gamma w, \rhd) x$, where γ is a real number. By successive applications of convexity, $y \succeq (p, \gamma w r, \rhd) x$, for all positive and rational r. By Archimedeanness, $x \succ (p, \alpha w + \gamma w, \rhd) y$, for all real α larger than some β. For some large r, there is $\alpha > \beta$ and such that $\gamma w r = \alpha w + \gamma w$. Therefore, we obtain a contradiction. □

Lemma 2 *Let $\succeq$ be an aggregation method satisfying the following conditions: completeness (A2), convexity (A3) and homogeneity (A18). For all $x \neq y$, there are real valued mappings $s^i_{xy}, i = 1 \dots k : (p, \rhd) \to s^i_{xy}(p, \rhd)$ such that*

- $x \succ (p, w, \rhd) y$ *and* $y \succ (p, w', \rhd) x \Rightarrow \sum_{i=1}^k s^i_{xy}(p, \rhd)[w'_i - w_i] \geq 0$,
- $x \succeq (p, u^i, \rhd) y$*, for all i, implies*
  - $s^i_{yx}(p, \rhd) = 1$ *if* $x \succ (p, u^i, \rhd) y$,
  - $s^i_{yx}(p, \rhd) = 0$ *if* $x \sim (p, u^i, \rhd) y$,
- $s^i_{yx}(p, \rhd) = -s^i_{xy}(p, \rhd)$,
- *if $x \succ (p, u^j, \rhd) y$ or $y \succ (p, u^j, \rhd) x$, for some j, then there is at least one criterion i such that $s^i_{yx}(p, \rhd) \neq 0$.*

Proof. By completeness, we can distinguish the two following cases.

1. $y \succ (p, u^j, \rhd) x$ for some j. Let

$$A_{xy}(p, \rhd) = \{w - w' : x \succ (p, w, \rhd) y \text{ and } y \succ (p, w', \rhd) x\}.$$

Let $B_{xy}(p, \rhd)$ be the convex hull of $A_{xy}(p, \rhd)$. Suppose that the k-dimensional vector 0 belongs to $B_{xy}(p, \rhd)$. Then there are weight vectors $w^1, \dots, w^M$ and $v^1, \dots, v^M$, with $x \succ (p, w^m, \rhd) y$ and $y \succ (p, v^m, \rhd) x, m = 1 \dots M$, such that the system

$$\sum_{m=1}^{M} \lambda_m (w^m_i - v^m_i) = 0, \forall i,$$

has non negative solutions with at least one λ_m strictly positive. Therefore,

$$\sum_{m=1}^{M} \lambda_m w^m = \sum_{m=1}^{M} \lambda_m v^m.$$

By convexity and homogeneity, $y \succ (p, \sum_{m=1}^M \lambda_m v^m, \rhd) x$ and $x \succ (p, \sum_{m=1}^M \lambda_m w^m, \rhd) y$. This is a contradiction. Hence, 0 does not belong to $B_{xy}(p, \rhd)$.

By the Supporting Hyperplane Theorem, we can choose $(s^1_{xy}(p, \rhd), \dots, s^k_{xy}(p, \rhd))$ in $(\Re^+)^C$ such that

- $s^i_{xy}(p, \rhd) \neq 0$, for some i,
- $x \succ (p, w, \rhd) y$ and $y \succ (p, w', \rhd) x \Rightarrow \sum_{i=1}^k s^i_{xy}(p, \rhd)[w'_i - w_i] \geq 0$,

To satisfy these conditions when the roles of x and y are reversed, we can simply let $s^i_{yx}(p, \rhd) = -s^i_{xy}(p, \rhd)$.

2. If $x \succeq (p, u^i, \rhd) y$, for all i, then we are free to choose

$$s^i_{yx}(p, \rhd) = \begin{cases} 1 & \text{if } x \succ (p, u^i, \rhd) y \\ 0 & \text{if } x \sim (p, u^i, \rhd) y. \end{cases}$$

□

Lemma 3 *Let $\succeq$ be an aggregation method satisfying the following conditions: completeness (A2), convexity (A3) and Archimedeanness (A5). Let $s^i_{xy}, i = 1 \ldots k$, satisfy the conditions of lemma 2.*

$$y \succ (p, w, \rhd) x \textit{ implies } \sum_{i=1}^k s^i_{xy}(p, \rhd) w_i \geq 0.$$

Proof. By lemma 1, homogeneity is satisfied as well. There are two possible cases.

1. There is j such that $x \succ (p, u^j, \rhd) y$. Then, by Archimedeanness, $y \succ (p, \alpha w + u^j, \rhd) x$, for some real number α. Hence,

$$\sum_{i=1}^k s^i_{xy}(p, \rhd)[\alpha w_i + u^j_i - u^j_i] \geq 0.$$

2. There is no j such that $x \succ (p, u^j, \rhd) y$. Then, by lemma 2, $s^i_{xy}(p, \rhd) \geq 0$, for all i. Therefore, $\sum_{i=1}^k s^i_{xy}(p, \rhd) w_i \geq 0$.

□

Lemma 4 *Let $\succeq$ be an aggregation method satisfying the following conditions: completeness (A2), convexity (A3), monotonicity (A4) and Archimedeanness (A5). Let $s^i_{xy}, i = 1 \ldots k$, satisfy the conditions of lemma 2.*

$$y \succeq (p, w, \rhd) x \textit{ implies } \sum_{i=1}^k s^i_{xy}(p, \rhd) w_i \geq 0.$$

Proof. There are two possible cases.

1. There is j such that $y \succ (p, u^j, \rhd) x$. By monotonicity, for any $\alpha > 0$, $y \succ (p, \alpha w + u^j, \rhd) x$. By lemma 3,

$$\sum_{i=1}^{k} s_{xy}^i(p, \rhd)[\alpha w_i + u_i^j] \geq 0.$$

   Hence,

$$\sum_{i=1}^{k} s_{xy}^i(p, \rhd) w_i \geq 0.$$

2. There is no j such that $y \succ (p, u^j, \rhd) x$. In other words, $x \succeq (p, u^i, \rhd) y$ for all i in C. Therefore, for all i, $x \sim (p, u^i, \rhd) y$ or $w_i = 0$. If this was not true, then there would be a criterion j such that $x \succ (p, u^j, \rhd) y$ and $w_j > 0$. By monotonicity, we would have $x \succ (p, w, \rhd) y$. This is a contradiction. By lemma 2, we know that $s_{xy}^i(p, \rhd) = 0$ for all i such that $w_i \neq 0$. Thus, $\sum_{i=1}^{k} s_{xy}^i(p, \rhd) w_i = 0$.

□

Lemma 5 *Let $\succeq$ be an aggregation method satisfying the following conditions: completeness (A2), convexity (A3) and Archimedeanness (A5). Let $s_{xy}^i, i = 1 \ldots k$, satisfy the conditions of lemma 2. If $y \succ (p, u^{i^*}, \rhd) x$ for some i^*, then there is a weight vector w^y such that*

$$y \succ (p, w^y, \rhd) x \text{ and } \sum_{i=1}^{k} s_{xy}^i(p, \rhd) w_i^y > 0.$$

Proof. By lemma 2, there is a criterion j such that $s_{xy}^j(p, \rhd) \neq 0$. By homogeneity and Archimedeanness, there is a positive real number α such that

$$y \succ (p, \alpha u^{i^*}, \rhd) x, \quad y \succ (p, \alpha u^{i^*} + u^j, \rhd) x, \quad y \succ (p, \alpha u^{i^*} + 2u^j, \rhd) x.$$

By lemma 3,

$$\sum_{i=1}^{k} s_{xy}^i(p, \rhd)[\alpha u_i^{i^*} + m u_i^j] \geq 0, \quad m \in \{0, 1, 2\}.$$

Let us rewrite this expression:

$$\sum_{i=1}^{k} s_{xy}^i(p, \rhd) \alpha u_i^{i^*} + m s_{xy}^j(p) \geq 0, \quad m \in \{0, 1, 2\}.$$

But $s_{xy}^j(p, \rhd) \neq 0$. This is possible only if

$$\sum_{i=1}^{k} s^i_{xy}(p, \rhd)\alpha u^{i^*}_i + s^j_{xy}(p, \rhd) > 0.$$

The proof is complete if we let $w^y = \alpha u^{i^*} + m u^j$. □

Proof of proposition 1, part 1. We need to prove that, if s^i_{xy}, $i = 1 \ldots k$, satisfy the conditions of lemma 2, then

$$\sum_{i=1}^{k} s^i_{xy}(p, \rhd) w_i \geq 0 \text{ implies } y \succeq (p, w, \rhd) x.$$

There are two cases.

1. There is j such that $y \succ (p, u^j, \rhd) x$. Let w^y be as in lemma 5. For any positive real number α,

$$\sum_{i=1}^{k} s^i_{xy}(p, \rhd)[\alpha w_i + w^y_i] > 0.$$

   By lemma 2,

$$\sum_{i=1}^{k} s^i_{yx}(p, \rhd)[\alpha w_i + w^y_i] < 0.$$

   By lemma 4, for all $\alpha > 0$, $y \succ (p, \alpha w + w^y, \rhd) x$. Consequently, it is not the case that $x \succeq (p, w, \rhd) y$. By completeness, $y \succeq (p, w, \rhd) x$.

2. There is no j such that $y \succ (p, u^j, \rhd) x$. Hence, $x \succeq (p, w, \rhd) y$ and, by lemma 4,

$$\sum_{i=1}^{k} s^i_{yx}(p, \rhd) w_i \geq 0.$$

   For all i such that $x \sim (p, u^i, \rhd) y$, we know by lemma 2 that $s^i_{yx}(p, \rhd) = 0$. Let $Q \subseteq C$, be the set of all criteria j such that $x \succ (p, u^j, \rhd) y$. For all j in Q, $s^j_{yx}(p, \rhd) > 0$. If, for all j in Q, $w_j = 0$, then, by convexity, $x \sim (p, w, \rhd) y$. If, on the contrary, $w_j > 0$ for some j in Q, then, by monotonicity, $x \succ (p, w, \rhd) y$. But

$$\sum_{i=1}^{k} s^i_{yx}(p, \rhd) w_i > 0.$$

   This is a contradiction. Hence, $w_j = 0$ for all j in Q and the proof is complete.

□

Proof of proposition 1, part 2. If independence of initial preferences (IIP) is satisfied, it is obvious that $s^i_{yx}(p, \rhd)$ depends only on p and not on $\rhd$. □

5.2 Simple weighted majority

Two more lemmas will be necessary before proving proposition 3.

Lemma 6 *Let $\succeq$ be an aggregation method satisfying the following conditions: IIC (A9), IIA (A10) and ordinality (A11). If $w_i = 0$ for all i such that $\succeq(p, u^i, \rhd)|_{xy} \neq \succeq(q, u^i, \rhd)|_{xy}$, then $\succeq(p, w, \rhd)|_{xy} = \succeq(q, w, \rhd)|_{xy}$.*

Proof. Let p' and q' be profiles such that

- $\delta^i(p'_i) = \delta^i(p_i)$ for all i in C,
- $\delta^i(q'_i) = \delta^i(q_i)$ for all i in C and
- $p'_i|_{xy} = q'_i|_{xy}$ for all i such that $\succeq(p, u^i, \rhd)|_{xy} = \succeq(q, u^i, \rhd)|_{xy}$.

These two profiles necessarily exist because of the fourth condition that we impose on the assessment structures. By ordinality, $\succeq(p, w, \rhd) = \succeq(p', w, \rhd)$ and $\succeq(q, w, \rhd) = \succeq(q', w, \rhd)$ for any w.

Let p'' and q'' be profiles such that

- $p''_i = p'_i$ and $q''_i = q'_i$ for all i such that $p'_i|_{xy} = q'_i|_{xy}$ and
- $p''|_{xy} = q''|_{xy}$ for all i in C.

By IIA, $\succeq(p'', w, \rhd)|_{xy} = \succeq(q'', w, \rhd)|_{xy}$. Let W be the set of all weight vectors such that $w_i = 0$ if $p'_i|_{xy} \neq q'_i|_{xy}$. By IIC, $\succeq(p', w, \rhd) = \succeq(p'', w, \rhd)$ and $\succeq(q', w, \rhd) = \succeq(q'', w, \rhd)$, for all $w \in W$. Therefore, $\succeq(p, w, \rhd)|_{xy} = \succeq(p', w, \rhd)|_{xy} = \succeq(p'', w, \rhd)|_{xy} = \succeq(q'', w, \rhd)|_{xy} = \succeq(q', w, \rhd)|_{xy} = \succeq(q, w, \rhd)|_{xy}$ for all $w \in W$. $\square$

Lemma 7 *Let $\succeq$ be an aggregation method satisfying the following conditions: weighted anonimity (A8), IIC (A9), IIA (A10) and ordinality (A11). If there is a permutation σ on C such that, for each criterion i,*

$$\succeq(p, u^i, \rhd)|_{xy} = \succeq(q, u^{\sigma(i)}, \rhd)|_{xy} \qquad \text{or} \qquad w_i = w_{\sigma(i)} = 0,$$

then $\succeq(p, w, \rhd)|_{xy} = \succeq(q, \sigma(w), \rhd)|_{xy}$.

Proof. Let p' be a profile such that

- $\succeq(p, u^i, \rhd)|_{xy} = \succeq(p', u^{\sigma(i)}, \rhd)|_{xy}$ for all i such that $w_i > 0$ and
- $\delta^i(p'_i) = \delta^{\sigma(i)}(p'_{\sigma(i)})$ for all i.

Such a profile p' necessarily exists, because of the fourth condition that we impose on the assessment structures. By lemma 6, $\succeq(p, w, \rhd)|_{xy} = \succeq(p', w, \rhd)|_{xy}$ and by weighted anonimity, $\succeq(p', w, \rhd) = \succeq(q, \sigma(w), \rhd)$. $\square$

Proof of Proposition 3. To complete the proof, all we need to do is to show that there are s^i_{yx} satisfying the conditions of lemma 2 and such that

$$s^i_{yx}(p,\rhd) = \begin{cases} 1 & \text{if } x\delta^i(p_i)y \quad \text{and NOT } y\delta^i(p_i)x, \\ 0 & \text{if } x\delta^i(p_i)y \quad \text{and } y\delta^i(p_i)x, \\ -1 & \text{if NOT } x\delta^i(p_i)y \quad \text{and } y\delta^i(p_i)x. \end{cases}$$

(a) If $x\delta^i(p_i)y$ for all i in C, then, by faithfulness, $x{\succeq}(p,u^i,\rhd)y$ for all i. By lemma 2, we find the following.

- $s^i_{yx}(p,\rhd) = 1$ if $x{\succ}(p,u^i,\rhd)y$. By faithfulness, $s^i_{yx}(p,\rhd) = 1$ if $x\delta^i(p_i)y$ and NOT $y\delta^i(p_i)x$.
- $s^i_{yx}(p,\rhd) = 0$ if $x{\sim}(p,u^i,\rhd)y$. By faithfulness, $s^i_{yx}(p,\rhd) = 0$ if $x\delta^i(p_i)y$ and $y\delta^i(p_i)x$.

(b) If we are not in case (a), then there are j and j^* such that NOT $y\delta^j(p_j)x$ and NOT $x\delta^{j^*}(p_{j^*})y$. By faithfulness, $x{\succ}(p,u^j,\rhd)y$ and $y{\succ}(p,u^{j^*},\rhd)x$. Suppose that $s^j_{yx}(p,\rhd) \neq -s^{j^*}_{yx}(p,\rhd)$. Then, by proposition 1, NOT $x{\sim}(p,u^j+u^{j^*},\rhd)y$. Let π be a permutation on X such that $\pi(x) = y$ and $\pi(y) = x$. By neutrality,

$$\succeq(\pi(p),w,\pi(\rhd))|_{xy} \text{ is the converse of } \succeq(p,w,\rhd)|_{xy}. \quad (1)$$

In other words, $x{\succ}(\pi(p),w,\pi(\rhd))y \Leftrightarrow y{\succ}(p,w,\rhd)x$ and $y{\succ}(\pi(p),w,\pi(\rhd))x \Leftrightarrow x{\succ}(p,w,\rhd)y$.

Let q be a profile such that $\delta^j(p_j)|_{xy} = \delta^{j^*}(q_{j^*})|_{xy}$ and $\delta^{j^*}(p_{j^*})|_{xy} = \delta^j(q_j)|_{xy}$. By faithfulness, $\succeq(p,u^j,\rhd)|_{xy} = \succeq(q,u^{j^*},\rhd)|_{xy}$ and $\succeq(p,u^{j^*},\rhd)|_{xy} = \succeq(q,u^j,\rhd)|_{xy}$. By lemma 7,

$$\succeq(p,u^j+u^{j^*},\rhd)|_{xy} = \succeq(q,u^j+u^{j^*},\rhd)|_{xy}. \quad (2)$$

Let us remark that $\succeq(q,u^j,\rhd)|_{xy} = \succeq(\pi(p),u^j,\pi(\rhd))|_{xy}$ and $\succeq(q,u^{j^*},\rhd)|_{xy} = \succeq(\pi(p),u^{j^*},\pi(\rhd))|_{xy}$. So, by lemma 6,

$$\succeq(q,u^j+u^{j^*},\rhd)|_{xy} = \succeq(\pi(p),u^j+u^{j^*},\pi(\rhd))|_{xy}. \quad (3)$$

If we combine equations (2) and (3), we obtain a contradiction with respect to equation (1), due to the fact that NOT $x{\sim}(p,u^j+u^{j^*},\rhd)y$.

Because of this contradiction, we know that $s^j_{yx}(p,\rhd) = -s^{j^*}_{yx}(p,\rhd)$. For any criterion g such that $x{\succ}(p,u^g,\rhd)y$, we can use j^* to find that $s^j_{yx}(p,\rhd) = s^g_{yx}(p,\rhd) = -s^{j^*}_{yx}(p,\rhd)$. And for any criterion g^* such that $y{\succ}(p,u^{g^*},\rhd)x$, we can use j to find that $s^j_{yx}(p,\rhd) = -s^{g^*}_{yx}(p,\rhd) = -s^{j^*}_{yx}(p,\rhd)$. Hence, $s^i_{yx}(p,\rhd)$ is either 0, a constant or the opposite of that constant. It is clear that the value of this constant is not important and we can choose it equal to 1. □

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