Outranking Methods

Denis Bouyssou

1 Introduction

A classical problem in the field of Multiple Criteria Decision Making (MCDM) is to build a preference relation on a set of multi-attributed alternatives on the basis of preferences expresses on each attribute and "inter-attribute" information such as weights. Based on this preference relation (or, more generally, on various relations obtained following a robustness analysis) a recommendation is elaborated (e.g. exhibiting of a subset likely to contain the "best" alternatives).

A common way [20] to do so is to attach a number $v(x)$ to each alternative $x \in X$ and to declare that x is at least as good as y if and only if $v(x) \geq v(y)$. The number $v(x)$ depends on the evaluations $x_1, x_2, ...x_n$ of x on the n attributes and we have $v(x) = V(x_1, x_2, ..., x_n)$.

The most common form for V is an additive value function in which $V(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} k_i v_i(x_i)$; in that case the task of the analyst reduces down to assessing the partial value functions v_i and the scaling constants k_i.

The preference relation that is built using this value function approach is a weak order, i.e. a complete and transitive binary relation. Using such information it is not difficult, in general, to elaborate a recommendation. The definition if the aggregation function V may not always be simple however. Making all alternatives comparable in a "nice transitive way" requires much information and, in particular, a detailed analysis of trade-offs between attributes.

Outranking Methods (OMs) were first developed in France in the late sixties following difficulties experienced with the value function approach in dealing with practical problems. They are closely associated with the name of Bernard Roy who developed the well-known family of ELECTRE Methods. A large part of the literature on OMs was written in French which has been prejudicial to their international diffusion; good accounts in English are [18, 37, 44, 51, 54] while detailed references in French include [24, 31, 39, 45, 46].

2 Basic Ideas

As in the value function approach, OMs build a preference relation, usually called an outranking relation, among alternatives evaluated on several attributes. B. Roy defines an outranking relation as a binary relation S on the set X of alternatives such that xSy if, given what is known about the preferences of the decision-maker and given the quality if the evaluations of the alternatives and the nature of the problem, there are enough arguments to declare that x is at least as good as y, while there is no essential reason to refute that statement.

In most OMs the outranking relation is built through a series of pairwise comparisons of the alternatives (this implies that these methods deal with finite sets of the alternatives; their underlying principles may however be adapted in order to deal with infinite sets [19]. Although pairwise comparisons can be done in many ways, the concordance-discordance principle is prevalent in most OMs (exceptions include [14, 48]). It consists in declaring that an alternative x is at least as good as an alternative $y(xSy)$ if:

- a *majority* of the attributes supports this assertion (concordance condition) and if
- the opposition of the other attributes—the *minority*—is not "too strong" (non-discordance condition).

This principle is at variance with the ones underlying the value function approach. It rests on a "voting" analogy and may be used without having recourse to a subtle analysis of trade-offs between attributes. It mainly uses ordinal considerations and has a strong non compensatory flavour [2, 10]. The application of this principle gives rise, in general, to binary relations which are neither complete (i.e. it is possible that $Not(xSy)$ and $Not(ySx)$ nor transitive (i.e. we may have xSy, ySz and $Not(xSz)$. Exploiting an outranking relation in order to arrive at a recommendation is therefore not an easy task and calls for the application of specific techniques [39, 51].

We briefly describe below ELECTRE I [34] which is the oldest and simplest OM before coming to some extensions and comments.

3 ELECTRE I

Consider a finite set of alternatives X evaluated in a family $N = \{1, 2, \ldots, n\}$ of attributes. A first step in the comparison of two alternatives $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ is to know how they compare on each attribute. ELECTRE I uses a traditional preference model for this purpose: a weak order(i.e. a complete and transitive binary relation) S_i is supposed to be defined on each $i \in N$, $x_i S_i y_i$ meaning that x is judged at least as good as y on attribute $i \in N$. Dealing with a finite set, it is not restrictive [15] to assume the existence of a real valued function g_i such that $x_i S_i y_i \Leftrightarrow g_i(x_i) \geq g_i(y_i)$. Quite often in practice, numbers are used to evaluate the alternatives on the various attributes and the relations S_i stem from the comparison of these numbers [3].

In ELECTRE I, the analysis of the proposition xSy rests on the partition of the set N of attributes into a concordant coalition $C(xSy) = \{j \in N : g_j(x_j) \geq g_j(y_j)\}$ and a discordant coalition $D(xSy) = \{j \in N : g_j(x_j) < g_j(y_j)\}$. The proposition xSy will be accepted if the concordant coalition $C(xSy)$ is "sufficiently important" (concordance condition) and if on any of the attributes in $D(xSy)$ the "difference of preference" in favour of y is not considered to be "large" (condition of non-discordance).

In order to implement the concordance condition, a positive weight k_j is assigned to each attribute $j \in N$ and the importance of a coalition supposed is represented by the sum of the weights of the attributes belonging to that coalition. Thus the concordance index $c(x,y) = \sum_{j \in C(xSy)} k_j \Big/ \sum_{j \in N} k_j$ represents the relative importance of the coalition $C(xSy)$ in the set N of all attributes; we have $c(x,y) \in [0;1]$. Whether or not $C(xSy)$ is "sufficiently important" is then judged comparing $c(x,y)$ to a concordance level $s \in [0;1]$. It is worth noting that the partition of N into $C(xSy)$ and $D(xSy)$ and the computation of the concordance index $c(x,y)$ rest on purely ordinal comparisons: altering the functions g_j without altering the binary relations S_j will not change the values of the concordance index.

Suppose that $c(x,y) \geq s$. Concluding that xSy would give no power to the attributes in $D(xSy)$. If on any of the attributes the, positive, preference difference between y and x is "large" there are good reasons to reject the proposition xSy. The definition of "large" preference differences is done in ELECTRE I via the definition of non negative veto thresholds v_j (which may vary with g_j) on each attribute; a preference difference is declared "large" as soon as $g_j(y_j) - g_j(x_j) > v_j$. It should be noticed that the implementation of the non-discordance principle through the definition of veto thresholds v_j linked to a particular functions g_j is a matter of commodity only; what is in fact looked for is a subset of the asymmetric part of S_j corresponding to "large" preference differences, which may be done independently of any numerical representation.

In summary, we have ELECTRE I:

$$xSx \Leftrightarrow \begin{cases} c(x,y) \geq s \\ \text{and} \\ g_j(y_j) - g_j(x_j) < v_j \text{ for all } j \in D(xSy) \end{cases}$$

When $s = 1$ (which amounts to requiring unanimity of the attributes in order to accept outranking) or $v_j = 0$ for all $j \in N$ (implying that all positive preference differences are "large"), the outranking relation S is nothing but the so-called dominance relation Δ defined by $x\Delta y \Leftrightarrow [x_j S_j y_j \text{for all} j \in N]$. It is not difficult to see that it is always true that $\Delta \subseteq S$. An outranking relation may be usefully seen as an enrichment of the dominance relation Δ in which unanimity of the attributes in not required and not all positive preference differences are considered "large"; decreasing the value of s and/or increasing the values of the v_j results in a richer but somewhat riskier outranking relation. Although, the dominance relation Δ is clearly reflexive and transitive (but not complete), simple examples, inspired by Condorcet's paradox [47], show that, in general, S is neither complete nor transitive when $s < 1$ and $v_j > 0$.

It is important to note that in ELECTRE I, the weights k_j cannot be interpreted as substitution rates or trade-offs; they are thus fundamentally different from the scaling constants that are used in the value function approach. In line with the voting analogy underlying the concordance-discordance principle, it is useful to interpret k_j as the "number of votes" given to attribute $j \in N$ (this number of votes being independent of the choice of the function g_j), the concordance threshold s specifying a level of "qualified majority".

ELECTRE I was originally designed to lead to "choice-type" results. Since S may not be complete or transitive, the set $\{x \in X : xSy \text{ for all } y \in X\}$ of maximal alternatives (in X given S) can be empty. In order to overcome this difficulty, ELECTRE I determine the minimal (w.r.t. inclusion) set of alternatives not outranking each other such that all the alternatives outside of this set are outranked by at least one alternative from this set. Technically, this leads to the determination of the *kernel* of the graph (X, S) after the detection and elimination by reduction of possible circuits (a well-known result in Graph Theory proves the existence and unicity of the kernel of a graph without circuit).

4 Extensions

Besides ELECTRE I, many other OMs have been proposed in the literature [12, 13, 23, 35, 38, 40, 43, 32, 55]. They mainly differ on:

- the type of result that is looked for (e.g. one may wish to use S to rank order alternatives or to sort them into pre-defined categories);
- the way the outranking relation is built. It is indeed possible to implement the concordance-discordance principle in various ways (e.g. allowing for synergy effects in $D(xSy)$). Moreover, varying the values of s and of the thresholds v_j lead to several different relations S; such variations are incorporated in methods which use several nested outranking relations as in [38, 40] or a fuzzy outranking relation in which a "credibility" is attached to each arc in the graph (X, S) as in [12, 13, 35, 55] (on the notion of fuzzy outranking relation see [17, 28]);
- the way alternatives are compared on each attribute. In ELECTRE I it is postulated that alternatives can be compared on each attribute according to a weak order. This traditional preference model may be inappropriate considering the inevitable elements of imprecision, uncertainty and inaccurate determination entering the evaluations of the alternatives. Indifference on each attribute might well not be transitive; moreover there may exist cases in which the transition from indifference to strict preference is not without ambiguity giving rise to model involving "weak preference" relations (such models involve *indifference* and/or *preference thresholds* [36, 33, 49, 50].

The following table adapted from [39] summarizes the main characteristics of the existing ELECTRE methods and might help in choosing an appropriate OM. We refer to [39, 54] for a complete description of these methods and of many others in a similar vein, in particular the TACTIC method [52] and the family of PROMETHEE methods [12, 13].

ELECTRE Methods	Preference Model	Use of Weights	# of relations	Result
I [34]	traditional	yes	1	Choice
IS [43]	non traditional	yes	1	Choice
II [38]	traditional	yes	2	Ranking (partial)
III [35]	non traditional	yes	1 (fuzzy)	Ranking (partial)
IV [40]	non traditional	no	up to 5	Ranking (partial)
TRI [39, 55]	non traditional	yes	1	Assignm. into pre-defined categories

5 Practical Considerations

We give here some indications on how to give a value to the parameters used in ELECTRE I: weights k_j, veto thresholds v_j and the concordance threshold s (they may be transposed to all ELECTRE methods; for a more detailed account see [24, 26, 39, 46] and for an alternative approach [21]). Before doing so, it is important to note that the underlying philosophy of OMs is not to describe as accurately as possible the preferences of a decision-maker.

This decision-maker is often a remote abstract entity (the State, the Region, the Firm); when this is not the case he/she is frequently not very accessible and his/her preferences may be only very partially structured. Searching for the "true" values of k_j, v_j or s makes little sense in these conditions. The concordance-discordance principle is best seen as a useful and easily understandable convention to help structuring preferences. The "assessment" of the parameters of the method should therefore aim at transforming what appears to be the stable basic judgements of the actors to be helped into numerical values. Needless to say that, under these conditions, the elaboration of a recommendation should be preceded by a thorough robustness analysis.

In order to give a numerical value to the weights k_j it is useful to envisage imaginary but realistic alternatives combining plausible evaluations on the various attributes. Consider two such alternatives x and y such that $g_j(x_j) > g_j(y_j)$ for all $j \in J \subset N$ and $g_i(y_i) > g_i(x_i)$ for all $i \notin J$. If the differences between the evaluations of x and y have been chosen in such a way as to avoid "large" preference differences and if it may be agreed that x is at least as good as y while y is not at least as good as x, we can then infer that $\sum_{j \in J} k_j \geq s$ and $\sum_{j \notin J} k_j < s$, supposing w.l.o.g. that $\sum_{j \in N} k_j = 1$.

Combining several questions of this type gives rise to a polyhedron of plausible values for k_j and s to be explored during the robustness analysis [41, 42]. It should be noted here that the precise numerical values of k_j and s are irrelevant in ELECTRE I as long as they imply a similar partition of subsets of attributes into "winning" coalitions (for which the sum of the weights exceed the concordance threshold) and "losing" ones.

Consider now two imaginary alternatives x and y such that $g_j(x_j) > g_j(y_j)$ for all $j \in N \setminus \{i\}$ and choose $g_i(y_i)$ to be one of the best evaluations on attribute $i \in N$ and $g_i(x_i)$ to be one of the worst. We have $D(xSy) = \{i\}$. If it can be accepted that xSy, then it is clear that no veto power should be conferred to attribute i, which amount to setting v_i to an arbitrarily large number. If not, attribute i has a veto power; in order to give value to v_i one can then increase $g_i(x_i)$ and/or decrease $g_i(y_i)$ till xSy is accepted. A slightly larger value than the difference $g_i(y_i) - g_i(x_i)$ leading to the acceptance of xSy gives a plausible value for v_i (note that before choosing a constant value of v_i it should be checked that the maximum difference $g_i(y_i) - g_i(x_i)$ on attribute i compatible with xSy does not vary along the scale of g_i; when this is the case variable thresholds can be easily used).

6 Theoretical appraisal

OMs have often been criticized for their lack of axiomatic foundations; ELECTRE I was proposed on a more or less *ad hoc* basis and subsequent methods aimed at extending it. The situation has changed dramatically in recent years giving rise to a variety of studies investigating the foundations of these methods. In particular, it is worth mentioning that:

- the links between concordance-discordance principle leading to possibly intransitive and incomplete outranking relations and classical aggregation problems in Social Choice Theory (exemplified by Arrow's Impossibility Theorem [47]) has been studied in depth [1, 4, 27];
- outranking methods may be axiomatised in more or less the same way as the various instances of the value function approach [2, 9, 10, 30, 52], the axioms emphasizing the "ordinal" and "non compensatory" features of the methods;
- the structural properties of outranking relations have been studied in depth [6], this problem having strong links with the classical problems of the construction of voting paradoxes [25] and the binary choice probabilities problem [16];
- various ways of exploiting outranking relations have been carefully analyzed and/or axiomatized [5, 7, 8, 11, 22, 29, 53].

This literature on the foundations of OMS while still being in its early stages has already greatly contributed to a better understanding of these methods and their underlying hypothesis.

7 Practical Applications

OMs have been applied in real-world studies since their creation. It is impossible to give here a complete list of applications and references. We only mention a few significant applications in various fields (detailed bibliographical indications may be found in [39]).

Environment Forestry management (Canada), Nuclear waste management (Belgium), Pollution prevention and control (France), Solid waste management (Finland, Greece), Water resource management (France, Hungary, USA);

Finance Allocation of grants (Belgium), Analysis of the international diversification of portfolios (Canada), Equitable burden sharing in international institutions (Belgium), Investment planning (France), Portfolio management (Canada);

Health Computer-aided diagnosis (France), Epidemiology (France), Identification of bacteria (Belgium), Management of hospitals (Canada);

Location Airports (Canada, the Netherlands), High voltage electric lines (France, Canada), Schools (France), Thermal power plants (Algeria);

Transportation Choice of a highway route (France), Planning the renovation of metro stations (France), Selection of suburban metro extensions projects (France);

Miscellaneous Analysis of tenders (France, Portugal), Choice between forecasting models (Belgium), Choice of marketing strategy (France), Inventory management (France), Production planning in a job-shop (Canada), Promotion of navy officers (Portugal), Regional planning (The Netherlands).

References

[1] K.J. Arrow and H. Raynaud. *Social choice and multicriterion decision making*. MIT Press, 1986.

[2] D. Bouyssou. Some remarks on the notion of compensation in MCDM. *European Journal of Operational Research*, 26:150–160, 1986.

[3] D. Bouyssou. Building criteria: A prerequisite for MCDA. In C.A. Bana e Costa, editor, *Readings in multiple criteria decision aid*, pages 58–80. Springer-Verlag, 1990.

[4] D. Bouyssou. On some properties of outranking relations based on a concordance-discordance principle. In L. Duckenstein, A. Goicoechea, and S. Zionts, editors, *Multiple criteria decision making*, pages 93–106. Springer-Verlag, 1992.

[5] D. Bouyssou. Ranking methods based on valued preference relations: A characterization of the net flow network. *European Journal of Operational Research*, 60:61–67, 1992.

[6] D. Bouyssou. Outranking relations: do they have special properties? *Journal of Multiple Criteria Decision Analysis*, 5:99–111, 1996.

[7] D. Bouyssou and P. Perny. Ranking method for valued preference relations: A characterization of a method based on leaving and entering flows. *European Journal of Operational Research*, 61:186–194, 1992.

[8] D. Bouyssou and M. Pirlot. Choosing and ranking on the basis of fuzzy preference relations with the 'min in favor'. In G. Fandel and T. Gal, editors, *Multiple criteria decision making- Proceedings of the twelfth international conference – Hagen (Germany)*, pages 115–127. Springer-Verlag, 1997.

[9] D. Bouyssou, M. Pirlot, and Ph. Vincke. A general model of preference aggregation. In M.H. Karwan, J. Spronk, and J. Wallenius, editors, *Essays in decision making*, pages 120–134. Springer-Verlag, 1997.

[10] D. Bouyssou and J.-C. Vansnick. Noncompensatory and generalized noncompensatory preferences structures. *Theory and Decision*, 21:251–266, 1986.

[11] D. Bouyssou and Ph. Vincke. Ranking alternatives on the basis of preference relations: a progress report with special emphasis on outranking relations. *Journal of Multiple Criteria Decision Analysis*, 6:77–85, 1997.

[12] J.-P. Brans and Ph. Vincke. A preference ranking organization method. *Management Science*, 31:647–656, 1985.

[13] J.P. Brans, B. Mareschal, and Ph. Vincke. Promethee: A new family of outranking methods in multicriteria analysis. In J.P. Brans, editor, *OR'84*, pages 408–421. North Holland, 1984.

[14] C.A. Bana e Costa. An additive value function technique with fuzzy outranking relation for dealing with poor intercriteria preference information. In C.A. Bana e Costa, editor, *Readings in multiple criteria decision aid*, pages 351–382. Springer-Verlag, 1990.

[15] P.C. Fishburn. *Utility theory for decision making*. Wiley, 1970.

[16] P.C. Fishburn. Induced binary probabilities and the linear ordering polytope: a status report. *Mathamatical Social Science*, 23:67–80, 1992.

[17] J. Fodor and M. Roubens. *Fuzzy preference modelling and multiple criteria decision support*. Kluwer, 1994.

[18] A Goicoechea, D.R. Hansen, and L. Duckstein. *Multiobjective decision analysis with engineering and business applications*. Wiley, 1982.

[19] A. Jaszkiewicz and R. Slowinski. The light beam search: outranking based interactive procedure for multiple-objective mathematical programming. In P.M. Pardalos, Y. Siskos, and C. Zopounidis, editors, *Advances in multicriteria analysis*, pages 129–146. Kluwer, 1995.

[20] R.L. Keeney and H. Raiffa. *Decisions with multiple objectives: preferences and value tradeoffs*. Wiley, 1976.

[21] L.N. Kiss, J.-M. Martel, and R. Nadeau. ELECCALC: an interactive software for modelling the decision-maker's preferences. *Decision Support Systems*, 12:311–326, 1994.

[22] Th. Marchant. Valued relations aggregation with the borda method. *Journal of Multiple Criteria Analysis*, 5:127–132, 1996.

[23] B. Matarazzo. A pairwise comparison approach: the MAPPAC and PRAGMA methods. In C.A. Bana e Costa, editor, *Readings in multiple criteria decision aid*, pages 253–273. Springer-Verlag, 1990.

[24] L.Y. Maystre, J. Pictet, and J. Simos. *Méthodes multicritères ELECTRE*. Presses Polytechniques et Universitaires Romandes, 1994.

[25] D. C. McGarvey. A theorem on the construction of voting paradoxes. *Econometrica*, 21:608–610, 1953.

[26] V. Mousseau. Eliciting information concerning the relative importance of criteria. In P.M. Pardalos, Y. Siskos, and C. Zopounidis, editors, *Advances in multicriteria analysis*, pages 17–43. Kluwer, 1995.

[27] P. Perny. Sur le non respect de l'axiome d'indépendance dans les méthodes de type ELECTRE. *Cahiers du CERO*, 34:211–232, 1992.

[28] P. Perny and B. Roy. The use of fuzzy outranking relations in preference modelling. *Fuzzy Sets and Systems*, 49:33–53, 1992.

[29] M. Pirlot. A characterization of 'min' as a procedure for exploiting valued preference relations and related results. *Journal of Multiple Criteria Decision Analysis*, 4:37–56, 1995.

[30] M. Pirlot. A common framework for describing some outranking methods. *Journal of Multiple Criteria Decision Analysis*, 6:86–92, 1997.

[31] J.-Ch. Pomerol and S. Barba-Romero. *Choix multicritère dans l'entreprise*. Hermès, 1993.

[32] M. Roubens. Preference relations on actions and criteria in multicriteria decision making. *European Journal of Operational Research*, 10:51–55, 1982.

[33] M. Roubens and Ph. Vincke. *Preference modelling*. Springer Verlag, 1985.

[34] B. Roy. Classement et choix en présence de points de vue multiples (la méthode electre. *RIRO*, 2:57–75, 1968.

[35] B. Roy. ELECTRE III : un algorithme de classement fondé sur une représentation floue des préférences en présence de critères multiples. *Cahiers du CERO*, 20:3–24, 1978.

[36] B. Roy. *Méthodologie multicritère d'aide à la décision, English translation: Multicriteria methodology for decision aiding, 1996, Kluwer*. Economica, 1985.

[37] B. Roy. The outranking approach and the foundations of ELECTRE methods. *Theory and Decision*, 31:49–73, 1991.

[38] B. Roy and P. Bertier. La méthode ELECTRE II : une application au media-planning. In M. Ross, editor, *OR'72*, pages 291–302. North Holland, 1973.

[39] B. Roy and D. Bouyssou. *Aide multicritère à la décision : méthodes et cas*. Economica, 1993.

[40] B. Roy and J.-Ch Hugonnard. Ranking of suburban line extension projects on the paris metro system by a multicriteria method. *Transportation Research*, 16A:301–312, 1982.

[41] B. Roy and V. Mousseau. A theoretical framework for analysing the notion of relative importance of criteria. *Journal of Multiple Criteria Decision Analysis*, 5:145–159, 1996.

[42] B. Roy, M. Present, and D. Silhol. A programming method for determining which paris metro station should be renovated. *European Journal of Operational Research*, 24:318–334, 1986.

[43] B. Roy and J.-M. Skalka. ELECTRE IS : aspects méthodologiques et guide d'utilisation. Document du lamsade, no. 30, Université de Paris-Dauphine, 1984.

[44] B. Roy and D. Vanderpooten. The european school of MCDA: emergence, basic features and current works. *Journal of Multiple Criteria Decision Analysis*, 5:22–37, 1996.

[45] A. Schärlig. *Décider sur plusieurs critères : panorama de l'aide à la décision multicritère.* Presses Polytechniques Romandes, 1985.

[46] A. Schärlig. *Pratiquer ELECTRE et PROMETHEE.* Presses Polytechniques et Universitaires Romandes, 1996.

[47] A. K. Sen. Social choice theory. In K. J. Arrow and M. D. Intriligator, editors, *Handbook of mathematical economics vol. III*, pages 1073–1181. Elsevier, 1986.

[48] J. Siskos. A way to deal with fuzzy preferences in multi-criteria decision problems. *European Journal of Operational Research*, 10:314–324, 1982.

[49] A. Tsoukiàs and Ph. Vincke. A survey on nonconventional preference modelling. *Ricerca Operativa*, 61:549, 1992.

[50] A. Tsoukiàs and Ph. Vincke. Extended preference structures in mcda. In J. Climaco, editor, *Multicriteria Analysis*, pages 37–50. Springer-Verlag, 1997.

[51] D. Vanderpooten. The construction of prescriptions in outranking methods. In C.A. Bana e Costa, editor, *Readings in Multiple Criteria Decision Aid*, pages 184–215. Springer-Verlag, 1990.

[52] J.-C. Vansnick. On the problem of weights in multiple criteria decision-making (the noncompensatory approach). *European Journal of Operational Research*, 24:288–294, 1986.

[53] Ph. Vincke. Exploitation of a crisp relation in a ranking problem. *Theory and Decision*, 32:221–240, 1992.

[54] Ph. Vincke. *Multicriteria decision-aid.* Wiley, 1992.

[55] W. Yu. *Aide multicritère à la décision dans le cadre de la problématique du tri: concepts, méthodes et applications.* PhD thesis, Université de Paris-Dauphine, 1992. Doctoral Dissertation.