BUILDING CRITERIA: A PREREQUISITE FOR MCDA

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1. INTRODUCTION

Following Roy (1985), we will say that decision-aid consists in trying to provide answers to questions raised by actors involved in a decision process using a clearly specified model. In order to do so, the analyst often has to compare "alternatives" (see Vincke, 1989). In an approach using several criteria, the analyst aims at establishing comparisons on the basis of the evaluation of the alternatives according to several criteria. In an approach using a single criterion, the analyst seeks to build a unique criterion taking into account all the relevant aspects of the problem. In either approach, the success of decision-aid crucially depends upon the way in which the unique criterion or the family of criteria has been built. The aim of this paper is to emphasize the importance of this phase by a number of frequently encountered difficulties and some techniques to overcome them.

The paper is organized as follows. We define the notion of criterion in section 2. In section 3, we use an example to show that building criteria is an important and difficult phase of the decision-aid process. Some standard techniques for constructing a criterion are presented in section 4. We conclude, in section 5,with some remarks concerning the choice of a consistent family of criteria.

2. WHAT IS A CRITERION?

a) Definition and remarks.

In what follows, we will call criterion a "tool" allowing to compare alternatives according to a particular "significance axis" or a "point of view[1]" (Roy, 1985)).

More precisely, a criterion is a real-valued function on the set A of alternatives, such that it appears meaningful to compare two alternatives a and b according to a particular point of view on the sole basis of the two numbers g(a) and g(b).

In a mono-criterion approach, the analyst builds a unique criterion capturing all the relevant aspects of the problem. The comparisons that are deduced from that criterion are to be interpreted as expressing "global preferences", i.e. preferences taking all the relevant points of view into account.

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[1] Though these two terms are not synonymous (an axis is the operational counterpart of a given point of view (see Roy, 1985)) we will use them interchangeably in this paper.

In a multiple criteria approach, the analyst seeks to build several criteria using several points of view. These points of view represent the different axes along which the various actors of the decision process justify, transform and argue their preferences. The comparisons deduced from each of these criteria should therefore be interpreted as partial preferences, i.e. preferences restricted to the aspects taken into account in the point of view underlying the definition of the criterion. Of course, speaking of partial preference implies the possibility of making *ceteris paribus* comparisons on the aspects that have not been taken into account in the definition of the criterion. This crucial hypotheses is central to MCDA. Its "test" would require the preferences of the actors of the decision process to be highly structured which is rather uncommon in decision-aid contexts.

However there are good reasons to believe that its adoption is not a severe restriction to the ability of MCDA to deal with real-world problems (see Roy and Bouyssou, 1988, chap. 2).

Our definition implies that a criterion is a model allowing to establish preference relations between alternatives. The quality of the construction of this model is crucial for the quality of decision-aid. An analogy will help understanding the importance of this phase. It is well known in Statistics that the implementation of sophisticated data analysis methods cannot compensate for the weaknesses of the phase consisting in gathering and preparing the data. The same is true for MCDA: applying sophisticated aggregation procedures is of little use if the criteria have been built in an unconvincing way.

This analogy with Statistics can be further pursued. If there are a number of standard techniques for analyzing various types of statistical data, this is not the case for the phase consisting in gathering and preparing the data. The choice of the statistical variables, the definition of the population, the redaction of a questionnaire, the coding of the data, ... are crucial problems the solution of which depends more on the art and the experience of the statistician than on "Science". In our opinion, the same is true in decision-aid for the construction of the criteria. In this paper, we will thus try more to warn the reader against common difficulties and to present a number of techniques that have proved-useful, than to present a well-established methodology that would allow to build criteria in all cases.

b) Some general guidelines for constructing a criterion.

When building a criterion, the analyst should keep in mind that it is necessary that all the actors of the decision process adhere to the comparisons that will be deduced from that model. This implies a number of important consequences.

i- The points of view underlying the definition of the various criteria should be understood and accepted by all the actors of the decision process, even if they disagree on the relative importance that they would like each of them to have in the aggregation model. These points of view should be familiar enough to these actors for them to be willing to discuss and argue on such a basis. Being able to associate to a given point of view a criterion having a clear physical unit may be seen as a great advantage in this respect.

ii- Once a point of view has been defined and accepted, the method allowing to arrive at the evaluation on the criterion for each alternative, should also be understood and accepted by all the actors of the decision process. The search for a simple and transparent model of evaluation should therefore be an important preoccupation of the analyst. Furthermore this method should be as free as possible from elements deeply linked to a particular value system. In fact, the presence of such elements may well lead some actors to question or even reject the validity of the comparisons made on the basis of the criterion.

iii- The choice of a particular way to build a criterion must take into account the quality of the "data" used to build it. In particular, the comparisons deduced from the criterion should take into account the elements of uncertainty, imprecision and/or inaccurate determination affecting the data used to build it.

c) Criteria, consequences and points of view.

The result of the implementation of an alternative can be modelled using a number of consequences or attributes (see Roy,1985, chap. 8). These consequences are, in general, numerous and concern many aspects: time, money security, quality, image, ...Conceptually it is possible to base the comparison of the alter-natives directly in terms of their consequences. However, due to the number of these consequences and to the fact that the evaluation of the alternatives on them often involves many elements of uncertainty, imprecision and inaccurate determination, this is, in general, difficult. A criterion thus appears as a tool allowing to "sum up" a set of evaluation on consequences related to a same point of view so as to be able to establish partial preferences. For instance, in a siting study, the analyst may want to build a criterion "impact on environment", taking into account consequences such as "impact on animal life", "impact on flora", "impact on landscape", ...

Building a criterion implies that one has chosen a point of view along which it seems adequate to establish comparisons. The determination of points of view that are understood and admitted by all actors is an important problem in MCDA. In order to do so, various techniques have been proposed. Roy (1985) considers that these points of view will emerge

after a thorough analysis of various class of consequences, taking into account the "culture" of the actors involved. Keeney and Raiffa (1976), Keeney (1988),Saaty (1980), Forman (1990), Belton and Vickers (1990) advocate a "hierarchical" way of building the criteria through the decomposition a unique point of view ("well-being", "social benefits", ...) into sub-points of view that are again decomposed, till the relevant points of view are reached. It is worth noting using a hierarchical approach, one often speaks of "criteria", "sub-criteria", "sub-sub-criteria" depending on the level of the hierarchy. Here, we will restrict the use of the world criteria for the models related to the upper levels of the hierarchy. It is worth noting that the down-up approach of Roy and the hierarchical approach are not exclusive.
Another important problem in an approach using multiple criteria is the choice of a family of criteria. We will turn to this problem in section 5. Let us only mention now that, for reasons related to cognitive limitations of the human mind (see Miller,1957) and to the necessity of gathering inter-criteria information for the implementation of aggregation procedures, it seems inadvisable to go much further than a dozen of criteria (defined at a high level of the hierarchy if such an approach is used).

d) The discriminating power of a criterion.

How to infer comparisons between alternatives on the basis of a criterion? In the most classical model, it is supposed that[2], for all a, b $\in$ A:

a P_g b $\Leftrightarrow$ g(a) > g(b) and

a I_g b $\Leftrightarrow$ g(a) = g(b),

where P_g (resp. I_g) is a binary relation that reads "is strictly preferred to (resp. indifferent to) considering the consequences taken into account in the definition of g". In this type of model called "true-criterion", any difference, as small as it maybe, between two evaluations implies a strict preference. As we will see, since the evaluations g(a) and g(b) are often obtained through a model that includes some arbitrariness and on the basis of imprecise, uncertain data, this model may lead to unconvincing comparisons. In fact, it is often reasonable to admit that "small" differences g(a) - g(b) are compatible with an indifference situation between a and b. This leads to the following model of comparison:

a P_g b $\Leftrightarrow$ g(a) - g(b) > q and

a I_g b $\Leftrightarrow$ $|g(a) - g(b)| \leq q$,

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[2] the direction of the inequality being conventional and notrestrictive

where q, the indifference threshold, is the largest difference g(a) - g(b) compatible with an indifference situation. Such a criterion is called a "quasi-criterion". In this model a difference greater than q implies a strict preference even if is very close to q. In order to avoid a sudden change from strict preference to indifference, it is possible to introduce a "buffer zone" in which there is an hesitation between the indifference and strict preference. Denoting this hesitation by a binary relation Q_g, often called "weak preference", we obtain a model with two thresholds, a preference threshold p and an indifference threshold q, called "pseudo-criterion", where[3]

$a\ P_g\ b \Leftrightarrow g(a) - g(b) > p$ and

$a\ Q_g\ b \Leftrightarrow q < g(a) - g(b) \leq p$

$a\ I_g\ b \Leftrightarrow |g(a) - g(b)| \leq q$,

It is not an easy task to give a value to these two thresholds (see Bouyssou and Roy, 1987). Yet, in many situations, every reasonable non-null value for p and q, leads to a model of preferences that seems more convincing than the one obtained by letting p = q = 0 as done with a true-criterion. However, this model is used in most methods for MCDA[4]. It is true that the elements of arbitrariness inherent to this model can be "corrected" by a sensitivity analysis. However, let us recall from the previous section that the family of criteria will only play its part in the decision-aid model if the comparisons that can be inferred on its basis are not subject to criticisms. The interest of the pseudo-criterion model is then seen more clearly, since the presence of criteria leading to unconvincing comparisons may lead some actors to reject the whole family of criteria. Further-more, it is worth noticing that the use of criteria with thresh-olds is of utmost importance if one wants to use afterwards an aggregation procedure based on a noncompensatory logic[5] as this is the case with the ELECTRE methods (Roy, 1990, Vanderpooten,1990), TACTIC (Vansnick, 1987) or ORESTE (Roubens, 1982, Pastijn and Leysen, 1989).

3. AN EXAMPLE

Suppose that an analyst wishes to build a criterion taking into account the impact in terms of noise of the construction of a new airport on the riparian population in a study aiming at prescribing one or several possible site for the construction of a new airport.

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[3] These thresholds may vary along the scale of the criterion, see Roy (1985).

[4] This has, perhaps, to do with the traditional "culture" of Operational Research.

[5] On this notion see Bouyssou and Vansnick (1986).

The objective of the construction of this criterion is to associate a figure to each site in such a way that this figure allows, at least, to determine if, from the point of view of the impact on the riparians, a site can be considered as preferable to another site. Though this study is conducted in a complex and conflictual context involving many actors (local and national authorities, air carriers, defense associations, public opinion),we will suppose that there is a consensus to take this point of view into account in order to choose a site.

The construction of this criterion takes place after a preliminary technical study that retained a small number of sites for a thorough analysis. In 1989, the time of the study, it is planned that the final decision will be taken in one or two years and that the construction will start within five years so that the airport could be fully operational around the turn of the century .

In order to build such a criterion, the analyst will probably try to estimate first the number of inhabitants that would be, in1989, affected by the installation of the new airport. Depending on time constraints, she may either count houses on a map or then multiply this number by an average number of inhabitants living in the region in different types of houses or conduct on-site studies. It should be clear that, whichever method is used, the number obtained will be highly imprecise. This imprecision due to the counting operations is however of limited importance compared to the multiple sources of inaccurate determination affecting the construction of this criterion. In particular, any mode of construction implies to take a definite position (explicitly or implicitly) on the following problems.

i- Where to place the boarder between "close" and "far" from the site? Ten years from now, will the airplanes be more or less noisy than they presently are? Will the problem of noise be more important than it presently is?

ii- The nuisance created to the riparians crucially depends on the position of the riparians relative to the runaways and the aerial corridors. At this stage of the study such information are probably not available yet. Should one consider a best guess hypothesis or some kind of average taking into account various possibilities or neglect this problem at that stage?

iii- What is a riparian? In particular, in the counting of the riparians should one give a particular weight (higher or lower?) to the second homes? Is it possible to make the hypothesis that the schools, hospitals, ... presently located near the site will move away from the site or, on the contrary, should these riparians be included in the total with a particular weight?

iv- Is it possible to make the hypothesis that the increase of the number of riparians will be identical on all sites during the period separating the study from the installation of the airport? On the contrary, should one consider that this increase depends on the distance between the projected site and the city center? If this is the case, should one envisage various scenarii for the growth of the population? Should one include in the model the probable consequences of the construction of the airport on the surrounding population (departure of the present riparians, installation of employees of the airport, installation of new firms near the site)? Should one take into account the fact that the construction of the airport will also imply the construction of new reads and railways that also create nuisance?

v- How to take into account the greater or lesser proximity of the riparians to the source of nuisance? Is the classical technique consisting in defining "zones" of noise and in giving a "weight"[6] to each zone satisfactory?

Building the criterion then implies to combine all this information in a formula allowing to compute a figure for each site. Depending on the options taken, one may use a criterion of one of the following types:

$$g(a) = \sum_{k=1}^{n} w_i h_i(s)\,,$$

$$g(a) = \sum_{k=1}^{n} w_i h_i(s)(1+\alpha_i)^d\,,$$

$$g(a) = \sum_{k=1}^{m_s} \left[\sum_{k=1}^{n} w_i h_i(s)(1+\alpha_i(k))^d \right] P(E_k)\,,$$

where there are n zones surrounding each site w_i, being the weight affected to the inhabitants of the i-th zone, $h_i(s)$ being the number of people living in the i-th zone of site s in 1989 (possibly corrected in order to take into account "privileged" inhabitants), α_i being the annual increase rate of that population for the next d years, $\alpha_i(k)$ being the same rate in scenario E_k having a probability $P(E_k)$, m_s scenarii being considered for the site s. Let us note that one may want to include in these formulas various scenarii concerning the orientation of the runaways or the possible date of installation of the airport.

Even if we complicated this example on purpose and if many problems would not be raised in practice either because of a lack of time or because the various sites are not different in these respects, we hope to have shown that building a criterion may be a long and difficult task.

This construction implies a large amount of work and during this work a number of crucial options are taken. Furthermore it is often impossible to avoid the introduction of elements of arbitrariness in the definition of a criterion. In our example, this arbitrariness concerns a

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[6] Usually inversely proportional to the distance, or the squared distance, between the zone and the site.

number of precise problems the solution of which does not seem to depend crucially on a particular value system. Therefore, in spite of this arbitrariness, it is not unlikely to be able to reach a consensus on a definition of this criterion. In our opinion this illustrates one of the major advantages of an approach using several criteria. In a mono-criterion approach, obtaining such a consensus is highly unlikely since elements dependent on a particular value system {e.g., the tradeoffs between transportation time and the amount of nuisance created to the riparians) are inextricably mixed to a large number of other options in a complicated and inevitably opaque model. In our example the construction of the criteria requires a vast amount of data in which imprecision, uncertainty and inaccurate determination are involved. Whatever the final definition of the "impact on riparians" criterion, one has to admit that it will only be an "order of magnitude" of what we were willing to capture[7]. It is crucial to keep this point in mind in the rest of the study if convincing comparisons are to be established. This explains the interest of pseudo-criterion type models. In our example, whatever formula is used at the end, it should be clear that any non-null and not unreasonable value for p and q (for instance, $p(g(s)) = 0.2g(s)$ and $q(g(s)) = 0.lg(s)$) will lead to more realistic comparisons than setting $p = q = 0$.

4. SOME TECHNIQUES FOR BUILDING CRITERIA

As we already mentioned, this section is not a catalogue of standard techniques where one could look for a "solution". More modestly and realistically, we tried to distinguish some simple cases that illustrate some important points. For more details on these various techniques we refer the reader to Roy (1985, chap.9) and to Bouyssou and Roy (1987).

a) Case of a criterion based on a single consequence.

Let us consider a criterion g that the analyst wants to base on a unique consequence, for instance the cost of a given project. Suppose first that the analyst considers that it is possible to neglect the elements of imprecision and/or uncertainty affecting the evaluation of the alternatives on that consequence. In such a situation it seems reasonable to build the criterion by letting for all $a \in A$:

$g(a) = c(a)$,

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[7] This is not to say that it is possible to give a precise definition of what we were willing to capture.

$c(a)$ being the evaluation of alternative a on the consequence. This technique is simple and, in general, leads to a criterion expressed in a clear physical unit. However, nothing implies that considering four alternatives a, b, c, d $\in$ A such that:

$g(a) - g(b) = g(c) - g(d)$,

one can "conclude" that the "preference difference" between a and b is similar to the preference difference between c and d (the difference between "very poor" and "poor" may well be different from the difference between "average" and "good").

It is easy to show that any monotonic transformation of such a criterion gives rise to a criterion leading to the same comparisons in terms of preference and indifference. It may therefore be interesting, especially if one wishes to use afterwards an aggregation procedure based on the idea of additive utility, to find among the strictly increasing monotonic transformation of g a transformation Ψ for which it seems legitimate to compare preference differences between alternatives as: $\Psi(g(a)) - \Psi(g(b))$ and $\Psi(g(c)) - \Psi(g(d))$.

Another classical situation concerns the case in which the evaluation of the alternatives on the unique consequence involves a best-guess evaluation $c(a)$, an optimistic evaluation $c^+(a)$ and a pessimistic one $c^-(a)$, a case in which each evaluation is surrounded by an interval of imprecision which is not necessarily symmetric. In such a situation it seems again reasonable to consider that:

$g(a) = c(a)$.

However it is no more possible to admit that a small difference between $g(a)$ and $g(b)$ implies a strict preference. It seems reasonable to consider in this situation that there is a strict preference for a over b only when $c^-(a) > c^+(b)$, a case in which the two imprecision intervals do not intersect[8]. When $c(b)$ increases, the two intervals intersect. This intersection can be interpreted as indifference when the best-guess evaluation of each alternative is contained in the imprecision interval of the other alternative[9]. The intermediate situation then corresponds to a hesitation zone that can be interpreted as a weak preference. It is possible to show (see Roy, 1985, chap. 9) that as long as the differences $c^-(a)$-$c(a)$ and $c^+(a)$-$c(a)$ only depend on the value of $c(a)$, this mode of comparison can be modelled using a pseudo-criterion. The reader might want to check that if for all a $\in$ A, we have:

$c^-(a) = c(a) - (\alpha' + \beta' c(a))$ and $c^+(a) = c(a) + (\alpha + \beta c(a))$, this mode of comparison defines a pseudo-criterion such that for all a $\in$ A,

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[8] Let us recall that the direction of the inequality is conventional.

[9] Other conventions are possible, see Siskos and Hunert (1983).

$g(a) = c(a)$,

$p(g(a)) = [\alpha + \alpha' + (\beta + \beta')g(a)] / (1 - \beta')$,

$q(g(a)) = \mathrm{Min}\, [\alpha + \beta g(a); (\alpha' + \beta' g(a)) / (1 - \beta')]$.

Another situation occurs when the evaluation of the alternatives on the consequence is "distributional". In others words, building a criterion amounts to compare distributions on the scale of the consequence. The necessity to consider distributions may come from several sources:

- the evaluation varies in time (problem of "actualization"),
- the evaluation varies in space (evaluation on a consequence of a "linear" project: highway (see Marchet and Siskos, 1979), high voltage line (see Grassin, 1986), distribution of the riparian population between the various zones of noise (in our example),
- the evaluation is uncertain and a probability (or plausibility) distribution is used.

In such a situation, a standard technique, called "point-reduction" in Roy (1985), to build a criterion consists in trying to sum up the distribution by a unique figure. This figure is usually a weighted average. More precisely, in order to build the criterion on the basis of a distribution consisting for an alternative a of a mass $f^a(x)$ associated to each level x of the consequence, a standard technique consists in setting:

$$g(a) = \int u(x)\, f^a(x)\, dx \qquad (1)$$

where u is some real-valued function on the set of levels of the consequence.

The most classical technique of point-reduction is "actualization". It consists in summing up a monetary distribution on a time scale. The criterion that is generally used is the Present Value of the distribution:

$$g(a) = \sum_{k=1}^{n} \frac{f^a(k)}{(1+i)^k},$$

where $f^a(k)$ is the cash flow generated by alternative a in period k, i being an actualization coefficient taking into account the fact that the importance of a flow depends on the period in which it is generated. Letting $u(x) = 1/(1+i)^x$, it is easily seen that actualization is a particular case of (1).

As far as distribution in space are concerned, the procedure we used in our example to define the criterion "impact on the riparians" is obviously a particular case of (1) in which u is defined for each zone by a weight that is a function of the distance between the zone and the airport. Similar techniques have been used in Grassin (1986) and Marchet and Siskos (1979).

When $f^a(x)$ can be interpreted as a probability, formula (1)defines what is usually called an expected utility criterion. When the scale of the consequences is a subset of R, letting u(x)= x in (1) amounts to using the expectation of the distribution as a criterion. Such a criterion would not allow to take into account crucial elements such as risk aversion, the risk of ruin, ... The introduction of the function u, called in that case a von Neumann-Morgenstern utility function, allows to take into account such phenomena. Suppose, for instance, that one of the actors of the decision process declares that she is a "risk averse" for money, i.e. that she has a definite preference for "nonrisky" investments. For instance, she prefers an investment with a certain net present value of 2 500.000 to an investment that may yield with equal chances either 2 000 000 or 0, even if the expected profit of the latter (1 000 000) is far greater than that of the former. In order to build a criterion taking into account such a preference, it suffices to choose u such that:

u(500 000) > ½u(0) + ½u(2 000 000).

There are a number of standard questioning techniques designed to assess a function u compatible with what can be perceived of the preference vis-à-vis risk of an actor (see Keeney and Raiffa,1976, chap. 4). This type of criterion has been extensively studied in literature. It has many interesting aspects[10] and has been frequently used in real-world situations (see, e.g., Keeney and Nair, 1977).

There is a theory (Expected Utility theory) aiming at justifying the use of such a criterion through an axiomatic analysis of the preferences compatible with formulations of type (1). However, the existence of such a theory does not oblige the analyst to use a formulation of type (1) if she thinks that another type of criterion could lead to more convincing comparisons[11]. Furthermore, the richness of formulation (1) should not lead one to forget that this type of criteria is based on probability distributions that are rarely the only sensible ones and on a function u which has been assessed through a questioning process that might have had an overwhelming influences on the shape of the function (see, e.g., McCord and de Neufville, 1983). Therefore it is often necessary to consider that criteria built in this way are not true-criteria and to use them surrounded by one or two discrimination thresholds.

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[10] For instance, it is easily shown that the shape of u (concavity or convexity) can be interpreted in terms of aversion of preference for risk (see Pratt, 1964).

[11] This is all the more true that there are many controversies concerning the normative, descriptive and prescriptive virtues of Expected Utility. For a recent overview of the debates, see,e.g., Munier (1988).

Either because it seems difficult to assess the function u or because the analyst does not wish to sum up a complex set of information by a single figure, it is possible not to use a point-reduction criterion and to sum up the distributional evaluation using several criteria. When the distribution is probabilistic, it is possible, for instance, to use two criteria: a criterion measuring a central tendency (expected value but also median or mode) and a dispersion criterion (variance or standard deviation, semi-variance, inter-quartile range, probability of ruin, probability of not reaching a specified target, ...), see Colson and Zeleny (1980) or Fishburn (1977).

Another mode of construction is to consider that the source of the distributional evaluation is the existence of various scenarii and to build a criterion per scenario without trying to aggregate them in this phase (see, e.g., Teghem and Kunsch, 1985).

b) Case of a criterion based on several consequences.

Either because of the size of the set of consequences, or because a hierarchical technique of construction is used, or because the actors of the decision process are used to thinking using concepts including several consequences, the analyst often have to build a criterion that takes into account more than one consequence. It is said that such a criterion sub-aggregates a set of consequences. In doing so, it is crucial to keep in mind that the result of this aggregation has to be accepted by all actors and sufficiently transparent and simple in order to be interpreted and easily discussed. That is to say that such a sub-aggregation should only concern a limited number of consequences that are "sufficiently" close from one another in order to keep the model simple and to avoid the introduction of "sensitive" information at this stage.

It is possible to use every aggregation procedure leading to the establishment of a unique criterion in order to build such a criterion. However, taking into account the proximity of the consequences and the necessary transparency of the model, simple methods are, in general, used: lexicographic aggregation, weighted average, sum of ranks, or any ad-hoc combination of these methods.

For instance, Roy et al. (1986), in order to build a criterion taking into account a point of view "level of discomfort" of the users of a subway station[12] chose to aggregate the consequences:

- climatic conditions in the station,
- noise in the station,
- "penibility" of the access to the station (presence of escalators, etc.),

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[12] In a study aiming at prescribing a renovation plan for these stations.

- time lost in going to one platform to another, and
- density of passengers on the trains,

using a three-point scale for each of these consequences:

0: "no problem has been observed",

1: "a minor problem exists",

3: "a major problem exists",

and defining the value of the criterion for a given station on the basis of the sum of the evaluations on the five consequences (after modifying this sum in an ad-hoc way to take into account particular circumstances).

5. CONCLUSION

The construction of criteria is a phase of the decision-aid process that takes place after a preliminary phase consisting in defining the set of alternatives, the problem formulation of the study and the strategy of intervention in the decision process. i. In a study aiming at prescribing a renovation plan for these stations. These two crucial phases represent, in real-world studies, the major part of the work of the analyst[13]. After the construction of the criteria, an alternative, i.e. in general, a complex project that may not be completely specified at the time of the study, will only be taken into account[14] through the vector $(g_1(a), g_2(a), ..., g_n(a))$. As we saw in the previous sections, it is essential that each component of this vector be a model that all actors understand and accept. But it is also important that this vector, as a whole, be a faithful representation of the alternative a. These two conditions seem to be a prerequisite for the application of MCDA techniques to be really useful in real-world problems.

We already mentioned two important qualities that a family of criteria should have:

- the "legibility", i.e. the family should contain a sufficiently small number of criteria so as to be a discussion basis allowing the analyst to assess inter-criteria information necessary for the implementation of an aggregation procedure,
- the "operationality", i.e. the family should be considered by all actors as a sound basis for the continuation of the decision-aid study.

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[13] Sometimes the analyst is only hired for these two phases, see,e.g. Grassin (1986).

[14] It is rather unusual, once the criteria have been built to go back to the initial data, i.e. , the evaluation of the alterna-tives on the various consequences.

However, as noticed by Roy and Bouyssou (1988, chap. 2), a family of criteria must also possess a number of technical properties, leading to the concept of "consistent family of criteria", to be able to be really useful for decision-aid purposes. In an informal way, we will say that a family of criteria is consistent if it is:

- exhaustive: the family should contain every important point of view. In particular this condition implies that if for all the criteria in the family we have $g_i(a) = g_i(b)$, every actor must agree to consider that a and b are indifferent,

-monotonic: the partial preferences that are modelled by each criterion have to be consistent with the global preferences expressed on the alternatives. This condition implies that if a is judged to be better than b taking into account all the points of view, the same judgment will hold for an alternative c that is judged at least as good as a on every criterion,

- minimal: for obvious reasons this condition implies not to include in the family unnecessary criteria, i.e. which suppression will lead to a family still satisfying the first two conditions.

Very often, the search for a legible, operational and consistent family of criteria leads the analyst to reconsider the definition of some criteria, to introduce new ones in the family, to aggregate some of them, etc. Thus the choice of a family of criteria interacts with the construction of the various criteria.

Let us finally mention that other desirable conditions can be imposed on a family of criteria (see Roy and Bouyssou, 1988,chap. 2). For instance, it seems reasonable to be willing to work with a family of criteria in which ceteris paribus comparisons on a sub-family of criteria (and not only on a single criterion) are legitimate and in which there are no functional relations between criteria. Without going into these questions in many details, let us only mention that it is not always possible to build a family that would satisfy these two conditions and still being legible and operational. When this is the case, the task of the analyst becomes very difficult in the aggregation phase either because she cannot implement aggregation procedures based on an addition operation (which is the case for nearly all aggregation procedures based on an addition operation (which is the case for nearly all aggregation procedures) or because some actors may accuse her of "double counting" a number of factors.

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