An axiomatic approach to outranking relations

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Introduction

Context

- preference modelling for MCDA

Two main traditions

- **Axiomatic**: conjoint measurement and additive value functions
  - firm theoretical background (Krantz et al., 1971)
  - implementation often delicate: requires a detailed analysis of preferences

- **Pragmatic**: dominance relation and refinements
  - outranking relations based on a concordance-discordance principle
  - intuitive… but often criticized for their lack of axiomatic foundations
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Outranking relations

**Roy (1968), ELECTRE I**

- alternative $x$ is “at least as good as” alternative $y$ if
  
  **Concordance condition** the set of attributes for which $x$ is at least as good as $y$ is “sufficiently important”

  **Non-discordance condition** there is no attribute on which $y$ is “far better” than $x$

**This type of comparison:**

- is, apparently, quite different from the one used in the additive value function model
- has a definite “ordinal” flavor
- may lead to intransitive/incomplete preference relations

$\Rightarrow$ usual conjoint measurement tools are not adequate
Roy (1968), ELECTRE I

- alternative \( x \) is “at least as good as” alternative \( y \) if
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⇒ usual conjoint measurement tools are not adequate
Objectives

Propose a general framework for conjoint measurement
- simple and intuitive
- nontrivial
- having a numerical representation
- tolerating incompleteness and intransitivity

Put this framework to work
- to characterize concordance relations (Brest talk)
- to characterize outranking relations (Today’s talk)
Objectives

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Put this framework to work
- to characterize concordance relations (Brest talk)
- to characterize outranking relations (Today’s talk)
1 Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2 Conjoint measurement framework
   - Model
   - Axioms
   - Results

3 Results
   - Concordance relations
   - Outranking relations

4 Discussion
Introduction

Outline

1. Definitions and notation
   - Setting
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Framework

Classical conjoint measurement setting

- \( N = \{1, 2, \ldots, n\} \): set of attributes
- \( X = \prod_{i=1}^{n} X_i \) with \( n \geq 2 \): set of alternatives
- notation: \( (x_{j}, y_{-j}) \) and \( (x_{i}, y_{-i}) \in X \)
- \( \succ \) asymmetric binary relation \( X \) “strict preference”

Remark

- we only study today asymmetric relations \( \succ \) à la TACTIC
- analysis can be extended to cover reflexive relations \( \succeq \) à la ELECTRE I
  - the introduction of discordance raises tricky duality problems however
Definitions and notation

Conjoint measurement framework

Results

Discussion

Setting

Concordance relations

Outranking relations

Example

Framework

Classical conjoint measurement setting

- \( N = \{1, 2, \ldots, n\} \): set of attributes
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Definition of strict concordance relations

Strict concordance relations (SCR)

\[ x \succ y \iff P(x, y) \succ P(y, x) \]

with \( P(x, y) = \{ i \in N : x_i \mathbin{P_i} y_i \} \) and

- \( P_i \): asymmetric binary relation \( X_i \)
- \( \succ \): binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

\[ A \succ B, \ C \supseteq A \text{ and } B \supseteq D \Rightarrow C \succ D \]
### Definition of strict concordance relations

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Definition of strict outranking relations

\[ x \succ y \iff [P(x, y) \triangleright P(y, x) \text{ and } V(y, x) = \emptyset] \]

with \( P(x, y) = \{i \in N : x_i P_i y_i\} \) and \( V(y, x) = \{i \in N : y_i V_i x_i\} \)

- \( P_i \): asymmetric binary relation \( X_i \)
- \( V_i \): a binary relation on \( X_i \) such that \( V_i \subseteq P_i \)
- \( \triangleright \): binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

\[ A \triangleright B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \triangleright D \]
Definition of strict outranking relations

**Strict outranking relations (SOR)**

\[ x \succ y \Leftrightarrow [P(x, y) \triangleright P(y, x) \text{ and } V(y, x) = \emptyset] \]

with \( P(x, y) = \{i \in N : x_i P_i y_i\} \) and \( V(y, x) = \{i \in N : y_i V_i x_i\} \)

- \( P_i \): asymmetric binary relation \( X_i \)
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\[ A \triangleright B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \triangleright D \]
Example

**TACTIC (Vansnick, 1986)**

\[ x \succ y \iff \left\{ \sum_{i \in P(x,y)} w_i > \rho \sum_{j \in P(y,x)} w_j + \varepsilon \right. \\
\text{and} \\
V(y, x) = \emptyset \]

with:
- \( \rho \geq 1 \) and \( \varepsilon \geq 0 \)
- \( P_i \): semiorder
- \( V_i \subseteq P_i \): semiorder
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Conjoint measurement framework

Model (M)

\[ x \succ y \iff F(p_1(x_1, y_1), p_2(x_2, y_2), \ldots, p_n(x_n, y_n)) > 0 \] (M)

with

- \( p_i \) skew symmetric \( p_i(x_i, y_i) = -p_i(y_i, x_i) \)
- \( F \) is odd \( F(x) = -F(-x) \)
- \( F \) is nondecreasing in all its arguments

Interpretation

- \( p_i \) measures preference differences between levels on attribute \( i \in N \)
- \( F \) synthesizes these preference differences

Many variants of model (M) not studied here
Conjoint measurement framework

**Model (M)**

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x \succ y \iff F(p_1(x_1, y_1), p_2(x_2, y_2), \ldots, p_n(x_n, y_n)) > 0 \quad (M)
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with

- \( p_i \) skew symmetric \( (p_i(x_i, y_i) = -p_i(y_i, x_i)) \)
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**Interpretation**

- \( p_i \) measures *preference differences* between levels on attribute \( i \in N \)
- \( F \) synthesizes these preference differences

Many variants of model (M) not studied here
Axioms

\[
\begin{align*}
(x_i, a_i) & \succ (y_i, b_i) \quad \text{and} \\
(z_i, c_i) & \succ (w_i, d_i) \\
\Rightarrow \quad \{ \quad & \{ \\
(x_i, c_i) & \succ (y_i, d_i) \\
(z_i, a_i) & \succ (w_i, b_i) \\
\} & \cup \\
\} & \Rightarrow \\
\} & \Rightarrow \\
\} & \Rightarrow \\
\}
\end{align*}
\]

\[
ARC_1 \iff ARC_1_i, \quad \forall i \in N
\]

\[
ARC_2 \iff ARC_2_i, \quad \forall i \in N
\]
Axioms

\[ (x_i, a_i) \succ (y_i, b_i) \quad \text{and} \quad (z_i, c_i) \succ (w_i, d_i) \implies \begin{cases} (x_i, c_i) \succ (y_i, d_i) \\ (z_i, a_i) \succ (w_i, b_i) \end{cases} \quad \text{ARC1}_i \]

\[ (x_i, a_i) \succ (y_i, b_i) \quad \text{and} \quad (y_i, c_i) \succ (x_i, d_i) \implies \begin{cases} (z_i, a_i) \succ (w_i, b_i) \\ (w_i, c_i) \succ (z_i, d_i) \end{cases} \quad \text{ARC2}_i \]

ARC1 iff \( \text{ARC1}_i, \forall i \in N \)
ARC2 iff \( \text{ARC2}_i, \forall i \in N \)
Axioms

\[
\begin{align*}
(x_i, a_i) &\succ (y_i, b_i) \\
&\text{and} \\
(z_i, c_i) &\succ (w_i, d_i)
\end{align*}
\implies
\begin{align*}
(x_i, c_i) &\succ (y_i, d_i) \\
&\text{or} \\
(z_i, a_i) &\succ (w_i, b_i)
\end{align*}
\Rightarrow
\begin{align*}
(z_i, a_i) &\succ (w_i, b_i) \\
&\text{or} \\
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\end{align*}
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\(ARC1_i\)

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\end{align*}
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\(ARC2_i\)

\(ARC1\ \text{iff}\ \ ARC1_i, \ \forall i \in N\)
\(ARC2\ \text{iff}\ \ ARC2_i, \ \forall i \in N\)
Theorem (B&P, 2002, JMP)

*When each $X_i$ is at most countably infinite*

A binary relation $\succ$ on $X$ has a representation in model $(M)$ iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC1$ and $ARC2$

Remark

- can be generalized to sets of arbitrary cardinality
Theorem (B&P, 2002, JMP)

[When each $X_i$ is at most countably infinite]
A binary relation $\succ$ on $X$ has a representation in model (M) iff
- $\succ$ is asymmetric
- $\succ$ satisfies ARC1 and ARC2

Remark
- can be generalized to sets of arbitrary cardinality
Remark

Model (M) contains as particular cases

- the additive value function model:

\[ x \succ y \iff \sum_{i=1}^{n} u_i(x_i) > \sum_{i=1}^{n} u_i(y_i) \]

- the additive difference model:

\[ x \succ y \iff \sum_{i=1}^{n} \Phi_i(u_i(x_i) - u_i(y_i)) > 0 \]

Coming next...

- model (M) also contains concordance relations
- model (M) also contains outranking relations
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Concordance relations

Observations
- if $\succ$ is a SCR, it satisfies $ARC1$ and $ARC2$
- if $\succ$ has a representation in model $(M)$ in which each $p_i$ takes at most three distinct values ($-k_i$, 0, $+k_i$), it is a SCR

Consequences
- model $(M)$ offers an adequate framework for characterizing concordance relations
- the distinctive feature of concordance relation is that they induce comparisons of preference differences that are coarse (at most three classes of preference differences)
Concordance relations

Observations

- If $\succ$ is a SCR, it satisfies $ARC1$ and $ARC2$.
- If $\succ$ has a representation in model (M) in which each $p_i$ takes at most three distinct values $(-k_i, 0, +k_i)$, it is a SCR.

Consequences

- Model (M) offers an adequate framework for characterizing concordance relations.
- The distinctive feature of concordance relation is that they induce comparisons of preference differences that are coarse (at most three classes of preference differences).
Axioms

\[
\begin{align*}
(x_i, a_{-i}) & \succ (y_i, b_{-i}) \\
\text{and} \\
(z_i, a_{-i}) & \succ (w_i, b_{-i}) \\
\text{and} \\
(z_i, c_{-i}) & \succ (w_i, d_{-i}) \\
\end{align*}
\]  \quad \Rightarrow \quad \left\{ \begin{array}{l}
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(x_i, c_{-i}) \succ (y_i, d_{-i}) \\
\end{array} \right. \quad \text{Maj1}_i

\[
\begin{align*}
(x_i, a_{-i}) & \succ (y_i, b_{-i}) \\
\text{and} \\
(w_i, a_{-i}) & \succ (z_i, b_{-i}) \\
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(z_i, c_{-i}) \succ (w_i, d_{-i}) \\
\end{array} \right. \quad \text{Maj2}_i

- \text{Maj1} \text{ if } \text{Maj1}_i, \forall i \in N
- \text{Maj2} \text{ if } \text{Maj2}_i, \forall i \in N
- RC1, RC2, \text{Maj1} \text{ and } \text{Maj2} \text{ are independent conditions}
Axioms

\[(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (z_i, a_{-i}) \succ (w_i, b_{-i}) \quad \text{and} \quad (z_i, c_{-i}) \succ (w_i, d_{-i}) \]

\[\Rightarrow \begin{cases} (y_i, a_{-i}) \succ (x_i, b_{-i}) \\ \text{or} \\ (x_i, c_{-i}) \succ (y_i, d_{-i}) \end{cases} \quad \text{Maj}1_i\]

\[(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (w_i, a_{-i}) \succ (z_i, b_{-i}) \quad \text{and} \quad (y_i, c_{-i}) \succ (x_i, d_{-i}) \]

\[\Rightarrow \begin{cases} (y_i, a_{-i}) \succ (x_i, b_{-i}) \\ \text{or} \\ (z_i, c_{-i}) \succ (w_i, d_{-i}) \end{cases} \quad \text{Maj}2_i\]

- \text{Maj}1 \text{ if } \text{Maj}1_i, \forall i \in N
- \text{Maj}2 \text{ if } \text{Maj}2_i, \forall i \in N
- \text{RC1, RC2, Maj}1 \text{ and Maj}2 \text{ are independent conditions}
Axioms

\[(x_i, a_{−i}) \succ (y_i, b_{−i}) \]
and
\[(z_i, a_{−i}) \succ (w_i, b_{−i}) \]
and
\[(z_i, c_{−i}) \succ (w_i, d_{−i}) \] \[\Rightarrow\] \[\{ (y_i, a_{−i}) \succ (x_i, b_{−i}) \]
or
\[(x_i, c_{−i}) \succ (y_i, d_{−i}) \] \[\text{Maj}_1_i\]

\[(x_i, a_{−i}) \succ (y_i, b_{−i}) \]
and
\[(w_i, a_{−i}) \succ (z_i, b_{−i}) \]
and
\[(y_i, c_{−i}) \succ (x_i, d_{−i}) \] \[\Rightarrow\] \[\{ (y_i, a_{−i}) \succ (x_i, b_{−i}) \]
or
\[(z_i, c_{−i}) \succ (w_i, d_{−i}) \] \[\text{Maj}_2_i\]

- \text{Maj1 if Maj}_1_i, \forall i \in N
- \text{Maj2 if Maj}_2_i, \forall i \in N
- \text{RC1, RC2, Maj1 and Maj2 are independent conditions}
Result model \((M)\)

**Theorem (B&P, 2002, JMP)**

A binary relation \(\succ\) on \(X\) has a representation in model \((M)\) iff

- \(\succ\) is asymmetric
- \(\succ\) satisfies \(ARC_1\) and \(ARC_2\)

**Remark**

- Model \((M)\) can be used to analyze other types of models (e.g., additive value functions or additive differences)
Theorem (B&P, 2005, EJOR)

A binary relation $\succ$ on $X$ is a SCR iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC1$ and $ARC2$
- $\succ$ satisfies $Maj1$ and $Maj2$

Remark

- model (M) can be used to analyze other types of models (e.g., additive value functions or additive differences)
Result SCR

Theorem (B&P, 2005, EJOR)

A binary relation \( \succ \) on \( X \) is a SCR iff

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- \( \succ \) satisfies Maj1 and Maj2

Remark

- model (M) can be used to analyze other types of models (e.g., additive value functions or additive differences)
Outranking relations

Observations

- if $\succ$ is an outranking relation, it satisfies $ARC1$ and $ARC2$
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it is the largest possible preference difference, so that $Maj1$ holds
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it may happen that $(y_i, x_i)$ is not the smallest possible preference difference

Consequences

- keep $ARC1$ and $ARC2$
- keep $Maj1$
- relax $Maj2$ in order to allow for veto effects
  - at most five classes of preference differences, the last one playing a very special rôle
Observations

- if $\succ$ is an outranking relation, it satisfies $ARC1$ and $ARC2$
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it is the largest possible preference difference, so that $Maj1$ holds
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it may happen that $(y_i, x_i)$ is not the smallest possible preference difference

Consequences

- keep $ARC1$ and $ARC2$
- keep $Maj1$
- relax $Maj2$ in order to allow for veto effects
  - at most five classes of preference differences, the last one playing a very special rôle
Axiom \textit{Maj}2

\[
\begin{align*}
(x_i, a_i) &\succ (y_i, b_i) \\
\text{and} \\
(w_i, a_i) &\succ (z_i, b_i) \\
\text{and} \\
y_i, c_i) &\succ (x_i, d_i)
\end{align*}
\]

\[
\Rightarrow \begin{cases} 
(y_i, a_i) \succ (x_i, b_i) \\
\text{or} \\
(z_i, c_i) \succ (w_i, d_i),
\end{cases}
\]

- \textit{Maj}3 \text{ if } \textit{Maj}3_i, \forall i \in N
- \textit{Maj}2_i \text{ implies } \textit{Maj}3_i
- \text{an outranking relation satisfies } \textit{Maj}3
- \textit{RC1, RC2, Maj1 and Maj3 are independent conditions}
- \text{condition } \textit{Maj}3 \text{ is inspired by GMS (2001)}
Axiom \textit{Maj}3

\[
\begin{align*}
(x_i, a_i) & \succ (y_i, b_i) \\
& \text{and} \\
(w_i, a_i) & \succ (z_i, b_i) \\
& \text{and} \\
(y_i, c_i) & \succ (x_i, d_i) \\
& \text{and} \\
(z_i, e_i) & \succ (w_i, f_i)
\end{align*}
\]

\[
\Rightarrow \begin{cases}
(y_i, a_i) \succ (x_i, b_i) \\
\text{or} \\
(z_i, c_i) \succ (w_i, d_i),
\end{cases}
\]

\textit{Maj}3_i

- \textit{Maj}3 if \textit{Maj}3_i, \forall i \in N
- \textit{Maj}2_i implies \textit{Maj}3_i
- an outranking relation satisfies \textit{Maj}3
- \textit{RC}1, \textit{RC}2, \textit{Maj}1 and \textit{Maj}3 are independent conditions
- condition \textit{Maj}3 is inspired by GMS (2001)
Axiom \( \text{Maj3} \)

\[
\begin{align*}
(x_i, a_i) \succ (y_i, b_i) \\
\text{and} \\
(w_i, a_i) \succ (z_i, b_i) \\
\text{and} \\
(y_i, c_i) \succ (x_i, d_i) \\
\text{and} \\
(z_i, e_i) \succ (w_i, f_i)
\end{align*}
\]

\[
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\Rightarrow \left\{ \begin{array}{l}
(y_i, a_i) \succ (x_i, b_i) \\
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\end{array} \right.
\end{align*}
\]

- \( \text{Maj3} \) if \( \text{Maj3}_i, \forall i \in N \)
- \( \text{Maj2}_i \) implies \( \text{Maj3}_i \)
- an outranking relation satisfies \( \text{Maj3} \)
- \( RC1, RC2, \text{Maj1} \) and \( \text{Maj3} \) are independent conditions
- condition \( \text{Maj3} \) is inspired by GMS (2001)
A binary relation \( \succ \) on \( X \) has a representation in model \((M)\) iff

- \( \succ \) is asymmetric
- \( \succ \) satisfies \( ARC1 \) and \( ARC2 \)
Result \textbf{SCR}

Theorem (B&P, 2005, EJOR)

A binary relation $\succ$ on $X$ is a \textbf{SCR} iff

- $\succ$ is asymmetric
- $\succ$ satisfies \textit{ARC1} and \textit{ARC2}
- $\succ$ satisfies \textit{Maj1} and \textit{Maj2}
Theorem (B&P, 2005, EJOR WP)

A binary relation \( \succ \) on \( X \) is a SCR SOR iff

- \( \succ \) is asymmetric
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Result SOR

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Outline

1 Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2 Conjoint measurement framework
   - Model
   - Axioms
   - Results

3 Results
   - Concordance relations
   - Outranking relations

4 Discussion
Summary

Model (M)

- is quite flexible but nontrivial
- has a simple and intuitive interpretation using preference differences
- has a simple axiomatic characterization
- allows to understand the main distinctive characteristics of concordance and outranking relations
  - in Brest we showed that the use of Fishburn’s “noncompensation” condition was not adequate to characterize concordance relations
  - the extension to outranking relations would have been impossible using the “noncompensation track”
What about SOR in which $S_i$ and $V_i$ have nice transitivity properties?

- add additional axioms
- these additional axioms are independent from the previous ones
- underlying model

\[ x \succ y \Leftrightarrow F(\varphi_1(u_1(x_1), u_1(y_1)), \ldots, \varphi_n(u_n(x_n), u_n(y_n))) > 0 \]

with $\varphi_i(\uparrow, \downarrow)$

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Extensions and future research

**Reflexive outranking relations à la ELECTRE I**
- no major problem: Bouyssou & Pirlot (2005)
  - duality: “veto” and “bonus” effects

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- extension to sorting models: Bouyssou & Marchant (2005)

**New models?**
- models using preference differences:
  - not as rich as in the additive value functions model
  - not as coarse as in outranking relations
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C'EST TOUT POUR AUJOURD'HUI.