An axiomatic approach to outranking relations

Denis Bouyssou\textsuperscript{1}  Marc Pirlot\textsuperscript{2}

\textsuperscript{1}CNRS
Paris, France

\textsuperscript{2}FPMs
Mons, Belgium

Luxembourg — EWG MCDA — March
Introduction

Context
- preference modelling for MCDA

Two main traditions
- Axiomatic: conjoint measurement and additive value functions
  - firm theoretical background (Krantz et al., 1971)
  - implementation often delicate: requires a detailed analysis of preferences
- Pragmatic: dominance relation and refinements
  - outranking relations based on a concordance-discordance principle
  - intuitive... but often criticized for their lack of axiomatic foundations
introduction

context

- preference modelling for MCDA

two main traditions

- axiomatic: conjoint measurement and additive value functions
  - firm theoretical background (Krantz et al., 1971)
  - implementation often delicate: requires a detailed analysis of preferences

- pragmatic: dominance relation and refinements
  - outranking relations based on a concordance-discordance principle
  - intuitive... but often criticized for their lack of axiomatic foundations
Roy (1968), ELECTRE I

- Alternative \( x \) is "at least as good as" alternative \( y \) if
  - Concordance condition: the set of attributes for which \( x \) is at least as good as \( y \) is "sufficiently important"
  - Non-discordance condition: there is no attribute on which \( y \) is "far better" than \( x \)

This type of comparison:

- is, apparently, quite different from the one used in the additive value function model
- has a definite "ordinal" flavor
- may lead to intransitive/incomplete preference relations

\( \Rightarrow \) usual conjoint measurement tools are not adequate
Outranking relations

Roy (1968), ELECTRE I

- alternative $x$ is "at least as good as" alternative $y$ if
  - **Concordance condition** the set of attributes for which $x$ is at least as good as $y$ is "sufficiently important"
  - **Non-discordance condition** there is no attribute on which $y$ is "far better" than $x$

This type of comparison:

- is, apparently, quite different from the one used in the additive value function model
- has a definite "ordinal" flavor
- may lead to intransitive/incomplete preference relations

$\Rightarrow$ usual conjoint measurement tools are not adequate
Propose a general framework for conjoint measurement

- simple and intuitive
- nontrivial
- having a numerical representation
- tolerating incompleteness and intransitivity

Put this framework to work

- to characterize concordance relations (Brest talk)
- to characterize outranking relations (Today’s talk)
Objectives

Propose a general framework for conjoint measurement
- simple and intuitive
- nontrivial
- having a numerical representation
- tolerating incompleteness and intransitivity

Put this framework to work
- to characterize concordance relations (Brest talk)
- to characterize outranking relations (Today’s talk)
Outline

1. Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2. Conjoint measurement framework
   - Model
   - Axioms
   - Results

3. Results
   - Concordance relations
   - Outranking relations

4. Discussion
Outline

1. Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2. Conjoint measurement framework
   - Model
   - Axioms
   - Results

3. Results
   - Concordance relations
   - Outranking relations

4. Discussion
Outline

1. Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2. Conjoint measurement framework
   - Model
   - Axioms
   - Results

3. Results
   - Concordance relations
   - Outranking relations

4. Discussion
Outline

1 Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2 Conjoint measurement framework
   - Model
   - Axioms
   - Results

3 Results
   - Concordance relations
   - Outranking relations

4 Discussion
Outline

1. Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2. Conjoint measurement framework
   - Model
   - Axioms
   - Results

3. Results
   - Concordance relations
   - Outranking relations

4. Discussion
Classical conjoint measurement setting

- $N = \{1, 2, \ldots, n\}$: set of attributes
- $X = \prod_{i=1}^{n} X_i$ with $n \geq 2$: set of alternatives
- notation: $(x_J, y_{-J})$ and $(x_i, y_{-i}) \in X$
- $\succ$ asymmetric binary relation $X$ “strict preference”

Remark

- we only study today asymmetric relations $\succ$ à la TACTIC
- analysis can be extended to cover reflexive relations $\sim$ à la ELECTRE I
- the introduction of discordance raises tricky duality problems however
Framework

Classical conjoint measurement setting

- \( N = \{1, 2, \ldots, n\} \): set of attributes
- \( X = \prod_{i=1}^{n} X_i \) with \( n \geq 2 \): set of alternatives
- notation: \((x_J, y_{-J})\) and \((x_i, y_{-i}) \in X\)
- \( \succ \) asymmetric binary relation \( X \) “strict preference”

Remark

- we only study today asymmetric relations \( \succ \) à la TACTIC
- analysis can be extended to cover reflexive relations \( \succeq \) à la ELECTRE I
  - the introduction of discordance raises tricky duality problems however
Definition of strict concordance relations

**Strict concordance relations (SCR)**

\[ x \succ y \iff P(x, y) \triangleright P(y, x) \]

with \( P(x, y) = \{ i \in N : x_i P_i y_i \} \) and

- \( P_i \): asymmetric binary relation \( X_i \)
- \( \triangleright \): binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

\[ A \triangleright B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \triangleright D \]
Definitions and notation

Conjoint measurement framework

Results

Discussion

Setting

Concordance relations

Outranking relations

Example

**Definition of strict concordance relations**

**Strict concordance relations (SCR)**

\[ x \succ y \iff P(x, y) \succ P(y, x) \]

with \( P(x, y) = \{ i \in N : x_i P_i y_i \} \)

- \( P_i \): asymmetric binary relation \( X_i \)

- \( \succ \): binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

\[ A \succ B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \succ D \]
Definition of strict outranking relations

\[ x > y \Leftrightarrow [P(x, y) \triangleright P(y, x) \text{ and } V(y, x) = \emptyset] \]

with \( P(x, y) = \{i \in N : x_i P_i y_i\} \) and \( V(y, x) = \{i \in N : y_i V_i x_i\} \)

- \( P_i \): asymmetric binary relation \( X_i \)
- \( V_i \): a binary relation on \( X_i \) such that \( V_i \subseteq P_i \)
- \( \triangleright \): binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

\[ A \triangleright B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \triangleright D \]
Definition of strict outranking relations

Strict outranking relations (SOR)

\[ x \succ y \iff [P(x, y) \succ P(y, x) \text{ and } V(y, x) = \emptyset] \]

with \( P(x, y) = \{ i \in N : x_i \ P_i y_i \} \text{ and } V(y, x) = \{ i \in N : y_i \ V_i x_i \} \)

- \( P_i \): asymmetric binary relation \( X_i \)
- \( V_i \): a binary relation on \( X_i \) such that \( V_i \subseteq P_i \)
- \( \succ \): binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

\[ A \succ B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \succ D \]
Example

**TACTIC (Vansnick, 1986)**

\[ x \succ y \iff \left\{ \begin{array}{l}
\sum_{i \in P(x,y)} w_i > \rho \sum_{j \in P(y,x)} w_j + \epsilon \\
\text{and} \\
V(y, x) = \emptyset
\end{array} \right. \]

with:
- \( \rho \geq 1 \) and \( \epsilon \geq 0 \)
- \( P_i: \text{semiorder} \)
- \( V_i \subseteq P_i: \text{semiorder} \)
Outline

1. Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2. Conjoint measurement framework
   - Model
   - Axioms
   - Results

3. Results
   - Concordance relations
   - Outranking relations

4. Discussion
Conjoint measurement framework

**Model (M)**

\[ x \succ y \iff F(p_1(x_1, y_1), p_2(x_2, y_2), \ldots, p_n(x_n, y_n)) > 0 \]  

(M)

with

- \( p_i \text{ skew symmetric} \) \( (p_i(x_i, y_i) = -p_i(y_i, x_i)) \)
- \( F \text{ is odd} \) \( (F(x) = -F(-x)) \)
- \( F \text{ is nondecreasing in all its arguments} \)

**Interpretation**

- \( p_i \text{ measures preference differences between levels on attribute} \)
  \( i \in N \)
- \( F \text{ synthesizes these preference differences} \)

Many variants of model (M) not studied here
Conjoint measurement framework

**Model (M)**

\[ x \succ y \iff F(p_1(x_1, y_1), p_2(x_2, y_2), \ldots, p_n(x_n, y_n)) > 0 \quad (M) \]

with

- \( p_i \) skew symmetric \( (p_i(x_i, y_i) = -p_i(y_i, x_i)) \)
- \( F \) is odd \( (F(x) = -F(-x)) \)
- \( F \) is nondecreasing in all its arguments

**Interpretation**

- \( p_i \) measures preference differences between levels on attribute \( i \in N \)
- \( F \) synthesizes these preference differences

Many variants of model (M) not studied here
Axioms

\[
\begin{align*}
(x_i, a_i) &\succ (y_i, b_i) \\
\text{and} \\
(z_i, c_i) &\succ (w_i, d_i) \\
\Rightarrow \\
(x_i, c_i) &\succ (y_i, d_i) \\
\text{or} \\
(z_i, a_i) &\succ (w_i, b_i)
\end{align*}
\]

\[ARC_1\]

\[
\begin{align*}
(x_i, a_i) &\succ (y_i, b_i) \\
\text{and} \\
(y_i, c_i) &\succ (x_i, d_i) \\
\Rightarrow \\
(z_i, a_i) &\succ (w_i, b_i) \\
\text{or} \\
(w_i, c_i) &\succ (z_i, d_i)
\end{align*}
\]

\[ARC_2\]

ARC1 iff \(ARC_1_i, \forall i \in N\)
ARC2 iff \(ARC_2_i, \forall i \in N\)
Axioms

\[
\begin{align*}
(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (z_i, c_{-i}) \succ (w_i, d_{-i}) \implies & \quad (x_i, c_{-i}) \succ (y_i, d_{-i}) \\
& \quad \text{or} \\
& \quad (z_i, a_{-i}) \succ (w_i, b_{-i}) \\
\end{align*}
\]

\[
\begin{align*}
(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (y_i, c_{-i}) \succ (x_i, d_{-i}) \implies & \quad (z_i, a_{-i}) \succ (w_i, b_{-i}) \\
& \quad \text{or} \\
& \quad (w_i, c_{-i}) \succ (z_i, d_{-i}) \\
\end{align*}
\]

\(ARC_1\) iff \(ARC_1\), \(\forall i \in N\)

\(ARC_2\) iff \(ARC_2\), \(\forall i \in N\)
Axioms

\[(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (z_i, c_{-i}) \succ (w_i, d_{-i}) \Rightarrow \begin{cases} (x_i, c_{-i}) \succ (y_i, d_{-i}) \\ (z_i, a_{-i}) \succ (w_i, b_{-i}) \end{cases} \]

\[ARC_1^i\]

\[(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (y_i, c_{-i}) \succ (x_i, d_{-i}) \Rightarrow \begin{cases} (z_i, a_{-i}) \succ (w_i, b_{-i}) \\ (w_i, c_{-i}) \succ (z_i, d_{-i}) \end{cases} \]

\[ARC_2^i\]

\[ARC_1 \iff ARC_1^i, \forall i \in N\]

\[ARC_2 \iff ARC_2^i, \forall i \in N\]
Theorem (B&P, \textit{JMP})

\[ \text{[When each } X_i \text{ is at most countably infinite]} \]

A binary relation $\succ$ on $X$ has a representation in model (M) iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC_1$ and $ARC_2$

Remark
- can be generalized to sets of arbitrary cardinality
Theorem (B&P, JMP)

[When each $X_i$ is at most countably infinite]
A binary relation $\succ$ on $X$ has a representation in model $(M)$ iff
- $\succ$ is asymmetric
- $\succ$ satisfies $ARC1$ and $ARC2$

Remark
- can be generalized to sets of arbitrary cardinality
Remark

Model (M) contains as particular cases

- the additive value function model:

\[ x \succ y \iff \sum_{i=1}^{n} u_i(x_i) > \sum_{i=1}^{n} u_i(y_i) \]

- the additive difference model:

\[ x \succ y \iff \sum_{i=1}^{n} \Phi_i(u_i(x_i) - u_i(y_i)) > 0 \]

Coming next...

- model (M) also contains concordance relations
- model (M) also contains outranking relations
Model (M) contains as particular cases

- the additive value function model:

\[ x \succ y \iff \sum_{i=1}^{n} u_i(x_i) > \sum_{i=1}^{n} u_i(y_i) \]

- the additive difference model:

\[ x \succ y \iff \sum_{i=1}^{n} \Phi_i(u_i(x_i) - u_i(y_i)) > 0 \]

Coming next... 

- model (M) also contains concordance relations
- model (M) also contains outranking relations
Remark

Model \((M)\) contains as particular cases

1. the additive value function model:

\[
x \succ y \iff \sum_{i=1}^{n} u_i(x_i) > \sum_{i=1}^{n} u_i(y_i)
\]

2. the additive difference model:

\[
x \succ y \iff \sum_{i=1}^{n} \Phi_i(u_i(x_i) - u_i(y_i)) > 0
\]

Coming next. . .

- model \((M)\) also contains concordance relations
- model \((M)\) also contains outranking relations
Outline

1. Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2. Conjoint measurement framework
   - Model
   - Axioms
   - Results

3. Results
   - Concordance relations
   - Outranking relations

4. Discussion
Concordance relations

Observations

- If \( \succ \) is a SCR, it satisfies \( ARC1 \) and \( ARC2 \)
- If \( \succ \) has a representation in model (M) in which each \( p_i \) takes at most three distinct values \( (-k_i, 0, +k_i) \), it is a SCR

Consequences

- Model (M) offers an adequate framework for characterizing concordance relations
- The distinctive feature of concordance relation is that they induce comparisons of preference differences that are coarse (at most three classes of preference differences)
Concordance relations

Observations

- If $\succ$ is a SCR, it satisfies $ARC_1$ and $ARC_2$.
- If $\succ$ has a representation in model (M) in which each $p_i$ takes at most three distinct values ($-k_i$, $0$, $+k_i$), it is a SCR.

Consequences

- Model (M) offers an adequate framework for characterizing concordance relations.
- The distinctive feature of concordance relation is that they induce comparisons of preference differences that are coarse (at most three classes of preference differences).
Axioms

\[
\begin{align*}
(x_i, a_i) &\succ (y_i, b_i) \\
\text{and} \\
(z_i, a_i) &\succ (w_i, b_i) \\
\text{and} \\
(z_i, c_i) &\succ (w_i, d_i)
\end{align*}
\]

\[
\Rightarrow \begin{cases}
(y_i, a_i) &\succ (x_i, b_i) \\
\text{or} \\
(x_i, c_i) &\succ (y_i, d_i)
\end{cases}
\]

\[
\begin{align*}
(x_i, a_i) &\succ (y_i, b_i) \\
\text{and} \\
(w_i, a_i) &\succ (z_i, b_i) \\
\text{and} \\
(y_i, c_i) &\succ (x_i, d_i)
\end{align*}
\]

\[
\Rightarrow \begin{cases}
(y_i, a_i) &\succ (x_i, b_i) \\
\text{or} \\
(z_i, c_i) &\succ (w_i, d_i)
\end{cases}
\]

\begin{itemize}
\item Maj1 if Maj1_i, \ \forall i \in N
\item Maj2 if Maj2_i, \ \forall i \in N
\item RC1, RC2, Maj1 and Maj2 are independent conditions
\end{itemize}
Axioms

\[
\begin{align*}
(x_i, a_{-i}) \succ (y_i, b_{-i})
\quad & \text{and} \\
(z_i, a_{-i}) \succ (w_i, b_{-i})
\quad & \text{and} \\
(z_i, c_{-i}) \succ (w_i, d_{-i})
\end{align*}
\]
\Rightarrow
\begin{align*}
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(x_i, c_{-i}) \succ (y_i, d_{-i})
\end{align*}
\tag{Maj 1_i}

\[
\begin{align*}
(x_i, a_{-i}) \succ (y_i, b_{-i})
\quad & \text{and} \\
(w_i, a_{-i}) \succ (z_i, b_{-i})
\quad & \text{and} \\
(y_i, c_{-i}) \succ (x_i, d_{-i})
\end{align*}
\]
\Rightarrow
\begin{align*}
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) \succ (w_i, d_{-i})
\end{align*}
\tag{Maj 2_i}

- \text{Maj 1 if Maj 1_i, } \forall i \in N
- \text{Maj 2 if Maj 2_i, } \forall i \in N
- RC1, RC2, Maj 1 and Maj 2 are independent conditions
Axioms

\[
\begin{align*}
(x_i, a_{-i}) & \succ (y_i, b_{-i}) \\
\text{and} & \\
(z_i, a_{-i}) & \succ (w_i, b_{-i}) \\
\text{and} & \\
(z_i, c_{-i}) & \succ (w_i, d_{-i})
\end{align*}
\]
\[
\Rightarrow \begin{cases} 
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} & \\
(x_i, c_{-i}) \succ (y_i, d_{-i})
\end{cases} \quad Maj_1_i
\]

\[
\begin{align*}
(x_i, a_{-i}) & \succ (y_i, b_{-i}) \\
\text{and} & \\
(w_i, a_{-i}) & \succ (z_i, b_{-i}) \\
\text{and} & \\
(y_i, c_{-i}) & \succ (x_i, d_{-i})
\end{align*}
\]
\[
\Rightarrow \begin{cases} 
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} & \\
(z_i, c_{-i}) \succ (w_i, d_{-i})
\end{cases} \quad Maj_2_i
\]

- Maj_1 if Maj_1_i, \forall i \in N
- Maj_2 if Maj_2_i, \forall i \in N
- RC_1, RC_2, Maj_1 and Maj_2 are independent conditions
Result model \((M)\)

**Theorem (B&P, , JMP)**

A binary relation \(\succ\) on \(X\) has a representation in model \((M)\) iff

- \(\succ\) is asymmetric
- \(\succ\) satisfies \(ARC1\) and \(ARC2\)

**Remark**

- model \((M)\) can be used to analyze other types of models (e.g., additive value functions or additive differences)
Theorem (B&P, EJOR)

A binary relation $\succ$ on $X$ is a SCR iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC_1$ and $ARC_2$
- $\succ$ satisfies $Maj_1$ and $Maj_2$

Remark

- model (M) can be used to analyze other types of models (e.g., additive value functions or additive differences)
Result SCR

**Theorem (B&P, EJOR)**

A binary relation $\succ$ on $X$ is a SCR iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC_1$ and $ARC_2$
- $\succ$ satisfies $Maj_1$ and $Maj_2$

**Remark**

- model (M) can be used to analyze other types of models (e.g., additive value functions or additive differences)
Observations

- if $\succ$ is an outranking relation, it satisfies $ARC1$ and $ARC2$
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it is the largest possible preference difference, so that $Maj1$ holds
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it may happen that $(y_i, x_i)$ is not the smallest possible preference difference

Consequences

- keep $ARC1$ and $ARC2$
- keep $Maj1$
- relax $Maj2$ in order to allow for veto effects
  - at most five classes of preference differences, the last one playing a very special rôle
Outranking relations

Observations

- If $\succ$ is an outranking relation, it satisfies $ARC1$ and $ARC2$.
- If the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it is the largest possible preference difference, so that $Maj1$ holds.
- If the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it may happen that $(y_i, x_i)$ is not the smallest possible preference difference.

Consequences

- Keep $ARC1$ and $ARC2$.
- Keep $Maj1$.
- Relax $Maj2$ in order to allow for veto effects.
  - at most five classes of preference differences, the last one playing a very special rôle.
Axiom \textit{Maj}2

\[
\begin{align*}
(x_i, a_{-i}) &\succ (y_i, b_{-i}) \\
\text{and} \\
(w_i, a_{-i}) &\succ (z_i, b_{-i}) \\
\text{and} \\
(y_i, c_{-i}) &\succ (x_i, d_{-i})
\end{align*}
\]

\[
\Rightarrow
\left\{ \begin{array}{c}
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) \succ (w_i, d_{-i})
\end{array} \right. \quad \text{Maj2}_i
\]

- \textit{Maj3} if \textit{Maj3}_i, \forall i \in N
- \textit{Maj2}_i \text{ implies } \textit{Maj3}_i
- an outranking relation satisfies \textit{Maj3}
- \textit{RC1}, \textit{RC2}, \textit{Maj1} and \textit{Maj3} are independent conditions
- condition \textit{Maj3} is inspired by GMS ( )
Axiom $\text{Maj3}$

\[
\begin{align*}
&(x_i, a_{-i}) \succ (y_i, b_{-i}) \\
&\quad \text{and} \\
&(w_i, a_{-i}) \succ (z_i, b_{-i}) \\
&\quad \text{and} \\
&(y_i, c_{-i}) \succ (x_i, d_{-i}) \\
&\quad \text{and} \\
&(z_i, e_{-i}) \succ (w_i, f_{-i})
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \begin{cases}
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) \succ (w_i, d_{-i}),
\end{cases}
\end{align*}
\]

- $\text{Maj3}$ if $\text{Maj3}_i$, $\forall i \in N$
- $\text{Maj2}_i$ implies $\text{Maj3}_i$
- an outranking relation satisfies $\text{Maj3}$
- $\text{RC1}$, $\text{RC2}$, $\text{Maj1}$ and $\text{Maj3}$ are independent conditions
- condition $\text{Maj3}$ is inspired by GMS (2001)
Axiom \textit{Maj3}

\[
\begin{aligned}
(x_i, a_{-i}) \succ (y_i, b_{-i}) \\
\text{and} \\
(w_i, a_{-i}) \succ (z_i, b_{-i}) \\
\text{and} \\
(y_i, c_{-i}) \succ (x_i, d_{-i}) \\
\text{and} \\
(z_i, e_{-i}) \succ (w_i, f_{-i})
\end{aligned}
\] \Rightarrow \begin{cases}
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
or \\
(z_i, c_{-i}) \succ (w_i, d_{-i}),
\end{cases}

\text{Maj} 3_i

\begin{itemize}
\item \textit{Maj3} if \textit{Maj} 3_i, \forall i \in N
\item \textit{Maj} 2_i \text{ implies } \textit{Maj} 3_i
\item an outranking relation satisfies \textit{Maj3}
\item \textit{RC1}, \textit{RC2}, \textit{Maj1} and \textit{Maj3} are independent conditions
\item condition \textit{Maj3} is inspired by GMS (2001)
\end{itemize}
Theorem (B&P, 2002, JMP)

A binary relation $\succ$ on $X$ has a representation in model (M) iff

- $\succ$ is asymmetric
- $\succ$ satisfies ARC1 and ARC2
**Theorem (B&P, EJOR)**

A binary relation $\succ$ on $X$ is a SCR iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC_1$ and $ARC_2$
- $\succ$ satisfies $Maj_1$ and $Maj_2$
Result **SCR SOR**

**Theorem (B&P, EJOR WP)**

A binary relation $\succ$ on $X$ is a **SCR SOR** iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC_1$ and $ARC_2$
- $\succ$ satisfies $Maj_1$ and $Maj_2$ and $Maj_3$
A binary relation $\succ$ on $X$ is a SOR iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC_1$ and $ARC_2$
- $\succ$ satisfies $Maj_1$ and $Maj_3$
Model (M)

- is quite flexible but nontrivial
- has a simple and intuitive interpretation using preference differences
- has a simple axiomatic characterization
- allows to understand the main distinctive characteristics of concordance and outranking relations
  - in Brest we showed that the use of Fishburn’s “noncompensation” condition was not adequate to characterize concordance relations
  - the extension to outranking relations would have been impossible using the “noncompensation track”
What about SOR in which \( S_i \) and \( V_i \) have nice transitivity properties?

- add additional axioms
- these additional axioms are independent from the previous ones
- underlying model

\[
x \succ y \iff F(\phi_1(u_1(x_1), u_1(y_1)), \ldots, \phi_n(u_n(x_n), u_n(y_n))) > 0
\]

with \( \phi_i(\uparrow, \downarrow) \)

---

What about SOR in which \( \succ \) has nice properties?

- add additional axioms
- these additional axioms are independent from the previous ones
What about SOR in which $S_i$ and $V_i$ have nice transitivity properties?

- add additional axioms
- these additional axioms are independent from the previous ones
- underlying model

$$x \succ y \iff F(\phi_1(u_1(x_1), u_1(y_1)), \ldots, \phi_n(u_n(x_n), u_n(y_n))) > 0$$

with $\phi_i(\uparrow, \downarrow)$

What about SOR in which $\succ$ has nice properties?

- add additional axioms
- these additional axioms are independent from the previous ones
Extensions and future research

Reflexive outranking relations à la ELECTRE I

- no major problem: Bouyssou & Pirlot (2005)
  - duality: “veto” and “bonus” effects

ELECTRE TRI

- extension to sorting models: Bouyssou & Marchant (2005)

New models?

- models using preference differences:
  - not as rich as in the additive value functions model
  - not as coarse as in outranking relations
- examples: models with “sophisticated discordance”
Extensions and future research

Reflexive outranking relations à la ELECTRE I
- no major problem: Bouyssou & Pirlot (2005)
  - duality: “veto” and “bonus” effects

ELECTRE TRI
- extension to sorting models: Bouyssou & Marchant (2005)

New models?
- models using preference differences:
  - not as rich as in the additive value functions model
  - not as coarse as in outranking relations
- examples: models with “sophisticated discordance”
Extensions and future research

Reflexive outranking relations à la ELECTRE I
- no major problem: Bouyssou & Pirlot (2005)
  - duality: “veto” and “bonus” effects

ELECTRE TRI
- extension to sorting models: Bouyssou & Marchant (2005)

New models?
- models using preference differences:
  - not as rich as in the additive value functions model
  - not as coarse as in outranking relations
- examples: models with “sophisticated discordance”